Relay-Aided Interference Neutralization for the Multiuser Uplink-Downlink Asymmetric Setting

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Abstract—In the context of multiuser relay-aided multi-way communications, we identify and meet the optimal degrees of freedom (DOF) for different multiuser uplink-downlink settings of practical importance. Under the imposed constraint of using simple linear techniques, the proposed solutions draw from interference-neutralization (IN) methods which linearly manipulate signals in time and space, and manage to reduce the effect of multiuser interference and of the half-duplex constraint. Focus is placed on asymmetric settings where the connectivity, size and rate of the uplink and downlink groups may vary.

I. INTRODUCTION

Interference and the half-duplex constraint are two limiting bottlenecks in networks having multiple users and nodes that wish to both transmit and receive information. Such models have been studied in different works such as [1], [3]–[5]. In such networks, which appear for example in uplink-downlink cellular systems, the two aforementioned problems often accept solutions drawing from network coding techniques [6], such as different interference neutralization methods [7], [8], which seek to properly combine signals arriving from different paths in such a way that the interfering signals are canceled while the desired signals are preserved. While such techniques offer implementation simplicity by consisting only of straightforward linear operations across the spatial dimension, their usefulness is limited to specific network topologies which conveniently allow for interference patterns that are treatable in a linear manner. This class of topologies may often involve relays which can be used to properly redirect interfering signals to be then encoded over the spatial (and here also over the temporal) dimension.

We here identify different scenarios which accept such linear treatment in a manner that can be optimal with respect to DOF, focusing on pertinent scenarios of practical importance in the context of uplink-downlink half-duplex constrained cellular systems. This involves identifying the outer bounds of the DOF for these networks, and then constructively meeting these bounds by proposing simple interference neutralization solutions which redirect signals both in spatial and, as proposed here, temporal dimensions. With time encoding in place, special attention is placed so that the linear solutions are also causal. Establishing the optimality of the proposed solutions is completed with an analysis of the signal-attenuating and noise-accentuating effects of the IN schemes.

A. Summary and Notations

In Section II we describe the pertinent scenarios and the corresponding channel and signal model. In Section III we describe the outer bound on the DOF region for different scenarios, and then propose different IN solutions that meet part or the entirety of these regions. Finally in Section IV we conclude by mentioning some salient features of the proposed IN solutions.

In this work, \((\bullet)^{-1}\), \((\bullet)^T\), \((\bullet)^\dagger\) and \(||\bullet||_F\) denote the inverse, transpose, conjugate transpose and Frobenius norm of matrix respectively, \((\bullet)^*\) denotes the complex conjugate, \(||\bullet||\) denotes Euclidean norm, \(j\) denotes \(j\)-th row of the matrix or column vector in the argument, and \((\bullet)_{ij}\) denotes the matrix element in the ith row and jth column. We use \(\approx\) to denote exponential equality, i.e., \(f(\rho) \approx \rho^d\) denotes \(\lim_{\rho \to \infty} \frac{\log f(\rho)}{\log \rho} = d\) and \(\leq\), \(\geq\), \(\prec\), \(\succ\) are similarly defined.

