A Matrix-Algebraic Approach to Successive Interference Cancellation in CDMA

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Abstract

In this paper we describe the linear SIC scheme based on matrix-algebra. We show that the linear SIC schemes (single- and multi-stage) correspond to linear matrix filtering that can be performed directly on the received chip-matched filtered signal vector without explicitly performing the interference cancellation. This leads to an analytical expression for calculating the resulting bit error rate which is of particular use for short-code systems. Convergence issues are discussed and it is shown that the simple implementation of the linear SIC provides similar or better performance than the decorrelator at only a few stages. The concept of ϵ-convergence is introduced to determine the number of stages required for practical convergence for both short and long codes. It has previously been observed that the linear SIC has an optimum number of stages for which the bit error rate is minimised. This behaviour is here related to the mean squared error which can be used to estimate the number of stages required to minimise the bit error rate.

I. Introduction

In a mobile communications system multiple access to the common channel resources is vital. In a system based on spread-spectrum transmission techniques code division provides simultaneous access for multiple users. By selecting mutually orthogonal codes for all users, they each achieve interference free single-user performance. It is however not possible to maintain orthogonal spreading codes at the receiver in a mobile environment and thus multiple-access interference (MAI) arises. Conventional single-user detection techniques are severely affected by MAI, making such systems interference limited [1]. Traditional matched filter receivers for CDMA also require strict power control in order to alleviate the near-far problem where a high-powered user creates significant MAI for low-powered users.

More advanced detection strategies can be adopted to improve performance. In [2] Verdú developed the optimal complexity-unconstrained maximum-likelihood (ML) detector for multiuser CDMA. This detector performs an exhaustive search over the constrained space of possible hypotheses. The inherent complexity however increases exponentially with the number of users, rendering the optimal ML detector impractical.

For practical implementation parallel and successive interference cancellation (SIC) schemes have been subject to most attention. These techniques rely on simple processing elements constructed around the matched filter concept. The first structure based on the principle of interference cancellation was the multi-stage detector in [3]. Here the cancellation is decision-directed (i.e., non-linear) and is done in parallel. In [4] Dent et al. proposed a serial approach, a single-stage non-linear SIC scheme. This scheme has been analysed in detail in [5]. In [6], Kawabe et al. suggested a multi-stage non-linear SIC technique. A closely related scheme was suggested by Sawahashi et al. in [7]. Johansson and Svensson have suggested both single- and multi-stage linear SIC detectors in [8], while Jamal and Dahlman have compared the performance of the linear and the non-linear SIC approaches in [9].

An algebraic approach to SIC was initially introduced in [10] and further developed in [11]. Closely related work by Elders-Boll et al. was presented in [12]-[14] where they suggest linear detectors based on the application of classic iterative techniques for solving linear systems. The Gauss-Seidel iteration was here identified as SIC. Iterative methods for linear detector design have also also been proposed by Juntti et al. in [15]. The equivalence to interference cancellation was however, not recognised.

In this paper we describe the linear SIC scheme based on matrix-algebra. We show that the linear SIC schemes (single- and multi-stage) correspond to linear matrix filtering that can be performed directly on the received chip-matched filtered signal vector without explicitly performing the interference cancellation. This leads to an analytical expression for calculating the resulting bit error rate (BER) which is of particular use for short-code systems. Convergence issues are discussed and it is shown that the simple implementation of the linear SIC provides similar or better performance than the decorrelator at only a few stages. The concept of ϵ-convergence is introduced to determine the number of stages required for practical convergence for both short and long codes. It has previously been observed that the linear SIC has an optimum number of stages for which the BER is minimised. This behaviour is here related to the mean squared error (MSE) which can be used to estimate the number of stages required to minimise the BER.

