EMBEDDED IMAGE COMPRESSION BASED ON
WAVELET PIXEL CLASSIFICATION AND SORTING

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ABSTRACT
A new embedded image compression algorithm is proposed, based on progressive Pixel Classification And Sorting (PCAS) in wavelet domain. To exploit the intraband and interband correlation in wavelet domain, EZW [1] and SPIHT [2] implicitly classify wavelet pixels as zerotree pixels or not, while MRWD[3], SLCCA[4], and EBCOT[5] implicitly classify wavelet pixels as neighbors of significant pixels or not. In this paper, the wavelet pixels to be encoded are explicitly and finely classified based on their predicted probabilities, which is more sophisticated and effective. Furthermore, wavelet pixel sorting is introduced to help improve rate-distortion performance within each bit-plane coding. The technique of pixel classification and sorting is simple, yet effective to produce the image code with excellent compression performance. In addition, our algorithm provides both SNR and resolution scalability.

1. INTRODUCTION
With the increasing demand of manipulation, storage and transmission of digital images, great efforts are made to seek for image compression algorithms which are of modest complexity, which exhibit efficient compression performance, and which provide scalability. Scalability can be achieved via an embedded compression algorithm, i.e., an algorithm which generates an encoded image bit stream that contains embedded subsets, each subset affording efficient compression of the original image at a reduced resolution (resolution or spatial scalability) or increased distortion (SNR or quality scalability) [5]. Embedded image coding provides the possibility to progressively reconstruct better and better reproductions of the image as more and more of the encoded image bit-stream is received. Our focus in this paper shall be upon embedded image compression algorithms that exploit the redundancy among wavelet coefficients obtained by applying the discrete wavelet transform to the image to be compressed. (We refer to these coefficients as wavelet pixels.) The best known such embedded compression algorithms are Shapiro’s EZW (Embedded Zerotree Wavelet) algorithm [1], Said and Pearlman’s SPIHT (Set Partitioning In Hierarchical Trees) [2], Servetto et al.’s MRWD (Morphological Representation of Wavelet Data) [3], Chai et al.s SLCCA (Significance-Linked Connected Component Analysis) [4], and Taubman’s EBCOT (Embedded Block Coding with Optimized Truncation) [5]. This paper presents an embedded image compression algorithm which we call the PCAS algorithm (wavelet Pixel Classification And Sorting algorithm), because it exploits a new method to classify and sort the wavelet pixels in order to effectively exploit intraband and interband correlation in the wavelet domain. Figure 1 illustrates the wavelet decomposition structure considered in this paper. Previous authors have customarily decomposed the wavelet transformed image into a variety of high-high, high-low, low-high, and low-low subbands. In Figure 1, we have given more suggestive notations to these subbands in keeping with the use we make of these subbands in our method: our H subbands (H standing for “horizontal”) are the low-high subbands of previous authors; our V subbands (V standing for “vertical”) are the former high-low subbands; our D subbands (D standing for “diagonal”) are the former high-high subbands; our A subband (A standing for “average”) is the former low-low subband.

It is well-known that pixels in spatial domain are correlated, which can be successfully reduced by wavelet transform. The linear correlation in wavelet domain is found negligible. However, non-linear correlation is still possible, such as magnitude and sign redundancy [3]. Sign coding has been introduced in [5] to exploit the sign redundancy among pixels, which is adopted by PCAS with some small modification. Our efforts concentrated on the exploitation for magnitude redundancy, a problem with which previous

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works are also concerned. PCAS offers an improved understanding of how to exploit the magnitude redundancy.

This paper is organized as follows. In section II, previous approaches to exploit the magnitude redundancy are reviewed. In section III significance templates are introduced to classify the wavelet pixels. In section IV fractional bit-plane coding and pixel sorting are introduced to help improve the rate-distortion performance, and the PCAS algorithm is developed. Performance evaluation is presented in section V. Section VI gives our conclusions.

2. PREVIOUS APPROACHES

The usual starting point in encoding the wavelet pixel magnitudes is to select a threshold $T$, and then to classify a wavelet pixel $x$ to be significant if its magnitude $|x|$ is $\geq T$, or insignificant if $|x| < T$. The magnitude redundancy includes both intraband correlation and interband correlation.

