Signal recognition: Fourier transform vs. Cosine transform

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Abstract

A new feature representation approach, the simultaneous usage of the real and imaginary Fourier components with taking into account the covariance between these components, was compared with the Cosine transform approach for Gaussian recognition.

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1. Introduction

A new generic approach for feature representation has been proposed by Gelman and Braun (2001) for those cases in which one- or multi-dimensional Fourier transforms are used for pattern recognition. The approach consists of simultaneously using two new recognition features: the real and imaginary components of the Fourier transform. This is in contrast to recognition applications (Oirrak et al., 2002; Ozaktas et al., 2000; Alam and Thompson, 1999; Wu and Sheu, 1998; Pinkowski, 1993, 1995, 1997; Goodman, 1996; Wilson and McCreary, 1995; Kauppinen et al., 1995; Moharir, 1993; Burdin et al., 1992; Reynolds et al., 1989; Arsenault et al., 1989; Reeves et al., 1988; Persoon and Fu, 1986; Shridhar and Badreddin, 1984; Duffieux, 1983; Price et al., 1981; Fukushima et al., 1980; Wallace and Mitchell, 1980; Gaskill, 1978; Granlund, 1972; Oppenheim and Lim, 1981) where power spectral density (PSD) or phase spectrum are used. As far as pattern recognition is concerned both approaches are heuristical.

It has been shown (Gelman and Braun, 2001; Gelman, 2002) that the proposed approach is more generic than the PSD, phase spectrum, and Hartley approaches, and provides better recognition effectiveness than the PSD and Hartley approaches.

However, the approach was investigated (Gelman and Braun, 2001; Gelman, 2002) only for a particular case, without taking into account the covariance between new features. Taking covariance between recognition features into account may improve the effectiveness of recognition.

In recent years, the Cosine transform (Rao et al., 2001; Hsu et al., 1983) has gained importance in image processing, pattern recognition, and applications. The purpose of this letter is to compare...
the recognition effectiveness of the new approach and of the Cosine transform approach taking into account the covariance between new features.

2. Theoretical analysis

We consider the following two-class recognition of stationary Gaussian narrowband centered signals $x(t)$:

- hypothesis $H_0$: signal variance is $\sigma_{00}^2$, normalized autocovariance function is $r_s$,
- hypothesis $H_1$: signal variance is $\sigma_{11}^2$, normalized autocovariance function is $r_s$.

The recognition information is contained in the short time Fourier transform at the frequency $\omega_s$. The proposed approach consists of using simultaneously the real $X_R$ and imaginary $X_I$ components of the short time Fourier transform as recognition features and taking into account the covariance between new features. The proposed features can be written as follows:

$$X_R(\omega_s, t_1) = \int_0^{t_1} x(t) \cos \omega_s t dt,$$

(1)

$$X_I(\omega_s, t_1) = \int_0^{t_1} x(t) \sin \omega_s t dt,$$

(2)

where $t_1$ is the duration of the signal.

We consider the general case of the short time Fourier transform ($t_1 \neq \infty$), which is the advanced time-frequency signal processing technique.

The recognition feature based on the short time Cosine transform can be written in the form (Rao et al., 2001):

$$X_C(\omega_s, t_1) = \int_0^{t_1} x(t) \cos \omega_s t dt = X_R,$$

(3)

Likelihood ratios (Young and Fu, 1986) of the features (1), (2) and (3) are defined respectively, as:

$$L_o(X_R, X_I) = \ln \frac{P(X_R, X_I|H_1)}{P(X_R, X_I|H_0)},$$

(4)

$$L_c(X_C) = \ln \frac{P(X_C|H_1)}{P(X_C|H_0)},$$

(5)

where $P(X_R, X_I|H_1)$, $P(X_C|H_j)$ are the conditional probability density functions of the features (1), (2) and (3), respectively, for hypothesis $H_j$, $j = 0, 1$.

Obviously, that the proposed features and Cosine based feature are Gaussian variables.