The paper is organised as follows. In Section II the uplink model is described and the techniques for SIC are briefly summarised. In Section III we introduce a matrix-algebraic approach for describing linear SIC, which allows for new insight into the behaviour of the schemes. The equivalent matrix filters for linear SIC are derived and convergence issues are discussed for multi-stage schemes, including the concept of ϵ-convergence for both short and long codes. An expression for the MSE of the scheme is derived to find the number of stages required to minimise the MSE which in most cases corresponds to minimising the BER. Numerical examples are presented in Section IV and conclusions are drawn in Section V. Throughout this paper scalars are lower-case, vectors are bold face lower-case, and matrices are bold face upper-case. Subscripting is dropped where no ambiguities arise. The symbols $(\cdot)^{\top}$, $(\cdot)^{\mathrm{H}}$, $(\cdot)^{-1}$ and $\|\cdot\|$ are the transposition, hermi-

tian, inversion and Euclidean vector-norm operators respectively, and the delimiter $\{\cdot\}^y$ defines a space of dimension y. All vectors are defined as column vectors with row vectors represented by transposition. $\mathbb{R}$ denotes the set of real numbers, and the following notation is used for the product of matrices,

$$\prod_{i=n_1}^{n_2} \mathbf{X}_i = \begin{cases} \mathbf{X}_{n_2}\mathbf{X}_{n_2-1}\cdots\mathbf{X}_{n_1+1}\mathbf{X}_{n_1} & \text{if } n_1 \leq n_2 \\ \mathbf{I} & \text{if } n_1 > n_2. \end{cases} \tag{1}$$

II. System Models

In this section, the model for the uplink of the CDMA communication system considered throughout this paper is briefly described. The uplink model is based on a discrete-time symbol-synchronous CDMA system assuming single-path channels and the presence of stationary additive white Gaussian noise (AWGN) with zero mean and variance $\sigma^2 = N_0/2$. Also, the linear SIC scheme is described and the relevant notation is introduced.

A. *Uplink Model*

A specific user in this K-user communication system transmits a binary information-symbol $d_k \in \{-1, 1\}$, by multiplying with a spreading code $\mathbf{s}_k \in \left\{\frac{-1}{\sqrt{N}}, \frac{1}{\sqrt{N}}\right\}^N$, of length N chips and then transmitting over an AWGN channel using BPSK[1]. The spreading codes transmitted by each user in any given symbol interval are assumed to be symbol-synchronous and the channel imposes no phase rotation on the transmitted signal. Symbol-synchronism is assumed for clarity. Similar arguments hold for the symbol-asynchronous case as demonstrated in [14]. Each user is received at a user-specific energy level c_k^2 that is assumed constant over one bit-interval. Note that we have assumed that $\mathbf{s}_k^\top \mathbf{s}_k = 1$. The output of a chip-matched filter is then expressed as a linear combination of spreading codes, specifically, the chip matched filtered received vector, $\mathbf{r}$, is a column vector of length N, encompassing the transmissions for all users. The received vector $\mathbf{r}$ is hence described through matrix-algebra as

$$\mathbf{r} = \mathbf{SCd} + \mathbf{n} \in \mathbb{R}^N, \tag{2}$$

where

$$\mathbf{S} = (\mathbf{s}_1, \mathbf{s}_2, \cdots, \mathbf{s}_K) \in \left\{\frac{-1}{\sqrt{N}}, \frac{1}{\sqrt{N}}\right\}^{N,K}, \tag{3}$$

$$\mathbf{C} = \text{diag}\,(c_1, c_2, \cdots, c_K) \in \mathbb{R}^{K,K}, \tag{4}$$

$$\mathbf{d} = (d_1, d_2, \cdots, d_K)^\top \in \{-1, 1\}^K. \tag{5}$$

To avoid rank deficiencies, we assume that $K \leq N$ and $\mathbf{S}$ has full rank, i.e., the spreading codes for all users are linearly independent. The sampled noise corrupting the output of the chip-matched filter is independent in each sample since the channel noise is assumed to be white and the chip waveforms are assumed to fulfill the Nyquist criterion (e.g. rectangular chip pulses). We

[1]Binary data and chip formats are assumed for clarity. All the presented concepts generalise to m-ary formats.

therefore obtain a noise vector $\mathbf{n}$ where each sample is Gaussian distributed with zero mean and variance $N_0/2$.