II. SIGNAL AND CHANNEL MODEL

We consider a setting where a node \(B\) with \(K\) antennas wants to receive from a set \(A\) of \(J\) single-antenna users, and transmit to a set \(C\) of \(L\) single antenna users, with the help of a \(K\) antenna relay \(R\), in the presence of the half-duplex constraint, and in the presence of unlimited channel state information at all transmitters and receivers (CSITR). In the context of cellular systems, \(B\) may play the role of a base station, and \(A\) and \(C\) the sets of uplink and downlink users respectively. Let \(b^{(t)}\), \(r^{(t)}\), \(a^{(t)}\) respectively denote the transmitted vectors sent at time \(t\) by \(B, R\) and by all of \(A\). Furthermore let \(y_B^{(t)}\), \(y_R^{(t)}\), \(y_C^{(t)}\) respectively denote the received signal and noise vectors at \(B, R\) and at all of \(C\). In the following we will ignore the time index if no ambiguity is caused. In the scale of interest we consider a uniform power constraint where \(E||a||^2 = E||b||^2 = E||r||^2 \leq \rho\), where \(\rho\) takes the role of the signal-to-noise ratio (SNR).
We let $h_{i,j}$ denote the channel fading coefficient between the $i$th user in $A$ and the receiver of the $j$th user in $C$, let $h_{A,j}$ denote the vector of fading coefficients between the entire set $A$ and the receiver of the $j$th user in $C$, and let $H_{A,C}, H_{A,R}, H_{R,C}, H_{R,B}, H_{B,C}$ denote the $A$-to-$C$, $A$-to-$R$, $R$-to-$C$, $R$-to-$B$, $B$-to-$R$, and $B$-to-$C$ collective channel fading matrices. Fading and additive noise coefficients are considered to be i.i.d. complex Gaussian $CN(0,1)$, and the fading is assumed to remain constant during the coherence period and to change independently afterwards. Regarding the different settings $(a), (b), (c)$ addressed here (cf. Figure 1), the channel model in $(a)$ is distinguished by the fact that $H_{B,C} = H_{A,B} = 0$, the channel model in $(b)$ by the fact that $H_{A,B} = 0$, and the channel model in $(c)$ corresponds to a concatenation of systems adhering to model $(b)$ as will be described later on. We are interested in analyzing the degrees of freedom, so a rate of $R$ bits per channel use (bpcu), corresponds to $d = \lim_{\rho \to \infty} \frac{R}{\log \rho}$ degrees of freedom.

III. PROPOSED IN SCHEMES AND DOF ANALYSIS

Before proceeding to establish and meet the DOF limits for the different scenarios shown in Figure 1, we provide the following necessary lemmas whose proofs are found in the Appendix.

Lemma 1: In a two-hop setting where the first (second) hop from node $S$ to node $R$ (from node $R$ to node $D$) is over a link with ergodic capacity $C_{SR}$ ($C_{RD}$), the $S$-to-$D$ ergodic capacity is $C_{SD} = C_{SR} + C_{RD}$.

Lemma 2: Let $\mathbf{E} \Delta = \sum_{i=1}^{N} \prod_{j=1}^{N} h_{i,j}^{-1}$, $\tau_{i,j} \in \{-1,1\}$, where the $h_{i,j}$ are independently drawn square random matrices each consisting of i.i.d. $CN(0,1)$ entries. Then for any $\epsilon > 0$ there exists a constant $d > 0$ such that $Pr(||E^{-1/2}||_F \geq \rho^d) \leq \rho^{-c/d}$.

We proceed with scenarios $(a)$ and $(b)$ depicted in Figure 1, and then briefly refer to the setting in $(c)$ where different systems, each adhering to model $(b)$, interfere with each other. Within the context of cellular networks, the latter case corresponds to the scenario of inter-cell interference with base station cooperation. We henceforth adopt the rate uniformity assumption where all users in set $A$ have the same rate $R_A$, and all users in $C$ have rate $R_C$, but where $R_A$ and $R_C$ are not necessarily equal. For simplicity we also limit exposition to the case where $J \leq L = K$.

A. Outer Bound

To derive the DOF outer bound we apply the cut-set theorem (cf. [9]), in the ergodic setting, as illustrated in Fig. 2. For the $A$-to-$B$ communication under model $(a)$, the pertinent cuts $\{A, C\} \rightarrow \{R, B\}$ and cut $\{A, R, C\} \rightarrow \{B\}$, together with Lemma 1 give that each user in $A$ can achieve no more than $d^{(a)}_A \leq \frac{K}{J+1}$ DOF. A similar approach shows the same for the $A$-to-$B$ communication under model $(b)$ where again each user in $A$ can achieve at most $d^{(b)}_A \leq K/J$ DOF.

