Transmit/Receive Beamforming and Interferences Cancellation Using Phased MIMO Radar with Full Waveform Diversity

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Abstract: A Phased MIMO radar with full waveform diversity is proposed which divides the transmit array into overlapping subarrays in such a way that the number of subarrays are equal to the number of transmit antennas. Each subarray sends out a unique wave form towards the target of interest and thus provides a full waveform diversity, which results in the virtual extension of received array. This virtual extension in received array along with the directional beams at the transmitter side provides some useful results by matched filtering for each waveform. The proposed Full Waveform Diverse Phased MIMO (FWD-PMIMO) radar not only provides some directional gain like a Phased array radar (PAR) but also provides the waveform diversity equivalent to that of MIMO radar. It has the capability of cancelling all the interferences like MIMO radar and can cancel more interferences than the phased array and the Phased MIMO radar. Finally its signal to interference plus noise ratio (SINR) is better than the three radars. The simulation results also exhibit the better performance of proposed radar compared to phased array radar, MIMO radar and phased MIMO radar.

Key words: Phased MIMO radar · Waveform diversity · Interference rejection · MVDR Beamformer
Transmit/receive beampattern

INTRODUCTION

Phased array antennas are widely used to send a directional beam towards the target of interest by applying the phase shifts across the elements [1-2]. The benefit of phased array is a very high gain towards the target of interest which allow a better detection at the receiver end. The target detection and parameter estimation of phased array radar using adaptive processing techniques have been discussed in literature [3]. Multiple Input and Multiple Output (MIMO) on the other hand transmit the orthogonal signals from each antenna in an omnidirectional manner and focused on exploiting this orthogonality between the signals.

MIMO with distributed antennas make use of spatial diversity to get a better radar cross section (RCS) of a target. The variation in RCS are analyzed for not only detection of the target but also the parameter estimation such as Doppler and direction of arrival [4-5]. MIMO with collocated antennas provide the waveform diversity which is very helpful to give increased resolution and better parameter identifiability [6-8]. One of the major advantages of collocated MIMO radar is the virtual extension of the aperture due to the orthogonal waveforms used at the transmitter, which can be achieved by matched filtering each waveform and stacking the data vector to get an extended data vector on the receiver end [9-11].

In recent years the hybrid of both phased array and MIMO radar were introduced which exploits the benefits of both radars [12-14]. The phased MIMO radars provides us a tradeoff between coherent gain and the interference rejection by carefully selecting the number of subarrays [13-14]. The structure used in these works consider a fully overlapped subarrays structure and transmit a unique waveform from each subarray. These subarrays independently can be considered multiple phased array radars transmitting a different waveform with certain gain.
The independence of the transmitted signals can be used at the receiver to get an extended data vector which helps in better resolution of target and rejection of the interferences in the area of interest. It is worth mentioning that the gain of this hybrid radar system is less than the phased array radar and can reject less no of the interferences than the MIMO radar.

In this paper new model is proposed for partitioning of transmit array, which divides the array in such a way that number of overlapped subarrays are equal to the number of elements in the whole array i.e. if there are N elements in a transmit array, then the number of subarrays are also N. Each subarray uses a unique waveform and thus gives us a full waveform diverse phased MIMO radar (FWD-PMIMO). Although MIMO radar also transmits N waveforms in case of N transmit antenna elements but FWD-PMIMO is slightly different from MIMO in a way that the MIMO uses only one antenna element to send out a waveform, whereas FWD-PMIMO uses different number of elements in each subarray for transmitting a waveform, thus giving multiple beams of different gains. The size of subarray in our design is variable ranging from 1 antenna element in first subarray to ‘N’ antenna elements in the last subarray.

The benefit obtained from proposed design is that it matches the MIMO in terms of interference suppression while maintaining a coherent gain at transmit side which is very useful for better target detection. The performance of proposed FWD-PMIMO is examined in an environment with large number of distributed interferers. The beampattern along with the output signal-to-interference-plus-noise ratio (SINR) of an adaptive beamformer, namely Capon beamformer or the minimum-variance distortionless-response (MVDR) [16] is used as a performance metric.

