The α-Cost Minimization Model for Capacitated Facility Location-Allocation Problem with Uncertain Demands

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Abstract

Facility location-allocation (FLA) problem, which has been proved to be a valuable method in siting service facility, is widely used in real life, such as emergency service systems, telecommunication networks, public services, etc. However, in real-world situations, the decision maker often lack the information about the customers' demands, which leads to the FLA problem in uncertain environment. Within the framework of uncertainty theory, this paper investigates the uncertain FLA problem. Due to the uncertain cost function, we introduce the optimistic criterion and establish the α-cost minimization model under uncertain environment. In order to simplify the uncertain model, we will give its equivalent crisp model and design a hybrid intelligent algorithm to compute it. Finally, a numerical example is presented to illustrate the uncertain model and the algorithm.

Keywords: Location-allocation problem; Uncertain measure; Uncertainty distribution function; Genetic algorithm

1 Introduction

With the rapid development of logistics and supply chain management in recent decades, strategic supply chain problem has become more and more important in various logistics and supply chain problems. As one of the most representative examples on strategic supply chain problem, the facility location-allocation (FLA) problem has received much attention from both researchers and practitioners.

Since FLA problem was initialized by Cooper [5] in 1963, it arise in many practical settings: emergency service systems, telecommunication networks, public services, etc. In 1982, Murtagh & Niwatitsayawong [28] proposed the capacitated FLA problem, which is considered as one of the most important researches in this field, specially focusing on facilities which have capacity constraints. Meanwhile, many different models have been carried out in the field of FLA theory (Badri [1], Hodey et al [14]) and a large amount of solution approaches for different models have been proposed (Kuenne & Soland [18], Murray & Church [27], Ernst & Krishnamoorthy [10], Gong et al [13]).

In real world, the precise demands of customers are usually very hard to present and thus are estimated from historical data. Traditionally, customers' demands in FLA problem are assumed to be random variables. Logendran & Terrell [19] firstly introduced the stochastic uncapacitated FLA model. Then many stochastic models are proposed including Zhou [34] and Zhou & Liu [35].

Although stochastic model has been accepted and accorded with the facts in widespread cases, it is not appropriate in a vast range of situations. As we know, the parameter can be estimated by the experiment data. In many practical situations, however, we are often short of the experiment data. In these cases, fuzzy set theory may do better in dealing with ambiguous information. Fuzzy set theory was initialized by Zadeh [38] and has been widely applied in many real problems. In the past decades many researchers have introduced fuzzy theory into FLA problem (Bhattacharya et al [2], Chen & Wei [4], Darzentas [8], Zhou & Liu [36] [37], Wen & Iwamura [33]).

We note that, the reason why we use the customer’s demand is about 100 to estimate the demand is that there are not enough (or even no) history data for probabilistic reasoning. So it is not appropriate
to regard it as random variable. If the quantity is assumed to be a fuzzy variable, we can obtain that the demand of the customer is exactly 100 thousands units with possibility measure 1. On the other hand, the opposite event of not exactly 100 thousands units has the same possibility measure. There is no doubt that nobody can accept this conclusion. This paradox shows that the customer’s demand is about 100 cannot be fuzzy concept. Yet, we know that such quantities behave neither like randomness nor like fuzziness. In order to formulate such subjective uncertainties as the customer’s demand is about 100, uncertainty theory was founded by by Liu [22] in 2007 and refined by Liu [24] in 2010. Uncertainty theory is a branch of mathematics based on an axiomatic system, in which a quantity the customer’s demand is about 100 may be formulated as an uncertain variable with some uncertainty distribution.

In this paper, we introduce uncertainty theory into capacitated FLA problem and lay down a foundation for FLA problem based on the assumption that each customer’s demand is uncertain variable. The remainder of this paper is organized as follows. In Section 2, some basic concepts and properties on uncertainty are introduced. An uncertain facility location-allocation model named $\alpha$-cost minimization model is given as well its equivalent crisp model is presented in Section 3. In order to solve this uncertain model, we embed the simplex algorithm in genetic algorithm to design a powerful hybrid intelligent algorithm in Section 4. Finally, Section 5 provides a numerical example to illustrate the performance and the effectiveness of the proposed model and algorithm.