The received vector is contained in a vector space of dimension N, $\mathbf{r} \in \mathbb{R}^N$. It is, however, only the part of $\mathbf{r}$ residing in the signal space[2] that is affecting the detector decision. The signal space is determined by span $\{\mathbf{S}\}$. If $N = K$, $\mathbf{r} \in$ span $\{\mathbf{S}\}$. In general however, $\mathbf{r} = \mathbf{r}_s + \mathbf{r}_{s^\perp}$ with $\mathbf{r}_s \in$ span $\{\mathbf{S}\}$ and $\mathbf{r}_{s^\perp} \in$ null $\left(\mathbf{S}^\top\right)$ where null $\left(\mathbf{S}^\top\right)$ denotes the nullspace of $\mathbf{S}^\top$ or equivalently, the orthogonal complement of $\mathbf{S}$. We can determine $\mathbf{r}_s$ and $\mathbf{r}_{s^\perp}$ by orthogonal projections.

$$\mathbf{r}_s = \mathbf{S}\left(\mathbf{S}^\top\mathbf{S}\right)^{\perp 1}\mathbf{S}^\top\mathbf{r} = \mathbf{P}_{\mathbf{S}}\mathbf{r} \tag{6}$$

$$\mathbf{r}_{s^\perp} = \left(\mathbf{I} - \mathbf{P}_{\mathbf{S}}\right)\mathbf{r} = \mathbf{P}_{\mathbf{S}^\perp}\mathbf{r}. \tag{7}$$

This will be of significance for convergence rate considerations.

B. *Linear Successive Interference Cancellation*

Successive interference cancellation schemes are best described by defining an interference cancelling unit (ICU) as shown in Figure 1. This unit is then used as a building-block in the multi-stage SIC scheme shown in Figure 2. The single-stage scheme is obtained by omitting all but the first stage. It is assumed that the K users are ordered according to their received signal power. The input residual signal vector to an ICU of user k at stage i is $\mathbf{e}_{i,k}$. For the first user in the first stage $\mathbf{e}_{1,1} = \mathbf{r}$. The contribution to be cancelled in the ICU of user k at stage i is $\mathbf{s}_k\mathbf{s}_k^\top\mathbf{e}_{i,k}$. In geometrical terms this is a projection of the residual received vector $\mathbf{e}_{i,k}$ onto the relevant spreading code $\mathbf{s}_k$. For the first stage $y_{0,k} = 0$ for all ICU blocks.

III. Matrix-Algebraic Approaches

Throughout this section the subscripting is according to the following conventions. Variables independent of the detector stage is, when needed, subscripted with a user index, i.e., x_k. The first subscript on variables dependent on the detector stage, $x_{i,k}$, denotes the current stage, the second subscript the user index.

A. *Linear Successive Interference Cancellation*

The first user in the first stage of the SIC scheme is operating directly on the received signal vector $\mathbf{e}_{1,1} = \mathbf{r}$. The first-stage filter output is $y_{1,1} = \mathbf{s}_1^\top\mathbf{e}_{1,1} = \mathbf{s}_1^\top\mathbf{r}$, leading to the resulting input vector for the next unit as

$$\mathbf{e}_{1,2} = \mathbf{e}_{1,1} - \mathbf{s}_1\mathbf{s}_1^\top\mathbf{e}_{1,1} = \left(\mathbf{I} - \mathbf{s}_1\mathbf{s}_1^\top\right)\mathbf{r}, \tag{8}$$

where $\mathbf{I}$ is an $N \times N$ identity matrix. The next step is then $y_{1,2} = \mathbf{s}_2^\top\mathbf{e}_{1,2} = \mathbf{s}_2^\top\left(\mathbf{I} - \mathbf{s}_1\mathbf{s}_1^\top\right)\mathbf{r}$, and

$$\mathbf{e}_{1,3} = \mathbf{e}_{1,2} - \mathbf{s}_2\mathbf{s}_2^\top\mathbf{e}_{1,2} = \left(\mathbf{I} - \mathbf{s}_2\mathbf{s}_2^\top\right)\left(\mathbf{I} - \mathbf{s}_1\mathbf{s}_1^\top\right)\mathbf{r}. \tag{9}$$

[2]The irrelevance theorem allows for the portion of the noise which lies outside of span $\{\mathbf{S}\}$ to be ignored [16].

