

# Macroeconomic Uncertainty and Asset Prices: A Stochastic Volatility Model\*

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## Abstract

In this paper we measure the time-varying uncertainty of macroeconomic fluctuations and study its link to asset returns via a consumption-based asset pricing model. To this end, we introduce a stochastic volatility model employing a latent nonstationary common volatility with two asymptotic regimes and smooth transition between them. We define the common volatility factor extracted from consumption and dividend growth rates as the macroeconomic uncertainty and analyze its effects on asset prices using a model with the Epstein-Zin-Weil preferences. The presence of smooth transition in our volatility model creates another layer of uncertainty in macroeconomic fluctuations, and thus provides an additional channel that generates a sizable risk premium for even a small amount of the consumption volatility. The channel can play an important role in determining asset prices, especially if the perceived macroeconomic uncertainty unravels slowly. Our estimates for the risk aversion and elasticity of intertemporal substitution are both around two, and the simulation results show that the model matches the first and the second moments of market return and the risk-free rate, hence the equity premium.

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# 1. Introduction

Time-varying macroeconomic uncertainty is an important ingredient for asset valuation. When there is a great deal of uncertainty about how an economy will evolve over time, this state is likely to be reflected in asset prices because financial markets demand premiums for bearing such non-diversifiable risk. The recent episode of financial crisis in 2008 shows that this is indeed a key link between macro variables and asset markets. However, the previous studies using macroeconomic models to explain asset prices did not pay much attention to this channel of generating risk premiums. Specifically, the most popular macroeconomic asset pricing models identify consumption growth as the link between macroeconomic variables and asset returns via a simple yet elegant Euler equation. Alas, the aggregate consumption behaves like a random walk and the size of unconditional variance of the consumption growth is fairly modest, which makes difficult measuring the macroeconomic uncertainty via the consumption growth. Moreover, even if the measurement issue is settled, the standard consumption-based models fail to justify high average equity premium with low and stable interest rates because of the small consumption volatility.<sup>1</sup> Consequently, the role of the stochastic macroeconomic uncertainty is minimal in this setting.

In this paper, we tackle this issue by introducing a novel stochastic volatility component in the consumption-based asset pricing model with a recursive preference function. Regarding the measurement of time-varying macroeconomic uncertainty, we set the volatility processes for consumption and dividend growths to be driven by non-linear functions of latent factors consisting of the common and idiosyncratic components. The common stochastic component is regarded as representing the “macroeconomic uncertainty”. We specify the common volatility factor to be persistent and let the actual volatilities be generated by a parametric logistic function. This setup yields volatility processes having two asymptotic levels (high and low uncertainty regimes) with smooth transitions between them. The existence of transition periods across the high and low regimes of volatilities implies that there is uncertainty about which regimes an economy will end up with in the next period and introduces uncertainty about uncertainty. Epstein and Zin (1989) show that economic agents prefer early resolution of uncertainty if the risk aversion parameter is larger than the reciprocal of the elasticity of intertemporal substitution for the Kreps-Porteus utility function. Equipped with this utility function, our volatility setup can create high risk premiums even with a modest level of consumption volatility since economic agents dislike uncertainty about regimes to which they may belong, especially if the perceived macroeconomic uncertainty unravels slowly.

For the statistical analysis, we formulate our shock processes as a state space model with stochastic volatility driven by nonstationary multiple latent factors. To effectively deal with such a model, we adopt the Bayesian approach and develop an algorithm to filter the stochastic volatilities using a Markov-Chain-Monte-Carlo (MCMC) method relying on the Gibbs sampler and Metropolis-Hasting algorithm. There are several other non-linear filtering techniques in the conventional approach that we may apply to estimate the state space models with non-linear measurement equations, such as the extended non-linear

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<sup>1</sup>These puzzles related to the consumption based asset pricing models have been one of the main research questions in finance and macroeconomics since Hansen and Singleton (1983) and Mehra and Prescott (1985).

Kalman filter and the density-based filter. The extended Kalman filter is not generally applicable in the presence of nonstationarity, since the Kalman gain may vanish in case the latent factors diverge without bounds. This motivates Kim, Lee and Park (2009) to develop the density-based filter for the estimation of nonstationary stochastic volatility models. On the other hand, it is extremely difficult and costly, if not impossible, to implement the density-based filter for large dimensional models like ours. The MCMC method we use to estimate our Bayesian model is significantly less prone to the problem of multi-dimensionality, because it utilizes a univariate conditional density function in every step. Furthermore, it appears that the Bayesian approach in general yields more stable estimates than the conventional approach, especially in the presence of nonstationary latent factors.

With the goal of evaluating the empirical performance of the proposed asset pricing model, we extract a common factor and idiosyncratic volatility factors from consumption and dividend growth rates. We find that the common factor delineating the macroeconomic uncertainty predicts post-war business cycle recessions quite well. In addition, our common volatility factor captures the great moderation periods beginning around 1984. Fama and French (1989) and many other studies report that the expected returns on stocks and long-term bonds have countercyclical variations. Thus, we believe that the close link between our macro uncertainty and business cycle can shed light on the dynamic behaviors of asset prices. To further analyze this problem, we estimate a consumption-based asset pricing model with the Epstein-Zin-Weil preferences incorporating this macroeconomic uncertainty. According to our estimation results, both the risk aversion and the intertemporal elasticity of substitution are estimated around two. Moreover our simulation results show that the model matches the first and the second moments of the stock returns and the risk-free rate hence, it explains the equity premium puzzle, the risk-free rate puzzle, and the volatility puzzle addressed in the literature. We also analytically show that this result stems from the existence of the smooth regimes of stochastic volatilities together with the non-indifferent preferences on the persistence of volatilities.

Our paper is related to at least three strands of literature. First and most directly related is the long-run risks asset price model. Bansal and Yaron (2004), and Hansen, Heaton, and Li (2005) set consumption and dividend growth processes to contain a small, but persistent process in their means, and show that they can explain many stylized facts in asset market. In their papers they emphasize the long-run risks channel, which is based on the common portion of the conditional expectations that vary slowly over time. Due to the long-run risks channel together with the preferences on the early resolution of uncertainty, these models can generate the sufficiently high risk premium. Bansal and Yaron (2004) also added a stochastic volatility term to their long-run risks model, but its role is mostly for explaining time-variability of risk premium. In the recent empirical paper by Bansal, Kiku, and Yaron (2007), the estimates of risk aversion and the elasticity of intertemporal substitution are around 10-15, and 0.5 respectively. Their results including the simulations vary little regardless of the existence of stochastic volatility. Thus, the role of stochastic volatility is restricted in their models. On the contrary, we focus on a more realistic and flexible volatility setup to see how the preferences on the timing of uncertainty resolution can produce a channel of uncertainty premium. Our empirical results show that the macroeconomic uncertainty is an important source of aggregate risk and properly valued in asset

markets.

Another important class of asset pricing models uses the habit formation preferences. For instance, Campbell and Cochrane (1999) can generate the high equity premium with low and stable interest rates via time-varying risk aversion. A major difference from our paper is that the habit formation emphasizes the near unit root behavior of consumption and assumes the constant conditional volatility for consumption growth. To produce time-varying expected returns, instead, relative risk aversion is changing over business conditions.<sup>2</sup> In addition, there are many papers that consider relaxing other assumptions of the standard consumption-based model to address various asset market behaviors. For example, Constantinides, Donalson, and Mehra (2002) report that the existence of limited participation in the asset market increases the equity premium. Our model can be extended to incorporate this feature without a major modification, but the availability of consumption data for asset market participants is somewhat limited. Another related literature would be recent works on rare events. Barro (2006) and Rietz (1988) report that the possibility of a rare disaster or event can make equity riskier.<sup>3</sup> This approach yields a fat-tail of asset returns by assigning small but positive probabilities to extremely rare events. Our model can also generate fat-tails due to the presence of nonstationary stochastic volatilities.<sup>4</sup>

The rest of the paper is organized as follows. In Section 2, we develop our asset pricing model. A new stochastic volatility model is proposed and integrated into our asset pricing model. Subsequently, we identify and analyze the macroeconomic uncertainty in Section 3. Furthermore, we develop a Bayesian procedure to estimate the unknown parameters and extract the latent volatility factors in our model. The estimation results and extracted factors as well as the data description, are also presented in the section. We present the empirical results on equity premium, interest rates, and volatilities in Section 4. Section 5 concludes the paper. Appendix includes more detailed derivations of our models, an exposition of the non-linear filtering algorithm applicable for our model and the additional tables and figures.

## 2. Asset Pricing with Time-Varying Uncertainty

### 2.1. Basic Model

We consider a simple closed economy in which the representative agent has a Epstein-Zin-Weil recursive preference (Epstein and Zin (1989) and Weil (1989)) given by

$$U_t = \left[ (1 - \delta) C_t^{\frac{1-\gamma}{\chi}} + \delta (\mathbb{E}_t U_{t+1}^{1-\gamma})^{\frac{1}{\chi}} \right]^{\frac{\chi}{1-\gamma}},$$

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<sup>2</sup>Given that our empirical result shows existence of non-trivial time-varying stochastic volatilities from consumption, it would be interesting to study the implications of our stochastic volatilities under the habit formation. However, we do not attempt to do this in the paper, since it is beyond the scope of our paper.

<sup>3</sup>Eraker and Shaliastovich (2008) theoretically study asset pricing implications of Epstein-Zin preferences with an affine jump diffusion. They emphasize the role of jumps to generate a volatility channel, similar to the rare events.

<sup>4</sup>The reader is referred to Park (2002) for more detailed explanations on how the presence of nonstationary stochastic volatilities can generate fat-tails.

where  $\chi = (1 - \gamma)/(1 - 1/\psi)$ ,  $\gamma \geq 0$  is the coefficient of relative risk aversion,  $\psi \geq 0$  is the elasticity of intertemporal substitution (EIS), and  $0 < \delta < 1$  is the time discount factor. Compared with a conventional power utility function, Epstein-Zin-Weil utility function permits more flexibility in that it breaks the tight link between the parameters of risk aversion  $\gamma$  and intertemporal substitution  $\psi$ . In case of the power utility function, EIS is the reciprocal of the risk aversion parameter so that  $\chi = 1$  should hold. Meanwhile, Epstein-Zin-Weil preferences can have both parameters larger than one, implying  $\chi < 0$  holds. Another useful property of this preference function is that the decision maker cares about timing for the resolution of uncertainty. It is well known that if  $\gamma > \psi^{-1}$  holds, then economic agents prefer early resolution of uncertainty.

The intertemporal budget constraint for the representative agent can be written as  $A_{t+1} = R_{a,t+1}(A_t - C_t)$ , where  $A_t$  is the wealth at time  $t$ , and  $R_{a,t+1}$  is the gross return on the portfolio of all invested wealth on consumption claims between  $t$  and  $t + 1$ . Epstein and Zin use dynamic programming to derive an Euler equation

$$1 = \mathbb{E}_t \left[ \delta^\chi \left( \frac{C_{t+1}}{C_t} \right)^{-\frac{\chi}{\psi}} R_{a,t+1}^{\chi-1} R_{i,t+1} \right] \quad (1)$$

for gross return  $R_{i,t+1}$  on asset  $i$  between  $t$  and  $t + 1$ . From the Euler equation (1) we have the logarithm of the intertemporal marginal rate of substitution (IMRS)

$$m_{t+1} = \chi \log \delta - \frac{\chi}{\psi} g_{c,t+1} + (\chi - 1) r_{a,t+1}, \quad (2)$$

where  $r_{a,t+1} = \log R_{a,t+1}$  is the log return on the portfolio of all invested wealth and  $g_{c,t+1}$  is the log consumption growth between  $t$  and  $t + 1$ . By defining  $R_{a,t+1}$  as the returns from holding the aggregate wealth which pays the consumption good as dividends, we can write it down as

$$R_{a,t+1} = \frac{C_t}{P_{a,t}} \left( 1 + \frac{P_{a,t+1}}{C_{t+1}} \right) \frac{C_{t+1}}{C_t},$$

where  $P_{a,t+1}$  is the price of a consumption claim at  $t + 1$ . Moreover, applying the log linearization method as in Campbell and Shiller (1988), we can find an approximate relationship among  $r_{a,t+1}$ , the price-consumption ratio, and consumption growth as

$$r_{a,t+1} \approx \phi_0(\bar{z}_c) + \phi_1(\bar{z}_c) z_{c,t+1} - z_{c,t} + g_{c,t+1}, \quad (3)$$

where  $z_{c,t} = \log(P_{a,t}/C_t)$  is the log price-consumption ratio.<sup>5</sup>

To further solve the model, we assume the consumption growth and dividend growth denoted by  $g_{c,t+1}$  and  $g_{d,t+1}$  have a conditional mean component, denoted as  $\mu + \nu_t$  and a time-varying volatility component driven by scalar processes  $x_{c,t}$  and  $x_{d,t}$  respectively.<sup>6</sup> Both of the volatility generating processes share a scalar common volatility generating

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<sup>5</sup>Coefficients of this approximation are as follows:  $\phi_0(\bar{z}) = \log(1 + \exp(\bar{z})) - \phi_1(\bar{z})\bar{z}$ ,  $\phi_1(\bar{z}) = \exp(\bar{z})/(1 + \exp(\bar{z}))$  for some value  $\bar{z}$ .

