Multi-Criteria Decision Model for e-learning Architecture Selection based on Utility Function and ELECTRE Method

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Abstract
An e-learning architecture selection has been analyzed. This is a typical problem when dealing with e-learning system selection. For each alternative of an e-learning architecture there is an evaluation of both cost and quality of service. The latter may include probabilistic delivery time and confidence in quality commitment. The decision-maker takes into account multi-criteria evaluation through ELECTRE method. Besides, each criterion is evaluated through a utility function. The model integrates both approaches to indicate the best e-learning architecture. This paper presents the formulation for the decision model and a numerical application to illustrate the use of the model.

Keywords: e-learning Architecture Selection; Multi-criteria Decision; Utility Theory; ELECTRE Method.

1. Introduction
E-learning refers to the use of electronic devices for learning, including the delivery of content via electronic media such as Internet/Intranet/Extranet, audio or video tape, satellite broadcast, interactive TV, CD-ROM, and so on (Kaplan-Leiserson, 2000). This type of learning moves the traditional instruction paradigm to a learning paradigm (Jönsson, 2005), thereby relinquishing much control over planning and selection to the learners. In addition, it offers the following advantages to learners: Cost-effectiveness, timely content, and access flexibility (Hong, Lai and Holton, 2003; Lorenzetti, 2005; Rosenberg, 2001).

The capability and flexibility of the web-based e-learning systems (WELSs) having been demonstrated in both training and education, resulted in their adoption by academia as well as industry. Since the commercial application package (or commercial off-the-shelf) strategy of system development is so widespread (Whittenet et al. 2004), the proliferation of WELS applications has caused confusion for the potential adopters having to make selective decisions from among candidate products or solutions. At the same time, organizations with adopted systems are faced with issues arising from the post-adoption phase.

Several studies have been conducted on the process of e-learning dealing with different aspects of this matter. Regarding the e-learning architecture selection little research has been found in the literature. Almeida (2001) has dealt with maintenance architecture selection based on a multi-criteria model, which uses contributions from
multi-attribute utility theory (MAUT). Dulmin and Mininno (2003) have presented a multi-criteria decision aid method, so called PROMETHEE/GAIA to approach a e-learning system provider’s selection model, which is applied in the context of the rail organization employees. Multi-criteria decision aid methods such as PROMETHEE/GAIA and MAUT allow the decision-maker to quantify multiple objectives even when these objectives contain conflicting attributes or when they are subjective.

This paper presents results of research dealing with a multi-criteria decision model for e-learning architecture selection, using contributions from utility theory associated with the ELECTRE method. The problem consists of selecting the most appropriate alternative for an e-learning architecture taking several criteria into account. Each architecture alternative implies a specific cost and service quality characteristics. Probabilistic delivery time, confidence commitment regarding deadlines and service quality are among these service quality characteristics. The decision-maker has to choose the best alternative taking into account the consequences modeled through a multi-criteria method. Utility function (Keeney and Raiffa, 1976) and ELECTRE method (Roy, 1996) have been taken into account to model this problem.

2. E-Learning Architecture Selection Problem

E-learning is a popular business strategy nowadays, and requires close attention to the appropriate architecture selection process. The e-learning implementation price is no longer the only aspect to be taken into account regarding decisions on e-learning architecture selection. That is, different aspects have to be considered by the decision-maker, such as cost of the architecture and performance of the service. In general, service delivery time is a formal commitment written in the e-learning implementation contract. Normally a different service delivery time implies a specific condition of resources, personal skills and service availability and may result in a different cost. The architecture is assumed to have basic variables related to multiple objectives. These multiple objectives may be represented through the performance objectives of a production strategy plan, such as quality, speed, dependability, flexibility, and cost (Slack et al., 1995). Other approaches for multiple objectives may be obtained from multidimensional quality views (Evans and Lindsay, 1989; Teboul, 1990; Garvin, 1988). The Action Space corresponds to the set of alternatives available to the decision-maker. An action element of the set is represented by \( a \). The set of all actions is discrete with \( m \) elements: \( \{a_1, a_2, \ldots, a_m\} \).