For the $B$-to-$C$ communication under model $(a)$, the pertinent cuts $\{B\} \rightarrow \{A, R, C\}$ and $\{A, B, R\} \rightarrow \{C\}$ together with Lemma 1 give that each user in $C$ (under the rate uniformity assumption) can achieve at most $d^{(a)}_C \leq \frac{1}{J}$ DOF, and for the $B$-to-$C$ communication under model $(b)$, the pertinent cut $\{A, B, R\} \rightarrow \{C\}$ implies that each user in $C$ has at most $d^{(a)}_C \leq \frac{1}{J}$ DOF. Towards tightening the outer bound of the DOF region we consider, for model $(b)$, the two-directional uplink-downlink cut $\{A, C, R\} \rightarrow \{B\}$, after which it is not difficult to show that in a half-duplex uplink-downlink setting, the sum ergodic capacity is equal to the maximum of the ergodic capacities of the uplink and of the downlink, which in turn implies that $d^{(b)}_A + d^{(b)}_C \leq 1$. The equivalent cut for model $(a)$ does not yield any tightening, and is thus ignored. $d^{(b)}_A$ and $d^{(b)}_C$ also serve as outer bounds for scenario $(c)$ which, as will be described later on is a concatenation of different systems adhering to model $(b)$. The above is summarized in the following Lemma and in Table I.

Lemma 3: For model $(a), (b), (c)$ an outer bound on the DOF is respectively given by

\[
\begin{align*}
&d^{(a)}_A \leq K/(K+J), \quad d^{(a)}_C \leq 1/2 \\
&d^{(b)}_A \leq K/(K+J), \quad d^{(b)}_C \leq 1, \quad Jd^{(b)}_A + Kd^{(b)}_C \leq K \\
&d^{(c)}_A \leq K/(K+J), \quad d^{(c)}_C \leq 1, \quad Jd^{(c)}_A + Kd^{(c)}_C \leq K.
\end{align*}
\]

TABLE I

<table>
<thead>
<tr>
<th></th>
<th>A → B</th>
<th>B → C</th>
<th>total</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d^{(a)}_A$</td>
<td>$K/(K+J)$</td>
<td>$1/2$</td>
<td>$-$</td>
</tr>
<tr>
<td>$d^{(b)}_A$</td>
<td>$K/(K+J)$</td>
<td>$1$</td>
<td>$Jd^{(b)}_A + Kd^{(b)}_C \leq K$</td>
</tr>
<tr>
<td>$d^{(c)}_A$</td>
<td>$K/(K+J)$</td>
<td>$1$</td>
<td>$Jd^{(c)}_A + Kd^{(c)}_C \leq K$</td>
</tr>
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B. Achievability and IN schemes

In what follows we describe two IN schemes $X_1$ and $X_2$ where $X_1$ concerns the general case of $J \leq K$ and applies with minor modifications to both models $(a)$ and $(b)$, and where $X_2$ is for $J = K$ and again applies with minor modifications to all models $(a), (b)$ and $(c)$. The following summarizes the performance of $X_1, X_2$ in different settings, as it will be shown later on.

**Proposition 1:** Scheme $X_1$ achieves $(d^{(a)}_A = K/(K + J), \quad d^{(a)}_C = 1/(K + J))$ and $(d^{(b)}_A = K/(K + J), \quad d^{(b)}_C = 1/(K + J))$ where $d^{(a)}_A = K/(K + J) \quad$ and $d^{(b)}_A = K/(K + J)$ are optimal. Scheme $X_2$ achieves $(d^{(a)}_A = 1/2, \quad d^{(c)}_A = 1/2)$ and achieves $(d^{(b)}_A + d^{(b)}_C = 1, \quad d^{(b)}_A \leq 1/2)$ and $(d^{(c)}_A + d^{(c)}_C = 1, \quad d^{(c)}_A \leq 1/2)$ which are optimal for the corresponding setting.
of $J = K^1$.

