Robust Indirect Adaptive Fuzzy Decentralized Control for a Class of Nonlinear Interconnected Systems

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Abstract—The problem of controlling a large scale uncertain nonlinear system consisting of interconnected subsystems is addressed. The interconnections between different subsystems are assumed to be bounded. Fuzzy logic systems are used to approximate the unknown dynamics of different subsystems. Then, robust indirect adaptive fuzzy decentralized controllers are derived. Moreover, the uncertainty bounds resulted from approximation are tuned adaptively so that global system stability is guaranteed and all closed loop signals are assured to be bounded. Simulation results are performed to validate the approach.

Keywords—Adaptive Control, Decentralized Fuzzy Systems, Nonlinear Systems

I. INTRODUCTION

The justification behind the introduction of decentralized control was to attack the problems encountered when controlling interconnected systems. Those include the complexity in designing centralized control methodologies, physical limitations in information exchange, and other reasons that may cause centralized control to be more complex. So, decentralized control would be more suitable for interconnected systems. However, the concept of adaptive control would be inevitable when parameters uncertainty is involved which motivated researchers to use adaptive control techniques in the decentralized control approach. Decentralized model reference adaptive control was successfully used and promising results were obtained [1-3]. Even though those approaches are limited to linear subsystem with possible nonlinear interconnections, they presented a good basis for later researches. Decentralized adaptive control techniques were successful used to handle the control of robotic manipulators [4, 5], and this was the spur for later researches on decentralized adaptive control of nonlinear subsystems (see [6, 7] and the references therein).

All the schemes above require the exact knowledge of the system dynamics, and this was the justification for using the fuzzy logic as a universal approximator for the case of unknown dynamics [8]. Decentralized adaptive fuzzy control approaches were suggested and promising results were obtained even if the dynamics of the subsystems is unknown (see [9-11] and references therein). However, till now the uncertainty resulted in functional approximation, in the decentralized adaptive fuzzy control, is not yet addressed.

In this paper, we present a new control scheme that takes the uncertainty resulted from the unknown functional approximation in decentralized adaptive fuzzy control into consideration. An indirect adaptive fuzzy decentralized control scheme is presented that guarantee global system stability performance.

The rest of the paper is organized as follows. Section 2 gives the problem statement. In section 3, fuzzy logic approximation is briefly described. The suggested control approach is explained in section 4, and section 5 gives the concluding remarks.

II. PROBLEM STATEMENT

Suppose that we have a large-scale nonlinear system composed of N subsystems, and each subsystem \( S_i \) is described by:

\[
\begin{align*}
\dot{X}_{i1} &= x_{i2} \\
\dot{x}_{i1} &= f_i(X_i) + g_i(X_i)u_i + \Delta_i(X_{1},X_{2},...,X_{N})
\end{align*}
\]

(2.1)

Where \( X_i = [x_{i1},x_{i2},...,x_{im}]^T \) is the \( m_i \)-dimensional state vector, \( u_i \in R \) is the control input, \( f_i(X_i) \) and \( g_i(X_i) \) are unknown but continuous functions, and \( \Delta_i(X_{1},X_{2},...,X_{N}) \) are the interconnections among the subsystems \( i = 1,2,...,N \).

The objective of this paper is to design robust indirect adaptive control laws \( u_i \) such that global stability is guaranteed and all closed loop signals are assured to be bounded. However, the following assumptions are needed to be satisfied:

A1. All references \( y_{ir} \) are assumed to be bounded and available for measurement. The references \( y_{ir} \) have up to \( (m_i - 1) \) bounded and measurable derivatives.

