Identifier Based Interval Type-2 Fuzzy Tracking Control

Tsung-Chih Lin¹
¹Department of Electronic Engineering
Feng-Chia University
Taichung, Taiwan, R.O.C.
tclin@fcu.edu.tw

I-Shin Liu³
³Department of Electronic Engineering
Feng-Chia University
Taichung, Taiwan, R.O.C.
allen00483@gmail.com

Chung-Ching Wang²
²Department of Electronic Engineering
Feng-Chia University
Taichung, Taiwan, R.O.C.
adidas2351@gmail.com

Valentina Emilia Balas⁴
⁴Department of Automation and Applied Informatics
Aurel Vlaicu University of Arad
Arad, Romania
balas@drbalas.ro

Abstract—Based on system identifier, interval type-2 fuzzy neural network (IT2FNN) tracking control for a class of unknown nonlinear dynamic system is developed in this paper. In order to fully handle or accommodate the linguistic and numerical uncertainties associated with dynamic unstructured environments, an IT2FNN controller equipped with a learning algorithm is developed. In the meantime, an IT2FNN identifier is incorporated into the IT2FNN controller to predict the system sensitivity of the unknown nonlinear dynamic system. The comparison between type-1 FNN (T1FNN) controller and IT2FNN controller is given to sufficiently illustrate the effectiveness of the proposed control scheme.

Keywords- Interval type-2 fuzzy neural network, identification, nonlinear systems.

I. INTRODUCTION

In the past decades, the fuzzy logic systems (FLS) are capable of uniformly approximating any nonlinear function over a compact set to any degree of accuracy, based on the universal approximation theorem [1]-[11]. By using a collection of the IF–THEN rules, a globally stable adaptive fuzzy controller is generally synthesized applicable to plants that are mathematically poorly understood and where experienced human operators are available for providing qualitative “rule of thumb” [1]-[11]. The free parameters of the adaptive fuzzy controller can be tuned by adaptive laws, based on the Lyapunov synthesis approach [2], [5]-[8], for the purpose of controlling a plant to track a reference trajectory.

In most of the industrial applications without uncertainties, the type-1 fuzzy logic control (FLC) and the interval type-2 FLC responses are similar [1]. However, in highly uncertain environments, it is better to choose interval type-2 FLC to fully handle or accommodate the linguistic and numerical uncertainties associated with dynamic unstructured environments. Many important research results have been announced in favor of interval type-2 FLCs [5]-[15] because of their potential to better model various structured and unstructured uncertainties.

In this paper, a hybrid unknown nonlinear dynamic system control is proposed based on the combination of IT2FNN identifier and IT2FNN tracking controller. First, the IT2FNN identifier is constructed to predict the behavior of the unknown nonlinear system, then the IT2FNN tracking controller is developed to force the system output to track the given reference trajectory.

This paper is organized as follows. The brief description of the interval type-2 FLS is given in section II. The interval type-2 FNN structure is described in section III. In section IV, the interval type-2 FNN control scheme which includes an interval type-2 FNN controller appended with an interval type-2 FNN identifier is constructed and the back propagation (BP) learning algorithm for FNN controller and identifier is introduced. Simulation example to demonstrate the performance of the proposed control scheme is provided in section V. Section VI lists the conclusions of the advocated design methodology.

II. THE BRIEF DESCRIPTION OF THE INTERVAL TYPE-2 FLS [5]-[8]

Owing to the complexity of the type reduction, the general type-2 fuzzy logic system (FLS) becomes computationally intensive and in order to make things simpler and easier to compute meet and join operations, the secondary MFs of an interval type-2 FLS are all unity which leads finally to simplify type reduction. The 2-D interval type-2 Gaussian membership function (MF) with uncertain mean \( m \in [m_1, m_2] \) and a fixed deviation \( \sigma \) is shown in Fig. 1.

\[
\mu_\mu(x) = \exp \left[ -\frac{1}{2} \frac{(x-m)^2}{\sigma} \right] \quad m \in [m_1, m_2] \tag{1}
\]

Fig. 1 Interval type-2 fuzzy set with uncertain mean.
It is obvious that the interval type-2 fuzzy set called a footprint of uncertainty (FOU) is in a region bounded by an upper MF and a lower MF denoted as $\mu_+(x)$ and $\mu_-(x)$, respectively. Moreover, the firing strength $F^i$ for the $i$th rule can be an interval type-1 set expressed as

$$F^i = \left[ \frac{f^i}{f^i} \right]$$

where

$$f^i = \mu_{\tilde{R}_i}(x_1) \cdots \mu_{\tilde{R}_i}(x_i) = \prod_{k=1}^n \mu_{\tilde{R}_i}(x_j)$$

$$f^i = \tilde{R}_i(x_1) \cdots \tilde{R}_i(x_i) = \prod_{k=1}^n \tilde{R}_i(x_j)$$

