Quasi-periodic transformations of nonlocal spatial solitons

Daniel Buccoliero[1,2] and Anton S. Desyatnikov[1]

[1]*Nonlinear Physics Center and* [2]*Laser Physics Center, Research School of Physics and Engineering, The Australian National University, Canberra ACT 0200, Australia*

dbu111@rsphysse.anu.edu.au

Abstract: We study quasi-periodic transformations between nonlocal spatial solitons of different symmetries triggered by modulational instability and resembling a self-induced mode converter. Transformation dynamics of solitons with zero angular momentum, e.g. the quadrupole-type soliton, reveal the equidistant spectrum of spatial field oscillations typical for the breather-type solutions. In contrast, the transformations of nonlocal solitons carrying orbital angular momentum, such as 2×3 soliton matrix, are accompanied by their spiralling and corresponding spectra of field oscillations show mixing of three characteristic spatial frequencies.

OCIS codes: (190.6135), Spatial solitons; (260.6042), Singular optics.

References and links

1. Yu. S. Kivshar and G. P. Agrawal, *Optical Solitons: From Fibers to Photonic Crystals* (Academic Press, 2003).
2. A. S. Desyatnikov, Yu. S. Kivshar, and L. Torner, "Optical vortices and vortex solitons," Prog. Opt. **47**, 291-391 (Ed. E. Wolf, Elsevier, 2005).
3. C. Conti, M. Peccianti, and G. Assanto, "Observation of Optical Spatial Solitons in a Highly Nonlocal Medium," Phys. Rev. Lett. **92**, 113902 (2004).
4. C. Rotschild, B. Alfassi, O. Cohen, and M. Segev, "Long-range interactions between optical solitons," Nat. Phys. **2**, 769-774 (2006).
5. W. Krolikowski, O. Bang, N.I. Nikolov, D. Neshev, J. Wyller, J.J. Rasmussen, and D. Edmundson, "Modulational instability, solitons and beam propagation in spatially nonlocal nonlinear media," J. Opt. B **6**, S288-S294 (2004).
6. D. Briedis, D. Petersen, D. Edmundson, W. Krolikowski, and O. Bang, "Ring vortex solitons in nonlocal nonlinear media," Opt. Express **13**, 435-443 (2005), http://www.opticsinfobase.org/oe/abstract.cfm?URI=oe-13-2-435
7. A. I. Yakimenko, Yu. A. Zaliznyak, and Yu. S. Kivshar, "Stable vortex solitons in nonlocal self-focusing nonlinear media," Phys. Rev. E **71**, 065603(R) (2005).
8. C. Rotschild, O. Cohen, O. Manela, M. Segev, and T. Carmon, "Solitons in Nonlinear Media with an Infinite Range of Nonlocality: First Observation of Coherent Elliptic Solitons and of Vortex-Ring Solitons," Phys. Rev. Lett. **95**, 213904 (2005).
9. C. Rotschild, M. Segev, Z. Xu, Y. V. Kartashov, L. Torner, and O. Cohen, "Two-dimensional multipole solitons in nonlocal nonlinear media," Opt. Lett. **31**, 3312-3314 (2006).
10. S. Lopez-Aguayo, A. S. Desyatnikov, Yu. S. Kivshar, S. Skupin, W. Krolikowski, and O. Bang, "Stable rotating dipole solitons in nonlocal optical media," Opt. Lett. **31**, 1100-1102 (2006).
11. S. Lopez-Aguayo, A. S. Desyatnikov, and Yu. S. Kivshar, "Azimuthons in nonlocal nonlinear media," Opt. Express **14**, 7903-7908 (2006), http://www.opticsinfobase.org/oe/abstract.cfm?URI=oe-14-17-7903
12. S. Skupin, M. Grech, and W. Krolikowski, "Rotating soliton solutions in nonlocal nonlinear media," Opt. Express **16**, 9118-9131 (2008), http://www.opticsinfobase.org/oe/abstract.cfm?URI=oe-16-12-9118
13. S. Skupin, M. Saffman, and W. Krolikowski, "Nonlocal stabilization of nonlinear beams in a self-focusing atomic vapor," Phys. Rev. Lett. **98**, 263902 (2007).
14. I. Kaminer, C. Rotschild, O. Manela, and M. Segev, "Periodic solitons in nonlocal nonlinear media," Opt. Lett. **32**, 3209-3211 (2007).
15. O. Cohen, H. Buljan, T. Schwartz, J. W. Fleischer, and M. Segev, "Incoherent solitons in instantaneous nonlocal nonlinear media," Phys. Rev. E **73**, 015601(R) (2006).