Let us define the output error of the \( i \)-th subsystem to be:

\[
e_{io} = y - y_{ir}
\]

(2.2)

Then we can easily find the error vector of the \( i \)-th subsystem to be:

\[
e_i = [e_{io}, e_{io}, ..., e_{io}^{(m_i - 1)}]^T
\]

(2.3)

Now, consider the following assumption concerning the interconnection.
functions of approximation to estimate those functions [8]. For each unknown, then we use the concept of fuzzy logic rule, single tone fuzzifier, center average defuzzifier, and single-output fuzzy logic system with the product-inference interconnection between the $i^{th}$ and $j^{th}$ subsystem. Then it is described as:

$$
\Delta_l(X_1, X_2, ..., X_m) \leq \sum_{j=1}^{m} q_{ij} \|v_j\|_2 \leq \sigma_i
$$

(2.4)

Where $\|\cdot\|_2$ is the Euclidean vector norm and $\sigma_i$ are positive constants. So, according to (2.4), the interconnections are assumed to be bounded and their bounds are known, say $\sigma_i$.

III. Fuzzy Logic Approximation

Since the functions $f_i(X_j)$ and $g_i(X_j)$ are assumed to be unknown, then we use the concept of fuzzy logic approximation to estimate those functions [8]. For each function of $f_i(X_j)$ and $g_i(X_j)$, consider an $m_i$-input and a single-output fuzzy logic system with the product-inference rule, single tone fuzzifier, center average defuzzifier, and Gaussian membership function given by $n_r$ fuzzy IF-THEN rules:

$$
R_i^r: \text{if } x_{i1} \text{ is } A_{i1}^r(x_{i1}) \text{ and } ... \text{ and } x_{im_i} \text{ is } A_{im_i}^r(x_{im_i}) \text{ then } v_i = b_i^r
$$

(3.1)

Where $R_i^r$ denotes the $r^{th}$ rule of the $i^{th}$ plant function to be approximated, $X_i = [x_{i1}, x_{i2}, ..., x_{im_i}]^T$ and $v_i$ are the input and output of the $i^{th}$ fuzzy logic respectively, $b_i^r$ is the fuzzy single tone for the output of the $r^{th}$ rule-$i^{th}$ plant function, and $A_{i1}^r(x_{i1}) ... A_{im_i}^r(x_{im_i})$ are fuzzy sets characterized by Gaussian membership functions:

$$
\mu_{\mathcal{A}_i}(x_i) = \exp \left\{ - \left( \frac{x_i - \sigma_i^r}{\sigma_i^r} \right)^2 \right\}
$$

(3.2)

where $\sigma_i^r$ and $\sigma_i$ are the center and width of membership function $\mu_{\mathcal{A}_i}(x_i)$. The output of the fuzzy logic system can be described as:

$$
v_i = [b_1^r b_2^r ... b_n^r] \begin{bmatrix} w_1^r(X_i) & w_2^r(X_i) & ... & w_n^r(X_i) \end{bmatrix}^T
$$

(3.3)

with

$$
w_i^r = \frac{\prod_{j=1}^{n_r} w_j(x_i)}{\sum_{j=1}^{n_r} w_j(x_i)}
$$

(3.4)

Or in matrix form:

$$
v_i = BW(X_i)
$$

(3.5)

where $B \in \mathbb{R}_m^{n \times m}$ is the parameter vector, and $W: X_i \rightarrow \mathbb{R}_m^{n \times m}$ are the fuzzy basis functions. The fuzzy rules described by (3.1) is said to be complete, if for any $X_i \in \mathbb{R}_m$, there is at least one fuzzy rule to be fired, i.e., $\sum_{r=1}^{n_r} \Pi_{i=1}^{n_i} \mu_{\mathcal{A}_i}^r(x_i) > 0$. It is well known that the fuzzy logic systems described by (3.5) are universal approximator and capable of approximating any continuous function, i.e., for any real functions $f_i: \mathbb{R}_m^{m_i} \rightarrow \mathbb{R}$ in a compact set $X_i \subset \mathbb{R}_m$ and any positive real constants $k_i$ there exist fuzzy logic systems such that [8]:

$$
sup |f_i(X_i) - f_i(X_i)| < k_i
$$

(3.6)