The defuzzified crisp output from an interval type-2 FLS is the average of $y_j$ and $y_u$, by using the center of sets type reduction, i.e.,

$$y(x) = \frac{y_j + y_u}{2}$$

where $y_j$ and $y_u$ are the left most and right most points of the interval type-1 set which can be obtained as

$$y_j = \frac{\sum_{i=1}^{m} f^i w'_i}{\sum_{i=1}^{m} f^i} = \frac{\sum_{i=1}^{m} f^i w'_i + \sum_{i=L+1}^{m} f^i w'_i}{\sum_{i=1}^{m} f^i + \sum_{i=L+1}^{m} f^i}$$

$$y_u = \frac{\sum_{i=L+1}^{m} f^i w'_i}{\sum_{i=L+1}^{m} f^i} = \frac{\sum_{i=L+1}^{m} f^i w'_i + \sum_{i=1}^{m} f^i w'_i}{\sum_{i=L+1}^{m} f^i + \sum_{i=1}^{m} f^i}$$

where $q'_i = f^i / D_i$, $\overline{q'_i}$ = $f^i / D_i$, and $D_i = (\sum_{i=1}^{m} f^i + \sum_{i=L+1}^{m} f^i)$.

Moreover

$$y_j = \frac{\sum_{i=1}^{B} f^i w'_i}{\sum_{i=1}^{B} f^i} = \frac{\sum_{i=1}^{B} f^i w'_i + \sum_{i=L+1}^{B} f^i w'_i}{\sum_{i=1}^{B} f^i + \sum_{i=L+1}^{B} f^i}$$

$$y_u = \frac{\sum_{i=L+1}^{B} f^i w'_i}{\sum_{i=L+1}^{B} f^i} = \frac{\sum_{i=L+1}^{B} f^i w'_i + \sum_{i=1}^{B} f^i w'_i}{\sum_{i=L+1}^{B} f^i + \sum_{i=1}^{B} f^i}$$

where $q'_i = f^i / D_j$, $\overline{q'_i}$ = $f^i / D_j$, and $D_j = (\sum_{i=1}^{B} f^i + \sum_{i=L+1}^{B} f^i)$ and $M$ is the total number of rules in the rule base of the interval type-2 fuzzy neural network. The weighting factors $w'_i$ and $w'_i$ of the consequent part represent the centroid interval set of the consequent type-2 fuzzy set of the $i$th rule. In the meantime, $R$ and $L$ can be determined by using the iterative Karnik-Mendel procedure [2], [5]-[8].

III. INTERVAL TYPE-2 FUZZY NEURAL NETWORK STRUCTURE

The structure of the proposed IT2FNN as shown in Fig. 2 [5] consists of four layers which include the input, the membership function, the rule and the output layers. The signal propagation and the interaction between each layer are given as follows:

Layer 1 - Input Layer: The nodes in this layer transmit input values to the next layer directly. For $k$th node in this layer, the input and output are described as:

Input variable of the $k$th node: $x^k_i(p)$

Output variable of the $k$th node:

$$F^k_i(p) = x^k_i(p), \quad k = 1,...,N$$

where $p$ is the number of iterations.

Layer 2 - Membership Layer: In this layer, each node represents the terms of respective linguistic variables and the interval type-2 Gaussian function is adopted as the membership function in (9).

$$N(m, \sigma, O_i^k) = \exp \left( -\frac{1}{2} \left( \frac{O_i^k - m}{\sigma} \right)^2 \right)$$

where

$$m_i = \left[ m_{i1}, m_{i2} \right], \quad j = 1,...,N_j$$

Meanwhile, for $k$th node and $j$th linguistic variable, the input and output are given as:

Input variable of $k$th node and $j$th linguistic variable:

$$u^k_j(p) = O^k_j(p)$$

Output variable of $k$th node and $j$th linguistic variable:

$$O^k_j = N(m_j, \sigma, u^k_j) = \exp \left( -\frac{1}{2} \left( \frac{u^k_j - m_j}{\sigma} \right)^2 \right)$$

where $j$ = 1,2, ..., $N_j$, $k$ = 1,2, ..., $N$, and $u^k_j$ is the lower and upper membership function, respectively and expressed as

$$
\begin{align*}
O^k_j &= \begin{cases} 
N\left(m_j, \sigma, u^k_j \right), & u^k_j \leq \frac{m_{ij} + m_{ij}}{2} \\
N\left(m_j, \sigma, u^k_j \right), & u^k_j > \frac{m_{ij} + m_{ij}}{2}
\end{cases}
\end{align*}
$$
where $\alpha$ is 1

$$\overrightarrow{O}_i = \begin{cases} 
N(m_{ij}, \sigma_{ij}; u_{ij}^*) , & u_{ij}^* \leq m_{ij} \\
1 , & m_{ij} \leq u_{ij}^* \leq m_{ij} \\
N(m_{ij}, \sigma_{ij}; u_{ij}^*) , & u_{ij}^* \geq m_{ij} 
\end{cases}$$ \hspace{1cm} (12)

