Abstract—This paper deals with the haptic rendering of collisions between a human operator and rigid objects in a virtual environment. The focus is on high-velocity impacts on a rigid wall. After discussing the importance of velocity measurements in contrast to force measurements, a new haptic device is introduced. The new device is capable of applying directly an ‘impulse’ to the operator, similar to collisions occurring in the real world. Based on a Poisson-model of the rigid virtual object, the necessary change of momentum of the operator’s hand is being estimated online. A momentum wheel, under velocity-control, is engaged through an electromagnetic toothed clutch, at the estimated instant of impact. The resulting immediate change in momentum is thought to be essential for giving interactions with rigid virtual objects a realistic feel. Rather than generating so-called ‘impulsive forces’ with big motors, this approach relies only on small motors and is thus intrinsically safer. Experiments with rigid virtual objects a realistic feel. Rather than generating so-called ‘impulsive forces’ with big motors, this approach relies only on small motors and is thus intrinsically safer. Experiments performed on a one-degree of freedom setup. Conclusions are drawn and future directions of this research are sketched.

I. INTRODUCTION

Lots of research has been done in the haptic field to solve the problem of rendering virtual objects with high stiffness. An unpredictable human, interfacing a discretized virtual world, sustained by high control gains, manipulating a haptic interface equipped with limited resolution encoders or noisy force sensor, tends to encounter unexpected behavior.

A first group of researchers focused on the stability issue of this control problem [1]. It became soon clear that traditional penalty-based control methods, making use of non-passive algorithms, were of no good. Constraint-based control methods were introduced by [2]. Next, attention turned towards control methods based on passivity theory, allowing for a decoupling between the design of the control method and that of the virtual world. [3] introduced a virtual coupling method, which is based on a frequency-domain analysis of the system’s passivity. More recently [4] presented a time-domain passivity based control method.

Meanwhile, a second group of researchers began implementing impulse-based control methods [5], [6], [7], [8]. They were influenced by the results obtained in the Computer Graphics Community [9], [10], where successfully thousands of simultaneous collisions could be simulated. In a sense these methods are closer to the real physical phenomenon [11], where Poisson’s equation governs the relation between the velocity of objects before and after collision. Recently, an extension to these methods was proposed in [12], [13]. They generate not only a high impulse upon intrusion of the surface, but a complete pre-computed force transient. Unfortunately, this asks for a large database of calculated force transients.

While these control efforts continued, other researchers developed new hardware, designed to render hard contacts; haptics equipped with controllable brakes, being inherently passive [14]; haptics with a hybrid active/passive actuator, combining motor and brake [15], solving the loss of actuation power in the direction of the brakes. Nonholonomic haptics, capable of creating adaptable constraints (and more complex than brakes allow), on the operator’s movements [16]. Also encountered type haptics [17] represent virtual objects through creation of real constraints.

The desire for specific hardware to render rigid virtual objects is justified. Despite all efforts from people working on control methods, it remains a fact that current available commercial haptic displays simply cannot render rigid objects realistically. The design of [8] is probably the best implementation of rigid virtual objects so far. They implemented an impulse-based control, applying ‘impulsive forces’ upon wall contact. To reach sufficient realism they were forced to equip their haptic with powerful motors, capable of supplying enough power to generate a big momentum change in a short period. By doing so, the user is at risk of being inflicted by a same amount of power, applied to him over a continuous and long period of time. Clearly this is also not an ideal situation.

In this paper we introduce a new haptic, designed for impulse-based control of rigid virtual environments. Rather than approximating the impulses of collision by so-called ‘impulsive forces,’ the new haptic directly applies a desired ‘impulse’ to the operator. Just as in [18] a momentum wheel is used to store momentum, but here the momentum is transmitted instantly at the moment of impact realizing the feeling of contact with a rigid object. Furthermore, the new haptic makes only use of small motors and is therefore intrinsically safer than e.g. [8]. For simplicity the following discussion treats only the one-dimensional case. A generalization to multiple degrees of freedom can be found in [8].

The discussion in this article is build up as follows, in the next section a brief overview of impulse-based control is given. Its strength and simplicity are explained. Section (III) introduces the concepts forming the basis of the new haptic device. Section (IV) describes a prototype of the new...
haptic. Some basic tests are included. An experiment of an interaction with a rigid virtual wall is presented in section (V). Conclusions are drawn in (VI) and further work is sketched.