<sup>6</sup>From now on, subscripts  $c$  and  $d$  refer to consumption growth and dividend growth.

process  $w_t$ , which we call the macroeconomic uncertainty process. We allow these volatility components to fluctuate between two regimes with smooth transition. Specifically, we have

$$g_{j,t+1} = \mu_j + \nu_{j,t} + \sqrt{f_j(x_{j,t})} \varepsilon_{j,t+1} \quad (4)$$

$$\nu_{j,t+1} = \rho_j \nu_{j,t} + \varphi_j \sqrt{f_j(x_{j,t})} \eta_{j,t+1} \quad (5)$$

$$x_{j,t} = \lambda_j w_t + e_{j,t} \quad (6)$$

$$w_t = w_{t-1} + u_t, \quad (7)$$

where

$$f_j(x_{j,t}) = \alpha_j + \frac{\beta_j}{1 + \exp[-(x_{j,t} - \kappa_j)]} \quad (8)$$

with  $\alpha_j > 0, \beta_j > 0$  for  $j = c, d$ . We let the error terms be characterized as

$$\begin{pmatrix} \varepsilon_{c,t} \\ \varepsilon_{d,t} \end{pmatrix} \sim iid \mathbb{N}(0, \Sigma), \quad \Sigma = \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix} \quad (9)$$

and  $e_{j,t}$  is *i.i.d.*  $\mathbb{N}(0, \sigma_j^2)$  for  $j = c, d$ , and  $u_t$  and  $\eta_{j,t}$  are *i.i.d.*  $\mathbb{N}(0, 1)$ . Moreover, we assume that  $\varepsilon_{j,t}, \eta_t, u_t$  and  $e_{j,t}$  are mutually independent of each other. The variances of  $\varepsilon_{j,t}, \eta_{j,t}$  and  $u_t$  are all set to be unity to identify the parameters  $\alpha_j, \beta_j, \varphi_j$  and  $\lambda_j$ . Furthermore,  $w_0$  is assumed to be independent of  $u_t, e_{j,t}$  and  $\varepsilon_{j,t}$ . In what follows, we denote by  $\theta_j = (\alpha_j, \beta_j, \kappa_j)$  the vector of unknown parameters in volatility function for  $j = c, d$ .

Equations (4) and (5) dictate that the conditional expectation of the consumption growth contains the time-varying component  $\nu_{c,t}$  and shocks to this component affects the future consumption profile. Therefore, if this shock is highly persistent, it may have a long-run effect. For this shock to be transmitted to financial markets, Bansal and Yaron (2004) and Hansen, Heaton, and Li (2005) additionally assume that the corresponding component of the dividend growth  $\nu_{d,t}$  equals a constant multiplication of  $\nu_{c,t}$  and this common component turns out to play an essential part in generating equity premium. This is so called the long-run risks channel following Bansal and Yaron (2004). Unlike their case, however, we do not assume a common component between  $\nu_{c,t}$  and  $\nu_{d,t}$  for the reasons we explain below.

First, it is one of our main objectives to examine whether the existence of the common volatility factor in consumption and dividend growth rates, which we interpret as the macroeconomic uncertainty, alone may explain the equity premium. Imposing the presence of commonality in the conditional means of consumption and dividend growth rates has a potential to have a fatal impact on the validity of our results. If either misspecified or imprecisely estimated, the common variation restriction on conditional mean would introduce a spurious common factor in volatility. Such a hazard is likely to occur in our analysis since it is well known to be hard to precisely estimate the conditional mean components of consumption and dividend growth rates. For this reason, we do not impose any restriction on the conditional means of the consumption and dividend growths and leave their specifications as flexible as possible.

Furthermore, the long-run risk models use a parameter, say  $\Gamma$ , connecting  $\nu_{c,t}$  and  $\nu_{d,t}$  ( $\nu_{d,t} = \Gamma \nu_{c,t}$ ) to make a common component in the conditional mean of the consumption

and dividend growths.<sup>7</sup> These models need a sufficiently large value, often greater than 3 to generate a sizable equity premium. We later show that relaxing the assumption may produce even a negative relationship between risk and return. Therefore, without a priori restriction on the conditional expectations except that they are time-varying and stochastic, our model allows us to analyze how important the fluctuations in macro volatilities is as a source of risk, in explaining the asset pricing anomalies, even if this common long-run risks channel is weak or not working properly.

Bearing this in mind, we now turn our attention to the stochastic volatilities. Equations (4) to (9) state that the stochastic volatilities in our model are generated by the latent factors  $x_{j,t}$ , which consists of two components, i.e., the common factor  $w_t$  and the idiosyncratic factor  $e_{j,t}$ . The common factor is set to be a random walk, whereas the idiosyncratic factor is assumed to be *i.i.d.* This specification is motivated by the facts that are well known in the literature on the stochastic volatilities of consumption and dividend growth rates and asset returns. The reader is referred to Jacquier, Polson, and Rossi (2004), Kim, Lee, and Park (2009) and Jeong, Kim, and Park (2009) for more details. First, it has been demonstrated clearly that the stochastic volatilities of these rates are non-stationary. Their AR coefficients are indeed all very close to unity across a different set of model choices.<sup>8</sup> Second, it is apparent that their stochastic volatilities have a common stochastic trend. Naturally, we may specify the presence of such a common stochastic trend as a cointegrating relationship, given the evidence of nonstationarity of individual stochastic volatilities.

The actual volatilities are generated by the parametric logistic function  $f_j$ . The logistic function has two asymptotes  $\alpha_j$  and  $\alpha_j + \beta_j$ , which represent the two asymptotic regimes, i.e., the low and high volatility regimes respectively. We impose the identifying restriction  $\beta_j > 0$ , so that a larger realized value of the latent volatility factor implies higher volatility. The parameters  $\kappa_j$  and  $\lambda_j$  in (6) characterize the transition between two regimes, i.e., the location and speed of the transition. As  $\lambda_j$  gets larger, the transition speed becomes faster and, to a larger extent, the actual volatilities are generated by one of the two asymptotic regimes. Indeed, we may view the regime switching model as the limiting case of our model with  $\lambda_j = \infty$ . Finally,  $\kappa_j$  designates the location of the transition. In our specification of  $w_t$  as a random walk with the starting value  $w_0$  independent of all other stochastic components in the model,  $\kappa$  and  $w_0$  are not individually identified. We must therefore set either  $w_0 = 0$  or  $\kappa_j = 0$ .

The smooth transition feature of our volatility model together with the presence of a persistent common factor provides us with some insight about the relationship between asset prices and the macroeconomic uncertainty. There is little doubt that the high volatility regime implies the larger amount of uncertainty and risk than that of the low volatility regime. However, if the level of volatility is in between these two regimes, it creates another

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<sup>7</sup>This is often called, consumption leverage parameter following Abel (1990) and Bansal and Yaron (2004). More generic form includes an additional factor as well as consumption and dividend growths. Menzly, Santos, and Veronesi (2004) find that this parameter may be close to 1 for some industry portfolios.

<sup>8</sup>The reader is referred to Park (2002) for the time series properties of the processes having non-stationary stochastic volatilities. In particular, he shows that they are more consistent with the properties of many of macroeconomic and financial time series, compared with stationary stochastic volatility models and other volatility models such as ARCH and GARCH type models.

layer of uncertainty because economic agents are not sure about which regime they may end up with in the future. Especially, if the adjustment speed is slow, this effect can be significant. Note that conventional regime shifting models allow only sudden changes and ignore the role of transition states. In addition, other stochastic volatility models have no boundaries, and hence are subject to the possibility of explosive dynamics. On the contrary, our setup not only resolves this issue but also predicts that an economy with relatively low-to-medium levels of the macroeconomic volatility (i.e. in a transition mode) can request a higher premium than otherwise due to this additional uncertainty different from the conventional sources of risk, as long as economic agents prefer a state of uncertainty to be revealed earlier than later. In sum, our stochastic volatility setup effectively prohibits the volatility from exploding over time, yet generates sufficient amount of risk and uncertainty due to the combination of the Epstein-Zin-Weil preferences and the period of smooth transitions between high and low volatility regimes.

We believe that this is a new way of understanding the time-varying uncertainty in macroeconomic variables. Similar to the conventional method of stochastic volatilities, the level of volatility matters to measure the degree of uncertainty plagued in an economy. However, our model says that the distance to the maximum or minimum uncertainty is also critical in quantifying the amount of risk and uncertainty that the economic agents have to put up with. To offer some theoretical explanations on this new channel, we solve and approximate the log price-consumption ratio  $z_{c,t}$  as

$$z_{c,t} = A_{0,c} + A_{1,c}\nu_{c,t} + A_{2,c}f_c(\lambda_c w_t) \quad (10)$$

with

$$\begin{aligned} A_{0,c} &= \frac{\chi \ln \delta + (1 - \gamma)\mu_c + \chi\phi_{0,c} + 0.5(\chi\phi_{1,c}A_{2,c})^2 [\xi_2^2 - (\xi_1 f_{c,0})^2]}{\chi(1 - \phi_{1,c})} \\ A_{1,c} &= \frac{1 - \gamma}{\chi(1 - \phi_{1,c}\rho_c)} \\ A_{2,c} &= \frac{1 - \phi_{1,c} + \sqrt{(1 - \phi_{1,c})^2 - 2(\phi_{1,c})^2\Upsilon [(1 - \gamma)^2 + (\chi\phi_{1,c}A_{1,c}\rho_c)^2]}}{2\chi(\phi_{1,c})^2\Upsilon}, \end{aligned}$$

where

$$\begin{aligned} \Upsilon &= \xi_1 (\xi_1 f_{c,0} + \xi_2) \\ \xi_1 &= \frac{\lambda_c}{\beta_c} (2\alpha_c + \beta_c - 2f_{c,0}) \\ \xi_2 &= \frac{\lambda_c}{\beta_c} [f_{c,0}^2 - \alpha_c (\alpha_c + \beta_c)], \end{aligned}$$

and  $f_{c,0} = f_c(x_{c,0})$ . The derivations for  $A_{0,c}$ ,  $A_{1,c}$ , and  $A_{2,c}$  are available in the Appendix A.1, but we want to mention that a crucial part of obtaining (10) involves approximating  $f_{c,t}$  as

$$f_{c,t+1} \approx f_{c,t} + (\xi_1 f_{c,t} + \xi_2)u_{t+1}.$$



Thus,  $(\xi_1 f_{c,t} + \xi_2)$  can be considered as a conditional volatility of  $f_{c,t+1}$  process, hence has the positive sign by construction. Additionally, if  $\gamma > 1$ ,  $\psi > 1$ , and  $\xi_1 > 0$ , then  $A_{1,c} > 0$  and  $A_{2,c} < 0$ .<sup>9</sup> This means that consumers prefer higher expected future growth, but do not like a rise in macroeconomic volatility. In addition to the conventional channel of volatility ( $f_c(\lambda_c w_t)$ ) in which higher  $\lambda_c w_t$  means higher uncertainty, we can observe from  $A_{2,c}$  that when  $\lambda_c$  gets smaller, i.e., when there exists a slower transition, the absolute value of  $A_{2,c}$  increases via decreases in  $\Upsilon$ . That is, given the traditional risk-return trade-off relationship, our model has the extra layer of the volatility channel: a low level of macroeconomic volatility does not necessarily mean that economic agents perceive the state of the economy as being safe. When uncertainty unfolds in a sluggish fashion and is in transit, they fear the possibility of being at the high volatility regime in the near future.