**Layer 3 - Rule Base and Inference Layer:** This layer constructs the fuzzy rules and realizes the fuzzy inference. Each node is corresponding to a fuzzy rule. The input links and the output links of each node represent the preconditions of the corresponding rule and the firing strength of the corresponding rule, respectively. The IF-THEN rule for interval real-time interval type-2 FNN can be expressed as:

$$R : \text{IF } O_1^i \text{ is } \tilde{F}_1^i, \text{ and } O_2^i \text{ is } \tilde{F}_2^i, \text{ and } \ldots, \text{ and } O_N^i \text{ is } \tilde{F}_N^i \text{ THEN } w_i = [w_{i1}, w_{i2}, \ldots, w_{i}}$$

where $i = 1, 2, \ldots, M$ is rule number, the $\tilde{F}_N^i$ is the interval type-2 fuzzy sets of antecedent part and $w_i$, $w_{i1}$, $w_{i2}$, $w_{i3}$, $w_{i4}$ are the output weight, and $q$ is the output number.

**Layer 4 - Output Layer:** In this layer, by using the type-reduced proposed by Karnik-Mendel [3], the crisp output can be obtained as

$$y_c = \frac{y_e + y_o}{2}, \quad c = 1, 2, \ldots, q$$ \hspace{1cm} (14)

where $y_e$ and $y_o$ are determined by (13). The proposed interval type-2 FNN map crisp inputs $x_i$ to crisp outputs $y_c$. The learning algorithm of system identifier and tracking controller for both nonlinear parameters of the antecedents and the linear parameters of the consequent of the rules will be derived in the next section.

IV. IDENTIFICATION BASED INTERVAL TYPE-2 FNN TRACKING CONTROL

In this section, the proposed identification based interval type-2 fuzzy tracking control scheme is constructed as shown in Fig. 3.

![Fig. 3 The identification based interval type-2 fuzzy tracking control scheme.](image)

An interval type-2 fuzzy identifier is incorporated into the IT2FNN controller to predict the system sensitivity of the unknown nonlinear dynamic system. According to the above analysis, the defuzzified outputs, i.e., the actual output, of the identifier and controller are determined from (14). The BP method is used to tune all parameters of the IT2FNN identifier and tracking controller.

For the $i$th rule, the detail learning algorithms used tune the parameters of each input antecedent fuzzy sets, i.e., $m_{ij}$, $m_{ij}$, and $\sigma^\prime$, as well as the consequent parameter of each crisp output, i.e., $w_{i1}$ and $w_{i2}$, will be derived in this section. Based on BP algorithm, for $P$ input/output training data pairs $(x^p, y_c^p)$, $p = 1, 2, \ldots, P$ the parameters mentioned above of the FNN can be adjusted such that the following error functions must be minimized:

**Identifier:**

$$e_e^p = \frac{1}{2} \left[ y_e^p - y_c(x^p) \right]^2, \quad p = 1, 2, \ldots, P$$ \hspace{1cm} (15)

**Tracking controller:**

$$e_e^p = \frac{1}{2} \left[ y_e^p - y_c(x^p) \right]^2, \quad p = 1, 2, \ldots, P$$ \hspace{1cm} (16)

Therefore, the BP algorithm may be written briefly as:

**Identifier:**

$$W_r(p + 1) = W_r(p) + \Delta W_r(p) = W_r(p) + \eta_r \left( -\frac{\partial e_e^p}{\partial W_r(p)} \right)$$ \hspace{1cm} (17)

**Tracking controller:**

$$W_r(p + 1) = W_r(p) + \Delta W_r(p) = W_r(p) + \eta_f \left( -\frac{\partial e_e^p}{\partial W_r(p)} \right)$$ \hspace{1cm} (18)

where $\eta_r$ and $\eta_f$ are the learning rates and $W_r$ and $W_f$ represent the tuning parameters, $m_{ij}$, $m_{ij}$, $\sigma^\prime$, $w_{i1}$ and $w_{i2}$, of the identifier and controller, respectively. Let’s reconsider $k$th input and $l$th rule, a Gaussian primary MF with standard deviation $\sigma_{ij}$ and an uncertain mean with interval value $[m_{ij}^l, m_{ij}^l]$, the upper and lower MFs can be described as