In general we get

$$\mathbf{e}_{1,k} = \prod_{j=1}^{k-1} \left(\mathbf{I} - \mathbf{s}_j \mathbf{s}_j^\top \right) \mathbf{r}, \tag{10}$$

and

$$y_{1,k} = \mathbf{s}_k^\top \prod_{j=1}^{k-1} \left(\mathbf{I} - \mathbf{s}_j \mathbf{s}_j^\top \right) \mathbf{r} = \mathbf{g}_{1,k}^\top \mathbf{r}. \tag{11}$$

A single-stage linear SIC scheme can therefore be represented as a linear filtering $\mathbf{g}_{1,k}$, of the received signal vector $\mathbf{r}$, where

$$\mathbf{g}_{1,k} = \prod_{j=1}^{k-1} \left(\mathbf{I} - \mathbf{s}_{k-j} \mathbf{s}_{k-j}^\top \right) \mathbf{s}_k. \tag{12}$$

The output vector $\mathbf{e}_{1,K+1}$ (see Figure 2) represents the residual received vector at the end of the first stage after all users have been processed. It is determined as

$$\mathbf{e}_{1,K+1} = \prod_{j=1}^{K} \left(\mathbf{I} - \mathbf{s}_j \mathbf{s}_j^\top \right) \mathbf{r}. \tag{13}$$

For notational purposes in what is to follow we define

$$\begin{aligned} \mathbf{\Phi}_k &= \left(\mathbf{I} - \mathbf{s}_k \mathbf{s}_k^\top \right) \left(\mathbf{I} - \mathbf{s}_{k-1} \mathbf{s}_{k-1}^\top \right) \cdots \left(\mathbf{I} - \mathbf{s}_1 \mathbf{s}_1^\top \right) \\ &\qquad \left(\mathbf{I} - \mathbf{s}_K \mathbf{s}_K^\top \right) \cdots \left(\mathbf{I} - \mathbf{s}_{k+1} \mathbf{s}_{k+1}^\top \right), \qquad \text{for } k \in \{1, 2, ...K\}. \\ &= \prod_{j=1}^{k} \left(\mathbf{I} - \mathbf{s}_j \mathbf{s}_j^\top \right) \prod_{m=k+1}^{K} \left(\mathbf{I} - \mathbf{s}_m \mathbf{s}_m^\top \right). \end{aligned} \tag{14}$$

Eqn. (13) then becomes $\mathbf{e}_{1,K+1} = \mathbf{\Phi}_K \mathbf{r}$. Furthermore, $\mathbf{e}_{1,K+1} = \mathbf{e}_{2,1}$ as indicated in Figure 2. The decision statistic is then

$$y_{2,1} = \mathbf{s}_1^\top \mathbf{e}_{2,1} + y_{1,1} = \mathbf{s}_1^\top \mathbf{\Phi}_K \mathbf{r} + \mathbf{s}_1^\top \mathbf{r} = \mathbf{s}_1^\top \left(\mathbf{\Phi}_K + \mathbf{I} \right) \mathbf{r}, \tag{15}$$

and the residual received vector for the next ICU is

$$\mathbf{e}_{2,2} = \mathbf{e}_{2,1} - \mathbf{s}_1 \mathbf{s}_1^\top \mathbf{e}_{2,1} = (\mathbf{I} - \mathbf{s}_1 \mathbf{s}_1^\top) \mathbf{e}_{2,1} = (\mathbf{I} - \mathbf{s}_1 \mathbf{s}_1^\top) \mathbf{\Phi}_K \mathbf{r}. \tag{16}$$

The decision statistic for user 2 at stage 2 is then

$$y_{2,2} = \mathbf{s}_2^\top (\mathbf{I} - \mathbf{s}_1 \mathbf{s}_1^\top) \mathbf{\Phi}_K \mathbf{r} + y_{1,2} = \mathbf{s}_2^\top (\mathbf{I} - \mathbf{s}_1 \mathbf{s}_1^\top) (\mathbf{\Phi}_K + \mathbf{I}) \mathbf{r}. \tag{17}$$