Once all the coefficients are verified, we can easily derive the innovations in the stochastic discount factor, or IMRS as

$$m_{t+1} - \mathbb{E}_t(m_{t+1}) \approx \Lambda_{m\varepsilon} \sqrt{f_c(x_{c,t})} \varepsilon_{c,t+1} - \Lambda_{m,\eta} \sqrt{f_c(x_{c,t})} \eta_{c,t+1} - \Lambda_{m,u}(t) u_{t+1} \quad (11)$$

with

$$\begin{aligned} \Lambda_{m\varepsilon} &\equiv -\gamma \\ \Lambda_{m,\eta} &\equiv (1 - \chi) \phi_{1,c} A_{1,c} \varphi_c \\ \Lambda_{m,u}(t) &\equiv (1 - \chi) \phi_{1,c} A_{2,c} (\xi_1 f_c(x_{c,t}) + \xi_2), \end{aligned}$$

where the coefficient terms are derived in Appendix A.2. Here the risk sources are represented by three terms, labeled as the short-run risk  $\varepsilon_{t+1}$ , the long-run risk  $\eta_{t+1}$ , and the common macroeconomic uncertainty  $u_{t+1}$ . Similar to (10), positive innovations in both short-term and long-term consumption growth lead to lower discount rates for futures, while higher volatility innovations refer to higher discount for futures again if  $A_{2,c} < 0$ .<sup>10</sup>

With the IMRS, we can express the risk-free rate as<sup>11</sup>

$$\begin{aligned} r_{f,t} &= -\log [\mathbb{E}_t \exp(m_{t+1})] \\ &= -\log \delta + \frac{1}{\psi} \mathbb{E}_t g_{c,t+1} + \frac{1 - \chi}{\chi} \mathbb{E}_t [r_{a,t+1} - r_{f,t}] - \frac{1}{2\chi} \text{Var}_t(m_{t+1}). \end{aligned} \quad (12)$$

The negative relation between the risk-free rate and the intertemporal elasticity of substitution is clear from the first term in (12). But as the risk-aversion coefficient becomes larger (i.e.,  $\chi$  being more negative), hence both the conditional mean of risk premium on wealth and the conditional variance of IMRS increase, the effect on the risk-free rate is not clear from (12) immediately because the last two terms have opposite signs. If the third term dominates the fourth term, which is usually the case, the risk-free rate decreases.

<sup>9</sup>By definition,  $0 < \phi_{1,c} < 1$  holds.  $\xi_1 > 0$  holds according to our estimates for the stochastic volatility.

<sup>10</sup>More specifically,  $\Lambda_{m,u}(t)$  is negative provided that  $\chi < 0$ ,  $A_{2,c} < 0$ , and  $\xi_1 f_c(x_{c,t}) + \xi_2 > 0$ . The last inequality holds by construction because the term is the approximated conditional volatility of  $f_c(x_{c,t})$ . It turns out that other restrictions hold as well, according to our estimates for the macroeconomic uncertainty.

<sup>11</sup>We derive the interest rate in terms of current value state variables as well. See the Appendix C. for derivation.

We will verify this via simulations later in Section 4. Before we move on to the empirical study however, we want to further examine (1) using (10) and (11) to provide some analytic explanations on the mechanism of our model.

## 2.2 Asset Returns Dynamics

Now we take a close look at the market return ( $r_{m,t}$ ) and analyzes how the non-linear, non-stationary common macroeconomic uncertainty non-trivially affects the asset return dynamics in equilibrium. For our estimation purpose, we numerically solve the model (1), (3), and (10) with the fundamental stochastic processes (4) - (9). However, the black-box nature of the Euler equation does not give much insight on the conditional covariance of the IMRS and asset returns. In this light, we approximate the solution for the market return as

$$\mathbb{E}_t[r_{m,t+1} - r_{f,t}] = \Phi_1 f_{c,t} + \Phi_2 (\xi_1 f_{c,t} + \xi_2)^2 - \frac{1}{2} \text{Var}_t(r_{m,t+1}) \quad (13)$$

with

$$\begin{aligned} \Phi_1 &= \frac{-\left(\gamma - \frac{1}{\psi}\right) \phi_{1,c} \phi_{1,m} (\varphi_c)^2}{\psi(1 - \phi_{1,c} \rho_c)(1 - \phi_{1,m} \rho_c)} \\ \Phi_2 &= \frac{(1 - \chi) \phi_{1,c} A_{2,c} \left[ \Xi - \sqrt{\Xi^2 - 4\Upsilon \phi_{1,m}^2 H} \right]}{2\Upsilon \phi_{1,m}} \\ \Xi &= 2\Upsilon (1 - \chi) \phi_{1,c} \phi_{1,m} A_{2,c} + 1 - \phi_{1,m}, \end{aligned}$$

where  $H$  is defined in Appendix B and see the derivation therein. The first term in the right hand side is basically the contribution by the long-run risks channel. However, note that  $\Phi_1 < 0$  with early resolution of uncertainty. This implies that this channel gives a negative risk-return trade-off with  $\chi < 0$ . This mainly comes from no restriction between consumption growth and dividend growth as previously mentioned. If we instead assume a long-run relationship as  $\nu_{d,t} = \Gamma \nu_{c,t}$  (i.e., the long-run risks channel), we obtain

$$\mathbb{E}_t[r_{m,t+1} - r_{f,t}] = \Phi_1 f_{c,t} + \Phi_2 (\xi_1 f_{c,t} + \xi_2)^2 - \frac{1}{2} \text{Var}_t(r_{m,t+1}) \quad (14)$$

with

$$\Phi_1 = \frac{\left(\gamma - \frac{1}{\psi}\right) \left(\Gamma - \frac{1}{\psi}\right) \phi_{1,c} \phi_{1,m} (\varphi_c)^2}{(1 - \phi_{1,c} \rho_c)(1 - \phi_{1,m} \rho_c)}$$

and  $\Phi_2$  and  $\Xi$  defined as for (13).<sup>12</sup> In this case,  $\Phi_1 > 0$  can hold if  $\Gamma > \psi^{-1}$  is satisfied. Existing studies use  $\Gamma$  greater than 3 to match the data. Note that we did not restrict the long-run relationship between two conditional expectations and solve most of our cases numerically. Thus, we do not resort to either of the above approximate solutions when we

<sup>12</sup>In this case, the formula for  $H$  is slightly different than (13). See the Appendix B for details.

evaluate the empirical performance of the model, but one can see that our model allows a case in which long-run risks channel is weak, or even reversed.

The main thrust of our model lies in the second term, which describes how the time-varying macro uncertainty is related to the expected excess returns for holding equity. Note that the second term includes the squared conditional volatility of the macroeconomic uncertainty and it is time-varying. This is due to the fact that the market price of uncertainty  $\Lambda_{m,u}(t)$  is time-varying and coincides with the uncertainty factor. One can see from the similarity of  $\Phi_2$  that this channel is robust regardless of the conditional mean specifications. An interplay between  $\chi < 0$  (preference for early resolution of uncertainty) and  $A_{2,c} < 0$  (our volatility specification) produces this uncertainty premium ( $\Phi_2 > 0$ ). Technically,  $\Xi < 0$  is sufficient to obtain a positive  $\Phi_2$ . Given  $A_{2,c} < 0$ , this occurs if the first term in  $\Xi$  dominates the second term and this is a quantitative concern like the long-run risk restriction. An empirical evaluation is therefore needed on this matter. However, we want to stress that a slow speed of adjustment (a small  $\lambda_c$ ) for the macroeconomic uncertainty can amplify  $\Phi_2$  by decreasing  $\Upsilon$  and thereby increasing the absolute value of  $A_{2,c}$ . That is, a fairly modest amount of the macroeconomic volatility does not necessarily imply a small risk premium requested by investors because there exists time-varying uncertainty about the volatility regimes.

### 3. Identifying the Macroeconomic Uncertainty

#### 3.1. A Bayesian Algorithm

According to the asset pricing model we developed in the previous section, asset returns evolve over time because of the time-varying uncertainty inherent in the macroeconomic variables. In this section, we propose an econometric procedure to identify the macroeconomic uncertainty process. For this purpose, we rewrite (4) as

$$y_{j,t+1} = \sqrt{f_j(x_{j,t})} \varepsilon_{j,t+1} \quad (15)$$

with  $y_{j,t+1} = g_{j,t+1} - \mu_j - \nu_{j,t}$  for  $j = c, d$ . Note that  $y_{j,t+1}$  is a martingale difference sequence defined from  $g_{j,t+1}$ , net of its slow moving conditional mean component  $\mu_j + \nu_{j,t}$ . In the paper, we use the Hodrick-Prescott filter to estimate the moving conditional mean component of  $g_{j,t+1}$ , which is subtracted from  $g_{j,t+1}$  to obtain the estimated  $y_{j,t+1}$ . We also experimented with the method suggested by Bansal and Yaron (2004) and Bansal, Kiku, and Yaron (2007), which regress the consumption growth onto the interest rate and the price-dividend ratio, exploiting the theoretical structures. Although both results are compatible with each other, we find that the statistical filtering seems to identify the long-run components better, especially with monthly frequency data.

Now with the estimates of  $y_{j,t}$  in hand, one can easily see that (15) as well as (5) to (8) form a usual state space model with latent factors. The resulting state space model, however, is nonstandard in two aspects: It is non-linear, and includes non-stationary latent factors. There are several methodologies that can be used to statistically analyze non-linear state space models with latent factors. The most conventional approach is to apply

the extended Kalman filter for the linearized version of the measurement equation (15) after taking squares and logs. However, we find that the conventional extended Kalman filter performs rather poorly in the presence of non-stationary latent factors.<sup>13</sup> To avoid this difficulty, Kim, Lee and Park (2009) have recently developed the density-based filter, relying on the updates, predictions and smoothings based on the estimated densities of latent factors, for the model similar to, but much simpler to, what we consider in the paper. At least in theory, it is straightforward to extend their filter to make it applicable for our model, as is shown in Appendix D. However, computationally it seems extremely difficult and time demanding to use this approach for the estimation of our model.

In the paper, we take the Bayesian approach. To introduce our approach, it will be convenient to define some additional notations. We let  $n$  be the sample size, and define  $Y = (y_1, \dots, y_n)$  with  $y_t = (y_{c,t}, y_{d,t})'$ , and  $L = (X, W)$  with  $X = (x_1, \dots, x_n)$ ,  $x_t = (x_{c,t}, x_{d,t})$  and  $W = (w_1, \dots, w_n)$ . Moreover, we define  $\Psi = (\theta, \rho, \lambda, \sigma^2)$  with  $\theta = (\theta_c, \theta_d)$ ,  $\lambda = (\lambda_c, \lambda_d)$  and  $\sigma^2 = (\sigma_c^2, \sigma_d^2)$ . Note that  $Y$ ,  $L$  and  $\Psi$  denote respectively the observed samples, latent factors and unknown parameters of our model. Moreover, we let  $D_t = \text{diag}(\sqrt{f_c(x_{c,t})}, \sqrt{f_d(x_{d,t})})$ , and signify the marginal and conditional densities by  $p(\cdot)$  and  $p(\cdot|\cdot)$  in what follows. Now we may easily deduce that the joint posterior density of the latent factors and unknown parameters is given by

$$p(L, \Psi|Y) \propto p(L, Y|\Psi)p(\Psi) \propto \left( \prod_{t=1}^n p(y_t|x_t, \Psi)p(x_{c,t}|w_t, \Psi)p(x_{d,t}|w_t, \Psi)p(w_t|w_{t-1}) \right) p(\Psi). \quad (16)$$

Following the usual Bayesian procedure, we will implement a Markov-Chain-Monte-Carlo (MCMC) method to sample  $(L, \Psi)$  from the joint posterior density  $p(L, \Psi|Y)$  in (16). Once the observations of  $(L, \Psi)$  are drawn, we may readily obtain any posterior sample moments of  $L$  and  $\Psi$  using the standard Monte-Carlo numerical integration.<sup>14</sup>

For our MCMC procedure, we use the Gibbs sampler and the Metropolis-Hastings (MH) algorithm within the Gibbs sampler.<sup>15</sup> The procedure allows us to effectively deal with the multi-dimensionality of our latent factors and parameters and difficulties in drawing samples from the complicated target densities. The Gibbs sampler simplifies the required sampling procedure by reducing our multi-dimensional transition to the composition of a sequence of uni-dimensional transitions. This is very helpful for the analysis of our model, which includes a large number of latent factors and unknown parameters. Subsequently to the application of Gibbs sampler, we rely on the MH algorithm in case it is difficult to proceed by sampling directly from the required transition. The MH algorithm allows us to implement the required transition simply by drawing samples from a candidate-generating density, together with

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<sup>13</sup>This is well expected, since some of the partial derivatives of our volatility function are integrable functions of non-stationary latent factors. Therefore, the Kalman gains get very close to zero, when the extracted factors take large values, providing no further updates for the latent factors.

<sup>14</sup>For related Bayesian procedures, the reader is referred to Jacquier, Polson and Rossi (1994, 2004), So, Lam, and Li (1998) and Geweke and Tanizaki (2001). Our procedure follows Jacquier, Polson, and Rossi (2004) most closely.

<sup>15</sup>As explained in Chib and Greenberg (1995), we may regard our entire procedure as the MH algorithm applied in turn to one variable at a time.

a random selection rule. See Chib and Greenberg (1995) for a nice introduction of the algorithm. For the choice of candidate-generating distributions, we follow the suggestion by Geweke and Tanizaki (2001). For the candidate-generating distributions of latent factors, we use the distributions given by the transition equation of our model. On the other hand, the prior distributions are used for the candidate-generating distributions for the unknown parameters. As is well known, our MCMC procedure is valid, since the underlying Markov chain is irreducible and aperiodic.