The paper is organized as follows. Section II will present some necessary details about phased array, MIMO radar. The idea of FWD-PMIMO will be elaborated in section III, while section IV will present the adaptive beamforming and signal to interference noise ratio (SINR) for proposed design. In section V, the simulation results will be given to compare the proposed scheme with Phased array radar and MIMO radar, followed by conclusion drawn in section VI.

Phased Array and Mimo Radar
Phased Array Radar: Consider a phased array radar consisting of P transmit and R receive antenna elements, then the signal transmitted by P transmit elements can be given as in [15].

\[
\mathbf{u}^*(\theta) \mathbf{s}(t)
\]

where \( s(t) \) is the transmitted signal and \( \mathbf{u}(\theta) \) is the transmit steering vector of look angle \( \theta \).

For a target at angle \( \theta \), in the far field, the signal can be given as

\[
\mathbf{u}^T(\theta) \mathbf{s}(t)
\]

The phased array will give a directional gain of P when \( \theta = \theta \). If there are Q interferences in the background of the target then the received signal can be written as

\[
g_{rad}(t) = \alpha_{\mathbf{v}} \mathbf{v}(\theta) \mathbf{s}(t) + \sum_{i=1}^{Q} \alpha_{i} \mathbf{u}^T(\theta) \mathbf{u}(\theta) \mathbf{v}(\theta) \mathbf{s}(t) + n(t)
\]

where \( \alpha_{\mathbf{v}} \) and \( \alpha_{i} \) are the complex amplitudes of target and interferences respectively, \( \mathbf{v}(\theta) t \) is the P×1 received steering vectors and \( n(t) \) is the noise term. The received signal is matched filter for transmitted waveform to get the data vector of dimension \( R \times 1 \) as given below

\[
d_{rad} = \alpha_{\mathbf{v}} \mathbf{z}(\theta) + \sum_{i=1}^{Q} \alpha_{i} \mathbf{z}(\theta) + \tilde{n}
\]

Where

\[
\mathbf{z}(\theta) = \eta \mathbf{v}(\theta)
\]

\( \mathbf{z}(\theta) \) is the virtual steering is vector of length \( R \times 1 \) and \( \eta \) is the directional gain.

MIMO Radar: A MIMO radar transmit multiple orthogonal signal from the transmit array in an omnidirectional fashion. This omnidirectional transmission does not allow us to form a beam towards a target. One can think of a MIMO radar as transmit array of P elements divided into P subarrays, where each subarray transmits an orthogonal waveform. Keeping this in view, let us consider a MIMO radar with P transmit antennas and R received antennas where each transmit antenna element transmit an orthogonal waveform \( \mathbf{s}_p(t), p = 1,2, \ldots, P \).

The orthogonality of signal is ensured through the following condition

\[
\int_{T_p} \mathbf{s}(t) \mathbf{s}^H(t) dt = \mathbf{I}_P
\]

Where \( T_p \) is the pulse width of radar, \( t \) is the time within that pulse. \( \mathbf{I}_P \) is the \( P \times P \) identity matrix and \( (\cdot)^H \) is the hermitian transpose. The signal reflected by a target and
Fig. 1: Full wave form diverse Phased MIMO (FWD-PMIMO) radar

Q interferences can be expressed at the receiver array as

\[ g_{mimo}(t) = \alpha_t \mathbf{v}(\theta) \mathbf{u}_t^T(\theta) s(t) + \sum_{i=1}^{Q} \alpha_i \mathbf{v}(\theta) \mathbf{u}_i^T(\theta) s(t) + n(t) \]  

(7)