2 Preliminaries

Uncertainty theory was founded by Liu [22] in 2007 and refined by Liu [24] in 2010. Nowadays uncertainty theory has become a branch of axiomatic mathematics for modeling human uncertainty. In this section, we will state some basic concepts and results on uncertain variables. These results are crucial for the remainder of this paper.

Let $\Gamma$ be a nonempty set, and $\mathcal{L}$ a $\sigma$-algebra over $\Gamma$. Each element $\Lambda \in \mathcal{L}$ is assigned a number $M(\Lambda) \in [0,1]$. In order to ensure that the number $M(\Lambda)$ has certain mathematical properties, Liu [22][24] presented the three axioms:

(i) $M(\Gamma) = 1$ for the universal set $\Gamma$.
(ii) $M(\Lambda) + M(\Lambda^c) = 1$ for any event $\Lambda$.
(iii) For every countable sequence of events $\Lambda_1, \Lambda_2, \cdots$, we have

$$M \left( \bigcup_{i=1}^{\infty} \Lambda_i \right) \leq \sum_{i=1}^{\infty} M(\Lambda_i)$$

The triplet $(\Gamma, \mathcal{L}, M)$ is called an uncertainty space. In order to obtain an uncertain measure of compound event, a product uncertain measure was defined by Liu [23], thus producing the fourth axiom of uncertainty theory:

(iv) Let $(\Gamma_k, \mathcal{L}_k, M_k)$ be uncertainty spaces for $k = 1, 2, \cdots, n$. Then the product uncertain measure $M$ is an uncertain measure satisfying

$$M \left( \prod_{k=1}^{n} \Lambda_k \right) = \min_{1 \leq k \leq n} M_k(\Lambda_k).$$

An uncertain variable is a measurable function $\xi$ from an uncertainty space $(\Gamma, \mathcal{L}, M)$ to the set of real numbers (Liu [22]). In order to describe an uncertain variable in practice, the concept of uncertainty distribution is defined as

$$\Phi(x) = M(\xi \leq x)$$

for any real number $x$. For example, the linear uncertain variable $\xi \sim \mathcal{L}(a, b)$ has an uncertainty distribution

$$\Phi(x) = \begin{cases} 0, & \text{if } x \leq a \\ (x - a)/(b - a), & \text{if } a \leq x \leq b \\ 1, & \text{if } x \geq b. \end{cases}$$
An uncertain variable $\xi$ is called zigzag if it has a zigzag uncertainty distribution
\[
\Phi(x) = \begin{cases} 
0, & \text{if } x \leq a \\
(x - a)/2(b - a), & \text{if } a \leq x \leq b \\
(x + c - 2b)/2(c - b), & \text{if } b \leq x \leq c \\
1, & \text{if } x \geq c 
\end{cases}
\tag{3}
\]
denoted by $Z(a,b,c)$ where $a, b, c$ are real numbers with $a < b < c$.

An uncertain variable $\xi$ is called normal if it has a normal uncertainty distribution
\[
\Phi(x) = \left(1 + \exp \left(\frac{\pi(e - x)}{\sqrt{3}\sigma}\right)\right)^{-1}
\tag{4}
\]
denoted by $N(e,\sigma)$ where $e$ and $\sigma$ are real numbers with $\sigma > 0$.

An uncertainty distribution $\Phi$ is said to be regular if its inverse function $\Phi^{-1}(\alpha)$ exists and is unique for each $\alpha \in (0, 1)$.

\[\textbf{Theorem 1 (Liu [24])}\]
Let $\xi_1, \xi_2, \cdots, \xi_n$ be independent uncertain variables with regular uncertainty distributions $\Phi_1, \Phi_2, \cdots, \Phi_n$, respectively. If $f$ is a strictly increasing function, then
\[
\xi = f(\xi_1, \xi_2, \cdots, \xi_n)
\tag{5}
\]
is an uncertain variable with inverse uncertainty distribution
\[
\Psi^{-1} = f(\Phi_1^{-1}(\alpha), \Phi_2^{-1}(\alpha), \cdots, \Phi_n^{-1}(\alpha)).
\tag{6}
\]

3 Uncertain Capacitated FLA Problem

The capacitated continuous FLA problem is to find the locations of $n$ facilities in continuous space in order to serve customers at $m$ fixed points as well as the allocation of each customer to the facilities so that total transportation costs are minimized. In order to model the capacitated FLA problem, firstly we make some assumptions:

1. Each facility has a limited capacity. Thus we need to select locations and decide the amount from each facility $i$ to each customer $j$.