Now we derive the conditional posteriors and the required sampling methods to implement the Gibbs sampler and MH algorithm, which are needed to execute our MCMC procedure. For the common latent factor  $w_t$ , we have for  $t = 1, \dots, n - 1$

$$\begin{aligned} p(w_t|X, W_{\setminus t}, Y, \Psi) &= p(w_t|x_{c,t}, x_{d,t}, w_{t+1}, w_{t-1}, \Psi) \\ &\propto p(x_{c,t}, x_{d,t}|w_t, w_{t+1}, w_{t-1}, \Psi)p(w_{t+1}, w_t|w_{t-1}) \\ &= p(x_{c,t}|w_t, \lambda_c, \sigma_c^2)p(x_{d,t}|w_t, \lambda_d, \sigma_d^2)p(w_{t+1}|w_t)p(w_t|w_{t-1}), \end{aligned} \quad (17)$$

where  $W_{\setminus t}$  denotes  $W$  with  $w_t$  deleted. We may readily deduce from (17) that the conditional distribution of  $w_t$  given  $X, W_{\setminus t}, Y$ , and  $\Psi$  is indeed  $\mathbb{N}(RS^{-1}, S^{-1})$ , where

$$R = \sum_j \frac{\lambda_j x_{j,t}}{\sigma_j^2} + w_{t+1} + w_{t-1} \quad \text{and} \quad S = \sum_j \frac{\lambda_j^2}{\sigma_j^2} + 2.$$

Therefore, it is quite straightforward to draw samples for the common factor given all values of other latent factors and unknown parameters.<sup>16</sup>

In case of the latent factor  $x_{j,t}$ , the conditional posterior density is given by

$$\begin{aligned} p(x_{j,t}|X_{\setminus j,t}, W, Y, \Psi) &= p(x_{j,t}|x_{\setminus j,t}, w_t, y_t, \Psi) \\ &\propto p(y_t|x_t, w_t, \Psi)p(x_t|w_t, \Psi) = p(y_t|x_t, \theta, \rho)p(x_{j,t}|w_t, \lambda_j, \sigma_j^2) \\ &= \frac{1}{2\pi} \det(D_t \Sigma D_t)^{-1/2} \exp\left(-\frac{y_t'(D_t \Sigma D_t)^{-1} y_t}{2}\right) \frac{1}{\sqrt{2\pi\sigma_j^2}} \exp\left(-\frac{(x_{j,t} - \lambda_j w_t)^2}{2\sigma_j^2}\right) \\ &\propto \det(D_t \Sigma D_t)^{-1/2} \exp\left(-\frac{y_t'(D_t \Sigma D_t)^{-1} y_t}{2} - \frac{(x_{j,t} - \lambda_j w_t)^2}{2\sigma_j^2}\right). \end{aligned} \quad (18)$$

Note that the conditional distribution of  $y_t$  given  $x_t, \theta, \rho$  and that of  $x_{j,t}$  given  $w_t, \lambda_j, \sigma_j^2$  are given by  $\mathbb{N}(0, D_t \Sigma D_t)$  and  $\mathbb{N}(\lambda_j w_t, \sigma_j^2)$  respectively. It is difficult to directly draw samples from the conditional density in (18), and therefore, we use the MH algorithm. In particular, we apply the algorithm with  $\mathbb{N}(\lambda_j w_t, \sigma_j^2)$  as the candidate-generating density.

Finally, we derive the conditional posterior distributions of  $(\theta, \rho, \lambda, \sigma^2)$ . The conditional

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<sup>16</sup>For  $t = n$ , we may similarly show that the conditional distribution of the common latent factor is also  $\mathbb{N}(RS^{-1}, S^{-1})$  with  $R = \sum_j (\lambda_j x_{j,n} / \sigma_j^2) + w_{n-1}$  and  $S = \sum_j (\lambda_j^2 / \sigma_j^2) + 1$ .

Table 1: Summary Statistics

	mean	max	min	std.	skewness	kurtosis
Consumption	0.0023	0.032	-0.026	0.0073	0.0089	4.309
Dividend	0.0037	0.0809	-0.126	0.0322	-0.825	4.40

*Notes:* In the table, “consumption” and “dividend” refer to monthly consumption and dividend growth rates respectively. Both series are per capita and adjusted in real terms.

posterior density of  $\theta_j$ ,  $j = c, d$ , is given by

$$\begin{aligned} p(\theta_j|X, W, Y, \Psi \setminus \theta_j) &\propto p(Y|X, \theta, \rho)p(\theta_j) \\ &= \prod_{t=1}^n \frac{1}{2\pi} \det(D_t \Sigma D_t)^{-1/2} \exp\left(-\frac{y_t'(D_t \Sigma D_t)^{-1} y_t}{2}\right) p(\theta_j). \end{aligned}$$

Similarly, the conditional posterior density of  $\rho$  is

$$\begin{aligned} p(\rho|X, W, Y, \Psi \setminus \rho) &\propto p(Y|X, \theta, \rho)p(\rho) \\ &= \prod_{t=1}^n \frac{1}{2\pi} \det(D_t \Sigma D_t)^{-1/2} \exp\left(-\frac{y_t'(D_t \Sigma D_t)^{-1} y_t}{2}\right) p(\rho). \end{aligned}$$

Moreover, the conditional posterior densities of  $\lambda_j$  and  $\sigma_j^2$  are given by

$$\begin{aligned} p(\lambda_j|X, W, Y, \Psi \setminus \lambda_j) &\propto p(X_j|W, \lambda_j, \sigma_j^2)p(\lambda_j) \\ &= \prod_{t=1}^n \frac{1}{2\pi\sigma_j^2} \exp\left(-\frac{(x_{j,t} - \lambda_j w_t)^2}{2\sigma_j^2}\right) p(\lambda_j) \end{aligned}$$

and

$$\begin{aligned} p(\sigma_j^2|X, W, Y, \Psi \setminus \sigma_j^2) &\propto p(X_j|W, \lambda_j, \sigma_j^2)p(\sigma_j^2) \\ &= \prod_{t=1}^n \frac{1}{2\pi\sigma_j^2} \exp\left(-\frac{(x_{j,t} - \lambda_j w_t)^2}{2\sigma_j^2}\right) p(\sigma_j^2) \end{aligned}$$

for  $j = c, d$ . We apply the MH algorithm to sample from the conditional posterior distributions of  $(\theta, \rho, \lambda, \sigma^2)$ , using their prior distributions as the candidate-generating densities. We use Gamma priors for the parameters  $\alpha, \beta, \sigma^2$  and  $\lambda$ , and normal priors for the parameters  $\kappa$  and  $\rho$ .

### 3.2 Data and Estimation Results

We use monthly consumption and dividend series from February 1959 to December 2006 to apply the method we developed. The consumption data for nondurable goods and service expenditure series are obtained from the Federal Reserve Banks of St. Louis web site (<http://research.stlouisfed.org>) and the dividend data from CRSP. Dividend growth is

Table 2: Estimation Results for Stochastic Volatility

Parameters	Priors	Posterior		Convergence
		Mean	Standard errors	Diagnostics
$\alpha_c$	$G(1, 2 \times 10^{-4})$	0.000003	0.0000011	0.171
$\beta_c$	$G(1, 4 \times 10^{-4})$	0.000022	0.0000081	-0.457
$\kappa_c$	$N(0, 1)$	-0.3256	0.8968	0.443
$\lambda_c$	$N(0.5, 1)$	0.3060	0.1621	-0.759
$\sigma_c^2$	$G(1, 2 \times 10^{-4})$	1.7333	0.4699	-1.091
$\alpha_d$	$G(1, 3 \times 10^{-4})$	0.000175	0.000032	-1.822
$\beta_d$	$G(5, 10^{-4})$	0.002831	0.0006273	2.643
$\kappa_d$	$G(0, 1)$	1.0930	0.8536	1.649
$\lambda_d$	$G(0.5, 1)$	0.7172	0.1573	0.221
$\sigma_d^2$	$G(1, 2)$	0.6429	0.3190	2.171
$\rho$	$N(0, 1)$	-0.0213	0.0442	-1.336

Notes:  $G(a, b)$  and  $N(a, b)$ , respectively, denote gamma and normal distributions with parameters  $a$  and  $b$ .  $G(a, b)$  has mean  $ab$  and variance  $ab^2$ , whereas  $N(a, b)$  has mean  $a$  and variance  $b$ .

generated from the seasonally-adjusted real dividend series, which is created using value-weighted returns with and without dividend.<sup>17</sup> Additionally, both series were adjusted as per capita values. There are total 575 observations. Table 1 provides summary statistics for the two series.

We draw 120000 samples for each parameters and latent variables by the Gibbs sampler and the MH algorithm, and discard the first 40000 samples, which are considered as samples in the burn-in period. Table 2 shows that most of the estimated parameters are well converged and significant. Though  $\beta_c$  and  $\alpha_d$  have relatively high convergence diagnostics<sup>18</sup>, their small standard errors show that sample series are quite concentrated after the burn-in period.<sup>19</sup> Jacquier, Polson, and Rossi (2004) warn that one must perform careful sampling

<sup>17</sup>Real dividend is computed using  $(\text{returns with dividend} - \text{returns without dividend}) \times \text{market price index} / \text{CPI}$ , where CPI is the consumer price index from the St. Louis Fed. We use cash dividends to measure dividend series and this may be insufficient to capture the total payouts distributed by corporations. Since we want to compare our results with the existing studies, we use the conventional definition of the dividend series. It would be interesting to analyze how the volatilities of the total payouts differ from those of the cash dividends.

<sup>18</sup>The measure of convergence diagnostic (CD) is given by

$$CD = \frac{\bar{\theta}_A - \bar{\theta}_B}{\sqrt{\frac{\hat{\omega}_A^2}{n_A} + \frac{\hat{\omega}_B^2}{n_B}}},$$

where  $A$  is the set of Gibbs samples with  $n_A$  iterations after burn-in period and  $B$  is the set of Gibbs samples with last  $n_B$  observations, and  $\hat{\omega}_A^2$  and  $\hat{\omega}_B^2$  are their HAC estimates obtained using Parzen window. By the convention we set  $n_A/n = 0.1$  and  $n_B/n = 0.5$ , where  $n$  denotes the number of Gibbs samples after burn-in periods. As shown in Geweke (1992), CD converges to the standard normal distribution as the number of samples goes to infinity, if the sequence of Gibbs samples for a parameter is stationary.

<sup>19</sup>Figures 5 and 6 in Appendix E illustrate this by including the burn-in period.

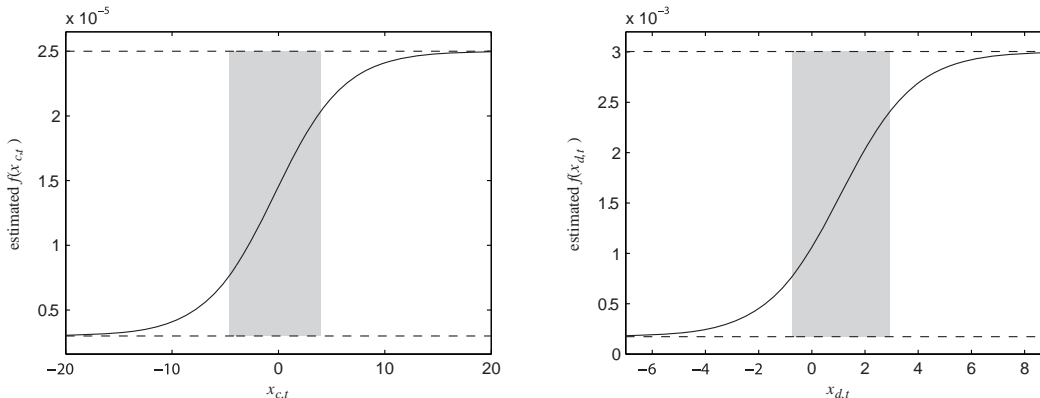


Figure 1: Estimated Logistic Volatility Functions for Consumption and Dividend.

*Notes:* The left panel is the estimated logistic volatility for consumption growth and the right panel is that of dividend growth. Horizontal axis stands for the latent factor  $x_{j,t}$  for  $j = c, d$ . Dashed lines represent asymptotes and the shaded area refers to transition period.

experiments to establish convergence across a wide range of empirically relevant parameter values. Our samples show that they wander a lot at the beginning, but converge after the burn-in period.