After passing this signal through matched filters, we get \( PR \times 1 \) data vector given as

\[ \mathbf{d}_{mimo} = \alpha_t \mathbf{x}_{mimo}(\theta) + \sum_{i=1}^{Q} \alpha_i \mathbf{x}_{mimo}(\theta) + \mathbf{n} \]  

(8)

Where

\[ \mathbf{x}_{mimo}(\theta) = \mathbf{u}(\theta) \otimes \mathbf{v}(\theta) \]  

(9)

In equation (9) we can see the virtual steering vector which is an extended steering vector as compared to the phased array counterpart. The virtual extension in data vector at the receiver is attributed to the \( P \) waveforms used at the transmit array. This extended data vector of length \( PR \times 1 \) helps in better resolution of the target and effective cancellation of interferences. In the next section we will give a design which will exploit this virtual extension and allow us to steer beams of variable size on the transmit side while maintaining the orthogonality among the waveform to suppress a lot of interferences at the receiver side.

Full Waveform Diverse-Phased MIMO radar (FWD-PMIMO)

**Idea and Design:** Phased MIMO radars with full waveform diversity is an idea that divide the transmit array into overlapped subarrays, where each subarray is modulated with a unique waveform. The number of subarrays are kept equal to total number of elements in the transmit array by subdividing in such a way that the size of each subarray is different from the other. The number of elements ranges from a single element in the first subarray to all the elements in the last subarray as shown in Figure 1. Since the number of subarray are equal to number of elements so the waveform diversity offered by proposed design is same as MIMO radar. In addition the transmit array also focuses on the target with beams of variable width which is helpful in target detecting at the receiver side.

Consider a transmit array of \( P \) elements divided into \( P \) subarrays through a slightly different way of array partitioning shown in Fig. 1. At the output of \( p^{th} \) subarray the complex envelop of the signals can be given as

\[ \mathbf{f}_p(t) = \rho s_p(t) \mathbf{w}_p, \quad p = 1, 2, \ldots, P \]  

(10)

Where \( \mathbf{w}_p \) represents the weight vector of \( P^{th} \) subarray containing the weights belonging to only the active elements of this particular array and \( \rho \) is the energy given to each subarray. \( \mathbf{w}_p \) vector contains beamforming weights corresponding to actual number of elements in each subarray i.e. first subarray has only one weight while the last subarray has \( P \) weights.

The signal reflected by a target in the far field at an angle \( \theta \) can be given by

\[ m_r(t, \theta) = \rho \alpha_\theta \sum_{p=1}^{P} \mathbf{w}_p^H \mathbf{u}_p(\theta) s_p(t) \]  

(11)

Where \( \alpha_\theta(\mathbf{l}) \) represents the reflection from the target and \( \mathbf{u}_p \) is the steering vector associated with \( N^{th} \) subarray. Since all the subarrays start at the same element i.e. the first element of the transmit array, Equation (11) can be written as

\[ m_r(t, \theta) = \rho \alpha_\theta(\mathbf{l}) (\mathbf{x}(\theta)^T) \mathbf{s}_p(t) \]  

(12)

where \( \mathbf{x}(\theta) \) and \( \mathbf{s}_l(\theta) \) are \( P \times 1 \) vectors given as

\[ \mathbf{x}(\theta) = [\mathbf{w}_1 \mathbf{u}_1, \mathbf{w}_2 \mathbf{u}_2, \ldots, \mathbf{w}_P \mathbf{u}_P]^T \]  

(13)

\[ \mathbf{s}_p(t) = [s_1(t), s_2(t), \ldots, s_p(t)]^T \]  

(14)

If the target angle is \( \theta \) and there are \( Q \) interferences in the background, then the signal received at an array of \( R \) elements will be
where \( v(\theta) \) is steering vector for receiver and \( \tilde{u} \) is the noise term. By matched filtering for the \( P \) different waveforms transmitted from transmitter side we will get a virtual data vector of length \( PR \times 1 \) given as

\[
d = \rho a_z z(\theta_r) + \sum_{i=1}^{Q} \rho a_z z(\theta_r) + \tilde{u}
\]

where

\[
z(\theta) = x(\theta) \otimes v(\theta)
\]

The data vector \( d \) is obtained by stacking the vectors of (15) and \( z(\theta) \) is the virtual steering vector of length \( PR \times 1 \). For a conventional phased MIMO in literature [14], the data vector can be obtained in the same way as above. However the length of data vector will be \( KR \times 1 \) where \( 1 < K < P \).