$$
\overrightarrow{O}_i = \begin{cases} 
N(m_{ij}^l, \sigma_{ij}; O_{ij}^l) , & O_{ij}^l < m_{ij}^l \\
1 , & m_{ij}^l \leq O_{ij}^l \leq m_{ij}^l \\
N(m_{ij}^l, \sigma_{ij}; O_{ij}^l) , & O_{ij}^l > m_{ij}^l 
\end{cases}$$ \hspace{1cm} (19)

and

$$\overrightarrow{O}_i = \begin{cases} 
N(m_{ij}^l, \sigma_{ij}; O_{ij}^l) , & O_{ij}^l \leq \frac{m_{ij}^l + m_{ij}^l}{2} \\
1 , & \frac{m_{ij}^l + m_{ij}^l}{2} \leq O_{ij}^l \leq \frac{m_{ij}^l + m_{ij}^l}{2} \\
N(m_{ij}^l, \sigma_{ij}; O_{ij}^l) , & O_{ij}^l > \frac{m_{ij}^l + m_{ij}^l}{2} 
\end{cases}$$ \hspace{1cm} (20)

where

$$N(m_{ij}^l, \sigma_{ij}; O_{ij}^l) = \exp \left( -\frac{1}{2} \left( \frac{O_{ij}^l - m_{ij}^l}{\sigma_{ij}} \right)^2 \right), \quad m_{ij}^l \in [m_{ij}^l, m_{ij}^l]$$ \hspace{1cm} (21)

The detail tuning procedure for all training parameters can be derived as follows:

**A.** By using (17) and chain rule, the BP learning algorithm of the identifier for tuning the end points of the centroid of the consequent type-2 fuzzy sets $w_{i1}$ and $w_{i2}$ (output weights) can be obtained as
\[w_i(p+1) = w_i(p) - \eta_i \left[ \frac{\partial e_i^p(p)}{\partial y} \frac{\partial y}{\partial \hat{v}_i} \frac{\partial \hat{v}_i}{\partial w_i} \right]\]  
(22)

and

\[w_i(p+1) = w_i(p) - \eta_i \left[ \frac{\partial e_i^p(p)}{\partial y} \frac{\partial y}{\partial \hat{v}_i} \frac{\partial \hat{v}_i}{\partial w_i} \right]\]  
(23)

where

\[\frac{\partial e_i^p(p)}{\partial y} = y_i(p) - y(p), \quad \frac{\partial y}{\partial \hat{v}_i} = \frac{1}{2}, \quad \frac{\partial \hat{v}_i}{\partial w_i} = \frac{1}{2}\]

and

\[\frac{\partial y}{\partial \hat{v}_i} = \frac{1}{2}, \quad \frac{\partial \hat{v}_i}{\partial w_i} = \frac{1}{2}\]

Similarly, the update laws for (1 ) where

\[\frac{\partial \hat{v}_i}{\partial w_i} = \frac{1}{2}\]

are given

\[\frac{\partial e_i^p(p)}{\partial y} = \frac{1}{2}, \quad \frac{\partial y}{\partial \hat{v}_i} = \frac{1}{2}\]

B. Using (17) and chain rule, the update laws of the identifier for \(m_{ij}^i\) and \(m_{ij}^i\) can be derived as

\[m_{ij}^i(p+1) = m_{ij}^i(p) - \eta_i \frac{\partial e_i^p(p)}{\partial m_{ij}^i} = m_{ij}^i(p) - \eta_i \times\]

\[\left[ \frac{\partial e_i^p(p)}{\partial y} \frac{\partial \hat{v}_i}{\partial m_{ij}^i} \frac{\partial m_{ij}^i}{\partial \hat{v}_i} \frac{\partial \hat{v}_i}{\partial y} \frac{\partial \hat{v}_i}{\partial m_{ij}^i} \frac{\partial \hat{v}_i}{\partial y} \right]\]  
(25)

Similarly, the update laws for \(m_{ij}^i(p+1)\) can be derived from (25) by replacing \(m_{ij}^i\) by \(m_{ij}^i\).