From (3.6), we can easily deduce that:

$$
f_i(X_i) = B_i^T W(X_i) + h_i
$$

(3.7)

Knowing that $k_i$ are the approximation errors of $f_i(X_i)$ satisfying:

$$
sup |k_i| < q_i
$$

(3.8)

$q_i$ are positive constants and $B_i^T$ are the optimal parameter vectors:

$$
B_i^* = \arg \min \{ sup |B_i^T W(X_i) - f_i(X_i)| \}
$$

(3.9)

Similarly,

$$
g_i(X_i) = B_g^* W(X_i) + h_i
$$

(3.10)

$k_i$ are the approximation errors of $g_i(X_i)$ satisfying:

$$
sup |k_i| < q_i
$$

(3.11)

$q_i$ are positive constants and $B_i^*$ are the optimal parameter vectors:

$$
B_i^* = \arg \min \{ sup |B_i^* W(X_i) - g_i(X_i)| \}
$$

(3.12)

Since $B_i^T W(X_i)$ and $B_i^* W(X_i)$ are unknown, then we can start with some known fuzzy approximators $B_i^T W(X_i)$ and $B_i^* W(X_i)$ such that:

$$
B_i^T W(X_i) = B_i^* W(X_i) + \rho_f
$$

(3.13)

$$
B_i^* W(X_i) = B_i^* W(X_i) + \rho_g
$$

(3.14)

Knowing that:

$$
sup |\rho_f| < p_f
$$

(3.15)

$$
sup |\rho_g| < p_g
$$

(3.16)

where $p_f$ and $p_g$ are positive constants. The nominal fuzzy logic system can be chosen as:
\[ f_i^o(X_i/\theta_i^p) = \theta_i^{\text{T}} \phi_i^o(X_i) \] (3.17)

\[ g_i^o(X_i/\theta_i^p) = \theta_i^{\text{T}} \phi_{gi}^o(X_i) \] (3.18)

Where \( \theta_i^p \) and \( \theta_i^o \) are the parameters vectors, and \( \phi_i^o(X_i) \) and \( \phi_{gi}^o(X_i) \) are the regressor vectors described by (3.4).

IV. ROBUST ADAPTIVE CONTROL DESIGN

To achieve the stated control objective, let us define the \( i^\text{th} \) filtered tracking error corresponding to the \( i^\text{th} \) subsystem:

\[ s_i(t) = \left( \frac{d}{dt} + \lambda_i \right)^{m_i-1} e_{io} \] (4.1)

with \( \lambda_i > 0 \) and \( e_{io} = y_{ir} - y_i \). Equation (4.1) can be rewritten as:

\[ s_i(t) = \Lambda_i^{T} e_i \] (4.2)

where:

\[ \Lambda_i^{T} = \left( \lambda_i^{m_i-1}, (m_i - 1)\lambda_i^{m_i-2}, ..., (m_i - 1)\lambda_i \right) \]

Taking the time derivative for equation (4.2), we obtain:

\[ \dot{s}_i(t) = \Lambda_i^{T} \dot{e}_i + f_i(X_i) + g_i(X_i)u_i + \Delta_i(X_i, X_{2, \ldots, n}) \]

\[ -y_{ir}^{(mi)} + u_{fi} \] (4.3)

\[ \dot{y}_{ir}^{(mi)} + u_{fi} \] (4.4)

\[ \dot{\phi}_{fi} = b_{fi}s_i(t) \] (4.5)

\[ \dot{k}_{fi} = b_{fi}s_i(t) \] (4.6)

\[ \dot{\phi}_{gi} = -b_{gi}s_i(t) \left( k_{di} \cdot s_i(t) + \Lambda_{vi}^{T} e_i + f_i^o(X_i/\theta_i^p) \right) - y_{ir}^{(mi)} + u_{fi} \] (4.7)

\[ \dot{k}_{gi} = -b_{gi}s_i(t) \left( k_{di} \cdot s_i(t) + \Lambda_{vi}^{T} e_i + f_i^o(X_i/\theta_i^p) \right) - y_{ir}^{(mi)} + u_{fi} \] (4.8)

Where:

\[ u_{fi} = \frac{1}{M_i} \left( \rho_i \cdot \text{sgn}(s_i(t)) + \dot{\phi}_{fi} + \dot{k}_{fi} \right) \]

\[ M_i = 1 + \frac{\rho_{gi} \cdot k_{gi}}{\theta_i^o(X_i/\theta_i^p)} \] (4.10)

achieve global system stability and guarantee the boundedness of all closed loop signals involved.