**Weight Matrix:** Since there are \( P \) different subarray in our proposed system so we will get a weight matrix in which the columns of matrix represent the vector corresponding to each subarray. The weight matrix is given as

\[
W = \begin{bmatrix}
w_{1,1} & w_{2,1} & \ldots & \ldots & w_{P,1} \\
0 & w_{2,2} & \ldots & \ldots & w_{P,2} \\
0 & 0 & \ldots & \ldots & \ldots \\
\vdots & \vdots & \ddots & \ddots & \ddots \\
0 & 0 & \ldots & \ldots & w_{P,P}
\end{bmatrix}
\]

where \( W \) is \( P \times P \) matrix of weights and \( w_{ij} \) is the weight of \( k^{th} \) element of \( i^{th} \) subarray.

For the transmit array to operate under this overlapped scheme, each antenna element should send the linear combinations of orthogonal waveforms. So the signal transmitted by \( P \) transmit antennas can be written as

\[
\gamma(t) = \rho W^* s_X(t)
\]

\[
\gamma(t) = \begin{bmatrix} \gamma_1(t) & \ldots & \gamma_P(t) \end{bmatrix}^T
\]

\( \gamma(t) \) is the \( P \times 1 \) vector of waveforms transmitted by \( P \) antenna elements.

\( W = [w_1, w_2, \ldots, w_K] \) is the weight matrix of \( P \times P \) and *'s* are transmitted waveforms. The signal used in the proposed scheme are orthogonal subarray-wise but it is not orthogonal element wise. The signal used for each subarray maintains orthogonality for various Doppler shifts and time delays by properly selecting \( T \) [17]. The signal used are

\[
\gamma(t) = \frac{1}{\sqrt{T}} \sum_{\mu=1}^{P} e^{j2\pi \mu \frac{f_c}{T}} P(t)
\]

In order to determine the signal transmitted by each antenna element we have to make linear combinations of different waveforms before transmission on a particular antenna.

The signal transmitted through each antenna element will be as:

**Antenna 1:**

\[
\gamma_1 = \rho(w_{1,1}^* s_1 + w_{2,1}^* s_2 + \ldots + w_{P,1}^* s_P)
\]

**Antenna 2:**

\[
\gamma_2 = \rho(w_{2,2}^* s_2 + w_{3,2}^* s_3 + \ldots + w_{P,2}^* s_P)
\]

**Antenna P-1:**

\[
\gamma_{P-1} = \rho(w_{P-1,1}^* s_{P-1} + w_{P,1}^* s_P)
\]

**Antenna P:**

\[
\gamma_P = \rho(w_{P,P}^* s_P)
\]

It can be noted that if a particular element is not the part of a subarray, then it will not send the waveform of that particular subarray due to zero weight.

**Beamforming and Sinr:** In this section we will discuss the beamforming using conventional and adaptive beamformer at the transmitter and receiver side respectively. An adaptive beamformer is used at the receiving side since we compare the proposed radar with PAR and MIMO in terms of interference cancellation.