In general the terms in the second stage are described by

$$\mathbf{e}_{2,k} = \prod_{j=1}^{k-1} \left(\mathbf{I} - \mathbf{s}_j \mathbf{s}_j^\top \right) \mathbf{\Phi}_K \mathbf{r}, \tag{18}$$

and

$$y_{2,k} = \mathbf{s}_k^\top \prod_{j=1}^{k-1} \left(\mathbf{I} - \mathbf{s}_j \mathbf{s}_j^\top\right) (\mathbf{\Phi}_K + \mathbf{I})\, \mathbf{r} = \mathbf{g}_{2,k}^\top \mathbf{r}, \tag{19}$$

where

$$\mathbf{g}_{2,k} = \left(\mathbf{\Phi}_K^\top + \mathbf{I}\right) \prod_{j=1}^{k-1} \left(\mathbf{I} - \mathbf{s}_{k-j} \mathbf{s}_{k-j}^\top\right) \mathbf{s}_k. \tag{20}$$

Continuing this way we obtain the general expressions for user k in stage i,

$$\mathbf{e}_{i,k} = \prod_{j=1}^{k-1} (\mathbf{I} - \mathbf{s}_j \mathbf{s}_j^\top) (\mathbf{\Phi}_K)^{i-1} \mathbf{r}, \tag{21}$$

and

$$y_{i,k} = \mathbf{s}_k^\top \prod_{j=1}^{k-1} (\mathbf{I} - \mathbf{s}_j \mathbf{s}_j^\top) \sum_{p=0}^{i-1} (\mathbf{\Phi}_K)^p \mathbf{r} = \mathbf{g}_{i,k}^\top \mathbf{r}, \tag{22}$$

where $(\mathbf{\Phi}_K)^0 = \mathbf{I}$ and

$$\mathbf{g}_{i,k} = \sum_{p=0}^{i-1} (\mathbf{\Phi}_K^\top)^p \prod_{j=1}^{k-1} \left(\mathbf{I} - \mathbf{s}_{k-j} \mathbf{s}_{k-j}^\top\right) \mathbf{s}_k. \tag{23}$$

So far we have considered users successively as suggested by the detector structure, deriving equivalent user specific matched filters, represented by $\mathbf{g}_{i,k}$. It is also possible to directly derive the matrix filter $\mathbf{G}_i = (\mathbf{g}_{i,1}, \mathbf{g}_{i,2}, ..., \mathbf{g}_{i,K})$. Writing out the decision statistic for user k as a function of the most recent decision statistics for the other users, we get

$$y_{i,k} = \mathbf{s}_k^\top \mathbf{r} - \sum_{j=1}^{k-1} \mathbf{s}_k^\top \mathbf{s}_j y_{i,j} - \sum_{m=k+1}^{K} \mathbf{s}_k^\top \mathbf{s}_m y_{i-1,m}. \tag{24}$$

Let $\mathbf{R} = \mathbf{S}^\top \mathbf{S} = \mathbf{I} + \mathbf{L} + \mathbf{L}^\top$ where $\mathbf{L}$ is the strict lower left triangular part of $\mathbf{R}$. Then (24) leads to

$$\mathbf{y}_i = \mathbf{y} - \mathbf{L}\mathbf{y}_i - \mathbf{L}^\top \mathbf{y}_{i-1} = (\mathbf{I} + \mathbf{L})^{\perp 1} (\mathbf{y} - \mathbf{L}^\top \mathbf{y}_{i-1}) = \mathbf{y}_1 - \mathbf{M}\mathbf{y}_{i-1}, \tag{25}$$

where $\mathbf{y}_i = (y_{i,1}, y_{i,2}, ..., y_{i,K})^\top$, $\mathbf{y} = \mathbf{S}^\top \mathbf{r}$, $\mathbf{y}_1 = (\mathbf{I} + \mathbf{L})^{\perp 1}\, \mathbf{y}$ and $\mathbf{M} = (\mathbf{I} + \mathbf{L})^{\perp 1}\, \mathbf{L}^\top$. Eqn. (25) is in fact identical to the Gauss-Seidel iteration [17] that is known to converge if $\mathbf{M}$ has a spectral radius less than one, i.e., eigenvalues with modulus less than one. This is guaranteed if $\mathbf{R}$ is symmetric positive definite, or equivalently, $\mathbf{S}$ has full rank which we have assumed. Therefore the linear SIC always converges. Furthermore, the Gauss-Seidel iteration was developed for matrix inversion so the linear SIC converges to the decorrelator. This was first pointed out in