Where \( b_{fi}, b_{gi}, b_{si}, \) and \( b_{ai} \) are positive constants, and \( |\rho_i| \geq \sigma_i \).

Proof: Consider the Lyapunov candidate:

\[ V = \frac{1}{2} \sum_{i=1}^{N} \left( s_i^2(t) + \frac{\dot{\phi}_{fi} \cdot \dot{k}_{fi}}{b_{fi}} + \frac{\dot{k}_{fi} \cdot \dot{k}_{fi}}{b_{fi}} + \frac{\dot{\phi}_{gi} \cdot \dot{\phi}_{gi}}{b_{gi}} \right) \] (4.11)

Taking the time derivative of (4.11), we obtain:

\[ \dot{V} = \sum_{i=1}^{N} \left( \dot{s}_i(t) \cdot s_i(t) + \frac{\dot{\phi}_{fi} \cdot \dot{k}_{fi}}{b_{fi}} + \frac{\dot{k}_{fi} \cdot \dot{k}_{fi}}{b_{fi}} + \frac{\dot{\phi}_{gi} \cdot \dot{\phi}_{gi}}{b_{gi}} \right) \] (4.12)

Substituting (4.4) into (4.12), we obtain:

\[ \dot{V} = \sum_{i=1}^{N} \left( s_i(t) \cdot \Delta_i(X_i, X_{2, \ldots, n}) - y_{ir}^{(mi)} \right) + \frac{\dot{\phi}_{fi} \cdot \dot{k}_{fi}}{b_{fi}} + \frac{\dot{\phi}_{gi} \cdot \dot{\phi}_{gi}}{b_{gi}} + \frac{\dot{\phi}_{gi} \cdot \dot{\phi}_{gi}}{b_{gi}} \] (4.13)
Using the facts given in equations (3.7), (3.10), (3.13), and (3.14), then we can easily deduce that:

\[ f_i(X_i) = \rho_{fi} + k_{fi} + \tilde{f}_i(X_i/\theta_{fi}^o) \]  
\[ g_i(X_i) = \rho_{gi} + k_{gi} + \tilde{g}_i(X_i/\theta_{gi}^o) \]

Substituting the control law given in (4.4) and equations (4.14), (4.15), (1.17), and (3.18) into (4.13), we obtain:

\[
\dot{V} = \sum_{i=1}^{N} s_i(t) \left[ A_{yi} e_i - y_{ir}^{(mi)} + \rho_{fi} + k_{fi} + \tilde{f}_i(X_i/\theta_{fi}^o) \right] \\
- \frac{\rho_{gi} + k_{gi} + \theta_{gi}^{\sigma r} \cdot \phi_{gi}^{\sigma r}(X_i)}{\theta_{gi}^{\sigma r} \cdot \phi_{gi}^{\sigma r}(X_i)} \left( A_{yi} e_i + \theta_{fi}^{\sigma r} \cdot \phi_{fi}(X_i) \right) \\
+ k_{dli} s_i(t) - y_{ir}^{(mi)} + u_{fi} + \Delta_i(X_1, X_2, \ldots, X_N) \\
+ \frac{\rho_{fi} \tilde{f}_i}{b_{li}} + \frac{k_{fi} k_{fi}}{b_{li}} + \frac{\rho_{gi} k_{gi}}{b_{li}} + \frac{k_{dli} k_{dli}}{b_{li}}
\]  
(4.16)