We can see the  $\alpha_c$  and  $\beta_c$  are smaller than  $\alpha_d$  and  $\beta_d$  due to the volatile dividend process. While the lower bounds of volatility for both consumption and dividend are  $\sqrt{\hat{\alpha}_c} = 0.02\%$  and  $\sqrt{\hat{\alpha}_d} = 1.19\%$ , the upper bounds of volatility are  $\sqrt{\hat{\alpha}_c + \hat{\beta}_c} = 0.49\%$  and  $\sqrt{\hat{\alpha}_d + \hat{\beta}_d} = 6.05\%$  in a month, respectively. The values of  $\hat{\lambda}_c$  and  $\hat{\lambda}_d$  indicate that the volatility generating processes  $x_{c,t}$  and  $x_{d,t}$  are scaled down compared with the magnitude of the macro uncertainty  $w_t$ . In addition, Figure 1 displays the estimated volatility functions of the consumption and dividend growths, and it shows that the volatilities in the macro level do not evolve over time via an explosive process without bounds, nor an abrupt regime-shifting process. Thus, when there is a shock to the macroeconomic uncertainty  $w_t$  which causes the volatilities of the consumption growth and the dividend growth to migrate into a transition period, the uncertainty about the consumption growth is likely to wander around for a longer period and this can increase the risk premium for bearing the macro level uncertainty, if early resolution of uncertainty is preferred. Regarding the volatility of idiosyncratic component,  $\hat{\sigma}_c^2$  is larger than  $\hat{\sigma}_d^2$ , which implies that the signal of the dividend process is stronger than that of consumption process such that the dividend volatility generating process  $x_{d,t}$  is closer to the common macro volatility generating process  $w_t$ . The differences in all these estimated parameters imply that the conventional assumption that the conditional volatility of the dividend is a simple constant multiplication of the conditional volatility of consumption may be somewhat overstated, though it is true that they have a common factor. The estimated value of  $\hat{\rho}$  indicates that the correlation between the short-run shocks is fairly small.

We also extract the latent common macroeconomic uncertainty process  $w_t$  and the



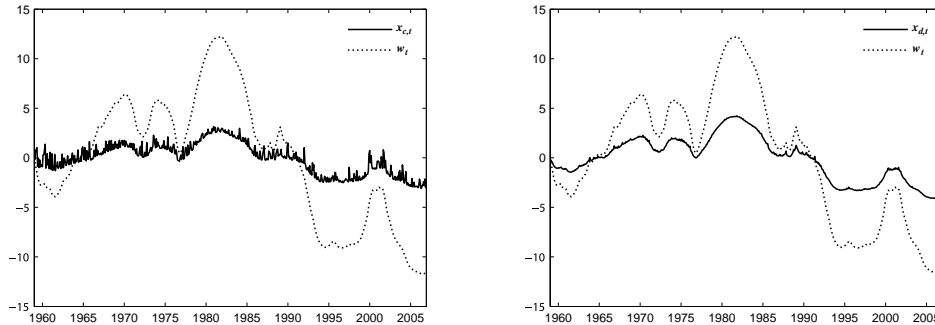


Figure 2: Volatility Factors for Consumption and Dividend.

*Notes:* Dotted lines in both panel displays the extracted common latent factor  $w_t$  and the other lines are the time series plots of the extracted factors for consumption and dividend growths,  $x_{j,t} = \lambda_j w_t + e_{j,t}$  for  $j = c, d$ .

volatility generating processes for the consumption and dividend growth rates  $x_{c,t}$ ,  $x_{d,t}$  as illustrated in Figure 2. All three volatility generating processes have a unit root, and the volatility generating processes of consumption and dividend are cointegrated with the common volatility generating process.<sup>20</sup> Figures 1 and 2 allow us to think about thresholds for a regime change. To be more specific, we regard the interval  $[\kappa - (1/\lambda) \log(2 - \sqrt{3}), \kappa - (1/\lambda) \log(2 + \sqrt{3})]$  as the transition period, where we have  $f'''(x) = 0$  at the endpoints of this interval. Then the area above the upper boundary can be regarded as the high volatility regime and the region below the lower boundary as the low volatility regime. We can interpret that the fundamental factors that generate volatility stayed in the low volatility regime in the 1960s, moved up into the transition period in the 1970s, then entered into the high volatility regime in the early 1980s. From the mid-1980s until the early 1990s the common factor has stayed in the transition period with sharply decreasing trends. Except during the 2001-2002 period, the US economy was in the low volatility period, often referred to as the great moderation period. In fact, the aggregate uncertainty in terms of consumption growth and dividend growth has been quite stable except in the late 1970s and early 1980s.

Notably, during most of the post-war period, the macro volatility was in transition periods. Thus, without incorporating the intermediate regimes, models trying to explain business cycle are likely to be misspecified. At the same time, the existence of the upper and lower boundaries is necessary to keep this volatility process from exploding, given its strong persistence. Knowing the maximum distance of consumption volatility from a current level of uncertainty, what matters to consumers would be how fast news on uncertainty evolves over time and how closely the current level of uncertainty approaches the maximum or the minimum level. In this regard, economic agents' preferences on different persistence of a

<sup>20</sup> ADF unit root test  $t$ -statistics are -1.2033, -1.2328 and -1.5258 respectively while the critical value for the 5% confidence level is -2.866. Residual based cointegration test  $t$ -statistics between  $w_t$  and  $x_{c,t}$  is -20.9900 and that between  $w_t$  and  $x_{d,t}$  is -4.0182, while the critical value for the 5% confidence level is -3.46.

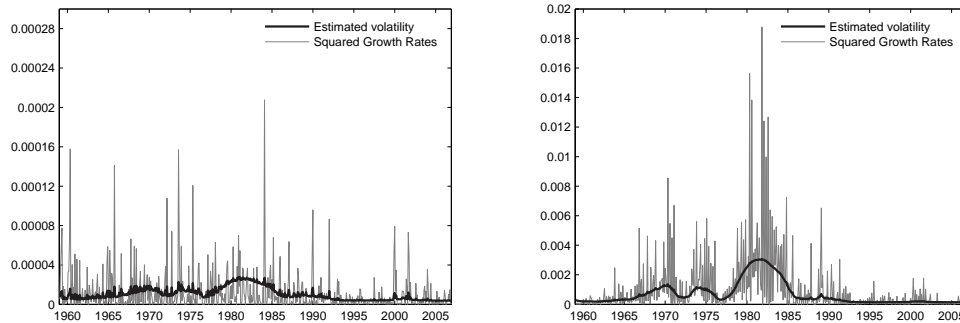


Figure 3: Squared Growth Rates and Estimated Variances of Consumption and Dividend.

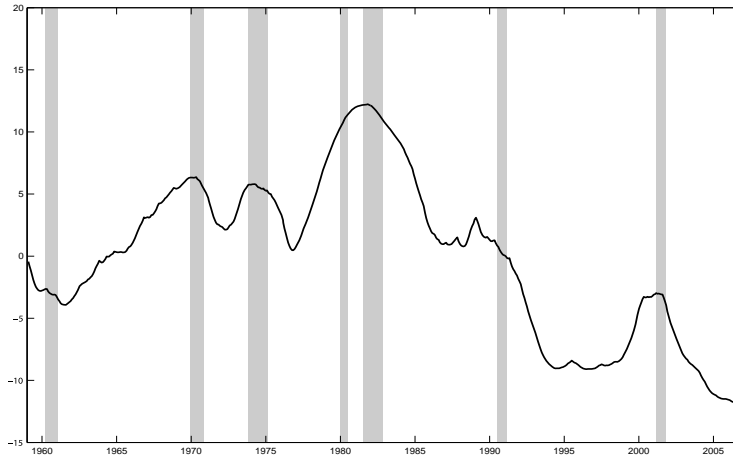
fundamental shock can be an important ingredient to account for the empirical evidence of high equity premium despite the small level of the consumption volatility. This view is consistent with the empirical result that the economy is mostly in transition periods with several observations of high volatilities. Figure 3 plots the estimated volatility and the realized volatility, which are obtained through the Monte-Carlo numerical integration. We see from Figure 3 that the estimated volatilities explain the realized volatilities quite well especially in light of their trend behaviors.

Finally, Figure 4 shows the relationship between the common volatility factor  $w_t$  and business cycle fluctuations recorded by the National Bureau of Economic Research (NBER). We find that the recession periods are well-matched with the local peaks of the common volatility generating process. In fact, the common volatility factor reached its local peak just before the entry into the 1970s, 1980s, the recent 2001 recessions, and even the 1991 mini credit crunch was predicted by a small, but a conspicuous increase in the macro uncertainty factor  $w_t$ . One of the most foundational links between asset returns and macroeconomic variables is that the expected asset returns are higher at business cycle troughs, reflecting the risk-return trade-offs with counter cyclical variations. Our macroeconomic uncertainty factor rises and reaches its local maximum when a recession begins, i.e. a bad economic time starts around the time of clear trend of rising uncertainty and the degree to which an economy is uncertain gets lower while the recession period passes by. Thus, we can expect that this is going to be well-connected to higher risk premia demanded by asset market participants. In the next section, we estimate this relationship using the identified state variables.

## 4. Implications on Asset Pricing Anomalies

In this section, we empirically study the implications of our model on asset prices. Toward this, we use the Generalized Method of Moment (GMM) to estimate two core preference parameters, the relative risk aversion coefficient  $\gamma$  and the intertemporal elasticity of substitution  $\psi$ . GMM uses only the moment conditions derived from the Euler equation (1) without additional assumptions on the probability distribution. Since the Euler equation

Figure 4: Business Cycles and the Macro Uncertainty Generating Process.



*Notes:* The solid line represents the common volatility generating process  $w_t$  and the shadow area stands for the recession period from the NBER.

is expressed as a conditional expectation of the product of the IMRS and the individual asset returns,  $\mathbb{E}_t(\exp(m_t)R_{i,t+1}) = 1$ , we can generate moment conditions by both considering multiple assets and multiple instrument variables which represent the conditioning information.

Our benchmark case employs multiple asset returns with the lagged consumption growth as the instrument and the results are presented in Table 3. We, in fact, experimented with various combinations of individual assets and the results are robust across each other. Among those we report three other cases in Appendix E. (Tables 5 and 6). These are categorized by the number of assets and the number of instruments considered. For the former, we have two cases: the “market returns” and “multiple asset returns” consisting of the returns from the market, large, small, growth and value stocks. The cases with multiple returns allow us to check if our asset pricing model has the ability of matching the cross-sectional features of the asset returns. The instruments that we use include the consumption growth, unemployment, inflation, and a nominal term spread (measured by the difference between the Treasury yields of one-year and three-month maturities). All instruments are lagged to make sure that they satisfy the orthogonality conditions. It turns out that our estimation results are not sensitive to the choices of instruments.<sup>21</sup> Each asset return series is obtained from the Fama and French data library (<http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data-library.html>), and the risk-free rate is the 3-month Treasury Bill rate obtained from the St. Louis Federal Reserve Bank.

<sup>21</sup>Specifically, we report the estimation results with two sets of the instruments. The first one uses only the consumption growth and the second one adopts all the instruments aforementioned. Only exception is the case with the market returns only. In this case, to check the overidentifying restriction, we include one more instrument, unemployment rate.

Table 3: GMM Estimation Result (Multiple Asset Returns)

Parameters	GMM Parameter Estimates			
	Estimates	Standard errors	t-stat	p-val
$\gamma$	2.111	0.380	5.56	0.0000
$\psi$	2.115	11.540	0.18	0.8572
	Selected Moment Conditions			
	Moment	Standard errors	t-stat	p-val
Small	0.003152	0.002374	1.33	0.1843
Large	0.000978	0.000955	1.02	0.3059
Value	-0.000767	0.000617	-1.24	0.2138
Growth	-0.000268	0.000328	-0.82	0.4134
Market	-0.001094	0.001386	-0.79	0.4300
$J$ -stat = 14.334		$\text{Prob}(\chi^2(8) > J) = 0.0735$		

*Notes:* The returns on small, large, growth, value, and market are used as individual assets satisfying the Euler equations. The panel of the selected moment conditions shows if the estimated moment conditions ( $\mathbb{E}_t(\exp(m_{t+1})R_{i,t}) - 1 = 0$ ) are close to zero. We use the lagged consumption growth as the instrumental variables in this estimation. Thus, we have five additional moment conditions which represent the orthogonality between each returns and the one period lagged consumption growth. Since we have more moment conditions than the number of parameters, we use an identity matrix as the initial weighting matrix to generate the optimal weighting matrix. We use the White heteroscedasticity correction for computing the weighting matrix.

These are all in monthly frequency and adjusted in real terms.

Table 3 shows that the estimate of the relative risk aversion parameter  $\gamma$  is 2.11 and accurately measured. Results from other cases report very similar results as well. Mehra and Prescott (1985) consider that a maximal level for relative risk aversion is around 10 and Barro (2006) argues that the usual view in the related literature is between 2 and 5. In this light, our result for the risk aversion parameter is reasonable and it is, in fact, a significant improvement over the existing studies which reported the estimated values of 15 to over 100.<sup>22</sup> For the elasticity of intertemporal substitution  $\psi$ , we have the estimates ranging between 1.2 and 3.5 and their measurements are somewhat noisy. Several papers consider that a plausible value of  $\psi$  lies in between 1 and 2. For example, Bansal and Yaron (2004) benchmark value is 1.5 and Hansen (1982), Attanasio and Weber (1989), and Vissing-Jorgensen (2002) estimate  $\psi$  to be well in excess of 1.5 respectively. In most studies, however, their estimates have large standard errors, and our case is not an exception. At least, we confirm that our finding for the EIS is similar to the results from the existing studies. One additional point related to our estimates is that since  $\hat{\gamma} > 1/\hat{\psi}$ , our estimates indicate that early resolution is preferred in this economy. Therefore, our volatility setup can generate a channel for risk premium as discussed in a previous section. In addition, Tables 3 and 5 show that our asset pricing model yields low pricing errors in terms of

<sup>22</sup>Although the external habit formation allows a low value of  $\gamma$ , the relative risk aversion in this setup is not  $\gamma$ , but  $\gamma/X_t$ , where  $X_t$  is a time-varying consumption surplus factor and the relative risk aversion is still very high.

