Blind Separation for Precoding-Based Blind Channel Estimation for MIMO-OFDM Systems

Song Noh and Michael D. Zoltowski
School of Electrical and Computer Engineering, Purdue University, West Lafayette, IN 47907, USA
Email: songnoh@purdue.edu, mikedz@ecn.purdue.edu

Abstract—In this paper, the problem of blind channel separation for precoding-based blind channel estimation is considered in multiple-input multiple-output (MIMO) orthogonal frequency-division multiplexing (OFDM) systems. In the new scheme, blind separation is achieved by implicitly “marking” the respective signal sent by each transmitter with a unique (known) circular time-shift during even-valued time slots. Separation of the blindly obtained channel mixtures is achieved through a combination of subspace alignment followed by SVD based computation of the intersection of two subspaces. Separability conditions required for the proposed precoding-based blind channel estimation are established. Numerical results are presented demonstrating the efficacy of the proposed algorithm.

I. INTRODUCTION

After its first introduction [1], semi-blind/blind channel estimation and equalization has been an active research area to improve spectral efficiency due to their advantage of not requiring pilot signals. Among the many blind channel estimation techniques for MIMO-OFDM systems, the precoding-based blind channel estimation method is particularly interesting and useful which exploits signal correlation induced by a non-redundant precoding [2]–[4]. In this method, the non-redundant linear precoding is applied to the information symbols to induce signal correlation across all of the subcarriers before inverse discrete Fourier transform (DFT) of OFDM modulation, and this operation is required for the blind channel identification method to perform properly.

However, the channel state could be estimated up to a unitary matrix ambiguity when the precoding-based blind channel estimation schemes are applied to the MIMO-OFDM systems [3]–[8]. Such a remaining matrix ambiguity is undesirable for many applications, there are some studies to reduce the matrix ambiguity based on cyclostationarity [9], linear prefiltering [10], and redundant linear precoding [11]. In this paper, we propose a new method of resolving the multidimensional ambiguity matrix which extends the existing linear precoding-based blind channel estimation methods in MIMO-OFDM systems [7], [8]. Our approach is based on the non-redundant circulant precoding to separate the desired channel state from the mixed channel estimates. We provide required conditions for the proposed precoder to resolve the matrix ambiguity.

Notations We will make use of standard notational conventions. Vectors and matrices are written in boldface with matrices in capitals. All vectors are column vectors. For a matrix $A$, $A^\dagger$, $A^T$, $A^H$, and $A^*$ indicate the Moore-Penrose pseudo inverse, transpose, Hermitian transpose, and the complex conjugate of $A$, respectively. Also, $A(\mathcal{I}, \mathcal{J})$ denotes the submatrix composed of rows $\mathcal{I}$ and columns $\mathcal{J}$ of $A$ for sets $\mathcal{I}$ and $\mathcal{J}$, and $: \text{ denotes the entire set. diag}(d_1, \ldots, d_n)$ is the diagonal matrix composed of elements $d_1, \ldots, d_n$ and $\text{diag}(A)$ gives a vector containing the diagonal elements of matrix $A$. $\mathcal{N}(A)$ represents the null space of $A$, i.e., the linear subspace spanned by the set of all vectors $x$ for which $Ax = 0$. $I_n$ stands for the identity matrix of size $n \times n$. $E\{x\}$ is the expectation of $x$, $\iota = \sqrt{-1}$, and $(\cdot)_N$ is the integer modulo $N$.