Each element of this set corresponds to a possible e-learning architecture to be adopted by the decision-maker that faces the problem. In this paper the e-learning architecture selection problem is analyzed with respect to the following criteria: cost, service delivery time and flexibility. Thus, for each action \( a_1 \) there is a related cost \( c_i \), and specific conditions associated to the service delivery time \( t_i \) and its flexibility \( d_i \). For each action \( a_1 \), \( c_i \) is assumed to be a constant value and \( t_i \) is a random variable. So, the decision model incorporates the uncertainty associated with \( t_i \) through the probability density function \( f_i(t_i) \). Flexibility is represented by the probability \( d_i \) for achievement of architecture conditions.
3. Multi-Criteria Decision Approaches

Several multi-criteria decision methods are available (Vincke, 1992; Brans and Mareschal, 2002; Belton and Stewart, 2002) to deal with this kind of problem. The method should be chosen considering the nature of the problem and the model building process. Regarding the model presented in this paper, two of these methods are briefly described. MAUT (Keeney and Raiffa, 1976; Belton and Stewart, 2002) allows the decision-maker to quantify and aggregate multiple objectives even when these objectives are composed of conflicting attributes. The decision-maker’s preferences are modeled in order to obtain a multi attribute utility function, for instance \( U(c_i, t_i, d_i) \). This function aggregates utility functions for all criteria or attributes. That is, an analytical function is obtained which combines all criteria through a synthesis function. Each particular analytical form for this function has preferential independence conditions to be evaluated, in order to guarantee that the decision maker’s preferences are associated to the basic axioms of the theory (Almedia, 2001; Keeney and Raiffa, 1976).

ELECTRE method provides a different approach. This method concentrates the analysis on the dominance relations among the alternatives. That is, this method is based on the study of outranking relations, exploiting notions of concordance (Roy, 1996, Vincke, 1992; Brans and Mareschal, 2002; Belton and Stewart, 2002). These outranking relations are built in such a way that it is possible to compare alternatives. The information required by ELECTRE consists of information among the criteria and information within each criterion (Roy, 1996). The method uses concordance and discordance indexes to analyze the outranking relations among the alternatives. These indexes are obtained through the following relations, considering two actions: \( a \) and \( b \): 

\[
C(a, b) = \frac{\sum (W^+ + W^-)}{\sum (W^+ + W^-)},
\]

\[
D(a, b) = \max \left[ \frac{(Z_{bk} - Z_{ak})}{Z_k^* - Z_k^-} \right], \text{ for all } k \text{ where } Z_{bk} > Z_{ak},
\]

\( a S b \) if \( C(a, b) \geq p \) and \( D(a, b) \leq q \), so called outranking relation, 

where, \( C(a, b) \) is the concordance index that action \( a \) outranks action \( b \), \( D(a, b) \) is the discordance index that action \( a \) outranks action \( b \), \( a S b \) corresponds to the outranking relation; it means that action \( a \) outranks \( b \), \( p \) is the concordance index threshold, \( q \) is the discordance index threshold, \( W^+ \) corresponds to the sum of weights for criteria where \( a \) is preferable to \( b \), \( W^- \) corresponds to the sum of weights for criteria where \( b \) is preferable to \( a \), \( Z_{ak} \) is the evaluation or utility of action \( a \) related to criteria \( k \), \( Z_k^* \) is the best degree of evaluation obtained for criteria \( k \) and \( Z_k^- \) is the worst degree of evaluation obtained for criteria \( k \). In order to facilitate the procedure, the evaluation of alternatives are normalized such that \( Z_k^* = 1 \) and \( Z_k^- = 0 \).
The outranking relation is obtained by applying both equation (3) and the procedure to obtain the kernel, which is the sub-set of the best alternatives. The kernel includes a sub-set of alternatives where any other alternative is outranked by at least one of the kernel and the alternatives of the kernel are incomparable.

4. Decision Model

The decision-maker’s preferences for each criterion are modeled in order to obtain the utility function for each objective of the architecture. Then, the utility function is obtained from the decision-maker for each consequence: \( U(t) \), \( U(c) \) and \( U(d) \). These utility functions are obtained by applying one of the classical elicitation procedures (Berger, 1985; Raiffa, 1970). The final solution depends on the utility function for each criterion. The analytical form of this utility function may represent one of the three basic conditions for the decision-maker behavior. That is, aversion, neutral or prone condition may be considered (Berger, 1985; Raiffa, 1970). For \( U(t) \) it is assumed that the decision-maker behavior is accordingly of exponential analytical form given below:

\[
U(t) = e^{-At}
\]  
(4)