Using the bivariate Gaussian conditional probability density function of the Gaussian random features (1), (2) and taking into account the covariance between features, we obtain the feature likelihood ratio (4) in the form (see Appendix A):

$$L_o = AX_R^2 + BX_I^2 + CX_RX_I + D,$$

(6)

where

$$A = \frac{\sigma_{R1}^2 - \sigma_{R0}^2}{2(1 - r^2)\sigma_{R0}\sigma_{R1}},$$

$$B = \frac{\sigma_{I1}^2 - \sigma_{I0}^2}{2(1 - r^2)\sigma_{I0}\sigma_{I1}},$$

$$C = \frac{r(\sigma_{R0}\sigma_{I1} - \sigma_{R1}\sigma_{I0})}{(1 - r^2)\sigma_{R0}\sigma_{R1}\sigma_{I0}\sigma_{I1}},$$

$$D = \ln \frac{\sigma_{R0}\sigma_{I0}}{\sigma_{R1}\sigma_{I1}},$$

$\sigma_{Rj}$ and $\sigma_{Ij}$ are standard deviations of the components $X_R$ and $X_I$ respectively for hypothesis $H_j$, $r$ is the correlation coefficient between short time Fourier transform features (1), (2).

This correlation coefficient is identical for hypothesis $H_j$, $j = 0, 1$, due to the identical normalized autocovariance functions of the signals for both hypothesis. Employing Gelman and Sadovaya (1980) we obtain: $(\sigma_{R1}/\sigma_{R0} = \sigma_{I1}/\sigma_{I0} = \sigma_{s1}/\sigma_{s0})$.

Using Eqs. (1)-(3) we find the likelihood ratio (5) of the Gaussian Cosine based feature in the form (see Appendix A):

$$L_c = EX_R^2 + J,$$

(7)

where

$$E = \frac{1}{2} \left( \frac{1}{\sigma_{R0}} - \frac{1}{\sigma_{R1}} \right), \quad J = \ln \frac{\sigma_{R0}}{\sigma_{R1}}.$$

One can see the difference between likelihood ratios (6) and (7). We find from Eqs. (6), (7) that the likelihood ratio (7) is not even the particular
case of the likelihood ratio (6), because always $B \neq 0$.

Now we estimate and compare the recognition effectiveness of the features (1), (2) and (3). We use an information criterion of recognition effectiveness, Fisher’s criterion (Young and Fu, 1986):

$$F_o = \frac{[m(L_o/H_1) - m(L_o/H_0)]^2}{\sigma^2(L_o/H_1) + \sigma^2(L_o/H_0)},$$

$$F_c = \frac{[m(L_c/H_1) - m(L_c/H_0)]^2}{\sigma^2(L_c/H_1) + \sigma^2(L_c/H_0)},$$

(8)

(9)

where $m(L_o/H_j)$ and $\sigma^2(L_o/H_j)$ are the mean and the variance respectively of the likelihood ratio (6) for hypothesis $H_j$; $m(L_c/H_j)$ and $\sigma^2(L_c/H_j)$ are the mean and the variance respectively of the likelihood ratio (7) for hypothesis $H_j$.

Using Eqs. (1)–(3), (6)–(9), after transformations we obtain (see Appendix A):

$$F_o = 1 - \frac{2}{b + \frac{1}{b}},$$

$$G = 2,$$

(10)

(11)

where

$$G = \frac{F_o}{F_c},$$

(12)

$b = (\sigma^2_{L1}/\sigma^2_{L0})$, parameter $b$ characterizes the difference of the signal variances, $G$ is the effectiveness gain.

We find from Eq. (10) that the recognition effectiveness of the proposed approach depends only upon the difference of the signal variances, and does not depend upon the correlation coefficient between the features (1), (2) and feature variances because the proposed approach takes into account the covariance between the features and the difference of the feature’s variances. It can be seen from Eqs. (11), (12) that use of the proposed approach provides essential constant recognition effectiveness gain $G$ in comparison with the Cosine transform approach for arbitrary values of correlation coefficient between the features (1), (2), signal variances, and feature variances. This indicates that the features (1), (2) have the essentially better recognition effectiveness than the Cosine based feature (3).