### Table II

<table>
<thead>
<tr>
<th>applicability</th>
<th>achievable DOF</th>
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<tbody>
<tr>
<td>$X_1$</td>
<td>$(d_A = \frac{K + 1}{2}, d_C = \frac{1}{K + J})$ optimal</td>
</tr>
<tr>
<td>$X_2$</td>
<td>$(d_A^{(2)} = \frac{K}{K + J} = \frac{1}{2}, d_C^{(2)} = \frac{1}{K + J})$ optimal</td>
</tr>
<tr>
<td>$X_2'$</td>
<td>$d_A^{(b)} + d_C^{(b)} = 1, d_A^{(b)} \leq 1/2$ optimal</td>
</tr>
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We will first present scheme $X_2$ simply because the derivation of the DOF, and specifically the analysis of the effects of signal attenuation and noise accentuation introduced by the IN solution, can be described in a more concise manner. The same techniques can then be applied with minor modifications to $X_1$.

1) Scheme $X_2$ - description and DOF analysis: For the case of $J = K$, we introduce scheme $X_2$ and analyze its DOF performance for model (b). Model (a) is handled similarly as was mentioned in [10], and model (c) will be discussed later on.

In the proposed two-phase scheme, each phase has duration equal to one channel use. In the first phase, A collectively transmits $\beta V_B b$ with $\beta = \left(\frac{\|V_B\|_2^2}{\|b\|_2^2}\right)^{-1/2}$ ensuring the power constraint $E\|b\|^2 \leq \rho$, and by construction we ask that $|b|^2$ be destined for the $j$th downlink user in $C$. The relay and set $C$ then respectively receive

$$ y_R = \beta H_{B,R} V_B b + H_{A,R} a + z_R, \quad y_C^{(1)} = \beta H_{BC} V_B b + H_{A,C} a + z_C^{(1)}. \quad (2) $$

During the second phase, node $R$ precodes, attenuates, and forwards the received signal, sending $r = \gamma V_R y_R$ where $\gamma = \left(\frac{\|V_A H_{A,R} a + z_R\|_2^2 + \frac{\|V_R H_{B,R} b + z_R\|_2^2 + \|V_R b\|_2^2}{\rho}}{\|V_B\|^2}\right)^{-1/2}$, and $B$ and $C$ respectively receive

$$ y_B = \gamma H_{R,B} V_R y_R + z_B, \quad y_C^{(2)} = \gamma H_{R,C} V_R y_R + z_C^{(2)}. \quad (3) $$

Towards neutralizing the interference experienced by an individual downlink user, where this interference contains (cf. 4) uplink signals as well as downlink signals intended for other users in $C$, we combine

$$ \tilde{y}_C = y_C^{(1)} + y_C^{(2)} = \gamma (H_{BC} V_R H_{B,R} V_B b + (H_{R,C} V_R H_{A,R} + H_{A,C}) a + H_{R,C} V_R z_R + z_C^{(1)} + z_C^{(2)}). \quad (4) $$

and remove all interference by setting

$$ V_R = -H_{R,C}^{-1} H_{A,C} H_{A,R}^{-1}, \quad V_B = (H_{BC} - H_{A,C} H_{A,R}^{-1} H_{B,R})^{-1} \quad (6) $$

to get

$$ \tilde{y}_C = \gamma \beta b + \gamma H_{R,C} V_R z_R + \gamma z_C^{(2)} \quad (7) $$

thus having the $j$th downlink user in $C$ observe an interference-free signal

$$ \tilde{y}_j = \gamma \beta b + \gamma H_{R,C} V_R z_R + \gamma z_C^{(2)}, \quad j = 1, \ldots, K, \quad (8) $$

albeit with a possibly attenuated signal $\gamma \beta b$ and accentuated noise $\tilde{y}_j \triangleq \gamma H_{R,C} V_R z_R + \gamma z_C^{(2)}$, both of which will be analyzed in what follows.