where

\[\frac{\partial \hat{v}_i^j(x_i')}{\partial m_{ij}^i} \frac{\partial \hat{v}_i^j(x_i')}{\partial m_{ij}^i} \frac{\partial \hat{v}_i^j(x_i')}{\partial m_{ij}^i} \frac{\partial \hat{v}_i^j(x_i')}{\partial m_{ij}^i} \frac{\partial \hat{v}_i^j(x_i')}{\partial m_{ij}^i} \frac{\partial \hat{v}_i^j(x_i')}{\partial m_{ij}^i} \]

are given as

\[\frac{\partial \hat{v}_i^j(x_i')}{\partial m_{ij}^i} = \begin{cases} \frac{(x_i'^2 - m_{ij}^i)^2}{(\sigma_i^j)^2}, & x_i'^2 < m_{ij}^i \\ 0, & m_{ij}^i \leq x_i'^2 \leq m_{ij}^i \\ 0, & x_i'^2 > m_{ij}^i \end{cases}\]  
(26)

\[\frac{\partial \hat{v}_i^j(x_i')}{\partial m_{ij}^i} = \begin{cases} \frac{(x_i'^2 - m_{ij}^i)^2}{(\sigma_i^j)^2}, & x_i'^2 < m_{ij}^i \\ 0, & m_{ij}^i \leq x_i'^2 \leq m_{ij}^i \\ 0, & x_i'^2 > m_{ij}^i \end{cases}\]  
(27)

C. By using (17) and chain rule, the standard deviation \(\sigma_i^j\) of the IT2FNN identifier can be adjusted as

\[\sigma_i^j(p+1) = \sigma_i^j(p) - \eta_i \frac{\partial e_i^p(p)}{\partial \sigma_i^j} = \sigma_i^j(p) - \eta_i \times\]

\[\left[ \frac{\partial e_i^p(p)}{\partial y} \frac{\partial \hat{v}_i}{\partial \sigma_i^j} \frac{\partial \hat{v}_i}{\partial y} \frac{\partial \hat{v}_i}{\partial \sigma_i^j} \frac{\partial \hat{v}_i}{\partial y} \right]\]  
(28)

According to the input of the interval type-2 FLS, there are sixteen possible combinations of \(\eta_i \frac{\partial e_i^p}{\partial \sigma_i^j}\)

\[\eta_i \frac{\partial e_i^p}{\partial \sigma_i^j}\]

and

\[\eta_i \frac{\partial e_i^p}{\partial \sigma_i^j}\]

as shown in Table I.

Similarly, for the \(i\)th rule of the IT2FNN tracking controller, the detail learning algorithms used to tune the parameters of
each input antecedent fuzzy sets, i.e., \( m_{ij}^{w} \), \( m_{ij}^{w} \) and \( \sigma_{ij}^{w} \), as well as the consequent parameter of each crisp output, i.e., \( \hat{w}_{i}^{w} \) and \( \hat{w}_{i}^{u} \), can be derived by replacing \( W_{i} \), \( \eta_{i} \) and \( e_{i}^{p} \) with \( W_{i} \), \( \eta_{i} \) and \( e_{i}^{p} \) as shown in (18).

In the equation (18),
\[
\frac{\partial e_{i}^{p}}{\partial W_{i}} = \frac{\partial e_{i}^{p}}{\partial \eta_{i}} = \frac{\partial y}{\partial u} =\left[ y - y_{r} \right] \frac{\partial y}{\partial u} \frac{\partial u}{\partial W_{r}}
\]
where \( u \) which is the output of the FNN controller is the control effort of the plant. \( \frac{\partial y}{\partial u} = y_{u} \) represents the system sensitivity. Due to \( y \) is an unknown system output, IT2FNN identifier is incorporated into the IT2FNN controller to predict the system sensitivity of the unknown nonlinear dynamic system, i.e., \( y_{u} \) can be approximated by \( \frac{\partial y}{\partial u} = y_{u} \) based on IT2FNN identifier. Let the following notations are defined as
\[
\begin{align*}
S_{1} &= \frac{\partial y_{1}}{\partial u} \frac{\partial y_{2}}{\partial \mu(u)} \frac{\partial N(m_{1})}{\partial m_{1}} , \quad S_{2} = \frac{\partial y_{1}}{\partial u} \frac{\partial y_{2}}{\partial \mu(u)} \frac{\partial N(m_{2})}{\partial m_{2}} , \\
S_{3} &= \frac{\partial y_{1}}{\partial u} \frac{\partial y_{2}}{\partial \mu(u)} \frac{\partial N(m_{3})}{\partial m_{3}} , \quad S_{4} = \frac{\partial y_{1}}{\partial u} \frac{\partial y_{2}}{\partial \mu(u)} \frac{\partial N(m_{4})}{\partial m_{4}} , \\
S_{5} &= \frac{\partial y_{1}}{\partial u} \frac{\partial y_{2}}{\partial \mu(u)} \frac{\partial N(m_{5})}{\partial m_{5}} , \quad S_{6} = \frac{\partial y_{1}}{\partial u} \frac{\partial y_{2}}{\partial \mu(u)} \frac{\partial N(m_{6})}{\partial m_{6}} , \\
S_{7} &= \frac{\partial y_{1}}{\partial u} \frac{\partial y_{2}}{\partial \mu(u)} \frac{\partial N(m_{7})}{\partial m_{7}} , \quad S_{8} = \frac{\partial y_{1}}{\partial u} \frac{\partial y_{2}}{\partial \mu(u)} \frac{\partial N(m_{8})}{\partial m_{8}} .
\end{align*}
\]

Therefore, for IT2FNN, there sixteen possible combinations of \( \frac{\partial y_{i}}{\partial u} \) are listed as shown in Table III.