#109908 - $15.00 USD Received 9 Apr 2009; revised 5 May 2009; accepted 20 May 2009; published 22 May 2009

16. C. Rotschild, T. Schwartz, O. Cohen, and M. Segev, "Incoherent spatial solitons in effectively instantaneous nonlinear media," Nat. Photon. **2**, 371-376 (2008).
17. A. V. Gorbach and D. V. Skryabin, "Cascaded Generation of Multiply Charged Optical Vortices and Spatiotemporal Helical Beams in a Raman Medium," Phys. Rev. Lett. **98**, 243601 (2007).
18. A. I. Yakimenko, V. M. Lashkin, and O. O. Prikhodko, "Dynamics of two-dimensional coherent structures in nonlocal nonlinear media," Phys. Rev. E **73**, 066605 (2006).
19. Y. V. Kartashov, L. Torner, V. A. Vysloukh, and D. Mihalache, "Multipole vector solitons in nonlocal nonlinear media," Opt. Lett. **31**, 1483-1485 (2006).
20. D. Buccoliero, A. S. Desyatnikov, W. Krolikowski, and Yu. S. Kivshar, "Laguerre and Hermite soliton clusters in nonlocal nonlinear media," Phys. Rev. Lett. **98**, 053901 (2007).
21. Y. V. Kartashov, V. A. Vysloukh, L. Torner, "Stability of vortex solitons in thermal nonlinear media with cylindrical symmetry," Opt. Express **15**, 9378-9384 (2007), http://www.opticsinfobase.org/oe/abstract.cfm?URI=oe-15-15-9378
22. F. Ye, Y. V. Kartashov, L. Torner, "Stabilization of dipole solitons in nonlocal nonlinear media," Phys. Rev. A **77**, 043821 (2008).
23. D. Buccoliero, A. S. Desyatnikov, W. Krolikowski, and Y. S. Kivshar, "Boundary effects on the dynamics of higher-order optical spatial solitons in nonlocal thermal media," J. Opt. A **11**, in press (2009).
24. E. G. Abramochkin and V. G. Volostnikov, "Generalized Gaussian beams," J. Opt. A **6**, S157-S161 (2004).
25. D. Buccoliero, A. S. Desyatnikov, W. Krolikowski, and Yu. S. Kivshar, "Spiraling multivortex solitons in nonlocal nonlinear media," Opt. Lett. **33**, 198-200 (2008).
26. S. Flach and A. Gorbach, "Discrete breathers-Advances in theory and applications," Phys. Rep. **467**, 1 (2008).
27. A. Snyder and J. Mitchell, "Accesible Solitons," Science **276**, 1538-1541 (1997).
28. S. Lopez-Aguayo and J. C. Gutiérrez-Vega, "Elliptically modulated self-trapped singular beams in nonlocal nonlinear media: ellipticons," Opt. Express **15**, 18326-18338 (2007), http://www.opticsinfobase.org/oe/abstract.cfm?URI=oe-15-26-18326.

1. Introduction

In local isotropic self-focusing media the spatial optical solitons [1] are, in general, unstable. If the collapse instability is removed by, e.g., saturation of nonlinearity, the fundamental bell-shaped solitons become stable while the higher-order solitons still break up because of modulational instability or phase-sensitive interactions [2]. The nonlocality of media response, such as in liquid crystals [3] and lead glases [4], allows to suppress collapse [5] as well as to stabilize vortex solitons [6–8], mulipoles [9], and azimuthons [10–13].

Nonlocal media supports great variety of spatial solitons and affects dramatically their stability. Most intriguing are periodic solitons [14], including multi-modal beams [15–17] and two-dimensional solitons transforming their symmetry during propagation [10, 18–20]. Quasi-periodic dynamics can be triggered by the internal modes of solitons when two different states, such as necklaces and matrices of solitons [20], can couple without violating conservation of power and Hamiltonian. In thermal media the stability of solitons is strongly affected by the boundaries [8, 9, 21, 22] and the transformation dynamics can be effectively controlled, e.g., by changing the aspect ratio of a rectangular sample [23].