Table 4: Model Implied Annual Asset Returns

	$\gamma$	$\psi$	$\mathbb{E}(r_f)$	$\sigma(r_f)$	$\mathbb{E}(r_m)$	$\sigma(r_m)$	$\mathbb{E}(r_m - r_f)$
Baseline	2.111	2.115	0.0187	0.0142	0.0880	0.1923	0.0693
Data			0.0351	0.0189	0.0860	0.1519	0.0509
Changes in $\gamma$	2.111	2.115	0.0187	0.0142	0.0880	0.1923	0.0693
	3		0.0138	0.0142	0.1107	0.2853	0.0968
	5		0.0043	0.0142	0.2073	0.4783	0.2030
	10		-0.0142	0.0142	0.7070	0.9040	0.7212
Changes in $\psi$	2.111	1.5	0.0227	0.0199	0.0888	0.1732	0.0662
		2.115	0.0187	0.0142	0.0880	0.1923	0.0693
		2.5	0.0170	0.0120	0.0879	0.1996	0.0710
		3	0.0154	0.0100	0.0888	0.2063	0.0734

matching the moment conditions related to the individual returns and it is not rejected by the over-identifying restrictions. In sum, our results suggest that a proper model of the time-varying macroeconomic uncertainty, together with sufficient persistence of the conditional mean part of the consumption growth, is quintessential in understanding the link between macroeconomic variables and asset returns.

To evaluate the performance of our asset pricing model, we simulate the 1000 data points series of consumption and dividend for 1000 times using the parameters obtained from the macroeconomic uncertainty estimation. In doing so, we use the parameter estimates in Table 3 as our baseline case. From these simulated data, we generate asset returns, which are reported as the baseline result in Table 4. Our simulated equity premium is 6.93% and the risk free rate is around 2% per annum. This is consistent with most of the empirical results and we can match the second moments of the equity premium and the risk free rate as well. When compared with the existing calibration and empirical literature, our model seems to perform quite well with both the risk aversion and EIS around two. We believe that the results mainly come from a realistic model of the time-varying macroeconomic uncertainty.

We also run the sensitivity check to understand the roles of the risk aversion coefficient and the EIS parameter. First, we increase the risk aversion coefficient from the baseline value of 2.111. Not so surprisingly, larger risk aversion parameters generate higher equity premium. In fact, the risk premium becomes too large compared with the actual data when the risk aversion is over 5. Next, we change the EIS parameter. We vary the EIS parameter between 1.5 and 3, which are in the range of our estimates. We find that its effect is relatively mild, compared with the relative risk aversion, and it is somewhat expected from the estimation results. That is, a weak identification problem appears to exist in case of identifying the parameter handling the intertemporal substitutability.<sup>23</sup> Overall, our asset pricing model accounts for the important characteristics of the stock return data with highly plausible values of the deep parameters.

<sup>23</sup>For a more discussion on the nature of the weak identification for the EIS in the context of Epstein-Zin-Weil preferences, see Jeong, Kim, and Park (2009).

## 5. Conclusion

Macroeconomic models have strong theoretical implications on asset prices. However, empirical links between macroeconomic and financial variables appear to be weak according to the existing studies which report the small amount of macroeconomic volatility as the main suspect. Although macroeconomic variables are observable, the macro uncertainty is inherently latent, especially to econometricians and therefore, measurement issue exists. Furthermore, a valid transmission mechanism is required to amplify its small and smooth dynamics into large fluctuations in financial variables which reflect future expectations about economic performances. In this paper we analyze how macroeconomic uncertainty affects asset prices by developing and estimating a consumption-based asset pricing model with an emphasis on stochastic volatilities. To quantify the macroeconomic uncertainty, we introduce a nonlinear nonstationary stochastic volatility model with consumption and dividend processes. Specifically, we capture the features of volatility observed in macroeconomic data by assuming that the volatility function takes a logistic form with a random walk common factor as well as a stationary idiosyncratic factor. That is, this setup has two asymptotic regimes with a continuum of smooth transitions and persistent fluctuations of the common latent factor. The main advantage of this setup is that volatility is highly persistent without explosive dynamics. This is not only realistic to describe macroeconomic volatilities but also relevant for producing uncertainty premium because it allows slow and potentially long transition periods which can be perceived to be uncertain by consumers, even if the level of volatility is not high. We develop a Bayesian algorithm and identify this model numerically to avoid the difficulties with multidimensional integrations. We find that the extracted volatility series explain well the realized volatility series of both the consumption and the dividend data. In addition, we see a counter-cyclical relationship of the extracted macroeconomic uncertainty. To make full use of our volatility setup, we then combine this stochastic volatility model with a recursive utility function which has a preference ordering on the persistency of shocks. Therefore, the key insight of our model is that news about how rapidly the macro uncertainty reaches its highest or lowest point can also be a source of risk because consumers are unsure about the future outlook on an economy if the current level of uncertainty is in a transition period.

We find that both the estimated risk-aversion coefficient and the intertemporal elasticity of substitution are around two, which are highly plausible according to both economics and finance literature. Recent works adopting Epstein-Zin-Weil preferences report the relatively high estimates of the risk aversion coefficient to generate high risk premium through the persistent long-run risk channel. However, our model produces high risk premium even with a moderate coefficient of the relative risk aversion. Based on our theoretical and empirical results, we argue that stochastic volatilities of macroeconomic variables are crucial in understanding the business cycle fluctuations and the behaviors of asset return dynamics. In addition, our volatility model is parsimonious yet flexible so that it is appropriate to tackle economic problems with multiple shock processes. For instance, incorporating our stochastic volatility setup into dynamic stochastic general equilibrium models can be a natural extension. Another interesting route is to embed learning mechanisms into our framework to see if the learning component interacts with our stochastic volatility model.

We leave these tasks to future works.

## Appendix

### A. The Aggregate Wealth and Stochastic Discount Factor

In this section, we derive a few equations related to the returns on the asset that pays the consumption good as dividends, which can be regarded as the aggregate wealth. We first derive the equation (10). Then, we plug it together with the consumption growth process into the Euler equation to solve for the returns on the aggregate wealth  $R_{a,t+1}$  using the method of undetermined coefficients. Next, based on the results, we derive the stochastic discount factor implied by the model as well.

#### A.1. The Returns on the Aggregate Wealth

We begin with the Euler equation for  $R_{a,t+1}$ .

$$\mathbb{E}_t \left[ \exp\left(\chi \ln \delta - \frac{\chi}{\psi} g_{c,t+1} + \chi r_{a,t+1}\right) \right] = 1$$

We will plug into the Euler equation the consumption growth process  $g_{c,t+1}$  and (3), the approximated return on aggregate wealth,

$$r_{a,t+1} \approx \phi_{0,c} + \phi_{1,c} z_{c,t+1} - z_{c,t} + g_{c,t+1}.$$

We conjecture the log price-consumption ratio  $z_{c,t} \equiv \log\left(\frac{P_{c,t}}{C_t}\right)$  is dependent on the state variable  $\nu_{c,t}$  and  $f_{c,t}$ , such that  $z_{c,t} = A_{0,c} + A_{1,c} \nu_{c,t} + A_{2,c} f_{c,t}$ . We use  $f_{c,t}$  to denote  $f_c(\lambda_c w_t)$  for a short-hand notation.

$$\mathbb{E}_t \left[ \exp \left( \chi \ln \delta + \left( \chi - \frac{\chi}{\psi} \right) g_{c,t+1} + \chi \{ \phi_{0,c} + \phi_{1,c} z_{c,t+1} - z_{c,t} \} \right) \right] = 1$$

We ignore the difference between  $f_c(x_{c,t})$  and  $f_c(\lambda_c w_t)$  given that  $x_{c,t}$  and  $w_t$  are cointegrated into each other and in particular, a logistic transformation makes two values similar in the high or low volatility regime at which the volatility mass will cluster asymptotically. Moreover, consumption volatility moves in a very small range between 0.000003 and 0.000025 according to our estimation. Using the conventional method of undetermined coefficients, we will verify the unknown coefficients for  $z_{c,t}$  process.

First, we rewrite  $f_{c,t+1}$  by

$$f_{c,t+1} = \alpha_c + \frac{\beta_c (f_{c,t} - \alpha_c)}{f_{c,t} - \alpha_c + (\alpha_c + \beta_c - f_{c,t}) \exp(-\lambda_c u_{t+1})}.$$

Then we linearize  $f_{c,t+1}$  around  $f_{c,t} = f_{c,0}$  after

$$\begin{aligned} f_{c,t+1} &\approx \alpha_c + \frac{\beta_c (f_{c,0} - \alpha_c)}{f_{c,0} - \alpha_c + (\alpha_c + \beta_c - f_{c,0}) \exp(-\lambda_c u_{t+1})} \\ &\quad + \frac{\beta_c^2 \exp(-\lambda_c u_{t+1})}{[f_{c,0} - \alpha_c + (\alpha_c + \beta_c - f_{c,0}) \exp(-\lambda_c u_{t+1})]^2} [f_{c,t} - f_{c,0}] \\ &= \alpha_c + K_1(u_{t+1}) + K_2(u_{t+1}) [f_c(\lambda_c w_t) - f_{c,0}]. \end{aligned}$$

We again linearize  $K_1(u_{t+1})$  and  $K_2(u_{t+1})$  around  $u_{t+1} \approx 0$

$$\begin{aligned} K_1(u_{t+1}) &\approx \frac{\beta_c (f_{c,0} - \alpha_c)}{f_{c,0} - \alpha_c + (\alpha_c + \beta_c - f_{c,0})} + \frac{\lambda_c \beta_c (f_{c,0} - \alpha_c) (\alpha_c + \beta_c - f_{c,0})}{[f_{c,0} - \alpha_c + (\alpha_c + \beta_c - f_{c,0})]^2} u_{t+1} \\ &= f_{c,0} - \alpha_c + \frac{\lambda_c}{\beta_c} (f_{c,0} - \alpha_c) (\alpha_c + \beta_c - f_{c,0}) u_{t+1} \\ K_2(u_{t+1}) &\approx \frac{\beta_c^2}{[f_{c,0} - \alpha_c + (\alpha_c + \beta_c - f_{c,0})]^2} + \frac{-\lambda_c \beta_c^4 + 2\lambda_c \beta_c^3 (\alpha_c + \beta_c - f_{c,0})}{[f_{c,0} - \alpha_c + (\alpha_c + \beta_c - f_{c,0})]^4} u_{t+1} \\ &= 1 + \frac{\lambda_c}{\beta_c} (\beta_c - 2f_{c,0} + 2\alpha_c) u_{t+1} \end{aligned}$$

Finally,  $f_{c,t+1}$  can be expressed in terms of  $f_{c,t}$  and  $u_{t+1}$ .

$$\begin{aligned} f_{c,t+1} &\approx f_{c,t} + \frac{\lambda_c}{\beta_c} (\beta_c - 2f_{c,0} + 2\alpha_c) f_{c,t} u_{t+1} \\ &\quad + \frac{\lambda_c}{\beta_c} [(f_{c,0} - \alpha_c) (\alpha_c + \beta_c + f_{c,0}) - \beta_c f_{c,0}] u_{t+1} \\ &= f_{c,t} + \xi_1 f_{c,t} u_{t+1} + \xi_2 u_{t+1} \end{aligned} \tag{19}$$

where  $\xi_1 = \frac{\lambda_c}{\beta_c} (2\alpha_c + \beta_c - 2f_{c,0})$  and  $\xi_2 = \frac{\lambda_c}{\beta_c} [f_{c,0}^2 - \alpha_c (\alpha_c + \beta_c)]$ . And we can linearize  $f_{c,t}^2$  around  $f_{c,t} \approx f_{c,0}$

$$f_{c,t}^2 \approx f_{c,0}^2 + 2f_{c,0} (f_{c,t} - f_{c,0}). \tag{20}$$

Using (2),(3), (10), (19) and (20), we rearrange the inside the exponential of the Euler equation (1) by a constant term,  $\nu_{c,t}$ , and  $f_{c,t}$  to obtain

$$\begin{aligned} &\chi \ln \delta + (1 - \gamma) \mu_c + \chi (\phi_{0,c} + (\phi_{1,c} - 1) A_{0,c}) + 0.5 (\chi \phi_{1,c} A_{2,c})^2 [\xi_2^2 - \xi_1^2 f_{c,0}^2] \\ &\quad + [(1 - \gamma) + \chi A_{1,c} (\phi_{1,c} \rho_c - 1)] \nu_{c,t} \\ &\quad + \left[ \chi^2 \phi_{1,c}^2 \Upsilon A_{2,c}^2 + \chi (\phi_{1,c} - 1) A_{2,c} + \frac{1}{2} \left\{ (1 - \gamma)^2 + (\chi \phi_{1,c} A_{1,c} \varphi_c)^2 \right\} \right] f_{c,t}. \end{aligned}$$

Since the Euler equation should hold for all values of the state variables, the coefficients



$A_{0,c}$ ,  $A_{1,c}$ , and  $A_{2,c}$  are verified as follows:

$$A_{1,c} = \frac{1 - \gamma}{\chi(1 - \phi_{1,c}\rho_c)} \quad (21)$$

$$A_{0,c} = \frac{\chi \ln \delta + (1 - \gamma)\mu_c + \chi\phi_{0,c} + 0.5(\chi\phi_{1,c}A_{2,c})^2 [\xi_2^2 - \xi_1^2 f_{c,0}^2]}{\chi(1 - \phi_{1,c})} \quad (22)$$

$$A_{2,c} = \frac{1 - \phi_{1,c} \pm \sqrt{(\phi_{1,c} - 1)^2 - 2\phi_{1,c}^2 \Upsilon \left[ (1 - \gamma)^2 + (\chi\phi_{1,c}A_{1,c}\varphi_c)^2 \right]}}{2\chi\phi_{1,c}^2 \Upsilon}, \quad (23)$$

where  $\Upsilon = \xi_1(\xi_1 f_{c,0} + \xi_2)$ . On  $A_{2,c}$ , we use the case with the positive sign in front of the square root term, which has the larger effect. Even if the negative sign is adopted, the sign of  $A_{2,c}$  does not vary because  $2\phi_{1,c}^2 \Upsilon \left[ (1 - \gamma)^2 + (\chi\phi_{1,c}A_{1,c}\varphi_c)^2 \right] > 0$  holds.