Both EZW [1] and SPIHT [2] try to exploit the intraband and interband correlation simultaneously with zerotree coding. A zerotree is a subset of spatial-frequency tree consisting entirely of insignificant wavelet pixels, where the spatial-frequency tree naturally comes from the spatial and frequency relationship in the hierarchical pyramid as shown in figure 1. With the identification of most significant wavelet pixels, where the spatial-frequency tree optimally consists entirely of insignificant wavelet pixels using smart zerotree coding, the uncertainty of significant pixels is reduced and coding efficiency is achieved. As mentioned in [3], significant pixels, generally resulting from the edge part of the image, tend to form irregular clusters. Consequently the significant parts within subband are also irregular. Both the irregular significant and insignificant parts cannot be effectively exploited by the regular structured zerotree, which limits the performance of zerotree methods.

Both MRWD [3] and SLCCA [4] try to exploit the intraband correlation by forming the within-subband irregular-shaped clusters which grow from the known significant pixel. The coding result proves that irregular clusters can well capture the significant pixels within subbands. EBCOT [5], which only exploits the intraband correlation, are based on the independent coding and optimization of code-blocks (say 64 x 64 pixels each) within subbands. The outstanding performance of EBCOT demonstrates that intraband correlation is strong enough to provide high coding efficiency if well exploited.

Simply to say, wavelet pixels are implicitly classified in EZW and SPIHT as zerotree pixels or not. While in MRWD, SLCCA, and EBCOT, wavelet pixels are explicitly classified as neighbors of significant pixels or not. In next section, more sophisticated and effective pixel classification is explicitly defined to exploit both intraband and interband correlation.

3. SIGNIFICANCE TEMPLATES FOR WAVELET PIXEL CLASSIFICATION

We envision a wavelet pixel encoding procedure taking place in several “threshold steps”. In each such step, there is a fixed threshold for that step, and the threshold on the next step will be one-half as much. In a given threshold step with threshold $T$, a wavelet pixel $x$ is called oversignificant if $|x| \geq 2T$, significant if $2T > |x| \geq T$, or insignificant if $|x| < T$. At the beginning of the given threshold step, all oversignificant pixels are known to encoder/decoder, each of the remaining wavelet pixels will be encoded with a bit in order to identify whether it is significant or not. The encoder will be an adaptive arithmetic coder [6]. The significance template, discussed in this section, assigns to each wavelet pixel to be encoded a “significance” ($S$), a nonnegative integer used as context by the encoder to compute the prediction probability that the arithmetic coder uses to encode that pixel (the significance will be computable by the decoder because it depends only on the previously decoded pixels and the current pixel’s location in subbands, not its value). Unknown pixels (pixels to be encoded) is classified by its significance, this is called pixel classification. Each significance ($S$) is determined by the intraband and interband significance pair $S_{\text{intra}}, S_{\text{inter}}$, where $S_{\text{intra}} \in \{0, 1, ..., N_{\text{intra}} - 1\}$ and $S_{\text{inter}} \in \{0, 1, ..., N_{\text{inter}} - 1\}$. Define $S = S_{\text{intra}} * N_{\text{inter}} + S_{\text{inter}}$. Accordingly, $S \in \{0, 1, ..., N - 1\}$, where $N = N_{\text{intra}} * N_{\text{inter}}$. In this paper, $N_{\text{intra}} = 15$, $N_{\text{inter}} = 3$, and $N = 45$.

Theoretically, all known pixels (oversignificant, significant, or insignificant) in A, H, V, or D subbands can provide the information for significance assignments. Therefore, the significant template set will include 12 intraband significance templates and 12 interband significance templates. The significance template that is employed can be determined experimentally from a training set of images, assuming that the image to be compressed is of the same type as those in the training set. The definition of significance template is based on the empirical “significance probability”, the probability of a pixel to be significant for a predefined position referring to the known pixel. Figure 2 shows the example of average “intraband significance probability” map. The training set here is the 5-scale wavelet decomposition of 512 x 512 grayscale “Lena” image with the popular Daubechies 9/7 bi-orthogonal wavelet filters. Figure 3 shows the intraband significance template for known oversignificant pixel in horizontal subbands. Lower intraband significance is assigned to lower intraband significance probability, etc. Some intraband significances are missing because they are reserved for other templates. Similarly, the empirical “interband significance probability” is used to define the interband significance template. No intraband or interband significance template is defined for known insignificant pixels, since they provide negligible information. The transposed intraband significance template for horizontal subbands is used for vertical subbands due to symmetry assumption. Average subband shares the same intraband template with diagonal subbands. All four types of subbands share the same interband template. Consequently, there are 4 intraband significance templates defined for oversignificant and significant pixels in H and D subbands. There are 2 interband significance templates defined for oversignificant and significant pixels.

