In this supplement we provide additional data for selected quantities. The data is partly related to considerations of technical nature.

With very large field amplitudes, the island becomes strongly deformed. In Fig. 1 we show examples of typical steady state configurations of \( N = 200 \) island in axis and diagonal field with amplitude \( E = 0.25 \) eV taken from the simulations of the MC model. For an axis-directed field, the island becomes a thin rod with an average thickness close to unity. Instead of a single rod, the island often consist of rods displaced by one nearest-neighbor distance (in this case 5 rods). For a diagonal field the island shape becomes a ”zig-zag” type but with lots of bulk atoms still remaining.

In addition to the width and average thickness of the islands, also the variance of these measures can be used. The variance \( \langle w^2 \rangle - \langle w \rangle^2 \) of the total width \( w \) is shown in Fig. 2. The variance follows very closely the total width. For very large islands, a local maximum appears at \( E \approx 0.23 \) eV and a local minimum at \( E \approx 0.03 \) eV.

As noted in the Article, for large fields \( (E \gg 0.01 \) eV) there is no longer simple (linear) relationship between the field amplitude and velocity of the island, instead complex non-linear behavior emerges. The correlation between the velocity, field amplitude and temperature is demonstrated in Fig. 3, where the sign of the minimum velocity \( \min(v(\tau)) \) in the electrophoretic ratchet is shown as a function of temperature and field amplitude for \( N = 10 \) and \( N = 12 \) for the ME model. To find phenomena such as current inversion and velocity increase reported in the Article, it is necessary to vary both temperature and the
FIG. 1: Examples of typical configurations of the MC model island with 200 atoms at $T = 600$ K with axis ($\alpha = 0^\circ$) and diagonal ($\alpha = 45^\circ$) fields with amplitude $E = 0.25$ eV. Note the different scales of the axis.

FIG. 2: (Color online) Variance of the island width perpendicular to the field for various island sizes at $T = 500$ K as a function of field amplitude ($\alpha = \gamma = 0^\circ$). The data is for the MC model.

field amplitude. For such an extensive scan of the parameter space the ME method is more suitable than the MC method.

It was noted that for $\gamma = 45^\circ$ and beyond linear-response regime, the two-maxima structure appears for velocity as a function of $\alpha$ (see Section III.C of the Article). However, the two-maxima structure appears also for large islands $N > 20$ in the case $\gamma = 0^\circ$ and fields near $E \approx 0.1$ eV and beyond. This transition from the single-maximum structure to double
FIG. 3: The sign of the minimum velocity in the electrophoretic ratchet in \((E_1, T)\)-plane with (a) \(N = 10\) and \(x = 1/4\) and (b) \(N = 12\) and \(x = 3/10\). Black color indicates positive velocity. The data is for the ME model.

FIG. 4: (Color online) The MC model with \(\gamma = 0\) and \(T = 600\) K. (a) Velocity increase for \(N = 41\) island for several field amplitudes \(E = 0.028 \ldots 0.118\) eV. (b) Optimal \(\alpha\) values that maximize the increase for several island sizes \(N = 20 \ldots 41\). The maxima is demonstrated for \(N = 41\) island in Fig. 4(a). The position of the maxima as a function of \(E\) is shown in Fig. 4(b) for several island sizes. The second maximum angle is found around \(\alpha \approx 35^\circ\) for all large islands.

In the Article, temporal switching of the field was given by a Markovian switching process for the ME model, whereas deterministic switching was used for the MC model. In Fig. 5 we show the difference between Markovian and deterministic switching for an 8-atom island for both pulsed and electrophoretic fields in temperature \(T = 600\) K using the ME model. With non-stochastic time-periods, deterministic switching creates more complex behavior,
such as more distinctive local maxima and minima. Stochastic switching creates a smoother response, but reproduces the overall structure of deterministic dynamics.

In the Article we have chosen for the ME model the Markovian switching, because it allows solving the steady state with iterative eigenstate solvers (Arnoldi iteration), which offers superior performance when compared to numerical integration tools. Because of the stiffness of the linear equation set of the island model, integration was found to be very slow and error-prone even when using specialized stiff integrators [1] and numerical matrix exponentiation methods [2]. Identification of sufficient convergence and high initial-state dependency of the results causes additional problems. For a comparison, obtaining the data for the deterministic case required of the order of $10^3$ times more computing time than that for the Markovian switching (solving steady state for the Markovian system takes only few seconds). For larger islands and lower temperatures, problems with the numerical integration escalates even further.

\[1\] Matlab’s (version 2010a) ODE15s and ODE23s functions.