The process of the dynamic reshaping of nonlocal solitons [20] resembles astigmatic transformations in mode converters. For linear Hermite-Laguerre-Gaussian beams [24] it can be described with the help of a single parameter, $0 \le \alpha \le \pi/4$, transforming between Hermite ($\alpha = 0$) and Laguerre ($\alpha = \pi/4$) profiles. The transformations of solitons is essentially more complex because the conservation of orbital angular momentum (OAM) leads to their spiralling in nonlinear media [25] and restricts conversion to the states with the same OAM. As a result, the lowest-order dipole [10] and tripole [25] spiralling solitons do not transform.

In this paper we analyze robust long-lived quasi-periodic transformations of nonlocal solitons, including transformations accompanied by spiralling. First we show that the field oscillations of solitons with zero OAM [20] exhibit equidistant spectrum characteristic for discrete breathers [26]. Second representative example is the 2×3 soliton matrix whose family includes the pure Hermite beam and the double-ring single-charge vortex soliton with non-zero OAM.

#109908 - $15.00 USD Received 9 Apr 2009; revised 5 May 2009; accepted 20 May 2009; published 22 May 2009

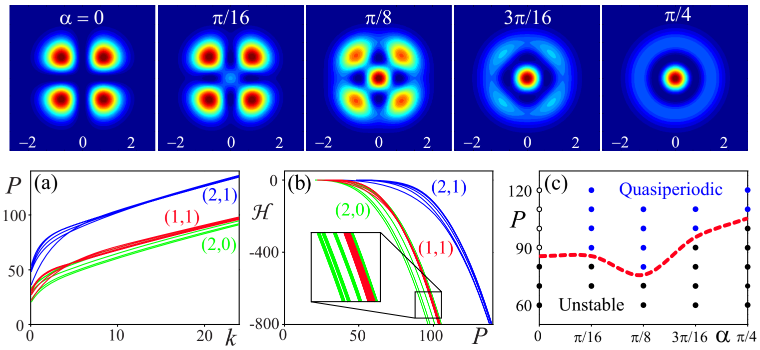

Fig. 1. Top row: intensities of generalized quadrupole GN_{11} for several values of α. (a) Power vs. propagation constant and (b) Hamiltonian vs. power for three different families GN_{mn} with indices (m,n) indicated next to the curves; for each family five curves are shown with $\alpha = \pi(0,1,2,3,4)/16$. (c) Stability domain of the generalized quadrupole.

The oscillation spectra of such spiralling soliton include discrete set of spatial frequencies produced by algebraic sum of soliton frequency, rotation rate, and transformation frequency.

We consider the system described by the nonlinear Schrödinger (NLS) equation [1]

$$i\frac{\partial E}{\partial z} + \frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial y^2} + E\int R\left(|\mathbf{r}-\mathbf{r}'|\right)|E\left(\mathbf{r}'\right)|^2 d\mathbf{r}' = 0, \tag{1}$$

where the medium response function is assumed to be Gaussian, $R(r) = \exp\left(-r^2\right)$. To find approximate soliton profiles we use variational principle with the generic ansatz, $E(x,y,z) = U(x,y)\exp(ikz)$, introducing soliton propagation constant k. Corresponding Lagrangian, $\mathscr{L} = -kP - \mathscr{H}$, is defined through intergrals of motion, the power $P = \int |U|^2 d\mathbf{r}$, and Hamiltonian

$$\mathscr{H} = \int |\nabla U|^2 d\mathbf{r} - \frac{1}{2}\int\int R\left(|\mathbf{r}-\mathbf{r}'|\right)|U(\mathbf{r})|^2|U(\mathbf{r}')|^2 d\mathbf{r}d\mathbf{r}'. \tag{2}$$

An important for the following integral of motion is the OAM, $M = \mathrm{Im}\int U^*(x\partial_y U - y\partial_x U)d\mathbf{r}$.

We construct different families of generalized nonlocal solitons, $U = \mathrm{GN}_{mn}$, using linear optical generalized Hermite-Laguerre-Gaussian modes $\mathrm{HLG}_{mn}(x,y;\alpha)$ [24]:

$$\mathrm{GN}_{mn}(x,y;\alpha) = A\exp\left(-\frac{x^2+y^2}{2a^2}\right)\sum_{j=0}^{n+m} C_{mn}^{j}(\alpha)\, H_{n+m-j}(x/b)\, H_j(y/b), \tag{3}$$

where H_j are Hermite polynomials and the coefficients $C_{mn}^{j}(\alpha)$ are expressed in terms of Jacobi polynomials $P_j^{(\mu,\nu)}$ as follows: $C_{mn}^{j}(\alpha) = i^j\cos^{n-j}\alpha\,\sin^{m-j}\alpha P_j^{(n-j,m-j)}(-\cos 2\alpha)$. Variational parameters are the amplitude A and soliton widths a and b.