The exponential utility function is a typical function often found in practice (Raiffa, 1970) for one-dimensional utility functions. In previous work (Almedia, 2001) the exponential utility function has been found for \( U(t) \) and \( U(c) \). This means that higher values of \( t \) or \( c \) are much more undesirable for the decision-maker. Thus,

\[
U(c) = e^{-Ac}
\]  
(5)

For \( U(d) \) is assumed to be a linear function:

\[
U(d) = A_d d
\]  
(6)

Therefore, the evaluation of variable \( t \) is given by the decision-maker through the utility function \( U(t) \). However, the evaluation of alternatives is based on the probabilistic characteristics of \( t \). Thus, a probability density function \( f(t) \) for \( t \) is taken into account. The assumption of \( f(t) \) implies different results, given the decision-maker’s preferences for this probabilistic criterion. Gamma probability function, with parameter \( n = 2 \), is assumed for \( f(t) \). This condition may be found in practical situations where delivery time is concentrated around a modal value. Thus,

\[
f(t) = u^2 te^{-ut}
\]  
(7)
Once $U(t)$ gives the evaluation for variable $t$, the evaluation of the alternatives is based on parameter $u$. Then, $U(u)$ is derived based on $U(t)$. Applying the linearity property of utility theory (Berger, 1985) for utility of $t$, it follows that

$$ E,U(u) = \int_{0}^{\infty} U(t)f(t)\, dt $$

(8)

Applying (4) and (7) into (8), it follows that

$$ E,U(u) = \int_{0}^{\infty} \left[u^2 e^{-ut} e^{-At}\right] dt,$$

$$ E,U(u) = \frac{u^2}{(A_i + u)^2} , $$

(9)

Therefore, in order to incorporate the probabilistic aspect related to $t$, this variable is represented by its parameter $u$. Thus, for each action $a_i$, a parameter $u_i$ is applied. Once the utility function is obtained for all criteria $U(u), U(d)$ and $U(c)$, the ELECTRE method may be applied. In this case, the decision-maker establishes the relative weights for the criteria, taking into account framework of the non-compensatory ELECTRE method (Roy, 1996; Vincke, 1992). In the MAUT approach the decision-maker preferences are modeled in order to obtain a multi-attribute utility function $U(u, c, d)$, when aggregates all utility functions $U(c), U(d)$ and $U(u)$. The function $U(u, c, d)$ has to be evaluated in order to guarantee that the axioms of the theory (MAUT) conform to the decision-maker’s preferences.

A different approach is employed by the ELECTRE method. This method exploits some characteristics of dominance regarding the multiple criteria analyzed. In this method a concordance notion allows the ranking of alternatives, analyzing outranking relations among alternatives (Vincke, 1992). This allows a decision support approach avoiding rigid assumptions required by MAUT from the decision-maker (Vincke, 1992). The ELECTRE method is based on the study of outranking relations, using a non-compensatory logic.

Each alternative of architecture can be evaluated through:
- Architecture cost $c$,
- Parameter $u$, associated to the probability density function of $t$, and
- Architecture flexibility $d$.

The ELECTRE method may now be applied to (5), (6) and (9) given to the decision-maker the best alternatives of architecture.

5. Numerical Application

In order to illustrate the use of the decision model, there follows a presentation of a numerical application. This application is based on a case regarding service outsourcing.
related to transportation. In this context the cost and the response time $t$ has characteristics suitable to the exponential utility function previously discussed.

The cost is given in monetary units for each architecture alternative as follows:

1) Action $a_1$ – most expensive ($c_1 = 100$) and reduced $t$, such that $u = 0.95; d = 0.95$.
2) Action $a_2$ – medium cost ($c_2 = 60$) and medium $t$, such that $u = 0.65; d = 0.90$.
3) Action $a_3$ – least expensive cost ($c_3 = 10$) and the large $t$, such that $u = 0.03; d = 0.75$.
4) Action $a_4$ – below medium cost ($c_4 = 30$) and $t$, such that $u = 0.15; d = 0.8$.
5) Action $a_5$ – below medium cost ($c_5 = 50$) and $t$, such that $u = 0.70; d = 0.75$.
6) Action $a_6$ – expensive cost ($c_6 = 85$) and reduced $t$, such that $u = 0.9; d = 0.8$.