II. SYSTEM MODEL

We consider a MIMO-OFDM system equipped with $N_t$ transmit and $N_r$ receive antennas. Denote the $i$-th data symbols from the $k$-th transmit antenna as $d^{(i)}_k = [d^{(i)}_{k,1}(1), \ldots, d^{(i)}_{k,N}(N)]^T$ where $N$ is the number of subcarriers. For blind channel estimation, this data symbols are first linearly precoded by the non-redundant precoding matrix $W_k$ to yield $x^{(i)}_k$, and then OFDM modulated. By applying an $N \times N$ precoder matrix $C_k$ to the OFDM modulated symbols at each transmit antenna, this system model includes the typical MIMO-OFDM system by setting $C_k = I_N$. After adding the cyclic prefix (CP), each precoded OFDM symbol is sent through a MIMO channel. We assume that the length of CP is equal to or longer than a MIMO channel order $L$ to avoid inter symbol interference and the channel is a quasi-static over an $N_s$ successive OFDM symbols. Let $h_{lk} = [h_{lk}(1), \ldots, h_{lk}(L)]^T$ be the channel vector in time domain from $k$-th transmit antenna to the $l$-th receive antenna. At the $l$-th receive antenna, the CP is first removed, and then $N$-point normalized DFT is applied as

$$\tilde{y}^{(i)}_l = \sum_{k=1}^{N_t} F H_{c,lk} C_k F^H x^{(i)}_k + \tilde{n}^{(i)}_l,$$  

where $x^{(i)}_k = W_k d^{(i)}_k$, $H_{c,lk}$ denotes a circulant matrix determined by $h_{lk}$ as a first column, and $F$ is an $N \times N$ normalized DFT matrix. In (1), $\tilde{n}^{(i)}_l$ is the additive white Gaussian noise vector with covariance matrix $\sigma^2 N$, and assume that the source covariance matrix is given by $E\{d^{(i)}_k d^{(i)H}_k\} = \sigma^2 I_N$ with $\sigma^2 = 1$. For simplicity, we will use the time index $(i)$ only if necessary.

A. Preliminary work: Blind estimation with non-redundant precoding for MIMO-OFDM systems

In this section, we briefly describe the blind channel estimation based on the non-redundant precoding matrix for MIMO-OFDM systems which provides a background to our
proposed method in Section III. Since the method is based on precoding and subspace technique with second-order statistic, we first write the received signal for $C_k = I_N$,

$$\tilde{y}_t = \sum_{k=1}^{N_t} \tilde{H}_{lk} x_k + \tilde{n}_t, \quad (2)$$

where $\tilde{H}_{lk}$ denotes the diagonal matrix with elements of $\tilde{h}_{lk} = \tilde{F}_{lk}$, and $\tilde{F}$ is the first $L$ columns of the normalized DFT matrix $\tilde{F}$. At $l$-th receive antenna, we compute the covariance matrix $R_l := E[\tilde{y}_t\tilde{y}_t^H]$ of the received signal (2) as

$$R_l = \sum_{k=1}^{N_t} \tilde{H}_{lk} \tilde{Q}_k \tilde{H}_{lk}^H + \sigma_n^2 I_N = \left( \sum_{k=1}^{N_t} \tilde{h}_{lk} \tilde{h}_{lk}^H \right) \otimes Q = \sigma_n^2 I_N,$$  \hspace{1cm} (3)$$

where $\otimes$ denotes the Hadamard product, $Q_k := W_k W_k^H$, and $Q_1 = \cdots = Q_{N_t} = Q$ called the precoder square matrix. Then, we can write the elements of $R_l$ as

$$[R_l]_{i,j} = \sum_{k=1}^{N_t} \tilde{h}_{lk}(i) \tilde{h}_{lk}(j) |Q|_{i,j} + \sigma_n^2 \delta(i-j), \quad (4)$$

where $\delta_A(x)$ denotes the indicator function, i.e., $\delta_A(x) = 1$ if $x \in A$ or zero otherwise. From (4), we obtain

$$\frac{[R_l]_{i,j} - \sigma_n^2 \delta(i-j)}{|Q|_{i,j}} = \sum_{k=1}^{N_t} \tilde{h}_{lk}(i) \tilde{h}_{lk}^*(j). \quad (5)$$

Since such a construction (5) is impossible if $|Q|_{i,j} = 0$, the authors proposed the following precoder square matrix $[3], [5]$.