V. SIMULATION EXAMPLE

In this section, we apply our identifier based IT2FNN controller to let the output of the Duffing force-oscillation system to track a sine-wave trajectory as well.

Example: The dynamic equations of the Duffing force-oscillation system are
\[
\begin{align*}
\dot{x}_{1} &= x_{2} \\
\dot{x}_{2} &= -0.1x_{2} - x_{1}^{3} + 12 \cos(t) + u + d
\end{align*}
\]
\[
y = [1 \quad 0] \begin{bmatrix} x_{1} \\ x_{2} \end{bmatrix}
\]

The proposed control scheme is applied to control system output \( y \) to track the reference trajectories, \( y_{r}(t) = \frac{\pi}{5} \sin(t) \).

Also, the external disturbance \( d \) is assumed to be a square wave with amplitude \( \pm 1 \) and period \( 2\pi \). The footprint of uncertainty of the initial interval type-2 membership function of the FNN identifier and FNN controller for \( x_{j} \), \( x_{k} \), \( k = 1, 2 \) are as shown in Fig. 4 and Fig 5, respectively. The all initial values are chosen as \( x_{1}(0) = 0 \), \( x_{2}(0) = 0 \).

The simulation results are described as following two cases: free of internal noise case and training data corrupted by white Gaussian noise (WGN) with signal-to-noise ratio (SNR) =20 dB case.

Case 1: Free of internal noise case: In this case, training data does not be corrupted with internal noise. Fig. 6 show output trajectories for type-1 FNN identifier and interval type-2 FNN identifier. Fig. 7 show output trajectories for type-1 FNN control scheme and interval type-2 FNN scheme. Control efforts of type-1 controller and interval type-2 controller are shown in Fig. 8.
Fig. 7 Output trajectories for type-1 FNN control scheme and interval type-2 FNN control scheme.

Fig. 8 Control efforts of type-1 controller and interval type-2 controller.

The footprint of uncertainty of the final interval type-2 membership function of the FNN identifier and FNN controller for $x_k, k = 1, 2$ are as shown in Fig. 9 and Fig 10, respectively.

Fig. 9 The final interval type-2 Gaussian membership function of the IT2FNN identifier for $x_k, k = 1, 2$.

Fig. 10 The final interval type-2 Gaussian membership function of the IT2FNN tracking controller for $x_k, k = 1, 2$.

From Fig. 6 and Fig. 7, we can see that the tracking performance and identification performance are almost the same under free of internal noise for T1FNN control scheme and IT2FNN control scheme.

**Case 2: Training data corrupted by WGN with SNR=20 dB case:** In order to show that the interval type-2 FLS can handle the measurement uncertainties, training data are corrupted by WGN with SNR=20 dB. Fig. 11 show output trajectories for type-1 FNN identifier and interval type-2 FNN identifier. Fig. 12 show output trajectories for type-1 FNN control scheme and interval type-2 FNN scheme. Control efforts of type-1 controller and interval type-2 controller are shown in Fig. 13.

Fig. 11 Output trajectories for type-1 FNN identifier and interval type-2 FNN identifier.

Fig. 12 Output trajectories for type-1 FNN control scheme and interval type-2 FNN.

Fig. 13 Control efforts of type-1 controller and interval type-2 controller.

The footprint of uncertainty of the final interval type-2 membership function of the FNN identifier and FNN controller for $x_k, x_k, k = 1, 2$ are as shown in Fig. 14 and Fig 15, respectively.
From Fig. 11 and Fig. 12, we can see that the tracking performance and identification performance of the IT2FNN control scheme are better than those of T1FNN control scheme, if training data corrupted by WGN with SNR = 20 dB case.

VI. CONCLUSION

In order to fully handle or accommodate the linguistic and numerical uncertainties associated with dynamic unstructured environments, an IT2FNN controller equipped with a learning algorithm is developed. In the meantime, an IT2FNN identifier is incorporated into the IT2FNN tracking controller to provide the system sensitivity of the unknown nonlinear dynamic system for tracking control learning algorithms. Simulation results show that the tracking performance and identification performance of the proposed control scheme are much better than those of the type-1 FNN control scheme when the FNN structures are corrupted with linguistic uncertainties and uncertainties which arise from the noisy training data.