In light of the fact that $\beta, \gamma$ and $E[\tilde{y}_j z_j^2]$ are random variables and functions of the fading, we proceed to identify problematic channel regions. Towards this, for some positive constant $\epsilon$, we consider regions $\mathcal{E}_1 = \{\beta^2 < \rho^{-\epsilon}\}$, $\mathcal{E}_2 = \{\gamma^2 < \rho^{-\epsilon}\}$ that may cause signal attenuation (cf. (9)), and region $\mathcal{E}_3 = \{\|H_{R,C} V_R\|_F^2 > \rho\}$ that may cause noise accentuation (cf. (9)). Lemma 2, and [10, Appendix B] tells us that for some $d_0 > 0$

$$ P(\mathcal{E}_1) \triangleq P(\|H_{BC} - H_{A,C} H_{A,R}^{-1} H_{B,R}\|_F^2 > \rho^2) \leq \rho^{-\delta}(10) $$

and from

$$ E[\tilde{y}_j z_j^2] = \gamma^2 \|H_{R,C} V_R z_R\|^2_2 + \gamma^2 + 1 \leq \gamma^2 \|H_{R,C} V_R\|^2_2 \gamma^2 + 1 \quad (12) $$

and [10, Lemma 2] we have that

$$ P(\mathcal{E}_3) = P(\|H_{R,C} V_R\|_F^2 > \rho) \leq \rho^{-\epsilon/2}, \quad (13) $$

and from

$$ I = \frac{1}{2} \log \left(1 + \frac{\gamma^2 \beta^2 \rho}{E[\tilde{y}_j z_j^2]}\right). $$

For $\mathcal{E} = (\mathcal{E}_1 \cup \mathcal{E}_2 \cup \mathcal{E}_3)$ and $d_m = \max\{2, d_0\}$, the corresponding outage probability is then bounded as

$$ P[I < R] = P[I < R | \mathcal{E}] P[\mathcal{E}] + P[I < R | \mathcal{E}^c] P[\mathcal{E}^c] $$

$$ \leq \sum_{i=1}^{3} P[\mathcal{E}_i] + P[I < R | \mathcal{E}] P[\mathcal{E}] $$

$$ \leq \sum_{i=1}^{3} \frac{1}{2} \log \left(1 + \frac{\gamma^2 \beta^2 \rho}{E[\tilde{y}_j z_j^2]}\right) \leq P[\mathcal{E}^c] $$

$$ \leq \rho^{-\frac{\epsilon}{2}} + P[I < R | \mathcal{E}] P[\mathcal{E}] $$

$$ = \rho^{-\frac{\epsilon}{2}} + P[I < R | \mathcal{E}] P[\mathcal{E}] $$

where, (a) follows from the union bound and (12), where (b) follows from (10), (11) and (13), and where $\mathcal{E}^c$ is the complement of $\mathcal{E}$. Similar treatment holds for the uplink

$^1$The optimal $d_C^{(b)} = 1$ can be trivially met by silencing $A$ and $R$. 
case where all interference is self-interference and where the signal attenuation and noise accentuation may be treated exactly as in the downlink case, in which case we conclude that the point $d_{A}^{(b)} = d_{B}^{(b)} = 1/2$ is reached. Time-sharing of $X_2$ with silencing $A$ achieves the optimal DOF region $(d_{A}^{(b)} + d_{B}^{(b)} = 1, d_{A}^{(b)} \leq 1/2).

2) Scheme $X_3$: description and DOF analysis: For the case of $J < K$, we proceed to describe scheme $X_3$ and analyze its DOF performance. We focus on model (a), with (b) handled similarly. We will proceed to show that each uplink user can communicate without interference $\frac{K-J}{K+J}$ symbols per channel use, and each downlink user can communicate without interference $\frac{J}{K+J}$ symbols per channel use. The signal attenuation and noise accentuation analysis follows directly from the previous subsection regarding $X_2$ and is hence omitted here.

