

A New Approach for Solving Type-2 Fuzzy Shortest Path Problem

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Abstract. In a network the arc lengths may represent time or cost. In practical situations, it is reasonable to assume that each arc length is a type-2 discrete fuzzy set. We called it the type-2 discrete fuzzy shortest path problem. In this paper we proposed an algorithm for finding shortest path and shortest path length from source node to destination node using type reduction method. We have compared our result with other measures like Hamming, Normalized Hamming, Exponential type distance measure also. An illustrative example also included to demonstrate our proposed approach.

Keywords: Type-2 fuzzy number, type-1 fuzzy number, distance measure, similarity measure, extension principle

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1. Introduction

In graph theory the shortest path problem is the problem of finding a path between two vertices such that the sum of the weights of the corresponding edges is minimized. It has applications in various fields like transportation, communication, routing and scheduling. In real world problem the arc length of the network may represent the time or cost which is not stable in the entire situation, hence it can be considered to be a fuzzy set. Fuzzy set was introduced by Zadeh in 1965.

Zadeh [20] proposed type-2 fuzzy sets as an extension of (type-1) fuzzy sets whose membership values are fuzzy sets on the interval [0,1]. There were further studies and applications of type-2 fuzzy sets, such as Mizumoto and Tanaka [14], Yager [19], Mendel [12], Jammeh [5], Mendoza [13] and Wagner and Hagra [18]. The membership function of a type-2 fuzzy set provides additional degree of freedom for modeling uncertainties so that type-2 fuzzy sets can better improve certain kinds of inference than do fuzzy sets with increasing imprecision, uncertainty and fuzziness in information.

Type reduction was proposed by Karnik and Mendel [7,8,9]. It is an 'extended version' [20] of type-1 defuzzification methods and is called type reduction because this

operation takes us from the type-2 output sets of the fuzzy logic system to a type-1 fuzzy set that is called “type reduction set”. There exist many kinds of type reduction such as centroid, centre –of- sets, heights and modified heights, the details of which are given in [7,8,9].

Computation of similarity between two or more kinds of information was very interesting for the fields of decision making, pattern classification, and so on [6,16,17]. The most obvious way of calculating similarity of fuzzy sets is based on their distance. This calculation is in two steps: In first part the distance between two fuzzy sets is obtained by a distance measure and in the second part one of the relationships between similarity and distance comes into play to reach at the degree of similarity. Various distance measures are present in literature. In this paper we have compared some existing distance measure with our proposed distance measure. Many works have been done on fuzzy graph some of them are given in [21-24].

The structure of the paper is following: In Section 2, we have some basic concepts required for analysis. Section 3, gives an algorithm is proposed to find shortest path and shortest path length using distance measure. Section 4 gives the network terminology. To illustrate the proposed algorithm the numerical example is solved in section 5. The obtained results are discussed in section 6.

2. Concepts

2.1. Type-2 fuzzy set

A type-2 fuzzy set denoted \tilde{A} , is characterized by a type-2 membership function $\mu_{\tilde{A}}(x, u)$ where $x \in X$ and $u \in J_x \subseteq [0, 1]$.

i.e., $\tilde{A} = \{ (x, u), \mu_{\tilde{A}}(x, u) \} / \forall x \in X, \forall u \in J_x \subseteq [0, 1]$ in which $0 \leq \mu_{\tilde{A}}(x, u) \leq 1$.

\tilde{A} can be expressed as

$$\tilde{A} = \int_{x \in X} \int_{u \in J_x} \mu_{\tilde{A}}(x, u) / (x, u) \quad J_x \subseteq [0, 1], \text{ where } \int \int \text{ denotes union over all}$$

admissible x and u . For discrete universe of discourse \int is replaced by \sum .

2.2. Type-2 fuzzy number

Let \tilde{A} be a type-2 fuzzy set defined in the universe of discourse R . If the following conditions are satisfied:

1. \tilde{A} is normal,
2. \tilde{A} is a convex set,
3. The support of \tilde{A} is closed and bounded, then \tilde{A} is called a type-2 fuzzy number.

2.3. Discrete Type-2 fuzzy number

The discrete type-2 fuzzy number \tilde{A} can be defined as follows:

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$\tilde{A} = \sum_{x \in X} \mu_{\tilde{A}}(x) / x$ where $\mu_{\tilde{A}}(x) = \sum_{u \in J_x} f_x(u) / u$ where J_x is the primary membership.