### TABLE I. TUNING ALGORITHM OF $m^i_o(x^i)$ BY $m^i_o(x^i)$ ACCORDING TO THE INPUT OF THE IT2FNN

<table>
<thead>
<tr>
<th>$x_i &lt; m^i_{i0}$</th>
<th>$i \leq L, i \leq R$</th>
<th>$i \leq L, i &gt; R$</th>
<th>$i &gt; L, i \leq R$</th>
<th>$i &gt; L, i &gt; R$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m^i_{i0} \leq x_i \leq m^i_{i0} + \frac{m^i_{i0}}{2}$</td>
<td>$\eta \left[ \frac{\partial I}{\partial y_j} \frac{\partial y_j}{\partial \mu} \frac{\partial \mu}{\partial m^i_{i0}} \right] + \eta \left[ \frac{\partial I}{\partial y_j} \frac{\partial y_j}{\partial \mu} \frac{\partial \mu}{\partial m^i_{i0}} \right]$</td>
<td>$\eta \left[ \frac{\partial I}{\partial y_j} \frac{\partial y_j}{\partial \mu} \frac{\partial \mu}{\partial m^i_{i0}} \right] + \eta \left[ \frac{\partial I}{\partial y_j} \frac{\partial y_j}{\partial \mu} \frac{\partial \mu}{\partial m^i_{i0}} \right]$</td>
<td>$\eta \left[ \frac{\partial I}{\partial y_j} \frac{\partial y_j}{\partial \mu} \frac{\partial \mu}{\partial m^i_{i0}} \right] + \eta \left[ \frac{\partial I}{\partial y_j} \frac{\partial y_j}{\partial \mu} \frac{\partial \mu}{\partial m^i_{i0}} \right]$</td>
<td>$\eta \left[ \frac{\partial I}{\partial y_j} \frac{\partial y_j}{\partial \mu} \frac{\partial \mu}{\partial m^i_{i0}} \right] + \eta \left[ \frac{\partial I}{\partial y_j} \frac{\partial y_j}{\partial \mu} \frac{\partial \mu}{\partial m^i_{i0}} \right]$</td>
</tr>
<tr>
<td>$m^i_{i0} + \frac{m^i_{i0}}{2} &lt; x_i \leq m^i_{i1}$</td>
<td>$\eta \left[ \frac{\partial I}{\partial y_j} \frac{\partial y_j}{\partial \mu} \frac{\partial \mu}{\partial m^i_{i0}} \right]$</td>
<td>$\eta \left[ \frac{\partial I}{\partial y_j} \frac{\partial y_j}{\partial \mu} \frac{\partial \mu}{\partial m^i_{i0}} \right]$</td>
<td>$\eta \left[ \frac{\partial I}{\partial y_j} \frac{\partial y_j}{\partial \mu} \frac{\partial \mu}{\partial m^i_{i0}} \right]$</td>
<td>$\eta \left[ \frac{\partial I}{\partial y_j} \frac{\partial y_j}{\partial \mu} \frac{\partial \mu}{\partial m^i_{i0}} \right]$</td>
</tr>
<tr>
<td>$x_i &gt; m^i_{i1}$</td>
<td>$\eta \left[ \frac{\partial I}{\partial y_j} \frac{\partial y_j}{\partial \mu} \frac{\partial \mu}{\partial m^i_{i0}} \right] + \eta \left[ \frac{\partial I}{\partial y_j} \frac{\partial y_j}{\partial \mu} \frac{\partial \mu}{\partial m^i_{i0}} \right]$</td>
<td>$\eta \left[ \frac{\partial I}{\partial y_j} \frac{\partial y_j}{\partial \mu} \frac{\partial \mu}{\partial m^i_{i0}} \right] + \eta \left[ \frac{\partial I}{\partial y_j} \frac{\partial y_j}{\partial \mu} \frac{\partial \mu}{\partial m^i_{i0}} \right]$</td>
<td>$\eta \left[ \frac{\partial I}{\partial y_j} \frac{\partial y_j}{\partial \mu} \frac{\partial \mu}{\partial m^i_{i0}} \right] + \eta \left[ \frac{\partial I}{\partial y_j} \frac{\partial y_j}{\partial \mu} \frac{\partial \mu}{\partial m^i_{i0}} \right]$</td>
<td>$\eta \left[ \frac{\partial I}{\partial y_j} \frac{\partial y_j}{\partial \mu} \frac{\partial \mu}{\partial m^i_{i0}} \right] + \eta \left[ \frac{\partial I}{\partial y_j} \frac{\partial y_j}{\partial \mu} \frac{\partial \mu}{\partial m^i_{i0}} \right]$</td>
</tr>
</tbody>
</table>
### Table II. Tuning Algorithm of $\sigma'_i(x^i)$ According to the Input of the IT2FNN