The proposed scheme has two phases, the first of duration $K$ and the second of duration $J$. During the first phase, i.e., for $t = 1, \cdots, K$, $R$ and $C$ listen, while at the same time node $B$ transmits a vector $\tilde{v}^j b_i$, where symbol $b_i$ is intended for the $(i = t)$th user in $C$, and where $\beta v_{R}$ precodes and handles the power constraint.

The received vectors at $R$ and at the $i$th user of $C$ are respectively
\[ y_R^{(t)} = H_{A,R} a^{(t)} + \beta H_{B,R} v B_i + z_R^{(t)}, \tag{14} \]
\[ y_i^{(t)} = h_{A,i} a^{(t)} + z_i^{(t)}, \tag{15} \]
where $a^{(t)} \triangleq [a_{1,t} \cdots a_{J,t}]^T$. The relay then proceeds to calculate
\[ \tilde{y}_{R}^{(t)} = H_{R}^{+} y_{R}^{(t)} = a^{(t)} + \beta H_{A,R}^{+} H_{B,R} v B_i + H_{A,R}^{+} z_R^{(t)}, \tag{16} \]
where $H_{A,R}^{+} = (H_{A,R} H_{A,R})^{-1} H_{A,R}$. Finally for $v = [v_1, \cdots, v_J]^T = \beta H_{A,R}^{+} H_{B,R} v B$ and for $z_R^{(t)} = [z_{R,1}^{(t)}, \cdots, z_{R,J}^{(t)}]^T = H_{A,R}^{+} z_R^{(t)}$ the relay collects the $J$-length vector
\[ \tilde{y}_{R}^{(t)} = [a_{1,t} + v_1 b_i + z_{R,1}^{(t)}, \cdots, a_{J,t} + v_J b_i + z_{R,J}^{(t)}]^T. \tag{17} \]

During the second phase, the relay reverts the dimension of time and space with respect to $\tilde{y}_{R}^{(t)}$, and for $t = K + 1, \cdots, K + J$ it broadcasts
\[ v^{(t=K+j)} = \gamma V_R [\tilde{y}_{R}^{(1)}], \cdots, [\tilde{y}_{R}^{(K)}]^T, \quad j = 1, \cdots, J, \]
where $\gamma V_R$ precodes and constrains the power. With $V_R = H_{R}^{-1} C$, at the end of the second phase, the $i$ user in $C$ collects the $J$-length observations
\[ y_i^{(K+j)} = a_{j,i} + v_j b_i + z_{R,i}^{(t)} + z_{i}^{(K+j)}, \quad j = 1, \cdots, J. \tag{18} \]
\[ y_i^{(t)} = \sum_{p=1}^{J} h_{p,i} g_{p,i} + z_{i}^{(t)} \tag{19} \]
received during $t = i$ and $t = K + 1, \cdots, K + J$, and calculates the interference free
\[ \tilde{y}_i = y_i^{(t)} - \sum_{j=1}^{J} h_{j,i} y_i^{(K+j)} = g_{i} b_i + \tilde{z}_i, \tag{20} \]
where
\[ g_i = -\sum_{j=1}^{J} h_{j,i} v_j \quad \text{and} \quad \tilde{z}_i = -\sum_{j=1}^{J} h_{j,i} (z_{R,i}^{(j)} + z_{i}^{(i)}) \]
describe the signal attenuation and noise accentuation, and which are analyzed similar to the case of $X_2$, so that we can conclude that each downlink user achieves $1/(K + J)$ degrees of freedom. The case of uplink is simple as all interference is self-interference, and the number of unknown symbols per channel use equals the dimension of the channel. This combined with noise accentuation analysis as in the case of $X_2$, gives us the achievable region of $(d_{A}^{(c)} + d_{C}^{(c)} = \frac{K}{K+J}, d_{A}^{(c)} < 1/2)$. The proof is omitted here.