2. Quasi-periodic transformations: quadrupole soliton

Quadrupole-type family of solutions GN_{11} can be described by the following ansatz:

$$\mathrm{GN}_{11}(x,y;\alpha) = A\exp\left(-\frac{x^2+y^2}{2a^2}\right)\left[\left(x^2+y^2-b^2\right)\sin 2\alpha - 2ixy\cos 2\alpha\right]/b^2, \tag{4}$$

and the variational solutions are derived numerically in the parameter domain (k,α), see Fig. 1. The family includes pure quadrupole beam for $\alpha = 0$, or 2×2 matrix of out-of-phase solitons,

#109908 - $15.00 USD Received 9 Apr 2009; revised 5 May 2009; accepted 20 May 2009; published 22 May 2009

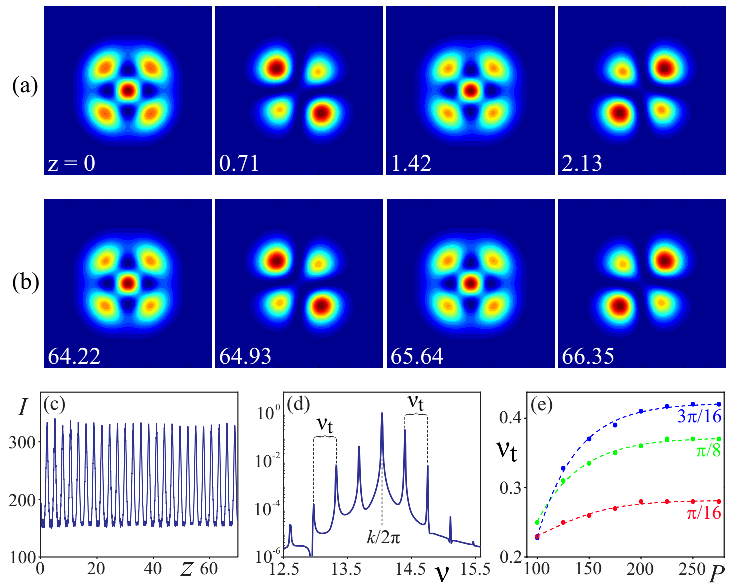

Fig. 2. Quasi-periodic breather-like dynamics of the GN_{11} soliton with $P = 200$ and $\alpha = \pi/8$ is shown at small (a, Media 1) and large (b, Media 2) propagation distances. At one spatial location in the transverse plane we show (c) the intensity and (d) the spectrum of oscillations of the field. (e) Transformation frequency ν_t vs. power P for $\alpha = \pi(1,2,3)/16$.

as well as the radial mode of a fundamental soliton for $\alpha = \pi/4$. Importantly, the OAM is zero for the whole family. For comparison, the families with nonzero OAM of the tripole soliton GN_{20} [25] and the 2×3 soliton matrix GN_{21} are also shown in the diagrams in Figs. 1(a)-1(b).

In direct simulations of the beam propagation we use the split-step FFT algorithm and the variational solutions Eq. (4) as input profiles. Varying α and P we investigate the stability properties of the quadrupole beams, the results are summarized in Fig. 1(c). The scaling of our model is such that the degree of nonlocality increases with power P [11, 20], so that at low power, closer to the local limit, we observe the solitons breaking up. At higher power this scenario changes dramatically and the beams evolve in distinctive ways depending on α.

An example of quasi-periodic behavior of the $GN_{11}(x,y;\pi/8)$ soliton is shown in Fig. 2. The rows of intensity snapshots in Fig. 2(a, Media 1) and 2(b, Media 2) are taken from the same simulation at different propagation distances. In the case shown ($P = 200$) the soliton constant is $k = 88.6$ so that soliton period $\pi/k = 0.036$, thus the row Fig. 2(b) is taken at the distances of about 1800 soliton periods. Although the fine details of the soliton structures are slightly different between Fig. 2(a, Media 1) and (b, Media 2) because the oscillations are not strictly periodic, the revival of the same states is remarkable. The period of oscillations can be roughly estimated from Figs. 2(a)-2(b) to be $T = 2.84$ and thus there were about 24 such oscillations.