II. IMPULSE-BASED CONTROL

This paragraph explains the impulse-based control method. First, the physical principles are shortly reviewed and current implementations of the control method are explained. Then, an analysis of a high-velocity collision with a real rigid wall is done. The strength of impulse-based control methods, in respect to methods that need force measurements such as those derived from [4], is shown.

A. Background

The basis for impulse-based control is the ‘Law of Conservation of Momentum.’ This law, applied to collisions of n ideal objects, says that

$$\sum_{i=1}^{n} M_i v_i(t_e) = \sum_{i=1}^{n} M_i v_i(t_i). \quad (1)$$

Which means that the sum of the momentums before the collisions, at time $t_i$, of the objects $i$, with inertia $M_i$ and generalized velocity $v_i$ at time $t_e$, after occurrence of the collisions, equals the sum of momentums before the collisions, at time $t_i$.

In the real world, collisions are not lossless. Depending on the shape of the contact surfaces, the magnitude of the relative velocities and so on [11], a certain amount of energy (almost constant ratio) will be dissipated during each collision. For a collision between two objects, this phenomenon is described by Poisson’s model of restitution, relating the velocities of the objects as

$$v_2(t_e) - v_1(t_e) = -e (v_2(t_i) - v_1(t_i)). \quad (2)$$

Here, the parameter $e$ stands for the restitution coefficient. It is a dimensionless value between 0 (inelastic impact) and 1 (perfectly elastic impact). For a collision between an object with velocity $v_h$, and a rigid wall, (2) can be further simplified to

$$v_h(t_e) = -ev_h(t_i), \quad (3)$$

because the wall’s mass is so high that its change in velocity is negligible.

B. Current Implementations of Impulse-based Control

Eq. (3) doesn’t tell us anything about the forces during the collision. However, recalling Newton’s Second Law, the change of momentum of an object with mass $M_h$ can be related as

$$M_h (v_h(t_e) - v_h(t_i)) = \int_{t_i}^{t_e} f_h(t)dt = p_h, \quad (4)$$

to the integral of the forces $f_h$ applied to it during the collision period $[t_i, t_e]$. This value is called the impulse $p_h$. In the case that both object and wall are infinitely rigid, the interval $[t_i, t_e]$ of collision will be infinitesimally short and the force will have to be infinitesimally high. In real interactions this is never the case, both the collision interval and the interaction forces will be bounded.

Current impulse-based control methods try to realize the desired impulse $p_h = p_{des}$ as fast as possible. Practically this means that upon intrusion into a wall, they will deliver a force $f(k)$ like

$$f(k) = \begin{cases} \frac{p_h(k)}{\Delta t} & \text{if } p_h(k) \leq f_{max}\Delta t \\ f_{max} & \text{if } p_h(k) > f_{max}\Delta t \end{cases} \quad (5)$$

with $p_h(0) = p_{des}$, $f_{max}$ the maximal motor output force and $\Delta t$ the period of the motor output. The forces generated according to equations (5), (6) are called ‘impulsive forces’ [7] and these control methods will be referred to as ‘impulse-based force control’ methods in the rest of the paper.

C. Interaction with a Real Wall: Basic Experiment

In the following basic experiment a one-degree of freedom haptic display, held by a human operator, is collided at high speed against a rigid wall (mechanical stopper). Fig.1 plots the measurements at 1024 Hz of position, velocity and force during this collision. The graphs at the left give an overview, while the right graphs zoom in on the actual collision-scene.

The overview figures show how the operator starts from a distance of about 10cm from the surface, gains momentum (speeding up to 1m/s) and hits the stopper at around 0.4s. According to the detailed position and velocity plots, the operator is bounced back from the surface and reaches a stable contact after some bounces. The velocity plot confirms Poisson’s equation (3).

However, when observing the force signals, it is almost impossible to come to this conclusion. It is not even clear that contact with the wall has been lost and the impact itself is observed too late. Moreover, by only looking at the force data, one could get the idea that this collision might be represented by typical haptic devices such as the PHANToMTM or Impulse EngineTM, whose output forces are not much below the measured forces.
In fact, the size of the interaction force is drastically underestimated. This can be verified through analysis of the velocity signal, changing $\Delta v = 1.7m/s$ over $1/1024s$ for a combined apparent inertia of the haptic device (0.088kg) and an estimate of the mass of the operator’s hand (0.06kg) a lower bound on the peak force can be found to be 257.8N.