C. Cooperation between interfering uplink-downlink systems

We briefly touch upon model (c), as depicted in Figure 1-(c), corresponding to the case where we have $M$ interfering systems (interfering cells), each adhering to model (b), where the downlink transmission of a cell interferes with the downlink users of one or more neighboring cells. For the case where the base stations of these systems can fully cooperate, entirely exchanging data and channel knowledge, it can be shown that under the assumption of $J = K$, employing a modified version of scheme $X_2$ achieves the optimal degrees of freedom $(d_{A}^{(c)} + d_{C}^{(c)} = \frac{K}{K+J}, d_{A}^{(c)} < 1/2)$ in the opposite direction. This latter rate-asymmetric case brought to the fore the role of multiple-antenna relays in not only handling uplink-downlink interference, but also in further reducing the negative effects of the half-duplex constraint - for example it allowed uplink users without base station connectivity to achieve closer to full-duplex rates.

The near-optimal solutions, at a cost of substantial CSITR, offer simplicity of implementation and reduced delay as compared to interference alignment techniques. A final salient
feature is that, even though transmitters encode over time, the solutions are causal, rendering them useful in cases where the different communication phases experience different fading.

V. ACKNOWLEDGMENTS

The research leading to these results has received funding from the Mitsubishi RD project Home-eNodeBS.

VI. APPENDIX

A. Proof of Lemma 1

Corresponding to the S-to-R-to-D half-duplex setting with communication over a large number $M$ of fading realizations \(\{H_{i,j}\}_{M=1}^{M}\), the rate is bounded as

\[
R_{SD} \leq \max \min_\{\Delta_{i,1}\} \left\{ \frac{1}{M} \sum_{i=1}^{M} I_{1,i}, \frac{1}{M} \sum_{i=1}^{M} (1 - \Delta_{i,1}) I_{2,i} \right\},
\]

where

\[
I_{1,i} = \max_{p(x)} I(x; y_{R}(H_{i}), I_{2,i} = \max_{p(x)} I(x; y_{R}(H_{i}), \Delta_{i,1} \text{ is the duration of the first phase (R in listening mode) during the }i\text{th fading realization, and where }p(x)\text{ denotes the joint probability distribution of the transmitted signals }x_{S}, x_{R} \text{ at } S \text{ and } R. \)

For \(C_{1} \triangleq 1 \sum_{i=1}^{M} \Delta_{i,1} I_{1,1} \), \(C_{2} \triangleq 1 \sum_{i=1}^{M} (1 - \Delta_{i,1}) I_{2,1}\), we rewrite the optimization problem in (21) in its Lagrangian form of maximizing \(C_{1} + \varphi(C_{2} - C_{1})\). We define a function \(g(s)\) as \(g(s) = 1, 0, 1/2\) for \(s > 0, s < 0\) and \(s = 0\) respectively and note that since \(\frac{d g}{d \varphi} = \frac{(1+s)+(1-s)}{2(1+s)}\), \(\varphi > 0\), the optimal \(\Delta_{i,1}\) takes the form \(\Delta_{i,1} = g\left(\frac{1+s}{1+s} - \varphi\right), \forall i\). In turn, the optimal value for \(\varphi\) is achieved when \(\varphi = \varphi^{*}\) such that \(C_{1} = C_{2}\), which implies that \(\Delta_{i,1} = g\left(\frac{1+s}{1+s} - \varphi^{*}\right), \forall i\). For \(\eta \triangleq \frac{C_{SR}}{C_{RD}}\) and for the optimal \(\varphi^{*}\) and optimal sequence \{\(\Delta_{i,1}\), the capacity \(C_{SD}\) takes the form

\[
C_{SD} = \frac{1}{1 + \eta} \left\{ C_{1} + \frac{\eta C_{2}}{C_{SR} + C_{RD}} \right\} = \frac{1}{1 + \eta} \sum_{j=1,0,1/2} P(A_{j}) \mathbb{E}f(A_{j}) = \frac{C_{SR} + \frac{\eta C_{RD}}{C_{SR} + C_{RD}}}{1 + \eta} = \frac{C_{SR} C_{RD}}{C_{SR} + C_{RD}},
\]

which serves as an upper bound on the ergodic capacity.