$$Q_N = (1-p)I_N + pee^T, \quad \frac{1}{N-1} < p < 1 \text{ (} p \neq 0 \text{),} \quad (6)$$

where $e$ is the column vector composed of all ones. However, since the precoding matrix $W := Q_N^{1/2}$ is dense in (6), then $W$ mixes the signals of all subcarriers, i.e., the data vector $d_k$ is mixed by an $N \times N$ matrix $W_k$ in (2). Then, a dense $W_k$ destroys the desirable feature of MIMO-OFDM systems that each subcarrier provides an independent flat-fading $N_c \times N_t$ MIMO channel [12]. To construct a sparse $W_k$, we proposed to use only a limited number of samples from $R_l$. This yields that we choose $T$ rows at $T$ columns from $R_l$ for $T \geq L$. Let $I_p$ and $I_q$ be the index sets of selected rows and columns, $(1 \leq p, q \leq N/T, p \neq q)$, such that

$$I_s = \{i_1, i_2, \ldots, i_T\}, \quad (7)$$

where $i_s = s + (t - 1)N/T$ and $s \in \{p, q\}$. Note that $I_p \cap I_q = \emptyset$, to avoid the diagonal elements of $R_l$ added by the unknown noise variance. From $[R_l]_{i_s,j_t} = \sum_{k=1}^{N_t} \tilde{h}_{lk}(i_s)\tilde{h}_{lk}(j_t) |Q|_{i_s,j_t} = \tilde{F}(i_s,:) \sum_{k=1}^{N_t} \tilde{h}_{lk}(j_t) |Q|_{i_s,j_t}$, for $i_s \neq j_t$, we define a new variable given by

$$v_t = \left[ [R_l]_{i_s,j_t}/|Q|_{i_s,j_t}, \ldots, [R_l]_{i_s,j_{T}}/|Q|_{i_s,j_{T}} \right]^T \quad (8)$$

$$= \tilde{F}(I_p,:) \sum_{k=1}^{N_t} \tilde{h}_{lk}(j_t), \quad (9)$$

where $i_s \in I_p$ and $j_t \in I_q$. Since $\tilde{F}(I_p,:)$, composed of the rows $I_p$ of the skinny DFT matrix $\tilde{F}$, is a $T \times L$ Vandermonde matrix with full column rank, we have $\tilde{F}(I_p, :)^H v_t = \sum_{k=1}^{N_t} \tilde{h}_{lk}(j_t)$. We now construct $J_{I_p,I_q}$ as

$$J_{I_p,I_q} = \sum_{k=1}^{N_t} \tilde{h}_{lk} \tilde{h}_{lk}^H = \tilde{F}(I_p, :)^H [v_1 \cdots v_T] (\tilde{F}(I_q,:))^{H}. \quad (10)$$

From (11), the channel is identifiable up to an $N_t \times N_t$ unitary ambiguity matrix [13] as

$$[b_{11}, \ldots, b_{N_t,N_t}] = U((1,1),(1,1),\ldots,(1,1))^1/2, \quad (12)$$

where $J_{I_p,I_q} = U A U^H$ from the eigenvalue decomposition. The optimal precoder square matrix under a sparse structure [7], [8], [14] is given by

$$Q_N = \begin{bmatrix} Q_{N/2} & Q_{N/2} & -I_{N/2} & 2I_{N/2} & -Q_{N/2} \end{bmatrix}, \quad (13)$$

where $Q_N \in \mathbb{R}^{N \times N}$ is symmetric positive definite and has non-zero off-diagonal elements only at $I_p \times I_q \cup I_q \times I_p$ with $\text{diag}(Q_N) = e$. An inherent problem of the blind channel estimation is that the channel states can only be estimated up to an $N_c \times N_t$ unknown matrix since the channel states are mixed together in (10). To resolve such an ambiguity matrix, we can use $N_t^2$ pilot signals at least or existing control signals in practice [15]. Our goal in this paper is to design a new precoder $C_k$ for the precoding-based blind channel estimation to reduce the matrix ambiguity up to a scalar ambiguity via the channel separation.