2. The path between any customer and facility is connected and transportation cost is proportionate to the quantity supplied and the travel distance.

3. Facility $i$ is assumed to be located within a certain region $R_i = \{(x_i, y_i)|g_i(x_i, y_i) \leq 0\}, i = 1, 2, \ldots, n$, respectively.

Then we will give the symbols and notations as follows:
\[i = 1, 2, \ldots, n: \text{index of facilities};\]
\[j = 1, 2, \ldots, m: \text{index of customers};\]
\[(a_j, b_j): \text{location of customer } j, 1 \leq j \leq m;\]
\[\xi_j: \text{uncertain demand of customer } j, 1 \leq j \leq m;\]
\[\Phi_j: \text{uncertainty distribution of } \xi_j, 1 \leq j \leq m;\]
\[s_i: \text{capacity of facility } i, 1 \leq i \leq n;\]
\[(x_i, y_i) \text{ is the decision variable which represents the location of facility } i, 1 \leq i \leq n;\]
\[z_{ij}: \text{quantity supplied by facility } i \text{ to customer } j, 1 \leq i \leq n, 1 \leq j \leq m.\]

For convenience, we also write
\[ \mathbf{Z}(\xi) = \begin{cases} 
\mathbf{z} | & \sum_{i=1}^{n} \sum_{j=1}^{m} z_{ij} = \xi_j, \ j = 1, 2, \ldots, m \\
\mathbf{z} | & \sum_{j=1}^{m} z_{ij} \leq s_i, \ i = 1, 2, \ldots, n 
\end{cases} \] \quad (7)

Then we can give the uncertain transportation cost with the best allocation \( \mathbf{z} \),
\[
C(x, y, \xi) = \min_{\mathbf{z} \in \mathbf{Z}(\xi)} \sum_{i=1}^{n} \sum_{j=1}^{m} z_{ij} \sqrt{(x_i - a_j)^2 + (y_i - b_j)^2}. \quad (8)
\]

If \( \mathbf{Z}(\xi) \) is an empty set for some \( \xi \), we can define
\[
C(x, y, \xi) = \max_{1 \leq i \leq n} \xi_j \sqrt{(x_i - a_j)^2 + (y_i - b_j)^2}. \quad (9)
\]

### 3.1 Method of Getting the Uncertainty distribution of a customer’s demand

Liu [24] proposed a questionnaire survey for collecting expert’s experimental data. It is based on experts’ experimental data rather than historical data. The starting point is to invite one expert who is asked to complete a questionnaire about the meaning of an uncertain demand \( \xi \) like "How many is the customer’s demand".

**Step 1:** An expert is asked to choose a possible value \( x \) of \( \xi \).

**Step 2:** “How likely is \( \xi \) less than or equal to \( x \)?”

**Step 3:** “The degree is \( \alpha \).”

**Step 4:** Obtain an expert’s experimental data.

**Step 5:** Repeat Steps 1 to 4 and obtain a set of experimental data:
\[
(x_1, \alpha_1), (x_2, \alpha_2), \cdots, (x_n, \alpha_n) \quad (10)
\]

that meet the following consistence condition (perhaps after a rearrangement)
\[
x_1 < x_2 < \cdots < x_n, \quad 0 \leq \alpha_1 \leq \alpha_2 \leq \cdots \alpha_n \leq 1. \quad (11)
\]