For a better characterization of the transformation dynamics we record during propagation the sequences of the values of complex field at randomly chosen points in the transverse plane. An example of the intensity at one such point is shown in Fig. 2(c), small variations of the amplitude are visible during 25 oscillations while the period seems to be constant. The logarithm of the Fourier transform of the recorded complex field, plotted in Fig. 2(d), reveals the fine structure of oscillations. The main peak at the frequency $\nu = 14.04$ corresponds to the soliton propagation constant, $k/2\pi$. The side peaks are equidistant from the main peak with frequency comb defined by the quasiperiodic transformations, $\nu_t = 0.357$; note that this value is in good

#109908 - $15.00 USD Received 9 Apr 2009; revised 5 May 2009; accepted 20 May 2009; published 22 May 2009

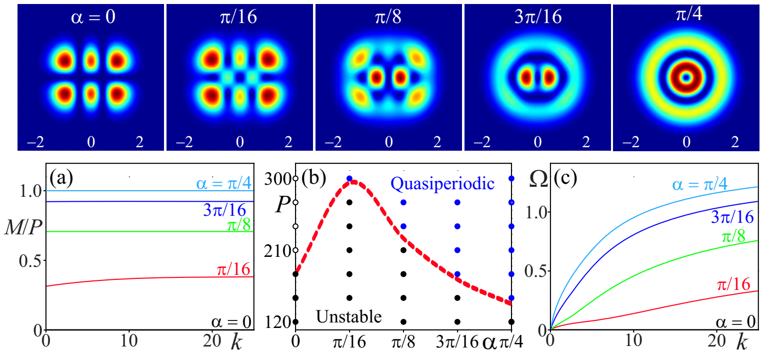

Fig. 3. Family of the 2×3 soliton matrix GN_{21}; top row: intensities for $P = 200$. (a) Normalized OAM, (b) stability diagram, and (c) the spiralling velocity vs. k.

agreement with directly observed inverse period of oscillations, $\nu_t \simeq 1/T = 0.352$. The clear "ladder"-like structure of the spectrum in Fig. 2(d) resembles an equidistant spectra of breather solutions in discrete nonlinear systems [26].

We examine the dependence of ν_t for the family of solitons, parameterized by α and P, by performing extensive numerical simulations of the beam propagation also applying initial random noise to confirm the robustness of results summarized in Fig. 2(e). We find that the transformation frequency ν_t increases monotonically with power, before saturating at high powers. In our model this observation points out that for larger scale of nonlocality the transformations are faster. In the limit of infinite nonlocality the linear Snyder-Mitchell model [27] approximates our system Eq. (1), thus beating (interference) of different linear modes of harmonic potential can be used to find related spatial frequencies [28].

3. Transformations and spiralling: 2×3 soliton matrix

We construct the family of the generalized 2×3 soliton matrix using the GN_{21} mode,

$$\mathrm{GN}_{21}(x,y;\alpha) = A\exp\left(\frac{-x^2-y^2}{2a^2}\right)\left\{y\left(x^2-y^2+b^2\right)\cos\alpha + y\left(3x^2+y^2-3b^2\right)\cos(3\alpha) + 2ix\sin\alpha\left[x^2+y^2-2b^2+\left(x^2+3y^2-3b^2\right)\cos(2\alpha)\right]\right\}/b^3. \quad (5)$$

Varying α from 0 to $\pi/4$ in Eq. (5) produces the family ranging from the 2×3 matrix to a double-ring single-charge vortex soliton, see Fig. 3. The OAM is largely defined by the modulation parameter α and practically does not change with power, as seen in Fig. 3(a). The stability diagram in Fig. 3(b) is qualitatively similar to the one for generalized quadrupole in Fig. 1(c), namely at low power the beams break up while for powers above a certain threshold (red dashed line in (b)) they transform and rotate with propagation. The rate of spiralling Ω [25] strongly changes with power in Fig. 3(c), in contrast to the OAM in Fig. 3(a), so that for larger scale of nonlocality the rotation is faster.