The following weights have been applied: for $c$: 0.40; for $d$: 0.25; for $t$: 0.35.

Table 1: Architecture Alternatives and their Related Normalized Utilities

<table>
<thead>
<tr>
<th>Action</th>
<th>Architecture alternatives</th>
<th>Normalized utilities for alternatives</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$u$</td>
<td>$C$</td>
</tr>
<tr>
<td>$a_1$</td>
<td>0.95</td>
<td>100</td>
</tr>
<tr>
<td>$a_2$</td>
<td>0.65</td>
<td>60</td>
</tr>
<tr>
<td>$a_3$</td>
<td>0.03</td>
<td>10</td>
</tr>
<tr>
<td>$a_4$</td>
<td>0.15</td>
<td>30</td>
</tr>
<tr>
<td>$a_5$</td>
<td>0.70</td>
<td>50</td>
</tr>
<tr>
<td>$a_6$</td>
<td>0.90</td>
<td>85</td>
</tr>
</tbody>
</table>

E-learning architecture alternatives are given in Table 1. Values of criteria are assigned to all alternatives $a_i$, where $i = 1, 2, \ldots, 6$. The utility function for cost is given in (5) with the parameter $A_2 = 0.02$. The utility function for $d$ is given in (6) with parameter $A_3 = 1$. The utility function for $t$ is given in (9) with parameter $A_1 = 0.1$. Applying these equations (5), (6) and (9), utilities are obtained for all alternatives. Table 1 also presents $U'(u)$, $U'(C)$ and $U'(d)$, corresponding to normalized values for utilities. The normalization procedure is based on a linear transformation. For instance $U'(u) = aU(u) + b$, such that $a>0$. According to the utility theory (Keeney and Raiffa, 1976) this linear transformation insures that $U'(u)$ is strategically equivalent to $U(u)$. That is, $U'(u)$ preserves the same properties and the preference structure of $U(u)$. Based on decision-maker’s preferences, the weights of criteria have been assigned as previously mentioned and the admissible levels (thresholds) for concordance index and discordance index are as follows: $p = 0.5$ and $q = 0.45$. Therefore, when (1) and (2) are applied, the concordance and discordance indexes are obtained. Then, the outranking relation is obtained by applying (3). Finally, by applying the procedure to obtain the kernel (Roy, 1996; Vincke, 1992; Belton and Stewart, 2002), alternatives $a_1$ and $a_4$ are identified. This result indicates that $a_1$ and $a_4$, although incomparable between themselves, are the two best alternatives for the preferences presented by the decision-maker and the assumptions underlying the model. A sensitivity analysis of weights and admissible levels for the concordance index and the discordance index (varying by 10%) shows that the result remains the same. This analysis indicates that the recommendation for action $a_1$ and $a_4$ are sufficiently robust, regarding the limits of variation mentioned above.
6. Conclusion

Two different multi-criteria approaches have been applied to deal with multiple criteria in similar problems: MAUT and the PROMETHEE method. The development of criteria scales to identify the intensity of preference for one alternative over another is based on deterministic way. The problem approached in this paper is related to the context of e-learning architecture, where uncertainties of some variables are relevant, such as: service delivery time $t_i$ and flexibility $d_i$, for a given alternative $a_i$. Then, a utility function is introduced in order to incorporate the uncertainty evaluation of those variables. Theses utility functions are integrated into the ELECTRE framework in order to obtain multi-criteria evaluation within a non-compensatory approach.

The main requirements of this theory imply a rationality that involves compensation among the criteria, which involves the procedure for aggregation of all criteria obtaining a synthesis multi-criterion utility function. This rationality is not always accepted by the decision-maker. The decision-maker rationality may require a non-compensatory method, where the decision support process does not require an aggregation of all criteria. The ELECTRE method may support decision process under this situation. The model proposed in this paper presents an alternative approach for analyzing an e-learning architecture selection problem. Using utility theory each criterion is represented by a utility function, incorporating the probabilistic structure of the problem. The probability function for the service delivery time is assumed to be gamma probability density function. The evaluation of the criteria represented by each utility function is analyzed through the ELECTRE method. The paper includes the structure of the decision model to support the decision-maker and a numerical application illustrates the use of the model.

7. Acknowledgments

This research has been supported by Mazandaran Telecommunication Research Center.

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