Based on those expert’s experimental data, Liu [24] suggested an empirical uncertainty distribution,
\[
\Phi(x) = \begin{cases} 
0, & \text{if } x \leq x_1 \\
\alpha_i + \frac{(\alpha_{i+1} - \alpha_i)(x - x_i)}{x_{i+1} - x_i}, & \text{if } x_i \leq x \leq x_{i+1}, \ 1 \leq i < n \\
1, & \text{if } x > x_n
\end{cases} \quad (12)
\]

Assume there are \( m \) domain experts and each produces an uncertainty distribution. Then we may get \( m \) uncertainty distributions \( \Phi_1(x), \Phi_2(x), \ldots, \Phi_m(x) \). The Delphi method was originally developed in the 1950s by the RAND Corporation based on the assumption that group experience is more valid than individual experience. Wang et al. [32] recast the Delphi method as a process to determine the uncertainty distribution. The main steps are listed as follows:

**Step 1:** The \( m \) domain experts provide their expert’s experimental data,
\[
(x_{ij}, \alpha_{ij}), \quad j = 1, 2, \cdots, n_i, \ i = 1, 2, \cdots, m. \quad (13)
\]
Step 2: Use the $i$-th expert’s experimental data $(x_{i1}, \alpha_{i1}), (x_{i2}, \alpha_{i2}), \ldots, (x_{in_i}, \alpha_{in_i})$ to generate the $i$-th expert’s uncertainty distribution $\Phi_i$.

Step 3: Compute $\Phi(x) = w_1\Phi_1(x) + w_2\Phi_2(x) + \cdots + w_m\Phi_m(x)$ where $w_1, w_2, \ldots, w_m$ are convex combination coefficients.

Step 4: If $|\alpha_{ij} - \Phi(x_{ij})|$ are less than a given level $\varepsilon > 0$, then go to Step 5. Otherwise, the $i$-th expert receives the summary ($\Phi$ and reasons), and then provides a set of revised expert’s experimental data. Go to Step 2.

Step 5: The last $\Phi$ is the customer’s uncertainty distribution.

3.2 The $\alpha$-Cost Minimization Model under Uncertain Environment

Chance-constrained programming (CCP), which was initialized by Charnes & Cooper [3], offers a powerful means for modelling stochastic decision systems. In [20][21], Liu and his coauthor extended the chance-constrained programming to fuzzy decision systems. The essential idea of chance-constrained programming is to optimize some critical value with a given confidence level subject to some chance constraints. Inspired by this idea, this paper will apply the CCP model to FLA problem and establish a new model under uncertain environment, named $\alpha$-cost minimization model. The FLA problem with uncertain demands is formulated as follows:

$$\begin{align}
\min_{x, y} \ f \\
\text{subject to :} \\
M\{C(x, y, \xi) \leq f\} \geq \alpha \\
g_i(x, y) \leq 0, i = 1, 2, \ldots, p
\end{align}$$

where $\alpha \in (0, 1)$. The model is different from traditional uncertain programming models because there is a sub-optimal problem in it, i.e.,

$$\begin{align}
\min_{z} \sum_{i=1}^{n} \sum_{j=1}^{m} z_{ij} \sqrt{(x_i - a_j)^2 + (y_i - b_j)^2} \\
\text{subject to :} \\
z_{ij} \geq 0, \ i = 1, 2, \ldots, n, \ j = 1, 2, \ldots, m \\
\sum_{i=1}^{m} z_{ij} = \xi_j, \ j = 1, 2, \ldots, m \\
\sum_{j=1}^{m} z_{ij} \leq s_i, \ i = 1, 2, \ldots, n
\end{align}$$

which aims to get the total cost $C(x, y, \xi)$.

It is obvious that the optimal value in (14) is increasing with respect to the confidence level $\alpha$.

3.3 Converting Uncertain FLA Programming Model to a Crisp Model

The uncertain FLA model (14) is an uncertain programming model, which is too complex to compute by traditional algorithm. Thus this section will convert it to a crisp model.