From above observations, it became clear that velocity measurements, although easily obtained, present us with invaluable information. Impulse-based control methods seem therefore well suited for applications needing high velocity interactions with rigid virtual objects. Force measurements, on the other hand should be treated with the necessary amount of suspicion. Control methods as [4] heavily depending on these force measurements, will only bring limited satisfaction in this case.

However, an important remark must be made. Stability of impulse-based methods is on its turn dependent on a correct estimation of the inertia of the colliding elements. It is clear that overestimation of these inertia leads to overreaction and could possibly lead to instability, since it could result in restitution coefficients exceeding 1. In such case, the ‘impulse-based force control’ methods of section II-B, equipped with motors capable of providing 257.8N continuously, could result in some major damage.

A new haptic display is presented next. The new hardware was designed to allow true ‘impulse-based control,’ generating big impulses relying only on small motors.

III. A TRULY IMPULSIVE HAPTIC: CONCEPTS

This section introduces the requirements and concepts at the basis of the new design. Following requirements were laid forward:

1) for safety reasons only small motors, as can be found in typical commercial haptic devices, should be used.
2) the system needs to render qualitative interactions with rigid virtual objects over a broad velocity-spectrum (from stable contact to high velocity impacts).

Following concepts formed the foundation of the design:

**Concept 1:** Distribution of Tasks. Two different tasks were identified to be necessary to fulfill above requirements:

- the generation of a continuous force output, and
- the rapid building-up of a certain amount of momentum, to realize an impulse.

To realize this first task, the small motor (requirement 1) should be equipped with a transmission element with a relatively high reduction ratio, to generate sufficient output force. For the second task, a transmission element with a low reduction ratio seems more appropriate at first sight\(^1\). To meet simultaneously with above contradictory tasks, it was decided to use two small motors for every one degree of freedom. The first being responsible for the force generation, the second is used for the impulse generation.

\(^1\)In such case the motor runs at high speeds and velocity control, through differentiation of the motor’s encoder, becomes relatively easy and accurate.

**Concept 2:** Separation in Time. Above tasks can be separated in time as follows: the generation of

- an impulse should be done at the exact instant of impact.
- a continuous force when stable contact is realized.

because the instant of impact is supposed to be very small, the contribution of the continuous force at this period becomes comparatively negligible. The desire to make a physical decoupling is a logical consequence. Basically, the proposed system exists of two haptics in one.

**Concept 3:** Integration of Stylus-based and Encountered-type Haptic. Above distribution of tasks, and separation in time, translates naturally to the use of both

- a stylus-based haptic: ideal for generation of continuous contacts with surfaces, these being rigid or deformable, simple or complex, equipped with necessary control methods.
- an encountered-based haptic: unnoticeable in free space\(^2\), but excellent for generating high-velocity impacts at localized contacts.

IV. PROTOTYPE OF A TRULY IMPULSIVE HAPTIC

A. Overview of the System

A prototype of an impulsive haptic was designed according to above requirements and concepts. The hardware is depicted in Fig.2. The system can be subdivided into three components.

a) ‘Force Generator’: This part corresponds to a typical stylus-based haptic display. A motor (Maxon Re25), equipped with position encoder (HEDS-5540, 500 cnts/round, quadrature encoding), actuates the stylus through a wire transmission and drive-angle (reduction ratio 9:1). The maximum force output at the tip of the handle is approximately 10N. A force sensor (Micro, BL-Tech\(^\text{TM}\), 0.05N resolution) is incorporated in the handle.

\(^2\)In this application free space extends towards all low impulsive situations.
b) ‘Coupling Part’: This element is responsible for transmitting the impulses towards the handle. An electromagnetically actuated toothed clutch (Ogura MZ2.5) was chosen to fulfill this task. Only at instants of impact, the connection is made, at other times both sides are completely disconnected. The clutch can transmit high impulses accurately. If the rotation speed is low enough practically no slip occurs. Engaging the clutch happens through off on switching of a 24V-source. Although the clutch is not so fast in its response, the response shows a good repeatability.

c) ‘Momentum Generator’: This part has to build up a desired amount of momentum. Since the clutch only engages well at low rotational speeds, we cannot connect the clutch directly to a motor operating at high speeds. Instead the motor is used to drive a momentum wheel through a gear train. The momentum stored in this momentum wheel is then transferred by engaging the clutch.

The haptic device is controlled in real-time (VxWorks™ at 1024Hz). Force and position measurements are measured, velocity is obtained through backwards differentiation of the encoder signals. Raw force data is being output as well as forces filtered through a 4th order Bessel filter (cutoff 50Hz).