[12]. From (25) it follows that

$$\begin{aligned}\mathbf{y}_i &= \sum_{m=0}^{i-1}(-\mathbf{M})^m\,(\mathbf{I}+\mathbf{L})^{\perp 1}\,\mathbf{y} = \left(\mathbf{I}-(-\mathbf{M})^i\right)(\mathbf{I}+\mathbf{M})^{\perp 1}\,(\mathbf{I}+\mathbf{L})^{\perp 1}\,\mathbf{S}^\top\mathbf{r} \\ &= (\mathbf{I}-(-\mathbf{M})^i)\mathbf{R}^{\perp 1}\mathbf{S}^\top\mathbf{r} = \mathbf{G}_i^\top\mathbf{r}. \end{aligned} \tag{26}$$

where it is clear that $\mathbf{G}_i^\top \to \mathbf{R}^{\perp 1}\mathbf{S}^\top$ (the decorrelator) for $i \to \infty$ since $\mathbf{M}$ has a spectral radius less than unity. Column k of $\mathbf{G}_i$ is obviously given by (23).

Since the detector is described by linear matrix filtering, a single-user matched filter approach with each matched filter given by $\mathbf{g}_{i,k}$ is possible. The complexity is then inherent in the filter computation. It should be noted however that it is not necessary to perform a matrix inverse to obtain the filter. It is also possible to analytically evaluate the BER performance using techniques similar to those used for the conventional detector. The BER for user k after i stages is thus

$$P_b(i,k) = \frac{1}{2^{K-1}}\sum_{\substack{\text{all } \mathbf{d}\\ d_k=1}} Q\left(\frac{\sqrt{2}\mathbf{g}_{i,k}^\top\mathbf{S}\mathbf{C}\mathbf{d}}{\sqrt{N_0\mathbf{g}_{i,k}^\top\mathbf{g}_{i,k}}}\right). \tag{27}$$

when using short codes. For long codes, the expression above must be averaged over all segments of length N.

B. *Convergence Rate*

When considering convergence rate, it is convenient to have the received vector $\mathbf{r}$ completely contained in the signal space. In case $K < N$ we will therefore consider $\mathbf{r}_s$ rather than $\mathbf{r}$. The starting vector is therefore[3] $\mathbf{e}_{1,1} = \mathbf{r}_s = \mathbf{P}_\mathbf{S}\mathbf{r}$.

In order to investigate the convergence rate, we re-consider Eqn. (21),

$$\mathbf{e}_{i,k} = \prod_{j=1}^{k-1}(\mathbf{I}-\mathbf{s}_j\mathbf{s}_j^\top)(\mathbf{\Phi}_K)^{i-1}\mathbf{r} = (\mathbf{\Phi}_{k-1})^{i-1}\prod_{j=1}^{k-1}(\mathbf{I}-\mathbf{s}_j\mathbf{s}_j^\top)\mathbf{r} = \mathbf{\Phi}_{k-1}\mathbf{e}_{i-1,k} = (\mathbf{\Phi}_{k-1})^{i-1}\mathbf{e}_{1,k}, \tag{28}$$

where we have made use of the definition of $\mathbf{\Phi}_k$ in (14). For convergence we have

$$y_{i,k} - y_{i-1,k} = \mathbf{s}_k^\top\mathbf{e}_{i,k} = \mathbf{s}_k^\top(\mathbf{\Phi}_{k-1})^{i-1}\mathbf{e}_{1,k} \to 0 \quad \text{for } i \to \infty \quad \text{for } \forall\, k. \tag{29}$$

That obviously corresponds to[4]

$$\mathbf{e}_{i,k} = (\mathbf{\Phi}_{k-1})^{i-1}\,\mathbf{e}_{1,k} \to \mathbf{0} \quad \text{for } i \to \infty \quad \text{for } \forall\, k. \tag{30}$$

Since $\mathbf{e}_{i,k}$ depends on the $(i-1)^{\text{th}}$ power of $\mathbf{\Phi}_{k-1}$, the convergence is dependent on the eigenvalues of $\mathbf{\Phi}_{k-1}$. From the definition of eigenvectors, it is easy to show that $\mathbf{\Phi}_k$ and $\mathbf{\Phi}_j$ have the same eigenvalues for all k and j. It is therefore sufficient to consider only one representative matrix

[3] This is merely an orthogonal projection for notational convenience and to simplify the convergence rate arguments. It is thus only for convergence considerations and has no effect on the detection process. Within the detector structure there is no need to pre-filter $\mathbf{r}$.