### III. PRECODING-BASED BLIND CHANNEL SEPARATION TECHNIQUES

In this section, we propose a new blind channel separation method for precoding-based blind channel estimation in MIMO-OFDM systems. We found that the unknown ambiguity matrix in (12) can be reduced to a scalar ambiguity by applying the non-redundant precoders on each transmit antenna at each even-valued time slots. Let us consider the multiple-input single-output (MISO) case which can be generalized to the MIMO case by regarding a MISO system as multiple MISO systems. (The received antenna indices are omitted for notational convenience in the MISO case.) Let $d_k^{(2i+1)}$ and $d_k^{(2i)}$ be the data symbols during odd and even-valued time slots, respectively. We consider a block-wise linear precoding using the precoder square matrix (13) for odd and even valued time slots as

$$\begin{bmatrix} x_k^{(2i+1)} \\ x_k^{(2i)} \end{bmatrix} = \begin{bmatrix} W_{1,1} & W_{1,2} \\ W_{2,1} & W_{2,2} \end{bmatrix} \begin{bmatrix} d_k^{(2i+1)} \\ d_k^{(2i)} \end{bmatrix}, \quad (14)$$

where

$$\begin{bmatrix} W_{1,1} & W_{1,2} \\ W_{2,1} & W_{2,2} \end{bmatrix} = \begin{bmatrix} Q_{1,1} & Q_{1,2} \\ Q_{2,1} & Q_{2,2} \end{bmatrix}^{1/2}, \quad (15)$$

and $W_{m,n}, Q_{m,n} \in \mathbb{R}^{N \times N}$ for $m, n \in \{1, 2\}$. The received signal $\tilde{y}^{(2i+1)}$ at odd-valued time index is same as (2) because we set $C_k = I_N$. The received signal $\tilde{y}^{(2i)}$ at even-valued time
index, on the other hand, is given by (1) for some $C_k$. We define the $2N \times 1$ received signal vector $\mathbf{y} := [y^{(2i+1)}', y^{(2i)}']'$, then compute the covariance matrix $\mathbf{R} = \bar{E} \{\mathbf{yy}'\}$, given by

$$
\mathbf{R} = \begin{bmatrix} R_{1,1} & R_{1,2} \\ R_{2,1} & R_{2,2} \end{bmatrix} = \left[ \sum_{k=1}^{N_t} \bar{H}_k Q_1^1 \bar{H}_k'^T + \sum_{k=1}^{N_t} \bar{H}_k Q_1^2 \bar{A}_k^T \right] + \sigma_n^2 \mathbf{I}_{2N_t}, \quad (17)
$$

where $\mathbf{A}_k := \mathbf{F} \mathbf{H}_e \mathbf{C}_k \mathbf{F}^H$. Note that the estimate of $[h_1, \ldots, h_{N_t}]$ can be obtained from the upper left sub-matrix $R_{1,1}$ as in (3) and (12), up to the $N_t \times N_t$ rotational ambiguity matrix.

In the new method for the channel separation, $C_k$ is a circulant matrix, given by

$$
\mathbf{C}_k = \begin{bmatrix} \mathbf{I}_{N_t} & \mathbf{I}_{k-1} \end{bmatrix}, \quad (18)
$$

where $k \in \{1, \ldots, N_t\}$. Thus, in (17) can be rewritten as

$$
\mathbf{A}_k = \mathbf{F} \mathbf{H}_e \mathbf{C}_k \mathbf{F}^H = \mathbf{D}_k \tilde{\mathbf{H}}_k, \quad (19)
$$

where $\mathbf{D}_k := \sqrt{N_t} \text{diag}(f_{i-k+1})$ and $\mathbf{F} = [f_1, \ldots, f_{N_t}]$. The received signal covariance $\mathbf{R}$ can be rewritten as

$$
\mathbf{R} = \left[ \sum_{k=1}^{N_t} \mathbf{D}_k \bar{H}_k Q_1^1 \bar{H}_k'^T + \sum_{k=1}^{N_t} \mathbf{D}_k \bar{H}_k Q_1^2 \bar{A}_k'^T \right] + \sigma_n^2 \mathbf{I}_{2N_t}. \quad (20)
$$