2.4. Extension principle

Let A_1, A_2, \dots, A_r be type-1 fuzzy sets in X_1, X_2, \dots, X_r , respectively. Then, Zadeh's Extension Principle allows us to induce from the type-1 fuzzy sets A_1, A_2, \dots, A_r a type-1 fuzzy set B on Y , through f , i.e., $B = f(A_1, \dots, A_r)$, such that

$$\mu_B(y) = \begin{cases} \sup_{x_1, x_2, \dots, x_n \in f^{-1}(y)} \min\{\mu_{A_1}(x_1), \dots, \mu_{A_n}(x_n)\} & \text{iff } f^{-1}(y) \neq \phi \\ 0, & f^{-1}(y) = \phi \end{cases}$$

2.5. Addition on type-2 fuzzy numbers

Let \tilde{A} and \tilde{B} be two discrete type-2 fuzzy number be $\tilde{A} = \sum \mu_{\tilde{A}}(x) / x$ and $\tilde{B} = \sum \mu_{\tilde{B}}(y) / y$ where $\mu_{\tilde{A}}(x) = \sum f_x(u) / u$ and $\mu_{\tilde{B}}(x) = \sum g_y(w) / w$. The addition of these two types-2 fuzzy numbers $\tilde{A} \oplus \tilde{B}$ is defined as

$$\begin{aligned} \mu_{\tilde{A} \oplus \tilde{B}}(z) &= \bigcup_{z=x+y} (\mu_{\tilde{A}}(x) \cap \mu_{\tilde{B}}(y)) \\ &= \bigcup_{z=x+y} ((\sum_i f_x(u_i) / u_i) \cap (\sum_j g_y(w_j) / w_j)) \\ \mu_{\tilde{A} \oplus \tilde{B}}(z) &= \bigcup_{z=x+y} ((\sum_{i,j} (f_x(u_i) \wedge g_y(w_j)) / (u_i \wedge w_j)) \end{aligned}$$

2.6. Maximum and minimum of two discrete fuzzy numbers

The maximum and minimum of fuzzy sets A and B is denoted by $\text{Max}(A, B), \text{Min}(A, B)$. The membership function of $\text{Max}(A, B)$ is given by

$$\text{Max}(A, B)(z) = \sup_{z=\max(x,y)} \text{Min}(A(x), B(y)), \quad \forall z \in \mathbb{R}$$

And $\text{Min}(A, B)(z)$ is given by

$$\text{Min}(A, B)(z) = \sup_{z=\min(x,y)} \text{Min}(A(x), B(y)), \quad \forall z \in \mathbb{R}$$

2.7. Similarity measure

If d is the distance measure between two fuzzy sets A and B on the universe X , then the following measures of similarity is presented respectively.

$$S(A, B) = \frac{1}{1 + d(A, B)}, \quad S(A, B) = 1 - d^N(A, B), \quad S_c(A, B) = 1 - d_c(A, B)$$

2.7. Distance based similarity measures for fuzzy sets

Various distance measures are available in literature. Here we are using the following distance measures for the proposed algorithm.

1. **The Hamming distance**

$$d(A, B) = \sum_{i=1}^n |A(x_i) - B(x_i)|$$

1. **Normalized Hamming distance**

$$d^N(A, B) = d(A, B)/n$$

2. **Normalized Exponential type distance**

$$d_e(A, B) = \frac{1 - \exp(-d^N(A, B))}{1 - \exp(-1)}$$

3. **Proposed distance**

$$d(A, B)(z) = \text{Sup}_{z=\text{Min}(x,y)} \left\{ |B(y_i) - A(x_i)| \right\} \quad \forall z \in \mathbb{R}$$

2.8. Centroid of type-2 fuzzy sets [Karnik and Mendel (10)]

The Centroid of a type-1 set A, whose domain, $x \in X$, is discretized into N points x_1, x_2, \dots, x_N , is given as

$$c_A = \frac{\sum_{i=1}^N x_i \mu_A(x_i)}{\sum_{i=1}^N \mu_A(x_i)}$$

Similarly, the centroid of a type-2 set \tilde{A} , $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) / x \in X\}$, whose x domain is

discretized into N points, so that $\tilde{A} = \sum_{i=1}^N \left[\sum_{u \in J_{x_i}} f_{x_i}(u)/u \right] / x_i$ can be expressed as,

$$C_{\tilde{A}} = \frac{\int_{\theta_1 \in J_{x_1}} \int_{\theta_2 \in J_{x_2}} \dots \int_{\theta_N \in J_{x_N}} [f_{x_1}(\theta_1) * f_{x_2}(\theta_2) * \dots * f_{x_N}(\theta_N)]}{\frac{\sum_{i=1}^N x_i \theta_i}{\sum_{i=1}^N \theta_i}}$$

$C_{\tilde{A}}$ is a type – 1 fuzzy set.