[4] If we consider $\mathbf{r}$ instead of $\mathbf{r}_s$ then $\mathbf{e}_{i,k} \to \mathbf{r}_{s^\perp}$ for $i \to \infty$.

and we select $\mathbf{\Phi}_K$. From (24) and (25) we have

$$\mathbf{e}_{2,1} = \mathbf{\Phi}_K \mathbf{r} = \mathbf{r} - \sum_{j=1}^{K} \mathbf{s}_j y_{1,j} = (\mathbf{I} - \mathbf{S}\,(\mathbf{I} + \mathbf{L})^{\perp 1}\,\mathbf{S}^{\top})\mathbf{r}, \tag{31}$$

i.e., we have that $\mathbf{\Phi}_K = (\mathbf{I} - \mathbf{S}\,(\mathbf{I} + \mathbf{L})^{\perp 1}\,\mathbf{S}^{\top})$. Considering only $\mathbf{\Phi}_K$, we focus on

$$\mathbf{e}_{i,1} = (\mathbf{\Phi}_K)^{i-1}\,\mathbf{e}_{1,1} = (\mathbf{\Phi}_K)^{i-1}\,\mathbf{r} \to \mathbf{0} \quad \text{for} \;\; i \to \infty \tag{32}$$

for convergence rate estimation. Following the approach for the derivation of the steepest descent algorithm [19], it is convenient to apply a similarity transformation to $\mathbf{\Phi}_K$. Since $\mathbf{\Phi}_K$ is not symmetric and is occasionally defective, i.e., $\mathbf{\Phi}_K$ is not always diagonalisable, we use the Jordan decomposition [18] which is applicable to all square matrices. For notational simplicity we let $\mathbf{\Phi}_K = \mathbf{\Phi}$ in the sequel.

$$\mathbf{\Phi} = \mathbf{X}\mathbf{J}\mathbf{X}^{\perp 1}, \tag{33}$$

where $\mathbf{X}$ is a non-singular $N \times N$ matrix, selected such that $\mathbf{J}$ is a Jordan matrix of the form

$$\mathbf{J} = \begin{pmatrix} \mathbf{J}_1(\lambda_1) & & & \bigcirc \\ & \mathbf{J}_2(\lambda_2) & & \\ & & \ddots & \\ \bigcirc & & & \mathbf{J}_P(\lambda_P) \end{pmatrix}. \tag{34}$$

Each block $\mathbf{J}_p(\lambda_p)$ is a Jordan block of the form

$$\mathbf{J}_p(\lambda_p) = \begin{pmatrix} \lambda_p & 1 & & & \bigcirc \\ & \lambda_p & 1 & & \\ & & \ddots & \ddots & \\ & & & & 1 \\ \bigcirc & & & & \lambda_p \end{pmatrix}, \tag{35}$$

where λ_p is the eigenvalue associated with the $(q_p + 1) \times (q_p + 1)$ Jordan block $\mathbf{J}_p(\lambda_p)$ of order $(q_p + 1)$. The number of Jordan blocks P, is the number of linear independent eigenvectors of $\mathbf{\Phi}$. The number of Jordan blocks corresponding to a given eigenvalue λ_p is the geometric multiplicity $m_g(\lambda_p)$ of λ_p while the sum of the orders of all Jordan blocks corresponding to that eigenvalue is the algebraic multiplicity $m_a(\lambda_p)$ of λ_p. If and only if $m_g(\lambda_p) = m_a(\lambda_p)$, then $\mathbf{\Phi}$ is diagonalisable and $\mathbf{J}$ strictly diagonal.