B. Physical Interpretation

When contacting a real surface an impulse will be felt, according to Poisson’s equation. Usually the rigid surfaces we contact are stationary and the relative velocity in (2) reduces to the absolute velocity of the operator (3). For a surface with restitution coefficient \( e_{surf} \) and an initial velocity \( v_h(t_i) \) of the operator, the velocity upon exit of the surface will be

\[
v_h(t_e) = -e_{surf} v_h(t_i).
\]

Eq. (7) is in fact the target of the control action. In this new approach, the actual collision occurs between the operator and a non-stationary element: the momentum wheel. The impulse itself is generated at the instant that the clutch engages. The human will feel the collision through the surface of the teeth of the clutch. By proper control of the speed of the momentum wheel, (7) can be realized. It can be expected that in this case the user will not be able to distinguish the collision from a normal one.

The necessary rotation speed of the momentum wheel can be determined as follows\(^3\). First, it is assumed that the clutch is properly engaged i.e. \( e_{real} = 0 \), which makes that

\[
v_h(t_e) - v_{wh}(t_e) = e_{real}(v_h(t_i) - v_{wh}(t_i))
\]

reduces to

\[
v_h(t_e) = v_{wh}(t_e).
\]

At the end of the collision \( t_e \), the operator’s hand moves at the same speed \( v_h \) as the momentum wheel \( v_{wh} \). In this case, no further impulse can be exchanged and the clutch can be safely disengaged.

\(^3\)For notational convenience \( M \) and \( v \) are used. They actually correspond to respectively inertia and rotational velocity.

According to the law of conservation of momentum,

\[
M_{wh} v_{wh}(t_i) + M_h v_h(t_i) = (M_{wh} + M_h) v_h(t_e)
\]

holds. Where \( M_{wh} \) represents the inertia of all moving parts of the Momentum Generator. And \( M_h \) is not only the inertia of the hand, but also of all moving parts of the Force Generator. Then filling (7) in, into (10) and solving for \( v_{wh}(t_i) \), the necessary rotation speed of the momentum wheel can be found as

\[
v_{wh}(t_i) = -\frac{e_{surf} M_{wh} + (1 + e_{surf}) M_h}{M_{wh}} v_h(t_i).
\]

To reach the special case of a complete stop after collision, \( e = 0 \), the necessary speed of the momentum wheel is

\[
v_{wh}(t_i) = -\frac{M_h}{M_{wh}} v_h(t_i).
\]

And to realize the special case of a perfect bounce, \( e = 1 \), a speed

\[
v_{wh}(t_i) = -\frac{M_{wh} + 2M_h}{M_{wh}} v_h(t_i).
\]

should be provided.

C. Basic Experiment

Following basic experiment shows the capability of the designed system to generate effectively impulses.

Experiment 1: Two excitations are compared. In the first case, the system is excited by an impulse delivered through the Momentum Generator. The velocity of the momentum wheel was controlled up to a speed of 30rad/s. Then, the supply was halted and the clutch engaged, generating the impulse. In the second case, a similar impulse is approximately realized by impulsive forces, according to (5), from the Force Generator\(^4\).

Figure (3) shows the resulting velocity patterns of both methods. It is clear that the new approach is more effective in transmitting the impulse. The irregularity in the velocity-pattern of the impulse-based method is thought to be caused by collisions between teeth of the clutch. Only after a few

\(^4\)Note that both Generators use the same 20W-motors and a transmission element with similar reduction ratio to actuate respectively momentum wheel and haptic handle.

![Fig. 3. Comparison of Velocity Patterns in Collision](image-url)
(typically 3) collisions, the clutch is fully engaged and the whole momentum is transmitted to the handle. The abrupt velocity-change at 0.18s and 0.215s, respectively, is caused by the collision with the joint limit (stopper).

From the figure an estimate can be made of the size of motor, necessary to obtain a response similar to that of the impulse-based method (i.e. $\Delta v = 1.4$ m/s in 2.9 ms), but when using an impulse-based force control method. To deliver a similar impulse by the Force Generator, or any typical haptic device, it must be equipped with a motor which is approximately 20 times bigger, then the current motor ($\Delta v = 1.15$ m/s in 56.4 ms).

V. EXPERIMENT

This section describes experiments obtained through implementation of the impulse-based control method on the newly developed haptic display. In all experiments the human operator, operating the haptic, was replaced by an equivalent mass (69.5g) fixed at the tip of the haptic handle. The operator only held the haptic to give it a sufficient high initial velocity before collision took place.