3. Algorithm

Karnik and Mendel [10] introduced the centroid of a type-2 fuzzy set. In this section we focus on centroid of a type-2 fuzzy set to convert the type-2 fuzzy set as type-1 fuzzy set. Using that we are finding the fuzzy shortest path length. We have proposed one distance measure to find the fuzzy shortest path through similarity measure with the help of fuzzy shortest path length. This algorithm finds the Shortest path Length and Shortest path from source node to destination node for a given network.

Algorithm for finding shortest path length procedure

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Step 1 : Form the possible paths from starting node to destination node and compute the corresponding path lengths, \tilde{L}_i $i = 1, 2, \dots, n$ for possible n paths.

Step 2 : Find $C_{\tilde{L}_i}$ using def 2.9

Step 3 : Set $C_{\tilde{L}} = C_{\tilde{L}_i}$

Step 4 : Let $i=2$

Step 5 : Compute $C_{\tilde{L}} = \text{Min} (C_{\tilde{L}}, C_{\tilde{L}_i})$ using def 2.6.

Step 6 : Set $i = i + 1$

Step 7 : If $i \leq n$ goto step 5

Step 8 : The shortest path length is $C_{\tilde{L}}$

Algorithm for finding shortest path

Step 1 : Compute the shortest path length is using shortest path length procedure

Step 2 : Let $j = 1$

Step 3 : Compute $d(C_{\tilde{L}}, C_{\tilde{L}_j})$ using def 2.8(4)

Step 4 : Compute $s_j(C_{\tilde{L}}, C_{\tilde{L}_j}) = \frac{1}{1 + d(C_{\tilde{L}}, C_{\tilde{L}_j})}$

Step 5 : If $j = 1$, Assign $s(C_{\tilde{L}}, C_{\tilde{L}_j}) = s_j(C_{\tilde{L}}, C_{\tilde{L}_j})$

Step 6 : Compute $s = \text{Max} (S, S_j)$

Step 7 : Put $j = j + 1$

Step 8 : If $j \leq n$ goto step 3.

Step 9 : The Highest Similarity degree is S and that corresponding path is the shortest path and $C_{\tilde{L}}$ is the shortest path length.

4. Network terminologies

Consider a directed network $G(V, E)$ consisting of a finite set of nodes $V = \{1, 2, \dots, n\}$ and a set of m directed edges $E \subseteq VXV$. Each edge is denoted by an ordered pair (i, j) , where $i, j \in V$ and $i \neq j$. In this network, we specify two nodes, denoted by s and t , which are the source node and the destination node, respectively. We define a path P_{ij} as a sequence $P_{ij} = \{i = i_1, (i_1, i_2), i_2, \dots, i_{l-1}, (i_{l-1}, i), i_l = j\}$ of alternating nodes and edges. The existence of at least one path P_{si} in $G(V, E)$ is assumed for every node $i \in V - \{s\}$.

\tilde{d}_{ij} denotes a Type-2 Fuzzy Number associated with the edge (i, j) , corresponding to the length necessary to transverse (i, j) from i to j . The fuzzy distance along the path P is denoted as $\tilde{d}(P)$ is defined as $\tilde{d}(P) = \sum_{(i, j \in P)} \tilde{d}_{ij}$

5. Numerical example

The problem is to find the shortest path and shortest path length between source node and destination node in the network having 6 vertices and 7 edges with type-2 fuzzy number.

