Autocorrelation-Based Decentralized Sequential Detection of OFDM Signals in Cognitive Radios

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Introduction

• Cognitive Radio Scenario
Outline

1. Motivation
2. Local autocorrelation based detection
   • Detection Problem in AWGN
   • Detection Problem in Multipath
3. Cooperative detection
   • Sequential detection test at fusion center
4. Simulation results
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Motivation

• Spectrum sensing is essential in cognitive radios in order to find unused frequency bands for agile spectrum use.

• In this paper, we consider the detection of Orthogonal Frequency Division Multiplexing (OFDM) systems. Examples:
  - IEEE 802.11a/g (WLAN), IEEE 802.16 (WiMAX)
  - 3GPP Long Term Evolution (LTE)
  - DVB terrestrial digital TV systems DVB-T, DVB-H, etc.

• Autocorrelation of CP-OFDM system is well-known property that has been exploited in equalization, channel estimation and synchronization.

• Cooperation among secondary users to detect the common primary user improves detection performance.

• Sequential processing reduces the delay and amount of data needed to make a reliable decision.
Detection Problem in AWGN

- Hypothesis test in an additive white Gaussian noise (AWGN) scenario is
  \[ H_0 : x(t) = w(t) \]
  \[ H_1 : x(t) = s(t) + w(t) \]

- \( s(t) \): transmitted OFDM signal, and
- \( w(t) \): AWGN process such that \( w(t) \sim \mathcal{N}_c(0, \sigma_w^2) \).

- Due to the central limit theorem (CLT), the OFDM signal is approximately Gaussian distributed for sufficiently large number of subcarriers in the OFDM system. Therefore \( s(t) \sim \mathcal{N}_c(0, \sigma_s^2) \).

- The distributions of the received signal under two hypotheses
  \[ H_0 : x(t) \sim \mathcal{N}(0, \sigma_w^2) \]
  \[ H_1 : x(t) \sim \mathcal{N}(0, \sigma_s^2 + \sigma_w^2) \]
Autocorrelation of the OFDM signal

- \( T_d \) is the length of the useful symbol data, \( T_c \) is the length of the cyclic prefix

\[
\begin{align*}
\text{CP} & \quad \vdots \\
T_c & \quad T_d
\end{align*}
\]

- Hypothesis test based on autocorrelation coefficient \( \rho \) at lags \( \tau = \pm T_d \)

\[
H_0 : \rho = 0 \\
H_1 : \rho = \rho_1 \neq 0
\]

where \( \rho_1 \triangleq \frac{T_c}{T_d+T_c} \frac{\sigma_s^2}{\sigma_s^2 + \sigma_w^2} = \frac{T_c}{T_d+T_c} \frac{\text{SNR}}{1+\text{SNR}} \). Also \( 0 < \rho_1 < 1 \).
Local Test for low SNR regime 1/2

- The maximum likelihood estimate of autocorrelation coefficient from $M + T_d$ observations $[x(0), \ldots, x(M + T_d - 1)]$ is given by

$$\hat{\rho}_{ML} = \frac{\sum_{t=0}^{M-1} \Re\{x(t)x^*(t + T_d)\}}{\sum_{t=0}^{M-1} x(t)x^*(t)}$$

where $\Re\{.\}$ is the real part of the complex number.

- Based on LLRT, we propose $\hat{\rho}_{ML}$ as the test statistic in the low SNR regime ($\sigma_n^2 \gg \sigma_s^2$).
Local Test for low SNR regime 2/2

• The distribution for the test statistics in low SNR can be approximated as

\[ H_0 : \hat{\rho}_{ML} \sim \mathcal{N}(0, \frac{1}{2M}) \]

\[ H_1 : \hat{\rho}_{ML} \sim \mathcal{N}(\rho_1, \frac{(1 - \rho_1^2)^2}{2M}) \].

• Constant false alarm rate (CFAR) test: Accept \( H_1 \) if \( \hat{\rho}_{ML} > \eta_l \).

• Here the threshold \( \eta_l \) is defined by the constraint on false alarm probability which is given by

\[ \alpha = P_{FA} = P(\hat{\rho}_{ML} > \eta_l | H_0) \].

• Probability of detection is given by

\[ P_d = P(\hat{\rho}_{ML} > \eta_l | H_1) \].
Detection Problem in Multipath

- Hypothesis test in multipath scenario is

\[ H_0 : x(t) = w(t) \]
\[ H_1 : x(t) = \sum_{l=0}^{P-1} h(l)s(t-l) + w(t) \]

where \( P \) is the multipath channel order.