In the case an impulse-based control method was applied, according to (11), the time of collision was estimated by evaluating $x_{t+\Delta T_{engage}}|t = x_{t} + v_{t}\Delta T_{engage}$, where $x_{t+\Delta T_{engage}}$ is the estimate of the position a period $\Delta T_{engage}$ later. The time necessary for engaging the clutch $\Delta T_{engage}$ was estimated to be $0.045 \pm 0.002$s. Due to the variability of $\Delta T_{engage}$ and also to the inaccuracy of $v_{t}$, slight differences in the location of the virtual walls can be seen. The implementation of a better observer belongs to the future work. At the moment only one impulse was generated by the Momentum Generator. The generation of a series of impulses, or the combination with a force-based wall implementation, remains for further work. Other parameters, used during the experiments, such as clutch timing constants, inertia of elements etc. can be found in table (I).

The Force Generator was used to deliver forces according to (5), (6), in the case of impulse-based force control. The continuous output force at the tip of the handle equals 3.1N, although temporarily higher forces 9N can be delivered.

**Experiment 1:** In this experiment two virtual walls, realizing a complete stop after collision ($e = 0$), and realizing a perfect bounce ($e = 1$), were generated. The virtual walls were located at 0m. This location corresponds to an upright position of the haptic handle. The results of the implementation according to the impulse-based control method (full line) and according to the impulse-based force control method (broken line) are depicted in Fig. (4(a)). For ease of comparison, the results are shifted in time, so that the instants of impact correspond more or less in the detailed figures. Remark that the initial velocities are slightly different, although this does not alter the interpretation of the results. In both cases, the impulse-based control method shows superior behavior. Its response is swift and reasonably accurate. The failure to cancel the velocity completely in the case $e = 0$ is thought to be caused by a small error in the estimated inertia of the device components. Nevertheless, the result is much better than that of the impulse-based force control method. The latter behaves very slowly and correspondingly a large intrusion into the virtual wall occurs. In the case $e = 1$ the handle hits the stopper on its return (at 9.16s and 9.32s). The plot gives so a view on an interaction with a real wall. Comparing these data with the data around 8.9s shows already good correspondence between the virtual and the real wall.

**Experiment 2:** In this experiment a virtual wall with $e = 0.57$ was implemented at 0.08m, just in front of the joint limit at 0.09m, which has a comparable restitution coefficient. The...
results of the interaction with the walls (stopper) are depicted in Fig. (5). A full line depicts the impulse-based control method, a semi-broken line shows the impulse-based force control method. The interaction with the stopper is drawn with a broken line, after translation over 0.01m, to coincide with the simulated responses. The slow response of impulse-based force control method, makes that its wall is not strong enough. For intrusion with a high velocity, the actual stopper is reached and this forces the handle back. The detailed view show how the response of the impulse-based control wall corresponds well with that of the real wall.

![Fig. 5. Real vs. Impulsive Wall, ε = 0.57](image)

### VI. CONCLUSION

This article addressed the problem of rendering rigid objects in a virtual world (focus on high-velocity impacts on a rigid wall). The importance of velocity measurement as compared to force measurements was stressed. It was argued that impulse-based methods are naturally suited for this application.

A new haptic device was designed and a prototype constructed. The prototype is capable of generating true ‘impulses’ and allows the implementation of an ‘impulse-based control’ method. This in contrast to current implementations which are in fact ‘impulse-based force control’ methods that approximate the impulse by an ‘impulsive force.’

The developed haptic furthermore only uses small motors. One of which is used to control the velocity of a momentum wheel. At the estimated instant of impact on the virtual wall, the momentum is transmitted to the operator by engaging an electromagnetic toothed clutch. This realizes a sudden change in velocity, corresponding to real world impacts. Current haptic displays, designed for ‘impulse-based force control’ methods, need to be equipped with heavy motors to obtain a similar quality of response. Reductions of motor sizes up to 20 times are thought to be possible, when using the new method.

After applying the impulse, the proposed mechanism needs a certain time to restore enough momentum for the next impulse. In case of instability this limits the damage done towards the user. The design is therefore intrinsically safer than mechanisms that use heavy motors, which can continuously apply a big force to the user.

Further work includes the online estimation of the operator’s mass, the development of an observer to estimate the instant of intrusion into the virtual wall. The actual collision mechanism of the clutch, will be studied, to get better accuracy and to realize a series of subsequent impulses.

### REFERENCES