We can now explain how the significance template works. At the very beginning of each given threshold step, the known pixel
set includes all the oversignificant pixels. All unknown pixels are initialized to have the minimum intraband and interband significance, $S = S_{\text{intra}} = S_{\text{inter}} = 0$. Exactly one pixel will be coded at each stage of given threshold step. Before each coding stage, the significance of each unknown pixel will be updated. Each known significant or oversignificant pixel will assign an intraband and interband significance to its neighbors or children according to the predefined significance templates. The current intraband or interband significance of each unknown pixel is the largest one assigned by all known pixels up to now. Consequently significance is obtained with $S = S_{\text{intra}} + N_{\text{intra}} + S_{\text{inter}}$. At each coding stage, one unknown pixel will be coded and identified with its current significance. Which unknown pixel will be coded depends on the algorithm you choose (see section IV for details). After each coding stage, the known pixel set will be updated. We summarize the pixel classification discussed as follows:

- Known wavelet pixels are classified into three types: insignificant, significant, and oversignificant.
- Unknown wavelet pixels are classified into $N$ groups based on their current intraband and interband significances.

### 4. PIXEL SORTING AND PCAS ALGORITHM

In embedded image compression, we want each added encoded bit contributing to the maximum possible distortion reduction. Averagely each significant or oversignificant pixel in higher bit-plane contributes more to distortion reduction than that in lower bit-plane. Hence, bit-plane by bit-plane coding is a natural choice for embedded image compression as in [1] [2] [3] [4] [5]. The efforts are made in [5] to produce good rate-distortion performance within each bit-plane coding. Each bit-plane coding pass is separated into several fractional passes. This is called fractional bit-plane coding. The information contributing more to rate-distortion performance is encoded and transmitted first such as “forward significance propagation pass” and “reverse significance propagation pass” in [5].

While EBCOT applies fractional bit-plane coding for each code-block, PCAS tries to code each subband with several significance scanning passes. This is called pixel sorting, which has the same effect as the fractional bit-plane coding. First, update the intraband and interband significances for each subband, then sort and scan the unknown pixels within each subband according to their significances. Consequently the whole bit-plane coding for each subband can be divided into $k + 1$ fractional bit-plane passes, including $k$ significance scanning passes $P_1, P_2, \ldots, P_k$ followed by the magnitude refinement pass $P_{k+1}$. For pass $P_i$ ($i = 1, 2, \ldots, k$), pixels with $S \geq S_i$ are identified and coded, and $S_j > S_k > \ldots > S_1 = 0$ (minimum significance). Note that each new significant pixel will update the significances for the unknown pixels. With $45$ significances available, $k \leq 45$ and $45$ significance scanning passes are possible, which provides perfect rate-distortion performance within each bit-plane coding.

In the following, we summarize the pixel classification and sorting with encoding process of PCAS algorithm, which is targeted to SNR scalability. If rearrange the subband scanning order, the code with resolution scalability can be easily obtained with the same algorithm.

1. **Initialization**
   
   Set the initial threshold $T = 2^m$ ($m$ is an integer) with $2T$ larger than the maximum magnitude in wavelet domain.

2. **Significance scanning passes**
   
   As indicated above, there are $45$ significance scanning passes in total with $S_i = 45 - i$ for $i = 1, 2, 3, \ldots, 45$). For significance scanning pass $P_i$, each unknown pixels with $S \geq S_i$ will be identified and coded. Following is the detail of significance scanning pass $P_i$ for each subband:

   (1) **Significance updating**
   
   Update the interband and intraband significances based on known pixels.

   (2) **Unknown pixel scanning**
   
   Horizontally scan each unknown pixels with $S \geq S_i$ until all of them are encoded. Code each unknown pixel with context model of its current significance. If a new significant pixel is identified, update the pixel significance and sort all unknown pixels if any pixel significance changed.