From (13) and (16), we know that all sub-matrices $Q_{m,n}$ have non-zero off-diagonal elements at $\mathcal{I}_p \times \mathcal{I}_q$ and also $\mathcal{I}_q \times \mathcal{I}_p$. Thus, the off-diagonal elements of $\mathbf{R}_{m,n}$ at $\mathcal{I}_p \times \mathcal{I}_q$ are given by

$$
[R_{m,n}]_{i,p,j,q} = \sum_{k=1}^{N_t} \mathbf{D}_k^{m-1} [i,k,j] [j,k] Q_{m,n} [i,j], \quad (C.1)
$$

where the matrix to the zeroth power is an identity matrix of the same size, i.e., $\mathbf{D}_k^0 = \mathbf{I}_N$. We now construct $\mathbf{v}_{i,(m,n)} \in \mathbb{C}^T$ for $m, n \in \{1, 2\}$ as

$$
\mathbf{v}_{i,(m,n)} = [\mathbf{r}_{m,n}]_{i,j} / [Q_{m,n}]_{i,j}, \ldots, [\mathbf{r}_{m,n}]_{i,j} / [Q_{m,n}]_{i,j} \bigg]_{N_t} \bigg]_{N_t} = \sum_{k=1}^{N_t} \mathbf{D}_k^{m-1} [i,p,j] \mathbf{F} [j,p] h_k h_k'^T [i,j] [j,i] \left[ \mathbf{D}_k^{n-1} \right]_{i,j}, \quad (C.1)
$$

where $i \in \mathcal{I}_p$, $j \in \mathcal{I}_q$. Then, the following equation holds:

$$
\mathbf{V}_{n,m} = \left[ \mathbf{v}_{1,(m,n)} \mathbf{v}_{2,(m,n)} \cdots \mathbf{v}_{T,(m,n)} \right] = \sum_{k=1}^{N_t} \left( \mathbf{D}_k^{m-1} [i,p,j] \mathbf{F} [j,p] h_k h_k'^T [i,j] [j,i] \left[ \mathbf{D}_k^{n-1} \right]_{i,j} \right)_{N_t}, \quad (21)
$$

By collecting all matrices $V_{m,n}$, we define a new variable for channel estimation as

$$
\mathbf{J} := \left( I_2 \otimes \mathbf{F}^T (I_{p,:,:}) \right) \mathbf{V}_{1,2} \left( I_2 \otimes \mathbf{F}^T (I_{q,:,:}) \right)^H \quad (a)
$$

and

$$
\mathbf{J} = \sum_{k=1}^{N_t} \mathbf{B}_{k,p} \mathbf{h}_k h_k'^T \sum_{k=1}^{N_t} \mathbf{B}_{k,q} \mathbf{h}_k h_k'^T B_{k,p}^H B_{k,q}^H \quad (b)
$$

where $\otimes$ denotes the Kronecker product. The equality $(a)$ follows $B_{k,p} := \mathbf{F}^T (I_{p,:,:}) D_{k} (I_{p,:}) D_{k} (I_{p,:})$ and $(b)$ holds by

$$
\mathbf{H}_o = \left[ h_1, h_2, \ldots, h_{N_t} \right], \quad \mathbf{H}_{k,p} = \left[ B_{1,p} h_1, B_{2,p} h_2, \ldots, B_{N_t,p} h_{N_t} \right]. \quad (23)
$$

From (22), the channel states can be identified up to the unknown matrix $\mathbf{M} \in \mathbb{C}^{N_t \times N_t}$ [13] as

$$
\left[ \begin{array}{c} \mathbf{H}_o \\ \mathbf{H}_{e,p} \end{array} \right] = \left( \mathbf{U}_c (1 : N_t) \mathbf{A} (1 : N_t, 1 : N_t) \right) \mathbf{M} \quad (25)
$$

and

$$
\mathbf{M} = \left[ \begin{array}{c} \mathbf{U}_c \\ \mathbf{U}_e \end{array} \right] \mathbf{M} \quad (26)
$$

where $\mathbf{J} = \mathbf{UAV}$ by the singular value decomposition (SVD) and the definitions of $\mathbf{U}_o, \mathbf{U}_e \in \mathbb{C}^{L \times N_t}$ are clear from (25).