3. Conclusion

The proposed new approach, the simultaneous usage of the real and imaginary components of the short time Fourier transform of signal with taking into account the covariance between these components, was compared with the Cosine transform approach, for recognition of the Gaussian signals with different variances and identical normalized autocovariance functions.

An information criterion of recognition effectiveness, Fisher’s criterion, was utilized for comparison. It was shown that recognition effectiveness of the proposed approach as well as recognition effectiveness of the Cosine approach depends only on the difference of the signal variances and does not depend on feature variances. The recognition effectiveness of the proposed approach does not depend on the covariance between new features.

It was shown that the Cosine based feature is not an optimal and does not represent even a particular case of the proposed approach. Use of the proposed approach provides essential constant effectiveness gain (given by expression (11)) in comparison with the Cosine approach for arbitrary values of the correlation coefficient between features, signal variances, and feature variances. This indicates that recognition is more effective when using the proposed features than when using the Cosine based feature.

Appendix A

Using the bivariate conditional probability density function of the Gaussian features (1), (2), $P(X_R, X_I/H_j)$ we obtain the likelihood ratio (4) in the form:

$$L_o = \frac{1}{2(1 - r^2)} \left[ \frac{X_R^2(\sigma^2_{R1} - \sigma^2_{R0})}{\sigma^2_{R0}\sigma^2_{R1}} + \frac{X_I^2(\sigma^2_{I1} - \sigma^2_{I0})}{\sigma^2_{I0}\sigma^2_{I1}} \right] + \frac{2rX_RX_I(\sigma_{R0}\sigma_{I0} - \sigma_{R1}\sigma_{I1})}{\sigma_{R0}\sigma_{I0}\sigma_{R1}\sigma_{I1}} \right] + \ln \frac{\sigma_{R0}\sigma_{I0}}{\sigma_{R1}\sigma_{I1}}.$$

(A.1)

The obtained likelihood ratio (A.1) after transformations can be written as Eq. (6).
Using one-dimensional conditional probability density function of the Gaussian Cosine based feature (3), \( P(X_C/H_j) \), we find the likelihood ratio (5) in the form:

\[
L_c = \frac{1}{2} \left[ \frac{X_C^2 (\sigma_{R1}^2 - \sigma_{R0}^2)}{\sigma_{R0}^2 \sigma_{R1}^2} \right] + \ln \frac{\sigma_{R0}}{\sigma_{R1}}.
\]  

(A.2)

The obtained likelihood ratio (A.2) after transformations can be written as Eq. (7).

The conditional mean and variance of the likelihood ratio (4) using Eq. (6) and taking into account the Gaussianity of the features (1), (2) can be written in the form:

\[
m(L_o/H_j) = A \sigma_{Rj}^2 + B \sigma_{ij}^2 + C \sigma_{Rj} \sigma_{ij} + D,
\]  

(A.3)

\[
\sigma^2(L_o/H_j) = 2A^2 \sigma_{Rj}^4 + 2B^2 \sigma_{ij}^4 + 4AB \sigma_{Rj}^2 \sigma_{ij}^2 + 4AC \sigma_{Rj}^3 \sigma_{ij} + 4BC \sigma_{Rj} \sigma_{ij}^3 + 4D \sigma_{ij}^4.
\]  

(A.4)

Using Eqs. (6), (A.3), (A.4), the Fisher criterion (8) after transformations can be written as Eq. (10).

The conditional mean and variance of the likelihood ratio (5) using Eq. (7) and taking into account the Gaussianity of the feature (3) can be written in the form:

\[
m(L_c/H_j) = E \sigma_{Rj}^2 + J,
\]  

(A.5)

\[
\sigma^2(L_c/H_j) = 2E^2 \sigma_{Rj}^4.
\]  

(A.6)

Using Eqs. (7), (A.5), (A.6), the Fisher criterion (9) after transformations can be written in the form:

\[
F_c = \frac{1}{2} \left( \frac{1}{b} - \frac{1}{b} \right).
\]  

(A.7)

Using Eqs. (10), (12) and (A.7), we obtain Eq. (11) of the effectiveness gain.

References