<table>
<thead>
<tr>
<th>$x_i &lt; m_{i0}^j$</th>
<th>$\frac{\partial}{\partial y_j} \frac{\partial y_i}{\partial y_j} \frac{\partial \mu_i}{\partial y_j} $</th>
<th>$\frac{\partial}{\partial y_j} \frac{\partial y_i}{\partial y_j} \frac{\partial \sigma_i(m_{i0})}{\partial y_j} $</th>
<th>$\frac{\partial}{\partial y_j} \frac{\partial y_i}{\partial y_j} \frac{\partial \sigma_i(m_{i0})}{\partial y_j} $</th>
<th>$\frac{\partial}{\partial y_j} \frac{\partial y_i}{\partial y_j} \frac{\partial \mu_i}{\partial y_j} $</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_{i0}^j \leq x_i \leq m_{i0}^j + m_{i1}^j$</td>
<td>$\frac{\partial}{\partial y_j} \frac{\partial y_i}{\partial y_j} \frac{\partial \mu_i}{\partial y_j} $</td>
<td>$\frac{\partial}{\partial y_j} \frac{\partial y_i}{\partial y_j} \frac{\partial \sigma_i(m_{i0})}{\partial y_j} $</td>
<td>$\frac{\partial}{\partial y_j} \frac{\partial y_i}{\partial y_j} \frac{\partial \sigma_i(m_{i0})}{\partial y_j} $</td>
<td>$\frac{\partial}{\partial y_j} \frac{\partial y_i}{\partial y_j} \frac{\partial \mu_i}{\partial y_j} $</td>
</tr>
<tr>
<td>$m_{i0}^j + m_{i1}^j &lt; x_i \leq m_{i0}^j + m_{i1}^j$</td>
<td>$\frac{\partial}{\partial y_j} \frac{\partial y_i}{\partial y_j} \frac{\partial \mu_i}{\partial y_j} $</td>
<td>$\frac{\partial}{\partial y_j} \frac{\partial y_i}{\partial y_j} \frac{\partial \sigma_i(m_{i0})}{\partial y_j} $</td>
<td>$\frac{\partial}{\partial y_j} \frac{\partial y_i}{\partial y_j} \frac{\partial \sigma_i(m_{i0})}{\partial y_j} $</td>
<td>$\frac{\partial}{\partial y_j} \frac{\partial y_i}{\partial y_j} \frac{\partial \mu_i}{\partial y_j} $</td>
</tr>
<tr>
<td>$x_i &gt; m_{i1}^j$</td>
<td>$\frac{\partial}{\partial y_j} \frac{\partial y_i}{\partial y_j} \frac{\partial \mu_i}{\partial y_j} $</td>
<td>$\frac{\partial}{\partial y_j} \frac{\partial y_i}{\partial y_j} \frac{\partial \sigma_i(m_{i0})}{\partial y_j} $</td>
<td>$\frac{\partial}{\partial y_j} \frac{\partial y_i}{\partial y_j} \frac{\partial \sigma_i(m_{i0})}{\partial y_j} $</td>
<td>$\frac{\partial}{\partial y_j} \frac{\partial y_i}{\partial y_j} \frac{\partial \mu_i}{\partial y_j} $</td>
</tr>
</tbody>
</table>

| $i \leq L, i \leq R$ | $i \leq L, i > R$ | $i > L, i \leq R$ | $i > L, i > R$ |

### Table III. There Sixteen Possible Combinations of $\frac{\partial y_i}{\partial y_j}$, System Sensitivity.

<table>
<thead>
<tr>
<th>$u &lt; m_{i0}^j$</th>
<th>$y_i = s_i + s_i$</th>
<th>$y_i = s_i + s_i$</th>
<th>$y_i = s_i + s_i$</th>
<th>$y_i = s_i + s_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_{i0}^j \leq u \leq m_{i0}^j + m_{i1}^j$</td>
<td>$y_i = s_i$</td>
<td>$y_i = s_i$</td>
<td>$y_i = s_i$</td>
<td>$y_i = s_i$</td>
</tr>
<tr>
<td>$m_{i0}^j + m_{i1}^j &lt; u \leq m_{i0}^j + m_{i1}^j$</td>
<td>$y_i = s_i$</td>
<td>$y_i = s_i$</td>
<td>$y_i = s_i$</td>
<td>$y_i = s_i$</td>
</tr>
<tr>
<td>$u &gt; m_{i1}^j$</td>
<td>$y_i = s_i + s_i$</td>
<td>$y_i = s_i + s_i$</td>
<td>$y_i = s_i + s_i$</td>
<td>$y_i = s_i + s_i$</td>
</tr>
</tbody>
</table>

### References