The simulations of Eq. (1) reveal complex dynamics of GN_{21} soliton with three simultaneous processes on different scales: fast breathing oscillations, spiralling, and transformations (see Fig. 4 and Media 3). It is impossible to distinguish different frequencies from direct observations and we resort to the analysis of oscillation spectra. Because of rotation, at any given point the field oscillates strongly without smooth structure as in Fig. 2(c). Therefore, we record sequences at ten different points, $E_n(z) = E(x_n, y_n, z)$, $n = 1, 2..10$, take the Fourier transform of each field, $\mathscr{F}_n = \mathscr{F}[E_n]$, and average them, $F = 0.1\sum_{n=1}^{10} |\mathscr{F}_n|^2$, see bottom frame of Fig. 4.

#109908 - $15.00 USD Received 9 Apr 2009; revised 5 May 2009; accepted 20 May 2009; published 22 May 2009

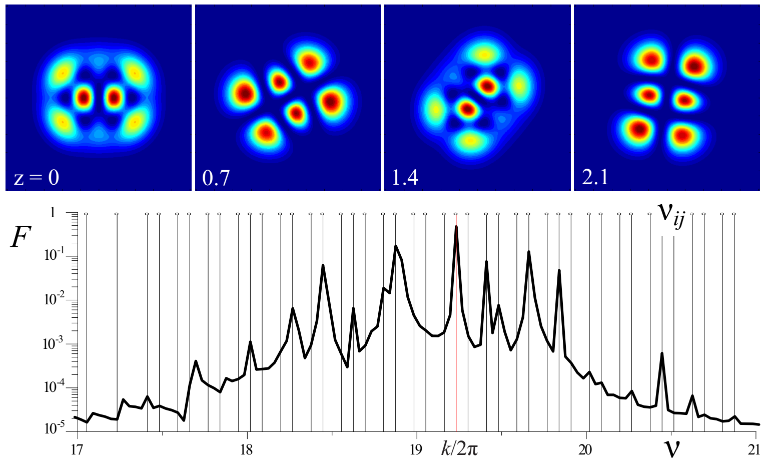

Fig. 4. Quasi-periodic transformations and spiralling of the $\mathrm{GN}_{21}(x,y;\pi/8)$ soliton with power $P = 300$ (Media 3). Bottom: the averaged spectrum of oscillations with vertical lines showing composite spatial frequencies ν_{ij}.

Analysis of the spectrum in Fig. 4 allows to draw several conclusions. The dominating frequency $\nu = 19.2$ is given by the soliton constant $k = 120.3$. Other side peaks are produced by the linear combination of two distinct frequencies (indices i and j are integer),

$$\nu_{ij} = k/2\pi + i\,\nu_t + j\,\nu_r, \tag{6}$$

namely the transformation frequency $\nu_t = 0.607$ (period $T = 1.647$), and the spiralling rate $\nu_r = 0.179$ (period of rotation $T_r = 5.587$ corresponds to angular velocity $\Omega = 2\pi/T_r = 1.125$, close to variational parameter $\Omega = 0.982$). We indicate in Fig. 4 with vertical lines the frequencies ν_{ij} obtained for $-3 \le (i,j) \le 3$. Remarkably, all major peaks in the spectrum belong to the ν_{ij} set while some combinations (i,j) are clearly missing. We conclude that, despite distortions of the profile during propagation, the major contributions to the complex dynamics of the GN_{21} soliton can be distinguished as spiralling and mode transformations.

4. Conclusions

We demonstrated, using generalized nonlocal solitons GN_{11} and GN_{21} as representative examples, that nonlocality of the nonlinear medium response supports quasi-periodic transformations between different symmetries of self-trapped optical beams. Despite fast breathing oscillations, typical for perturbed solitons, the oscillation spectra contain discrete set of spatial frequencies of transformation and spatial rotation. The missing composite frequencies ν_{ij} in Fig. 4 indicate that only specific combinations (i,j) in Eq. (6) contribute to the final spectrum, reflecting specific (unknown) structure of the soliton. Note that transformations can not be described in linearized perturbation analysis because they are not small. On the other hand, the periods of rotation and transformation are at least one order of magnitude lower than soliton period. Therefore, the ratio ν_t/k can be used as a small adiabatic parameter to address the question of whether truly periodic two-dimensional solitons exist.

Acknowledgment

Authors appreciate critical reading of this manuscript by Yu. S. Kivshar and W. Krolikowski. This work is supported by the Australian Research Council through the Discovery Project.

#109908 - $15.00 USD Received 9 Apr 2009; revised 5 May 2009; accepted 20 May 2009; published 22 May 2009