Note that $B_{k,p}$ resulting from $C_k$ in (19) affects channel identifiability up to a scalar ambiguity. To guarantee the separation of the channel state, we have the following constraints for $B_{k,p}$:

(C.1) $B_{k,p}$ is non-singular for $1 \leq k \leq N_t$.

(C.2) $B_{k,p} \neq \alpha B_{k',p}$ for $k \neq k'$ and some $\alpha \in \mathbb{C}$.

Constraint (C.1) is for the invertibility of $B_{k,p}$. Constraint (C.2) guarantees that $[\mathbf{H}_o, B_{k,p}^{-1} \mathbf{H}_e, \mathbf{p}]$ has only one-dimensional null space. From the constraints (C.1) and (C.2), the condition for the channel separation in this case is given by

$$
\mathcal{N} \left( [\mathbf{H}_o, B_{k,p}^{-1} \mathbf{H}_e, \mathbf{p}] \right) = \mathcal{N} \left( [\mathbf{U}_o \mathbf{M}, B_{k,p}^{-1} \mathbf{U}_e \mathbf{M}] \right). \quad (27)
$$

Since each matrix $[\mathbf{H}_o, B_{k,p}^{-1} \mathbf{H}_e, \mathbf{p}]$ has a unique null vector from its definition, the subspace equivalence in (27) can be rewritten as

$$
\mathbf{H}_o, B_{k,p}^{-1} \mathbf{H}_e, \mathbf{p}] z_k = [\mathbf{U}_o \mathbf{M}, B_{k,p}^{-1} \mathbf{U}_e \mathbf{M}] z_k \quad (28)
$$

and

$$
= \mathbf{U}_o m_k - B_{k,p}^{-1} \mathbf{U}_e m_k \quad (29)
$$

where the equality (29) follows $z_k = [e_k^1, e_k^2]'$, $e_k \in \mathbb{R}^{N_t}$ and $m_k = \left[ m_1, m_2, \ldots, m_{N_t} \right]'$. From (29), the estimate of $m_k$ (denoted as $\hat{m}_k$) can be obtained from the eigenvector of

$\hat{e}_k$. denotes the vector whose components are all zero except for the $k$-th component which is equal to one.
U_{H_1} U_{B_1} \text{ which corresponds to the smallest eigenvalue. Thus, the estimate of } H_0 \text{ is achieved from (26) and (30), given by}
\begin{equation}
\tilde{H}_0 D_T = U_o [\tilde{m}_1, \ldots, \tilde{m}_{N_c}] D_T,
\end{equation}
where \( \tilde{H}_0 = [\tilde{h}_1, \ldots, \tilde{h}_{N_c}] \) is the estimate of \( H_0 \) and \( D_T \in \mathbb{C}^{N_c \times N_c} \) is the unknown diagonal matrix which yields a scalar ambiguity for each channel estimate \( \tilde{h}_k \).

Based on the constraints (C.1) and (C.2), we now provide required conditions of the proposed method on channel identifiability up to a scalar ambiguity. Note from (7) that each element of \( \tilde{F}(I_p,:) \) is given by
\begin{equation}
[\tilde{F}(I_p,:) ]_{t,l} = \frac{1}{\sqrt{N}} e^{-j\frac{2\pi}{N}(t-1)(l-1)},
\end{equation}
where \( i_t = p + (t-1)N/T \in I_p, 1 \leq t \leq T, \) and \( 1 \leq l \leq L \). From (33), \( \tilde{F}(I_p,:) \) can be decomposed into two matrices
\begin{equation}
\tilde{F}(I_p,:) = \sqrt{\frac{T}{N}} F_T \text{diag}(\phi_{I_p}),
\end{equation}
where \( \phi_{I_p} = [1, e^{-j\frac{2\pi}{N}(1-1)}, e^{-j\frac{2\pi}{N}(2-1)}, \ldots, e^{-j\frac{2\pi}{N}(L-1)}]^T \).