   (3) **Sign coding**
   
   Each new identified significant pixel is followed by its sign coding. To exploit the redundancy in pixel sign, context models same to [5] are used for the sign coding of vertical subbands and horizontal subbands. Another $5$ context models same to [5] are used for the sign coding of average subband and diagonal subbands. The modification to [5] is based on the fact that sign correlation in vertical or horizontal subbands is different from that in average or diagonal subbands.

   (3) **Magnitude refinement pass for each subband**
   
   For each oversignificant pixel in the subband, output the refined bit for current threshold. Two context models are used to exploit the redundancy in the refined bits. One is for the first refinement bits. The other is for the rest refinement bits.

   **Threshold updating**
   
   Update threshold $T = T/2$, go to step 2. The process will repeated until the target bit rate is achieved.

   The decoding process is straightforward and can be obtained by simply reversing the encoding process. One of the most important characteristic of PCAS is that the pixel classification and sorting are progressive, since each new identified significant pixel will update the significances of all unknown pixels. The main contribution of PCAS is the clear description of pixel classification and sorting. As a novel technique, pixel classification and sorting is simple, yet efficient for embedded image compression. Some
optimization may be introduced to best exploit the benefit of pixel classification and sorting. Finally, “pixel classification and sorting technique” can also be used for DCT based image coding, wavelet-packet based image coding, etc.

5. PERFORMANCE EVALUATION

PCAS algorithm is evaluated on 2 natural 512 x 512 grayscale images, “Lena” and “Goldhill”. The performance is compared with EZW [1], SPIHT [2], MRWD [3], SLCCA [4], and EBCOT [5]. SPIHT algorithm is the one with arithmetic coding. MRWD result is from algorithm 2, which produces the embedded code. SLCCA result is for the target bit rate, i.e., the set of rate-distortion results may not come from the same image code. EBCOT result is from the generic case, which provides the best SNR scalability. PCAS result is in actual size and obtained from the same embedded code. The unit for rate is bit per pixel (bpp). The distortion is measured by the peak signal to noise ratio in dB: $PSNR = 10\log_{10}(255^2/MSE)$, where $MSE$ denotes the mean-square-error between the original and reconstructed 8-bit grayscale image. For two test images, we use the popular Daubechies 9/7 bi-orthogonal wavelet filters with a 5-scale wavelet decomposition. “Periodic” extension mode is used at the image boundaries.

Two results are provided for PCAS. PCAS I is the case to exploit both the interband and intraband correlation with pixel classification and sorting based on both interband and intraband significances. PCASI is the case to exploit the intraband correlation only; the unknown pixels within subbands are classified into 15 groups based on intraband significances only. As a result, the subbands can be separately coded in the second case, which will simplify the coding and help prevent the error propagation from subband to subband if carefully handled. Table 1 shows the $PSNR$ comparison on “Lena” and “Goldhill” images at different rates.

For “Lena” and “Goldhill”, PCAS I consistently outperforms EZW, SPIHT, MRWD, SLCCA, and EBCOT algorithms. PCAS II provides about same excellent performance with average 0.07 dB loss in $PSNR$ compared to PCAS I, which means interband correlation only contributes about 0.07 dB improvement to the $PSNR$ performance averagely. This helps prove that interband correlation is stronger and more important than interband correlation and PCAS exploits the intraband correlation successfully. For convenience, table 1 also includes the result from EQ algorithms [7], a state-of-the-art non-embedded image code. Averagely, EQ outperforms PCAS in $PSNR$ by 0.06dB (for “Goldhill”) and 0.23dB (for “Lena”). This 0.06dB or 0.23dB $PSNR$ penalty is negligible compared to the most important embedding property for some applications such as wireless communication and Internet browsing.

6. CONCLUSIONS

We propose a new embedded image compression algorithm (PCAS) based on progressive pixel classification and sorting in wavelet domain. The PCAS algorithm offers excellent compression performance with both SNR scalability and resolution scalability. The concepts of pixel classification and sorting are introduced to effectively exploit both the intraband and interband correlation in wavelet domain. While no optimization is involved, the PCAS algorithm produces finely embedded image code with bit-plane based coding and fractional bit-plane coding naturally from pixel classification.

Table 1. $PSNR$ results for various images and bit-rates

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7. ACKNOWLEDGMENTS

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8. REFERENCES