## A.2. The Stochastic Discount Factor

This subsection derives the stochastic discount factor implied by the model. This coincides with the intertemporal marginal rate of substitution (IMRS). The conditional mean of IMRS is

$$\begin{aligned} \mathbb{E}_t(m_{t+1}) &= \chi \ln \delta - \gamma\mu_c + (\chi - 1)(\phi_{0,c} + (\phi_{1,c} - 1)A_{0,c}) \\ &\quad + [(\chi - 1)(\phi_{1,c}\rho_c - 1)A_{1,c}] \nu_{c,t} + [(\chi - 1)A_{2,c}(\phi_{1,c} - 1)] f_{c,t}. \end{aligned}$$

Then the innovations in IMRS is

$$m_{t+1} - \mathbb{E}_t(m_{t+1}) = \Lambda_{m\varepsilon} \sqrt{f_{c,t}} \varepsilon_{c,t+1} - \Lambda_{m,\eta} \sqrt{f_{c,t}} \eta_{c,t+1} - \Lambda_{m,u}(t) u_{t+1}, \quad (24)$$

where the prices of risk for each sources of risk are defined as

$$\begin{aligned} \Lambda_{m\varepsilon} &\equiv -\gamma \\ \Lambda_{m,\eta} &\equiv (1 - \chi)\phi_{1,c}A_{1,c}\varphi_c \\ \Lambda_{m,u}(t) &\equiv (1 - \chi)\phi_{1,c}A_{2,c}(\xi_1 f_{c,t} + \xi_2). \end{aligned}$$

## B. The Return from holding the Stock Market

This section derives an approximated version of the market returns as a function of the aggregate state variables. This equation provides a pedagogically useful representation of the anticipated returns from holding risky assets. First, we can write down the gross market return as  $R_{m,t+1} = \frac{D_{t+1} + P_{m,t+1}}{P_{m,t}}$ . Then, we log-linearize the returns on the market portfolio as follows:

$$r_{m,t+1} \approx \phi_{0,m}(\bar{z}_m) + \phi_{1,m}(\bar{z}_m) z_{m,t+1} - z_{m,t} + g_{d,t+1}, \quad (25)$$

where  $z_{m,t} = \log(P_{d,t}/D_t)$  is the log price-dividend ratio.

To solve for the market return, we use (15), the process of dividend growth  $g_{d,t+1}$  with the process of long run component of dividend  $\nu_{d,t+1}$ ,

$$\nu_{d,t+1} = \rho_d \nu_{d,t} + \varphi_d \sqrt{f_d(x_{d,t})} \eta_{d,t+1}. \quad (26)$$

We can use the method of undetermined coefficient by guessing approximately  $z_{m,t} \approx A_{0,m} + A_{1,m}^c \nu_{c,t} + A_{1,m}^d \nu_{d,t} + A_{2,m} f_{c,t}$ . Note that the relevant state variable is  $\nu_{c,t}, \nu_{d,t}$  and  $w_t$  because dividend process also is affected by consumption. First, approximate  $f_d(\lambda_d w_t)$  in terms of  $f_{c,t}$  at  $f_{d,0}$

$$\begin{aligned} f_d(\lambda_d w_t) &\approx \alpha_d + \frac{\beta_d (f_{d,0} - \alpha_c)^\lambda \exp(\lambda \kappa_c - \kappa_d)}{(f_{d,0} - \alpha_c)^\lambda \exp(\lambda \kappa_c - \kappa_d) + (\alpha_c + \beta_c - f_{d,0})^\lambda} \\ &\quad + \frac{\lambda \beta_c \beta_d (f_{d,0} - \alpha_c)^{\lambda-1} (\alpha_c + \beta_c - f_{d,0})^{\lambda-1} \exp(\lambda \kappa_c - \kappa_d)}{\left[ (f_{c,0} - \alpha_c)^\lambda \exp(\lambda \kappa_c - \kappa_d) + (\alpha_c + \beta_c - f_{d,0})^\lambda \right]^2} (f_{c,t} - f_{d,0}) \\ &= \pi_1 + \pi_2 (f_{c,t} - f_{d,0}) = (\pi_1 - \pi_2 f_{d,0}) + \pi_2 f_{c,t}, \end{aligned}$$

where  $\lambda = \lambda_d/\lambda_c$  and  $\pi_1$  and  $\pi_2$  are defined appropriately.

Here we ignore the difference between  $f_d(x_{d,t})$  and  $f_d(\lambda_d w_t)$  given the small contribution from the idiosyncratic component. To derive expressions for the undetermined coefficients  $A_{0,m}, A_{1,m}^c, A_{1,m}^d$ , and  $A_{2,m}$ , use the Euler equation again for the market return  $r_{m,t+1}$ :

$$\mathbb{E}_t \left[ \exp \left( \chi \ln \delta - \frac{\chi}{\psi} g_{c,t+1} + (\chi - 1) r_{a,t+1} + r_{m,t+1} \right) \right] = 1.$$

Expanding  $r_{a,t+1}$  and  $r_{m,t+1}$  processes, we have

$$\mathbb{E}_t \left[ \exp \left( \begin{aligned} &\chi \ln \delta - \gamma g_{c,t+1} + (\chi - 1) (\phi_{0,c} + \phi_{1,c} z_{c,t+1} - z_{c,t}) \\ &+ \phi_{0,m} + \phi_{1,m} z_{m,t+1} - z_{m,t} + g_{d,t+1} \end{aligned} \right) \right] = 1.$$

Now we collect the terms inside the exponential in light of the state variables  $\nu_{c,t}, \nu_{d,t}, f_{c,t}$ , and constants. Note that there is one cross product term  $-\gamma \sqrt{f_{c,t}} \sqrt{f_{d,t}} \rho$  due to the correlation of short-run shocks. Alternatively, we can assume a relationship between  $f_{c,t}$  and  $f_{d,t}$  such as  $f_{d,t} = \hat{k} f_{c,t}$  for some constant  $\hat{k}$ . Either way, empirically  $\rho$  is close to zero, hence this term is negligible. Then, we can find the coefficient terms as follows:

$$\begin{aligned} A_{0,m} &= \frac{[\chi \ln \delta - \gamma \mu_c + \mu_d + (\chi - 1) (\phi_{0,c} + \phi_{1,c} A_{0,c} - A_{0,c}) + \phi_{0,m} + 0.5 H_1]}{1 - \phi_{1m}} \\ A_{1,m}^c &= \frac{-\frac{1}{\psi}}{1 - \phi_{1,m} \rho_c} \\ A_{1,m}^d &= \frac{1}{1 - \phi_{1,m} \rho_d} \\ A_{2,m} &= \frac{\Xi \pm \sqrt{\Xi^2 - 4 \Upsilon \phi_{1,m}^2 H}}{2 \Upsilon \phi_{1,m}^2}, \end{aligned}$$

where  $\Pi = \pi_1 - \pi_2 f_{d,0}$  and

$$\begin{aligned}\Xi &= 2\Upsilon(1 - \chi)\phi_{1,c}\phi_{1,m}A_{2,c} + 1 - \phi_{1,m} \\ H_1 &= \left[1 + (\phi_{1,m}A_{1,m}^d\varphi_d)^2\right]\Pi + [(\chi - 1)\phi_{1,c}A_{2,c} + \phi_{1,m}A_{2,m}]^2(\xi_2^2 - \xi_1^2 f_{c,0}^2) \\ H &= \Upsilon(\chi - 1)^2\phi_{1,c}^2A_{2,c}^2 + (\chi - 1)A_{2,c}(\phi_{1,c} - 1) \\ &\quad + 0.5\left[\gamma^2 + \left[1 + (\phi_{1,m}A_{1,m}^d\varphi_d)^2\right]\pi_2 + [(\chi - 1)\phi_{1,c}A_{1,c} + \phi_{1,m}A_{1,m}^c]^2\varphi_c^2\right].\end{aligned}$$

Once all the coefficients are verified, we can easily derive the conditional mean of equity premium from the fact that risk premium of any asset is negatively related to both the conditional covariance between the return and intertemporal marginal rate of substitution and variance of the return.

Then innovation to the return  $r_{m,t+1}$  is

$$\begin{aligned}r_{m,t+1} - \mathbb{E}_t(r_{m,t+1}) &= \varphi_d\sqrt{f_{d,t}}\varepsilon_{d,t+1} + \phi_{1,m}A_{1,m}^c\varphi_c\sqrt{f_{c,t}}\eta_{c,t+1} + \phi_{1,m}A_{1,m}^d\varphi_d\sqrt{f_{d,t}}\eta_{d,t+1} \\ &\quad + \phi_{1,m}A_{2,m}(\xi_1 f_{c,t} + \xi_2)u_{t+1}\end{aligned}$$

and its conditional variance  $\text{Var}_t(r_{m,t+1})$  is

$$\text{Var}_t(r_{m,t+1}) = \left(1 + (\phi_{1,m}A_{1,m}^d\varphi_d)^2\right)f_{d,t} + (\phi_{1,m}A_{1,m}^c\varphi_c)^2f_{c,t} + (\phi_{1,m}A_{2,m}(\xi_1 f_{c,t} + \xi_2))^2,$$

and the risk premium for  $r_{m,t+1}$  is equal to

$$\begin{aligned}\mathbb{E}_t[r_{m,t+1} - r_{f,t}] &= -\text{Cov}_t[m_{t+1} - \mathbb{E}_t(m_{t+1}), r_{m,t+1} - \mathbb{E}_t(r_{m,t+1})] - \frac{1}{2}\text{Var}_t(r_{m,t+1}) \\ &= \Lambda_{m,\eta}\phi_{1,m}A_{1,m}^c\varphi_c f_{c,t} + \Lambda_{m,u}(t)\phi_{1,m}A_{2,m}(\xi_1 f_{c,t} + \xi_2) - \frac{1}{2}\text{Var}_t(r_{m,t+1}).\end{aligned}$$

Now we assume that the long-run component of dividend is strongly related to the long-run component of consumption following Bansal and Yaron (2004), and let

$$\nu_{d,t} = \Gamma\nu_{c,t}.$$

Although our model does not impose a priori such a relation in our model and estimation, this assumption is plausible because actual data seem to share a similar trend. We then can reduce the number of the state variable and verify undetermined coefficients as follows:

$$A_{0,m} = \frac{[\chi \log \delta - \gamma\mu_c + \mu_d + (\chi - 1)(\phi_{0,c} + \phi_{1,c}A_{0,c} - A_{0,c}) + \phi_{0,m} + 0.5H_3]}{1 - \phi_{1,m}} \quad (27)$$

$$A_{1,m} = \left(\frac{\Gamma - \psi^{-1}}{1 - \phi_{1,m}\rho_c}\right) \quad (28)$$

$$A_{2,m} = \frac{\Xi \pm \sqrt{\Xi^2 - 4\Upsilon\phi_{1,m}^2 H}}{2\Upsilon\phi_{1,m}^2},$$

where

$$\begin{aligned} H_3 &= \Pi + ((\chi - 1) \phi_{1,c} A_{2,c} + \phi_{1,m} A_{2,m})^2 (\xi_2^2 - \xi_1^2 f_{c,0}^2) \\ H &= \Upsilon (\chi - 1)^2 \phi_{1,c}^2 A_{2,c}^2 + (\chi - 1) A_{2,c} (\phi_{1,c} - 1) \\ &\quad + 0.5 \left( \gamma^2 + \pi_2 + [(\chi - 1) \phi_{1,c} A_{1,c} + \phi_{1,m} A_{1,m}]^2 \varphi_c^2 \right). \end{aligned}$$

Regarding  $A_{2,m}$ , we choose to use the one for our illustration with the negative sign in front of the square root term in the numerator, although both cases have the same sign for  $A_{2,m} < 0$  as long as  $2\Upsilon (1 - \chi) \phi_{1,c} A_{2,c} \phi_{1,m} + 1 < \phi_{1,m}$  holds.