- The distribution for the test statistics in low SNR can be approximated as

\[ H_0 : \hat{\rho}_{ML} \sim \mathcal{N}_r(0, 1/2M) \]
\[ H_1 : \hat{\rho}_{ML} \sim \mathcal{N}_r(\rho_2, (1 - \rho_2^2)^2/2M). \]

where \( \rho_2 \triangleq \frac{T_c}{T_d + T_c} \frac{\delta \sigma_s^2}{\delta \sigma_s^2 + \sigma_w^2} \). Also \( 0 < \rho_2 < 1 \). Here \( \delta = \sum_{l=0}^{P-1} E[|h(l)|^2] \).
Cooperative Detection Model

- Secondary users evaluate the autocorrelation based local LLRs and send them to the fusion center (FC), which combines them to make the final decision.
- Tests at fusion center
  - Sequential detection test.
  - Fixed sample size test.
- Optimal Fusion Rule: Sum of LLRs.
Sequential Detection At Fusion Center

- Fusion center collects LLRs from the users sequentially.
- Sequential test at the fusion center for using $k$ statistics is given by

$$\sum_{n=1}^{k} L_n \leq \log B, \quad \text{Decide } H_0;$$
$$\sum_{n=1}^{k} L_n \geq \log A, \quad \text{Decide } H_1;$$

Otherwise, Take Next Users Statistics;

where $A = \frac{1-\beta}{\alpha}$ and $B = \frac{\beta}{1-\alpha}$, $\beta =$Probability of missed detection.

- Here $n^{th}$ LLR statistic is given by

$$L_n = -M \log(1 - \rho_n^2) + \frac{2M\rho_n(\hat{\rho}_n - \rho_n)}{1 - \rho_n^2}.$$

where $\hat{\rho}_{ML}$ is denoted by $\hat{\rho}_n$ for $n^{th}$ statistic for convenience.
Fixed Sample Size Test At FC

- Fusion center performs following test at the fusion center after collecting LLRs from fixed number of secondary users $K_f$

$$T_f = \sum_{n=1}^{K_f} L_n < \eta_t, \quad \text{Decide } H_0;$$
$$T_f = \sum_{n=1}^{K_f} L_n \geq \eta_t, \quad \text{Decide } H_1;$$

where $\eta_t$ is the threshold of the Neyman Pearson detector.
Simulation results 1/6

• Performance of local autocorrelation-based detector in AWGN:
  - $T_d = 32, T_c = 8, M = 100(T_d + T_c)$. 1000 realizations.
  - Probability of detection vs. SNR (dB) for $\alpha = 0.05$.
  - Receiver Operating Characteristic for SNR = -10 dB.
Simulation results 2/6

- Performance of local detector for different wireless standards for AWGN channel and $\alpha = 0.05$. $\mu = \frac{T_c}{T_c + T_d}$ determines the performance.
  - DVB-T: $T_d = 8192$, $T_c = 1024$, $M = 5(T_d + T_c)$, $\mu = 0.111$.
  - LTE: $T_d = 512$, $T_c = 36$, $M = 100(T_d + T_c)$, $\mu = 0.067$.
  - WLAN: $T_d = 52$, $T_c = 13$, $M = 1000(T_d + T_c)$, $\mu = 0.2$. 
Simulation Parameters

- For different channel condition and for cooperative detection
- At each local detector: \( T_d = 32, T_c = 8, M = 100(T_d + T_c) \).
- 1000 realizations.
- Multipath:
  - Exponential decaying Rayleigh channel
  - Channel Order \( P = 6, \delta = 1 \).
- Shadowing:
  - Log-normal distribution.
  - Mean SNR=-10dB, Std Dev= 5dB.
- Each sensor sees independent channel. No correlation has been assumed.
Simulation results 3/6

- Performance of local detector in different channel conditions
- Cooperative Detection in different channel condition
- $P_d$ vs. $P_{FA} = \alpha$ for SNR=-10dB.
Simulation results 4/6

- Comparison of sequential detection scheme with fixed sample size test: Number of secondary user statistics vs. SNR (dB).
- Probability of false alarm $\alpha = 0.05$, Probability of missed detection $\beta = 1 - P_d = 0.05$, AWGN Channel.
Simulation results 5/6

- Comparison of SD with FSS in AWGN: Relative Efficiency (RE=$K_{fss}/K_{sd}$) vs. SNR (dB).

\[ \alpha = 0.05, P_d = 0.95 \]
Simulation results 6/6

- Comparison of SD with FSS in Shadowing: Relative Efficiency (RE) vs. SNR (dB).

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Conclusions : Local Detection

• We have presented a local autocorrelation coefficient based test for detecting CP-OFDM systems.

• Performance of the local detector is analyzed and validated by simulations. Although analysis is done under low SNR regime, the performance remains good even for high SNR.

• The scheme has a good performance for different channel conditions.
Conclusions: Cooperative Detection

- Cooperative detection improves detection performance and mitigates multipath and shadowing effects.
- The proposed distributed sequential detection, on average, gives impressive relative efficiency of 1.6 to 2 for different channel conditions when compared with fixed sample size test for the same error probabilities of false alarm and missed detection.
- As shown in results, SD gives high relative efficiency as compared to FSS, even in the practical scenario of few SUs cooperating.
Thank You

• Questions?
• Thank You for your interest!