**Conventional Beamforming:** The conventional beamforming on the transmitter side have the weight vector given as

\[
w_p = \frac{u_p(\theta)}{\|u_p(\theta)\|}, \quad p = 1, 2, \ldots, P
\]
The normalized beam pattern for the transmit side is as
\[
B(\theta) = \left| w_p^H u_p(\theta) \right|^2
\]  

(23)

Adaptive Beamforming: At the receiving end an adaptive beamformer is used. The weight vector for receiver side can be written as
\[
w_r = z(\theta) = x(\theta) \otimes v(\theta)
\]  

(24)

Minimum variance distortionless response (MVDR) beamformer is used as an adaptive beamformer to give the distortionless response in target direction and minimize the interference plus noise. The optimization problem is as follows
\[
\min_{w_r} w_r^H C_{i+n} w_r \quad \text{subject to} \quad w_r^H z(\theta) = 1
\]  

(25)

The solution of (25) is given in [16] as
\[
w_r = \frac{C_{i+n}^{-1}(\theta) z(\theta)}{z^H(\theta) C_{i+n}^{-1}(\theta) z(\theta)}
\]  

(26)

\(C_{i+n}\) is the interference plus noise covariance matrix for proposed FWD-P MIMO radar subarray, which can be estimated as \(\hat{C} = \sum_{j=1}^J d_j d_j^H\), where \(d_j\) is the snapshots of data obtained from \(J\) different radar pulses.

The beampattern on the receiver can be given as
\[
B(\theta) = \left| w_r^H z(\theta) \right|^2
\]  

(27)

Signal to interference plus noise ratio can be given as
\[
\text{SINR}_{FWD-PMIMO} = \frac{\rho^2 \beta_i^2 \left| w_p^H u_p(\theta) \right|^2}{w_r^H C_{i+n} w_r}
\]  

(28)

The \(C_{i+n}\) matrix can be given as
\[
C_{i+n} = \sum_{i=1}^O \rho^2 \beta_i^2 z(\theta) z_i^H(\theta) + \beta_n^2 I
\]  

(29)

By substituting \(C_{i+n}\) and \(w\) in equation (28), we get
\[
\text{SINR}_{FWD-PMIMO} = \frac{\rho^2 \beta_i^2 \left| w_p^H u_p(\theta) \right|^2}{w_r^H C_{i+n} w_r} = \frac{\rho^2 \beta_i^2 \left| w_p^H u_p(\theta) \right|^2 \left| z(\theta) \right|^2}{z^H(\theta) \sum_{i=1}^O \rho^2 \beta_i^2 z_i(z_i^H(\theta) + \beta_n^2 I) z(\theta)}
\]  

(30)

where \(\beta_i^2\) and \(\beta_n^2\) are the variances of reflection coefficient from target and noise, respectively. After simplifying (30) we will get the final relation for the SINR given as
\[
\text{SINR}_{FWD-PMIMO} = \frac{\rho^2 \beta_i^2 \rho^2 R^2}{\sum_{i=1}^O \rho^2 \beta_i^2 \left| z^H(\theta) z(\theta) \right|^2 + \beta_n^2 R^2}
\]  

(31)

For interference dominant case we consider only the interference part, so the SINR can be given as
\[
\text{SINR}_{FWD-PMIMO} = \frac{\rho^2 \beta_i^2 \rho^2 R^2}{\sum_{i=1}^O \rho^2 \beta_i^2 \left| z^H(\theta) z(\theta) \right|^2}
\]  

(32)

Now the SINR of phased array and MIMO radar given in [14] can be written as given below
\[
\text{SINR}_{PAR} = \text{SINR}_{MIMO} = \frac{\beta_i^2 \rho^2 R^2}{\sum_{i=1}^O \beta_i^2 \left| z^H(\theta) z(\theta) \right|^2}
\]  

(33)

It is interesting to see that both phased array and MIMO radar have same SINR, but this is the case when only a small number of interferences are present in area of interest. In case of greater no of interferences the MIMO radar outperform the phased array radar. It is also worth mentioning that the \(z(\hat{e})\) in equation (32) and equation (33) are of different from each other. For PAR, \(z(\hat{e})\) has dimension of RX1 and for MIMO and FWD-PMIMO it has dimension of PRX1. However, MIMO has no gain at the transmit side while FWD-PMIMO does have some gain at transmit side which is clear from its better SINR plot in next section. The phased MIMO radar existing in literature also has lesser dimension compared to proposed radar since the number of subarrays in Phased MIMO radar is always less than the proposed radar. As a result the proposed radar outperform Phased array, MIMO radar and phased MIMO with equal subarrays in terms of SINR for larger number of interferences.
RESULTS AND DISCUSSION