Then innovation to the return  $r_{m,t+1}$  is

$$\begin{aligned} r_{m,t+1} - \mathbb{E}_t(r_{m,t+1}) \\ = \varphi_d \sqrt{f_{d,t}} \varepsilon_{d,t+1} + \phi_{1,m} A_{1,m} \varphi_c \sqrt{f_{c,t}} \eta_{c,t+1} + \phi_{1,m} A_{2,m} (\xi_1 f_{c,t} + \xi_2) u_{t+1} \end{aligned}$$

and its conditional variance  $\text{Var}_t(r_{m,t+1})$  is

$$\text{Var}_t(r_{m,t+1}) = \varphi_d^2 f_{d,t} + \phi_{1,m}^2 A_{1,m}^2 \varphi_c^2 f_{c,t} + \phi_{1,m}^2 A_{2,m}^2 (\xi_1 f_{c,t} + \xi_2)^2,$$

and the risk premium for  $r_{m,t+1}$  is equal to

$$\begin{aligned} \mathbb{E}_t[r_{m,t+1} - r_{f,t}] &= -\text{Cov}_t[m_{t+1} - \mathbb{E}_t(m_{t+1}), r_{m,t+1} - \mathbb{E}_t(r_{m,t+1})] - \frac{1}{2} \text{Var}_t(r_{m,t+1}) \\ &= \Lambda_{m,\eta} \phi_{1,m} A_{1,m} \varphi_c f_{c,t} + \Lambda_{m,u}(t) \phi_{1,m} A_{2,m} (\xi_1 f_{c,t} + \xi_2) - \frac{1}{2} \text{Var}_t(r_{m,t+1}), \end{aligned}$$

where

$$\begin{aligned} \Lambda_{m\varepsilon} &\equiv -\gamma \\ \Lambda_{m,\eta} &\equiv (1 - \chi) \phi_{1,c} A_{1,c} \varphi_c \\ \Lambda_{m,u}(t) &\equiv (1 - \chi) \phi_{1,c} A_{2,c} (\xi_1 f_{c,t} + \xi_2). \end{aligned}$$

### C. The Risk-Free Rate

The risk-free rate can be derived from the Euler equation as

$$\begin{aligned} r_{f,t} &= -\log [\mathbb{E}_t \exp(m_{t+1})] \\ &= -\log \delta + \frac{1}{\psi} \mathbb{E}_t g_{c,t+1} + \frac{1 - \chi}{\chi} \mathbb{E}_t [r_{a,t+1} - r_{f,t}] - \frac{1}{2\chi} \text{Var}_t(m_{t+1}). \end{aligned}$$

Here we have three terms in which to elaborate. First, the conditional mean of consumption growth  $E_t g_{c,t+1}$  is simply  $\mu_c + \nu_{c,t}$ . Secondly, the conditional mean of excess return on the wealth  $E_t [r_{a,t+1} - r_{f,t}]$  can be solved by the conditional covariance between  $r_{a,t+1}$  and  $m_{t+1}$  and the conditional variance of  $r_{a,t+1}$ . Using the fact that  $r_{a,t+1} - \mathbb{E}_t r_{a,t+1} = \sqrt{f_{c,t}} \varepsilon_{t+1} + \frac{\Lambda_{m,\eta}}{1-\chi} \sqrt{f_{c,t}} \eta_{t+1} + \frac{\Lambda_{m,u}(t)}{1-\chi} u_{t+1}$ , we have

$$\begin{aligned} \mathbb{E}_t [r_{a,t+1} - r_{f,t}] &= -\text{Cov}_t [m_{t+1} - \mathbb{E}_t(m_{t+1}), r_{a,t+1} - \mathbb{E}_t(r_{a,t+1})] - \frac{1}{2} \text{Var}_t(r_{a,t+1}) \\ &= \left[ \gamma + \frac{\Lambda_{m,\eta}^2}{1 - \chi} - \frac{1}{2} \left( \frac{\Lambda_{m,\eta}^2}{(1 - \chi)^2} + 1 \right) \right] f_{c,t} + \frac{\Lambda_{m,u}^2(t)}{(1 - \chi)^2} \left( \frac{1}{2} - \chi \right). \end{aligned}$$

Lastly, the conditional variance of IMRS is derived from (24) and given by

$$\text{Var}_t(m_{t+1}) = \Lambda_{m,\varepsilon}^2 f_{c,t} + \Lambda_{m,\eta}^2 f_{c,t} + \Lambda_{m,u}^2(t).$$

#### D. Density-Based Nonlinear Filtering Algorithm

If we apply the non-linear filtering algorithm of Tanizaki (1996) to our model, we may deduce the prediction step

$$\begin{aligned} p(w_t|\mathcal{F}_{t-1}) &= \int p(w_t, w_{t-1}|\mathcal{F}_{t-1}) dw_{t-1} \\ &= \int p(w_t|w_{t-1})p(w_{t-1}|\mathcal{F}_{t-1}) dw_{t-1} \\ p(x_{c,t}, x_{d,t}|\mathcal{F}_{t-1}) &= \int p(x_{c,t}, x_{d,t}, w_{t-1}|\mathcal{F}_{t-1}) dw_{t-1} \\ &= \int p(x_{c,t}, x_{d,t}|w_{t-1})p(w_{t-1}|\mathcal{F}_{t-1}) dw_{t-1}. \end{aligned}$$

and the updating step

$$\begin{aligned} p(w_t|\mathcal{F}_t) &= p(w_t, y_{c,t}, y_{d,t}|\mathcal{F}_{t-1})/p(y_{c,t}, y_{d,t}|\mathcal{F}_{t-1}) \\ &= p(y_{c,t}, y_{d,t}|w_t)p(w_t|\mathcal{F}_{t-1})/p(y_{c,t}, y_{d,t}|\mathcal{F}_{t-1}) \\ p(x_{c,t}, x_{d,t}|\mathcal{F}_t) &= p(x_{c,t}, x_{d,t}, y_{c,t}, y_{d,t}|\mathcal{F}_{t-1})/p(y_{c,t}, y_{d,t}|\mathcal{F}_{t-1}) \\ &= p(y_{c,t}, y_{d,t}|x_{1t}, x_{2t})p(x_{1t}, x_{2t}|\mathcal{F}_{t-1})/p(y_{c,t}, y_{d,t}|\mathcal{F}_{t-1}), \end{aligned}$$

where

$$\begin{aligned} p(y_{c,t}, y_{d,t}|w_t) &= \iint p(y_{c,t}, y_{d,t}, x_{c,t}, x_{d,t}|w_t) dx_{c,t} dx_{d,t} \\ &= \iint p(y_{c,t}, y_{d,t}|x_{c,t}, x_{d,t})p(x_{c,t}, x_{d,t}|w_t) dx_{c,t} dx_{d,t}. \end{aligned}$$

We may apply these steps recursively to our model and estimate the unknown parameters and extract the latent factors similarly as in Kim, Lee, and Park (2009).

## E. Additional Figures and Tables

Figure 5: Gibbs Samples for the Parameters of Consumption Growth.

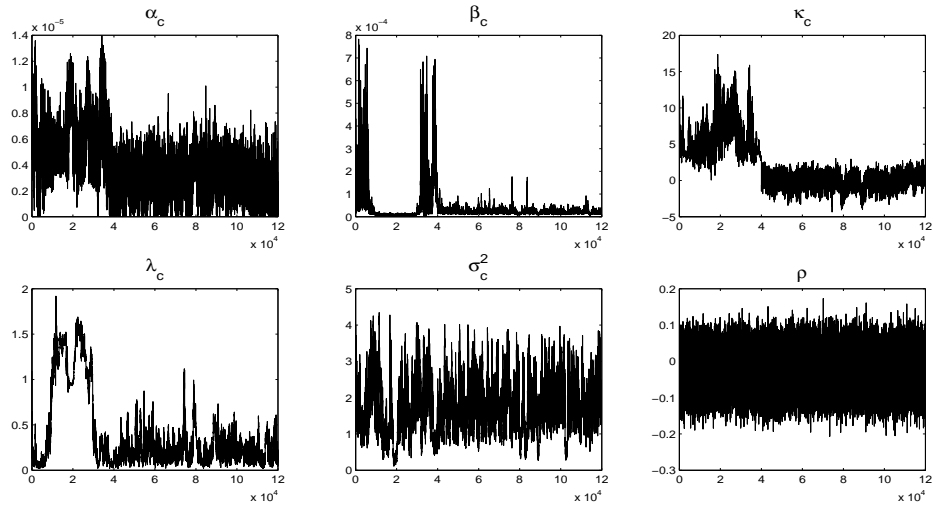


Figure 6: Gibbs Samples for the Parameters of Dividend Growth.

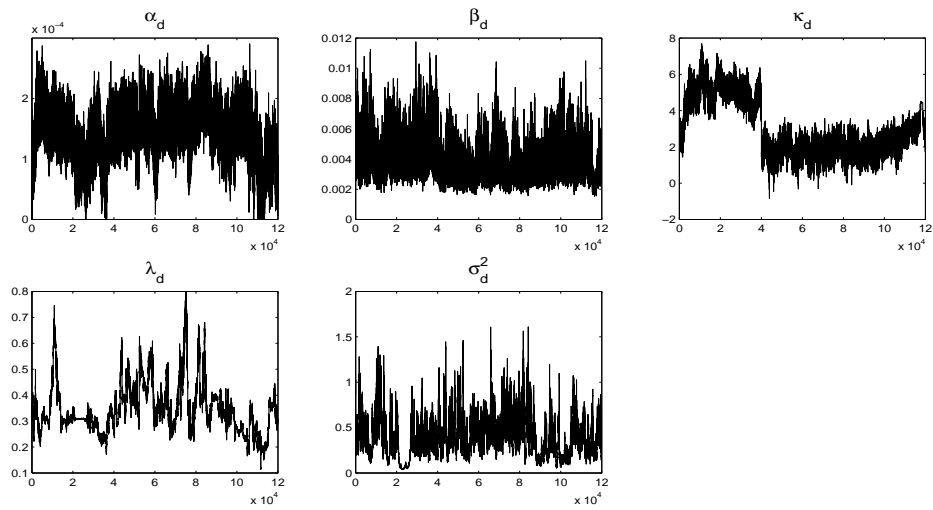


Table 5: GMM Estimation Result (Multiple Asset Returns; Alternative Instruments)

Parameters	GMM Parameter Estimates			
	Estimates	Standard errors	t-stat	p-val
$\gamma$	2.090	0.221	9.46	0.000
$\psi$	3.450	7.565	0.46	0.647
	Selected Moment Conditions			
	Moment	Standard errors	t-stat	p-val
Small	0.003499	0.002348	1.49	0.1362
Large	0.001712	0.001227	1.39	0.1630
Value	0.000210	0.000133	1.58	0.1139
Growth	0.000198	0.000123	1.61	0.1084
Market	0.000006	0.000006	1.06	0.2873
$J$ -stat = 25.07		Prob( $\chi^2(8) > J$ ) = 0.3468		

*Notes:* In this table, we use the lagged consumption growth, lagged unemployment, lagged inflation, and a lagged term spread as the instrumental variables in this estimation. Thus, we have fifteen additional moment conditions which represent the orthogonality between each returns and each of the instrumental variables.

Table 6: GMM Estimation Results (Market Returns Only)

Panel A				
Parameters	Estimates	Standard errors	t-stat	p-val
$\gamma$	2.029	0.278	7.29	0.000
$\psi$	3.311	8.400	0.39	0.693
$J$ -stat = 0.002		Prob( $\chi^2(1) > J$ ) = 0.967		
Panel B				
Parameters	Estimates	Standard errors	t-stat	p-val
$\gamma$	1.901	0.262	7.27	0.000
$\psi$	1.225	1.043	1.17	0.241
$J$ -stat = 4.048		Prob( $\chi^2(3) > J$ ) = 0.256		

*Notes:* This table refers to the GMM estimation results with the market returns only. Panel A. reports the results with the lagged consumption growth and the lagged unemployment rate as instrument variables, and the estimation in Panel B. uses the lagged inflation, and a lagged term spread as well as the lagged consumption growth and the lagged unemployment rate.

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