After several mathematical manipulations on (4.16) and using the facts that:

\[ \tilde{\rho}_{fi} = \tilde{\rho}_{fi} - \rho_{fi} \]
\[ \tilde{\rho}_{gi} = \tilde{\rho}_{gi} - \rho_{gi} \]
\[ \tilde{k}_{fi} = \tilde{k}_{fi} - k_{fi} \]
\[ \tilde{k}_{gi} = \tilde{k}_{gi} - k_{gi} \]

we can obtain:

\[
\dot{V} = \sum_{i=1}^{N} s_i(t) \left[ -u_{fi} + \tilde{\rho}_{fi} + \tilde{k}_{fi} - \frac{\tilde{\rho}_{gi} + \tilde{k}_{gi}}{\theta_{gi}^{\sigma r} \cdot \phi_{gi}^{\sigma r}(X_i)} \left( A_{yi} e_i \right) \\
+ \tilde{f}_i(X_i/\theta_{fi}^o) + y_{ir}^{(mi)} + k_{dli} s_i(t) + u_{fi} \right] \\
+ \Delta_i(X_1, X_2, \ldots, X_N) \right] + \tilde{\rho}_{fi} \left( \frac{\tilde{f}_i}{b_{li}} - s_i(t) \right) \\
+ \tilde{k}_{fi} \left( \frac{\tilde{k}_{fi}}{b_{li}} - s_i(t) \right) - \frac{\tilde{\rho}_{gi} + \tilde{k}_{gi}}{\theta_{gi}^{\sigma r} \cdot \phi_{gi}^{\sigma r}(X_i)} \left( A_{yi} e_i \right) \\
+ \tilde{f}_i(X_i/\theta_{fi}^o) - y_{ir}^{(mi)} + u_{fi} \right] + \tilde{k}_{gi} \left( \frac{\tilde{g}_i}{b_{li}} + \frac{s_i(t)}{\theta_{gi}^{\sigma r} \cdot \phi_{gi}^{\sigma r}(X_i)} \right) \left( A_{yi} e_i + f_{ir}^o(X_i/\theta_{fi}^o) - y_{ir}^{(mi)} + u_{fi} \right)
\]  
(4.17)

Using the parameters updates laws given in equations (4.5), (4.6), (4.7), and (4.8) in equation (4.17), we get:

\[
\dot{V} = -\sum_{i=1}^{N} \left[ k_{dli} s_i(t) + \Delta_i s_i(t) - \rho_{fi} s_i(t) \cdot sgn(s_i(t)) \right]
\]  
(4.18)

From (4.18) it is clear that \( s_i(t) \in L_2 \cap L_{\infty} \) and \( \tilde{\rho}_{fi}, \tilde{\rho}_{gi}, \tilde{k}_{fi}, \tilde{k}_{gi} \in L_{\infty} \). Since \( \rho_{fi}, \rho_{gi}, k_{fi}, \text{ and } k_{gi} \) are bounded, then \( \tilde{\rho}_{fi}, \tilde{\rho}_{gi}, \tilde{k}_{fi}, \text{ and } \tilde{k}_{gi} \) are also bounded. From equation (4.4) (4.14) and (4.15), we can easily conclude that \( s_i(t) \in L_{\infty} \). Since \( s_i(t) \in L_1 \cap L_{\infty} \) and \( \tilde{\phi}_{gi}(s_i(t)) \rightarrow 0 \) as \( t \rightarrow \infty \). This would make \( e_i(t) \) converging to \( \Omega_{e_i} \) as \( t \rightarrow \infty \).

V. Conclusion

In this paper, we have presented a new control approach for controlling nonlinear interconnected systems with uncertain dynamics. We suggested indirect adaptive fuzzy decentralized control for attacking the control problem in hand. Fuzzy function approximators were used to approximate the unknown dynamics of the subsystems. The uncertainty arising from the functional approximation was compensated and an efficient approach is resulted.

REFERENCES