**Theorem 2** Assume the constraint function $C(x, y, \xi_1, \xi_2, \ldots, \xi_m)$ is strictly increasing with respect to $\xi_1, \xi_2, \ldots, \xi_n$ and strictly decreasing with respect to $\xi_{n+1}, \xi_{n+2}, \ldots, \xi_m$. If $\xi_1, \xi_2, \ldots, \xi_m$ are independent uncertain variables with uncertainty distributions $\Phi_1, \Phi_2, \ldots, \Phi_m$, respectively, then the chance constraint function

$$M\{C(x, y, \xi_1, \xi_2, \ldots, \xi_m) \leq 0\} \geq \alpha$$

5
holds if and only if
\[ C(x, y, \Phi_1^{-1}(\alpha), \ldots, \Phi_n^{-1}(\alpha), \Phi_{n+1}^{-1}(1-\alpha), \ldots, \Phi_m^{-1}(1-\alpha)) \leq 0. \] 

Since the uncertain transportation cost \( C(x, y, \xi_1, \xi_2, \ldots, \xi_m) \) is strictly increasing with respect to \( \xi_1, \xi_2, \ldots, \xi_m \), the uncertain capacitated FLA model is equivalent to
\[
\begin{align*}
\min_{\mathbf{x}, \mathbf{y}} & \quad f \\
\text{subject to :} & \quad C(x, y, \Phi_1^{-1}(\alpha), \ldots, \Phi_n^{-1}(\alpha)) \leq f \\
& \quad g_i(x, y) \leq 0, i = 1, 2, \ldots, p
\end{align*}
\]
which can be write as
\[
\begin{align*}
\min_{\mathbf{x}, \mathbf{y}} & \quad C(x, y, \Phi_1^{-1}(\alpha), \ldots, \Phi_n^{-1}(\alpha)) \\
\text{subject to :} & \quad g_i(x, y) \leq 0, i = 1, 2, \ldots, p.
\end{align*}
\]

The cost function \( C(x, y, \Phi_1^{-1}(\alpha), \ldots, \Phi_n^{-1}(\alpha)) \) is determined by the following model:
\[
\begin{align*}
\min_{\mathbf{x}, \mathbf{y}, \mathbf{z}} & \quad \sum_{i=1}^{n} \sum_{j=1}^{m} z_{ij} \sqrt{(x_i - a_j)^2 + (y_i - b_j)^2} \\
\text{subject to :} & \quad z_{ij} \geq 0, \quad i = 1, 2, \ldots, n, \quad j = 1, 2, \ldots, m \\
& \quad \sum_{i=1}^{n} z_{ij} = \Phi_j^{-1}(\alpha), \quad j = 1, 2, \ldots, m \\
& \quad \sum_{j=1}^{m} z_{ij} \leq s_i, \quad i = 1, 2, \ldots, n.
\end{align*}
\]
Models(19) and (20) can be rewrite as follows:
\[
\begin{align*}
\min_{\mathbf{x}, \mathbf{y}, \mathbf{z}} & \quad \sum_{i=1}^{n} \sum_{j=1}^{m} z_{ij} \sqrt{(x_i - a_j)^2 + (y_i - b_j)^2} \\
\text{subject to :} & \quad z_{ij} \geq 0, \quad i = 1, 2, \ldots, n, \quad j = 1, 2, \ldots, m \\
& \quad \sum_{i=1}^{n} z_{ij} = \Phi_j^{-1}(\alpha), \quad j = 1, 2, \ldots, m \\
& \quad \sum_{j=1}^{m} z_{ij} \leq s_i, \quad i = 1, 2, \ldots, n \\
& \quad g_i(x, y) \leq 0, i = 1, 2, \ldots, p
\end{align*}
\]
in which \( \alpha \in (0, 1) \).