For achievability we set \(\Delta_{i,1} = 1, 1 - \Delta_{2,i}, \forall i\) and divide the total duration of transmission into \(M_{1}\) blocks of \(M_{2} = M/M_{1}\) sub-blocks, where each sub-block has two phases of fractional duration of \(\frac{1}{1+\eta}\) and \(\frac{1}{1-\eta}\), where both \(M_{1}, M_{2}\) are sufficiently large, and where the channel changes independently with each sub-block. During the first phase of each sub-block of the \(j\)th block, \(S\) transmits while \(R\) listens, and then \(R\) waits to transmit this decoded message during the second phase of the sub-blocks of the next \((j + 1)\)th block, during which \(S\) is silent. As a result \(C_{1} = \frac{1}{1+\eta} \mathbb{E}I_{1,1} = \frac{C_{SR} + \frac{\eta C_{RD}}{C_{SR} + C_{RD}}}{C_{SR} + C_{RD}}\) and \(C_{2} = \frac{\eta}{1+\eta} \mathbb{E}I_{2,1} = \frac{C_{SR} + \frac{\eta C_{RD}}{C_{SR} + C_{RD}}}{C_{SR} + C_{RD}}\), which proves the achievability and the result.

B. Proof of Lemma 2

We wish to bound the probability that the inverse of \(E = \sum_{i=1}^{N} \prod_{j=1}^{M} H_{ij}\) has large power. To begin with, split exponent indexes into sets \(S_{i}(z) = \{j|z_{i,j} = z\}\) and let \(\{x\}\) be the collection of all random variables, entries in any \(H_{ij}\).

Now recalling that \(H_{ij}^{-1} = \text{adj}(H_{ij})\), where \(\text{adj}(H_{ij})\) denotes the matrix of cofactors of \(H_{ij}\), and setting \(S_{i,j} = \{H_{ij}\}\) when \(j \in S_{i}(1), \) else setting \(S_{i,j} = H_{ij}\), we have that

\[
E = \sum_{i=1}^{N} \prod_{j=1}^{M} \text{det}(H_{ij}), \text{det}(H_{ij}) = \prod_{i} \prod_{j \in S_{i}(1)} \text{det}(H_{ij}) - \prod_{i} \prod_{j \in S_{i}(1)} \text{det}(H_{ij}).
\]

Defining the numerator and denominator in the above equation respectively as \(E\) and \(C\), we have that \(E^{-1} = \frac{E}{C}\) and that

\[
||E^{-1}||_{F}^{2} = \frac{||C||_{2}^{2} \sum_{m,n} ||\text{adj}(E)||_{m,n}^{2}}{||\text{det}(E)||^{2}}.
\]

Defining the numerator and denominator in the above equation respectively as \(f_{1}(\{x\}), f_{2}(\{x\})\), we note that both \(E\) and \(C\), and consequently both \(f_{1}(\{x\})\) and \(f_{2}(\{x\})\) are, with probability 1, positive exponent polynomials in \(\{x\}\) without constant terms. Hence the result in [11, Lemma 2.10] applies to tell us that there exists a constant \(d > 0\) such that

\[
P(f_{2}(\{x\}) \leq \rho^{-d}) \leq \rho^{-e/d}.
\]

Combining this with the straightforward fact that \(P(f_{2}(\{x\})) > \rho^{0}\) completes the proof. \(\square\)

REFERENCES