Lemma 1:
\begin{equation}
[B_{k,I_p}]_{m,n} = \frac{1}{T} e^{j\frac{2\pi}{N}((1-k)N+(m-n))(p-1)} \sum_{t=1}^{T} e^{j\frac{2\pi}{N}(l-1)(t-1)},
\end{equation}
where \( m, n \in \{1, 2, \ldots, L\} \).

The following proposition provides a property of the precoder for blind channel identification.

**Proposition 1:** Given \( I_p, I_q \) in (7), \( B_{k,I_p} \) resulting from \( C_k \) in (18) satisfies the constraints (C.1) and (C.2) if \( T = L \), under the assumption that \( T \) divides \( N \) exactly and \( N \) is even.

Proof: Case 1) \( T > L \): Suppose that \( k = IT + s \) where \( l \in \{1, 2, \ldots, N/T - 1\} \) and \( s \in \{1, 2, \ldots, T\} \). From Lemma 1, \( B_{k,I_p} \) is an \( L \times L \) Toeplitz matrix. Since \( T \) divides \( N \) exactly, each summand on the right hand side (RHS) in (35) is given by
\begin{equation}
e^{j\frac{2\pi}{N}((1-k)N+(m-n))} = e^{j\frac{2\pi}{N}(-(s-1)+(m-n))}.
\end{equation}
From (36), the \( s \)-th lower off-diagonal elements of \( B_{k,I_p} \) have non-zero values as
\begin{equation}
B_{k,I_p} = e^{j\frac{2\pi}{N}((1-k)N+(k-1))} [I_{L-(s-1)}, 0_{s-1}],
\end{equation}
where \( s \in \{1, 2, \ldots, L\} \) and \( 0_{s-1} \) denotes an all-zero matrix of size \( s \times s \). For \( s \in \{L + 1, L + 2, \ldots, T + L - 1\} \), \( B_{k,I_p} \) is given by an \( L \times L \) zero matrix because \( (s-1) \neq (m-n) \) on the RHS in (36), \( m - n \in \{-L + 1, \ldots, L - 1\} \).

Since \( (1-k)_N = N - IT - s + 1 \) for \( s \in \{T - L + 2, T - L + 3, \ldots, T\} \), (36) is given by
\begin{align*}
e^{j\frac{2\pi}{N}((1-k)_N+(m-n))} &= e^{j\frac{2\pi}{N}((N-IT)-s+1+(m-n))} \\
&= e^{j\frac{2\pi}{N}((N-IT))} e^{j\frac{2\pi}{T}(-s+1+(m-n))} \\
&= e^{j\frac{2\pi}{N}(r-T+(m-n))} = e^{j\frac{2\pi}{N}(r+(m-n))},
\end{align*}
where \( r := -s + T + 2 \) and \( r \in \{2, 3, \ldots, L\} \). From (38), the \( r \)-th upper off-diagonal elements of \( B_{k,I_p} \) have non-zero values as
\begin{equation}
B_{k,I_p} = e^{j\frac{2\pi}{N}((1-k)_N-(1-k)r)(p-1)} [I_{L-(r-1)}, 0_{r-1}],
\end{equation}
where \( s \in \{T - L + 2, T - L + 3, \ldots, T\} \). Thus, if \( T > L \), \( B_{k,I_p} \) violates the constraint (C.1).

Case 2) \( T = L \): Now consider \( k = lT + s \) where \( l \in \{0, 1, \ldots, N/L - 1\} \) and \( s \in \{1, 2, \ldots, L\} \). From Lemma 1, each summand on the RHS in (35) is given by
\begin{equation}
e^{j\frac{2\pi}{N}((1-k)_N+(m-n))} = e^{j\frac{2\pi}{N}(-(s-1)+(m-n))},
\end{equation}
In this case, the \( L \times L \) Toeplitz matrix \( B_{k,I_p} \) has non-zero elements at the \( s \)-th lower diagonal elements from \( s = 1 \) to \( m - n \), and also has the \((1 - s)L + 1\)-th upper diagonal elements. Thus, we have
\begin{equation}
B_{k,I_p} = e^{j\frac{2\pi}{N}((1-k)_N-(1-k)r)(p-1)} \left( \phi_{I_p} \phi_{I_p}^T \right) \circ \left[ I_{L-(k-1)L} I_{k-1L} \right].
\end{equation}
Since \( B_{k,I_p} \) in (41) satisfies the constraints (C.1) and (C.2) for \( 1 \leq k \leq L \), the claim follows.