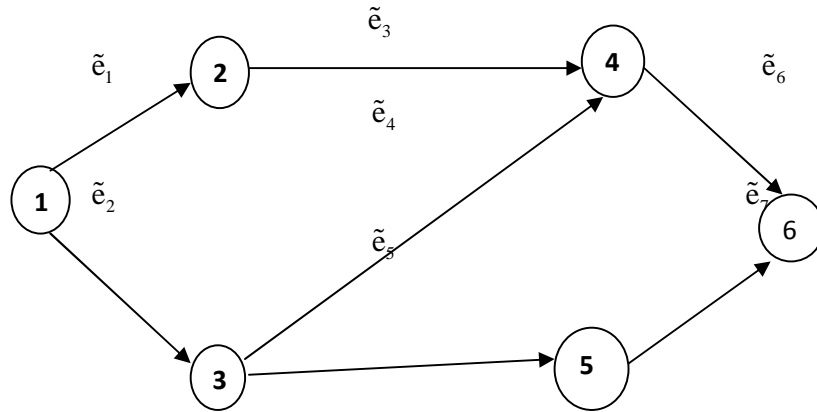


Figure 1:

The edge weights are

$$\tilde{e}_1 = (0.5/0.8 + 0.4/0.7)/2 + (0.4/0.8)/3$$

$$\tilde{e}_2 = (0.3/0.8 + 0.8/0.7)/1 + (0.2/0.2)/3$$

$$\tilde{e}_3 = (0.7/0.8)/2 + (0.9/0.6 + 0.7/0.5)/4$$

$$\tilde{e}_4 = (0.6/0.8)/4$$

$$\tilde{e}_5 = (0.9/0.6 + 0.7/0.5)/3 + (0.4/0.3)/5$$

$$\tilde{e}_6 = (0.8/0.7 + 0.4/0.5)/2$$

$$\tilde{e}_7 \tilde{e}_7 = (0.6/0.6)/2 + (0.7/0.5 + 0.5/0.4)/4$$

Algorithm for finding shortest path length

Step 1: Form the possible paths from starting node to destination node and compute the corresponding path lengths, \tilde{L}_i $i = 1, 2, \dots, n$ for possible n paths.

$$\tilde{p}_1 = \tilde{e}_1 + \tilde{e}_3 + \tilde{e}_6 = 1 - 2 - 4 - 6$$

$$\tilde{p}_2 = \tilde{e}_2 + \tilde{e}_4 + \tilde{e}_6 = 1 - 3 - 4 - 6$$

$$\tilde{p}_3 = \tilde{e}_2 + \tilde{e}_5 + \tilde{e}_7 = 1 - 3 - 5 - 6$$

$$\tilde{L}_1 = (0.5/0.7 + 0.4/0.5)/6 + (0.4/0.7 + 0.4/0.5)/7 + (0.5/0.6 + 0.5/0.5)/8 + (0.4/0.6 + 0.4/0.5)/9$$

$$\tilde{L}_2 = (0.6/0.7 + 0.4/0.5)/7 + (0.2/0.2)/9$$

$$\tilde{L}_3 = (0.6/0.6 + 0.6/0.5)/6 + (0.2/0.5 + 0.2/0.4)/8 + (0.2/0.3)/10 + (0.2/0.2)/12$$

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Step 2 : Find $C_{\bar{L}_i}$ using def 2.9

$$C_{\bar{L}_1} = 0.04/7.3 + 0.04/7.4 + 0.04/7.5 + 0.032/7.6$$

$$C_{\bar{L}_2} = 0.12/7.4 + 0.08/7.6$$

$$C_{\bar{L}_3} = 0.0048/8.1 + 0.0048/8.3$$

Step 3 : Set $C_{\bar{L}} = C_{\bar{L}_1}$

Step 4 : Let $i=2$

Step 5 : Compute $C_{\bar{L}} = \text{Min} (C_{\bar{L}}, C_{\bar{L}_i})$ using def 2.6.

$$C_{\bar{L}} = 0.04/7.3 + 0.04/7.4 + 0.04/7.5 + 0.032/7.6$$

Step 6 : Set $i = i + 1$

Step 7 : If $i \leq n$ goto step 5

Step 8 : The shortest path length is $C_{\bar{L}}$

$$C_{\bar{L}} = 0.0048/7.3 + 0.0048/7.4 + 0.0048/7.5 + 0.0048/7.6$$

Shortest path procedure

Step 1 : Compute the shortest path length is using shortest path length procedure

Step 2 : Let $j = 1$

Step 3 : Compute $d(C_{\bar{L}}, C_{\bar{L}_j})$ using def 2.8(4)

$$d(C_{\bar{L}}, C_{\bar{L}_1}) = 0.0352$$

Step 4 : Compute $s_j(C_{\bar{L}}, C_{\bar{L}_j}) = \frac{1}{1 + d(C_{\bar{L}}, C_{\bar{L}_j})}$

$$s_1(C_{\bar{L}}, C_{\bar{L}_1}) = 0.966$$

Step 5 : If $j = 1$, Assign $s(C_{\bar{L}}, C_{\bar{L}_j}) = s_j(C_{\bar{L}}, C_{\bar{L}_j})$

$$s(C_{\bar{L}}, C_{\bar{L}_1}) = 0.966$$

Step 6 : Compute $s = \text{Max} (S, S_j)$

$$s(C_{\bar{L}}, C_{\bar{L}_1}) = 0.966$$

Step 7 : Put $j = j + 1$

$$j = 2$$

Step 8 : If $j \leq n$ goto step 3.