### 4 Hybrid Intelligent Algorithm

Generally speaking, uncertain programming models are difficult to solve by traditional methods due to its complexity. Moreover the FLA problem has been proved to be NP-hard [N. Megiddo & K.J. Supowit [30]]. Heuristic methods have been shown to be the best way to tackle larger NP-hard problems. Modern heuristics such as simulated annealing, tabu search, genetic algorithms (GA), variable neighborhood search, and ant systems increase the chance of avoiding local optimality. In this paper, we use GA which was showed useful and effective in solving engineering design and optimization problems by numerous experiments (Gen & Cheng [11][12]) to compute the FLA problem. And we use simplex algorithm to solve the sub-optimal problem (8) in uncertain expected value model. In this paper, we integrate the simplex algorithm, Mote Kaio simulation and genetic algorithm to produce a hybrid intelligent algorithm for solving the uncertain FLA model. We describe the algorithm as the following procedure:
Step 1. From the potential region \( \{(x, y)| g_i(x, y) \leq 0, i = 1, 2, \ldots, n\} \), initialize \( \text{pop\_size} \) chromosomes 
\( V_k = (x^k, y^k) = (x_1^k, x_2^k, \ldots, x_n^k, y_1^k, y_2^k, \ldots, y_n^k), \ k = 1, 2, \ldots, \text{pop\_size} \), which denote the locations of all the facilities.

Step 2. Calculate the objective values \( U^k \) for all chromosomes \( V_k, k = 1, 2, \ldots, \text{pop\_size} \), where the simplex algorithm is used to solve (20) to get the optimal cost \( C(x, y, \Phi_1^{-1}(\alpha), \ldots, \Phi_n^{-1}(\alpha)) \).

Step 3. Compute the fitness of all chromosomes \( V_k, k = 1, 2, \ldots, \text{pop\_size} \). The rank-based evaluation function is defined as

\[
\text{Eval}(V_k) = \beta(1 - \beta)^{k-1}, \ k = 1, 2, \ldots, \text{pop\_size} \tag{22}
\]

where the chromosomes \( V_1, V_2, \ldots, V_{\text{pop\_size}} \) are assumed to have been rearranged from good to bad according to their objective values \( U^k \) and \( \beta \in (0, 1) \) is a parameter in the genetic system.

Step 4. Select the chromosomes for a new population. The selection process is based on spinning the roulette wheel characterized by the fitness of all chromosomes for \( \text{pop\_size} \) times, and each time we select a single chromosome. Thus we obtain \( \text{pop\_size} \) chromosomes, denoted also by \( V_k, k = 1, 2, \ldots, \text{pop\_size} \).

Step 5. Renew the chromosomes \( V_k, k = 1, 2, \ldots, \text{pop\_size} \) by crossover operation. We define a parameter \( P_c \) of a genetic system as the probability of crossover. This probability gives us the expected number of \( \text{pop\_size} \) chromosomes undergoing the crossover operation.

Step 6. Update the chromosomes \( V_k, k = 1, 2, \ldots, \text{pop\_size} \) by mutation operation. The parameter \( P_m \) is the probability of mutation, which gives us the expected number of \( \text{pop\_size} \) chromosomes undergoing the mutation operations.

Step 7. Repeat the second to the sixth steps for a given number of cycles.

Step 8. Report the best chromosome \( V^* = (x^*, y^*) \) as the optimal locations.

5 A Numerical Example

Consider a company that wishes to locate four new facilities within the square region \([0, 100] \times [0, 100]\). Assume that there are 20 customers whose demands \( \xi_i \) are zigzag uncertainty variables. The location \( (a_i, b_i) \) and the uncertain demand \( \xi_i \) of the customer \( i, i = 1, 2, \ldots, 20 \) are given in Table 1. The capacities \( s_i \) of the four facilities are 100, 110, 120 and 130, respectively.