**Corollary 1:** Given \( I_p, I_q \) in (7) and \( T = L \), \( B_{k,I_p} \) is a scaled version of \( B_{k',I_p} \) if \( (k-1)_T = (k'-1)_T \) when \( N_k > L \).

Proof: From (41), \( B_{k,I_p} \) is given by
\begin{equation}
B_{k,I_p} = e^{j\frac{2\pi}{N}((1-k')_N-(1-k')r)(p-1)} \left( \phi_{I_p} \phi_{I_p}^T \right) \circ \left[ I_{L-(k-1)L} I_{k-1L} \right].
\end{equation}
Since the second and third terms on the RHS in (42) are the same as the terms in (41) where \( (k-1)_T = (k'-1)_T \), the only difference between (41) and (42) is a phase rotation.

**Corollary 1** describes a property of the constraint (C.2) for the case \( (1-k)_N = (1-k')_N \). In this case, the corresponding null vectors in (28) are given by
\begin{equation}
z_k = [e_k^T, -e_k^T]^T \text{ and } [e_k', -ae_k']^T \in \mathbb{N} \left( [H_k, B_{k,I_p}^T H_{e,I_p}] \right).
\end{equation}
where \( e_k \in \mathbb{R}^{N_k} \) and \( a = e^{j\frac{2\pi}{N}((1-k)_N-(1-k')_N)(p-1)} \). This yields the remaining multidimensional ambiguity for such channels \( h_k \) and \( h_{k'} \).

**IV. Numerical Results**

In this section, we provide some numerical results. We consider the MIMO-OFDM system with \( N_t = N_r = 2 \), and the number of subcarriers is \( N = 64 \). The input symbols
were independent and identically-distributed (i.i.d.) with 16-ary quadrature amplitude modulation (QAM) symbols. We assume that the channel had four taps \( L = 4 \). The channel coefficients are randomly generated, given by \( b_{11} = [-0.92 - \iota 0.12, -0.43 - \iota 0.92, -1.05 - \iota 0.63, 0.39 - \iota 0.69]^{T} \), \( b_{12} = [0.91 + \iota 0.41, -0.27 - \iota 1.14, 0.01 - \iota 1.06, 0.63 + \iota 0.41]^{T} \), \( b_{21} = [0.63 - \iota 0.82, -0.53 - \iota 0.84, 0.62 - \iota 0.90, -0.21 + \iota 1.05]^{T} \), and \( b_{22} = [-0.22 - \iota 1.02, 0.61 + \iota 0.79, 0.94 + \iota 0.34, -0.11 - \iota 0.95]^{T} \). The noise variance \( \sigma_n^2 \) is determined according to the SNR, i.e., \( \text{SNR} = \frac{\sigma_n^2}{\sigma^2} \). Fig. 1 shows the detected constellation for each transmitter antenna. As expected, the constellations have an arbitrary phase rotation resulting from a scalar ambiguity introduced by blind channel estimation. For coherent processing at the receiver, such a scalar ambiguity associated with the channel estimate can be resolved by a few pilot signals or control signals in real systems.

V. Conclusions

We have proposed a blind approach for the channel separation based on the precoding-based blind channel estimation methods in MIMO-OFDM systems. We have shown that blind channel identification up to a scalar ambiguity is identifiable by applying the circular precoding for each transmit antenna during even-valued time slots. The proposed algorithm jointly exploits subspace alignment and SVD based computation to separate the blindly obtained channel mixtures. Required conditions on identifiability are provided under the proposed circulant precoder.

REFERENCES