For simulation we assume $P=5$ antenna elements on the transmitter divided into $P$ fully overlapped subarrays as discussed in section III. Receiver array also contains $R=5$ antenna elements and both transmitter and receiver elements are at half wavelength from each other. We choose small transmit array in order to get a better understanding of interference cancellation capability of proposed system. Throughout our simulation we assume a target at angle of $0^\circ$ but the interferences are assumed at many different angles. The number of interferences are kept high since the proposed scheme is designed for large number of interferences around the target. Complex Gaussian noise with zero mean is considered as the additive noise.

**Part 1:** The first part of simulation give results on the transmit side to show the maximum gain beampattern, waveform diversity beampattern and the overall beampattern of phased array radar (PAR), MIMO radar, Phased MIMO (PMIMO) radar and Full Waveform diverse Phased MIMO (FWD-PMIMO) radar. Figure 2 show the maximum gain provided by all the radar systems. It can be observed that phased array has the best gain due to focusing of all the energy in one direction. FWD-PMIMO and Phased MIMO (PMIMO) has lesser gain compared to phased array but a lot better gain than the MIMO radar which show a flat line because of no transmit gain. The gain provided by PMIMO and FWD-PMIMO is because of the subarrays made at the transmitter which give focused beams instead of omnidirectional transmission.

The waveform diversity beampattern is shown in Figure 3 which show that the diversity of both MIMO and FWD-PMIMO is equal and at its maximum but phased array radar offer no diversity at all. Whereas, the PMIMO gives a diversity gain better then PAR but less than MIMO and FWD-PMIMO. The reason for flat graph of phased array radar is that it use phased shifter versions of only one waveform at the transmitter. On the other hand MIMO and FWD-PMIMO transmit $P$ unique waveforms and PMIMO used less than $P$ unique waveforms.

The overall beampattern depicted in Figure 4 is resultant of both gain beampattern and diversity beampattern. The corresponding values of both beampattern are multiplied (added in dB) to get the overall beampattern which can be named as Gain Diversity product.

![Fig. 2: Transmit gain beampattern of PAR, MIMO, Phased MIMO and FWD-PMIMO](image1)

![Fig. 3: Waveform diversity beampattern of PAR, MIMO, PMIMO and FWD-PMIMO](image2)

![Fig. 4: Overall beampattern of PAR, MIMO, PMIMO and FWD-PMIMO](image3)
Fig. 5: MVDR Beampattern of Phased array radar (PAR), MIMO, PMIMO and FWD-PMIMO for $Q=8$ interferences

Fig. 6: SINR of MVDR Beamformer for Phased array radar (PAR), MIMO, PMIMO and FWD-PMIMO for $Q=8$ interferences

gain and diversity. It is also interesting to note that the overall beampattern of phased array and MIMO radar is exactly the same due to same contribution in first two plots i.e. Phased array contribute with directional gain of 5 elements while MIMO contribute with five orthogonal waveforms. The PMIMO also contribute to both the plots but its diversity is less than proposed radar.

**CONCLUSION**

A new type of array partitioning is proposed to exploit the waveform diversity to cancel the interferences more efficiently than the phased array and MIMO radar. The array is divided into number of subarrays and each subarray is modulated with a unique waveform to exploit this uniqueness on the receiver side to get better detection and interference cancellation. Since each subarray provide different gain in our proposed scheme so the gain along with waveform diversity make FWD-PMIMO better choice for interference cancellation compared to phased array radar, MIMO radar and Phased MIMO radar with equal subarrays.

**REFERENCES**