<table>
<thead>
<tr>
<th>Customer j</th>
<th>location</th>
<th>demand</th>
<th>Customer j</th>
<th>location</th>
<th>demand</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (28,42)</td>
<td>( \mathcal{Z}(14, 15, 17) )</td>
<td>11 (14,78)</td>
<td>( \mathcal{Z}(13, 15, 17) )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 (18,50)</td>
<td>( \mathcal{Z}(13, 14, 18) )</td>
<td>12 (90,36)</td>
<td>( \mathcal{Z}(11, 14, 17) )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3 (74,34)</td>
<td>( \mathcal{Z}(12, 14, 16) )</td>
<td>13 (78,20)</td>
<td>( \mathcal{Z}(13, 15, 19) )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4 (74,6)</td>
<td>( \mathcal{Z}(17, 18, 20) )</td>
<td>14 (24,52)</td>
<td>( \mathcal{Z}(11, 13, 16) )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5 (70,18)</td>
<td>( \mathcal{Z}(21, 23, 26) )</td>
<td>15 (54,6)</td>
<td>( \mathcal{Z}(20, 24, 26) )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6 (72,98)</td>
<td>( \mathcal{Z}(24, 26, 28) )</td>
<td>16 (62,60)</td>
<td>( \mathcal{Z}(16, 18, 23) )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7 (60,50)</td>
<td>( \mathcal{Z}(13, 15, 16) )</td>
<td>17 (98,14)</td>
<td>( \mathcal{Z}(18, 19, 22) )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8 (36,40)</td>
<td>( \mathcal{Z}(12, 14, 17) )</td>
<td>18 (36,58)</td>
<td>( \mathcal{Z}(13, 14, 17) )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9 (12,4)</td>
<td>( \mathcal{Z}(13, 15, 17) )</td>
<td>19 (38,88)</td>
<td>( \mathcal{Z}(16, 17, 20) )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10 (18,20)</td>
<td>( \mathcal{Z}(22, 24, 26) )</td>
<td>20 (32,54)</td>
<td>( \mathcal{Z}(19, 22, 25) )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
In this example, the $\alpha$-cost minimization model (21) can be write as

$$
\begin{align*}
\min_{x,y,z} & \sum_{i=1}^{4} \sum_{j=1}^{20} z_{ij} \sqrt{(x_i - a_j)^2 + (y_i - b_j)^2} \\
\text{subject to:} & \\
& z_{ij} \geq 0, \ i = 1, 2, \ldots, 4, \ j = 1, 2, \ldots, 20 \\
& \sum_{i=1}^{4} z_{ij} = \Phi^{-1}_j(\alpha), \ j = 1, 2, \ldots, 20 \\
& \sum_{j=1}^{20} z_{1j} \leq 100 \\
& \sum_{j=1}^{20} z_{2j} \leq 110 \\
& \sum_{j=1}^{20} z_{3j} \leq 120 \\
& \sum_{j=1}^{20} z_{4j} \leq 130 \\
& 0 \leq x_i \leq 100, \ i = 1, 2, 3, 4 \\
& 0 \leq y_i \leq 100, \ i = 1, 2, 3, 4.
\end{align*}
$$

(23)

In order to solve the model (23), the hybrid intelligent algorithm has been run with 1000 generations in GA.

The results for different $\alpha$ are shown in Table 2. From Table 2, we can get that the minimal cost is increasing when the confidence level $\alpha$ is increasing. This phenomenon is coincide with the properties of the model (14).

Table 2: The results of the example with different $\alpha$

<table>
<thead>
<tr>
<th>$\alpha$-cost</th>
<th>Locations of facilities</th>
<th>Minimal cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.6</td>
<td>(78,20),(62,61),(21,17),(29,53)</td>
<td>5781</td>
</tr>
<tr>
<td>0.7</td>
<td>(78,20),(62,60),(21,16),(31,53)</td>
<td>5939</td>
</tr>
<tr>
<td>0.8</td>
<td>(78,20),(61,65),(21,16),(29,53)</td>
<td>6010</td>
</tr>
<tr>
<td>0.9</td>
<td>(78,20),(62,60),(21,17),(29,53)</td>
<td>6251</td>
</tr>
</tbody>
</table>

The locations of customers and the optimal locations of the facilities are in Figure 1, in which the points represent the locations of the customers and the diamonds represent the optimal locations of the facilities.

6 Conclusion

Many researches have been done in the FLA problem because it has widely practical backgrounds. In this paper, the $\alpha$-cost minimization model with uncertain demands is proposed. Since the uncertain FLA model is too complex to compute by traditional algorithm, we have given a method of converting it to a crisp model. The hybrid algorithm combining simplex algorithm and genetic algorithm is designed to produce the uncertain FLA model. The computational results of the numerical experiment imply that the proposed algorithm is effective to solve the uncertain model.

References

Figure 1: Locations of customers and facilities, where \( \cdot \) denotes location of customer and \( \Diamond \) denotes location of facility