$$2 \leq 3 \text{ goto step 3}$$

Step 3 : Compute $d(C_{\bar{L}}, C_{\bar{L}_j})$ using def 2.8(4)

$$d(C_{\bar{L}}, C_{\bar{L}_2}) = 0.1152$$

Step 4 : Compute $s_j(C_{\bar{L}}, C_{\bar{L}_j}) = \frac{1}{1 + d(C_{\bar{L}}, C_{\bar{L}_j})}$

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$$s_2(C_{\bar{L}}, C_{\bar{L}_2}) = 0.896$$

Step 5 : If $j = 1$, Assign $s(C_{\bar{L}}, C_{\bar{L}_j}) = s_j(C_{\bar{L}}, C_{\bar{L}_j})$

Here $j = 2$

Step 6 : Compute $s = \text{Max}(S, S_j)$
 $S = 0.966$

Step 7 : Put $j = j + 1$
 $j = 3$

Step 8 : If $j \leq n$ goto step 3.
 $3 \leq 3$ goto step 3

Step 3 : Compute $d(C_{\bar{L}}, C_{\bar{L}_j})$ using def 2.8(4)

$$d(C_{\bar{L}}, C_{\bar{L}_3}) = 0.0048$$

Step 4 : Compute $s_j(C_{\bar{L}}, C_{\bar{L}_j}) = \frac{1}{1 + d(C_{\bar{L}}, C_{\bar{L}_j})}$

$$s_3(C_{\bar{L}}, C_{\bar{L}_3}) = 0.995$$

Step 5 : If $j = 1$, Assign $s(C_{\bar{L}}, C_{\bar{L}_j}) = s_j(C_{\bar{L}}, C_{\bar{L}_j})$

Here $j = 3$

Step 6 : Compute $s = \text{Max}(S, S_j)$
 $S = 0.995$

Step 7 : Put $j = j + 1$
 $j = 4$

Step 8 : If $j \leq n$ goto step 3.
 $3 \leq 4$ stop the procedure

Step 9 : The Highest Similarity degree is $S = 0.995$ and that corresponding path $1 - 3 - 5 - 6$ is the Shortest path
and $C_{\bar{L}} = 0.0048/7.3 + 0.0048/7.4 + 0.0048/7.5 + 0.0048/7.6$ is the shortest path length.

6. Results and discussion

In this section we have discussed our proposed distance and some existing distance measures. Our proposed system is make the calculation part as simplest one while comparing with other distances like Hamming, Normalized hamming etc., and we are getting the same shortest path in both proposed and some existing distance measures also.

Here we have compared existing distance measures with our proposed distance measures. The highest similarity measure gives the shortest path from source node to destination node in the network.

Table 6.1 :

Possible Paths	Similarity Measure	Hamming distance	Normalized Hamming distance	Normalized exponential type	Proposed distance	Classification of Shortest Path (as
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				distance		per Similarity Measure)
1-2-4-6	$s_1(C_{\bar{L}}, C_{\bar{L}_1}) =$	0.883	0.9668	0.948	0.966	2
1-3-4-6	$s_2(C_{\bar{L}}, C_{\bar{L}_2}) =$	0.83	0.95	0.923	0.896	3
1-3-5-6	$s_3(C_{\bar{L}}, C_{\bar{L}_3}) =$	0.97	0.995	0.99	0.995	1

Hence the path $P_2 = 1 - 3 - 5 - 6$ is having the highest similarity degree $s_2(C_{\bar{L}}, C_{\bar{L}_2}) = 0.995$

7. Conclusion

The fuzzy shortest path problem in a network has been investigated in numerous articles because of its importance to various applications. It aims to offer a decision maker the shortest path length and the shortest path in a network with fuzzy arc lengths. For this purpose, in this paper, we proposed new approach to determine the shortest path length and the shortest path in a fuzzy network. Here type-2 fuzzy number has been reduced to type-1 fuzzy number using type reduction method.

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