The Jordan matrix can be expressed as a sum of a diagonal matrix of eigenvalues $\mathbf{\Lambda}$ and a nilpotent matrix $\mathbf{N}$ such that $\mathbf{J} = \mathbf{\Lambda} + \mathbf{N}$. The diagonal matrix of eigenvalues is defined as $\mathbf{\Lambda} = \text{diag}\,(\lambda_1, \lambda_2, ..., \lambda_N)^{\top}$ where the $q_p + 1$ identical eigenvalues of $\mathbf{J}_p(\lambda_p)$ are numbered consecutively. Assuming that the largest Jordan block is of order $q + 1$ and that $i > q + 1$, then $\mathbf{N}^{q+1} = \mathbf{0}$ and

$$\mathbf{J}^{i-1} = \sum_{j=0}^{q} \binom{i-1}{j} \mathbf{\Lambda}^{i-1-j} \mathbf{N}^{j}. \tag{36}$$

For $i > q+1$,

$$\mathbf{e}_{i,1} = \mathbf{X}\mathbf{J}\mathbf{X}^{\perp 1}\mathbf{e}_{i-1,1} \tag{37}$$

$$\Rightarrow \quad \mathbf{X}^{\perp 1}\mathbf{e}_{i,1} = \mathbf{J}\mathbf{X}^{\perp 1}\mathbf{e}_{i-1,1}$$

$$\Rightarrow \quad \mathbf{v}_{i,1} = \mathbf{J}\mathbf{v}_{i-1,1} = \mathbf{J}^{i-1}\mathbf{v}_{1,1}, \tag{38}$$

where

$$\mathbf{v}_{i,1} = \mathbf{X}^{\perp 1}\mathbf{e}_{i,1} \tag{39}$$

$$\Rightarrow \quad \mathbf{e}_{i,1} = \mathbf{X}\mathbf{v}_{i,1} = \sum_{n=1}^{N} \mathbf{x}_n v_{i,1}(n), \tag{40}$$

$\mathbf{X} = [\mathbf{x}_1, \mathbf{x}_2, ..., \mathbf{x}_N]$ and $v_{i,1}(n)$ denotes individual elements of $\mathbf{v}_{i,1}$. To get a simple relationship between $\mathbf{e}_{i,1}$ and $\mathbf{v}_{1,1}$, we consider the impact of a Jordan block. For a Jordan block of order $q_p + 1$ at location n to $n + q_p$, we can combine (36) and (38) to express the elements $v_{i,1}(n)$ to $v_{i,1}(n + q_p)$ as

$$v_{i,1}(n+l) \quad = \quad \sum_{j=0}^{q_p - l} \binom{i-1}{j} \lambda_p^{i-1-j} v_{1,1}(n+j), \quad l = 0, 1, 2, ..., q_p. \tag{41}$$

The vector $\mathbf{v}_{i,1}$ can be divided into segments where each segment is only affected by one specific Jordan block, i.e., all elements of $\mathbf{v}_{i,1}$ can be expressed as indicated above. As we know the linear SIC scheme converges when $\mathbf{S}$ has full rank, then all eigenvalues have modulus less than one. To make conclusive remarks about the convergence rate however, we have to know the eigenvalues of the individual Jordan blocks.

Considering the eigenvalues of $\mathbf{\Phi} = \mathbf{I} - \mathbf{S}\left(\mathbf{I} + \mathbf{L}\right)^{\perp 1}\mathbf{S}^{\top}$, we firstly examine $\mathbf{S}\left(\mathbf{I} + \mathbf{L}\right)^{\perp 1}\mathbf{S}^{\top}$. The non-zero eigenvalues of $\mathbf{S}\left(\mathbf{I} + \mathbf{L}\right)^{\perp 1}\mathbf{S}^{\top}$ are obviosly the same as the eigenvalues of $\left(\mathbf{I} + \mathbf{L}\right)^{\perp 1}\mathbf{S}^{\top}\mathbf{S}$. We further have that $\text{rank}(\left(\mathbf{I} + \mathbf{L}\right)^{\perp 1}\mathbf{S}^{\top}\mathbf{S}) = \text{rank}(\mathbf{R}) = K$ so $\mathbf{S}\left(\mathbf{I} + \mathbf{L}\right)^{\perp 1}\mathbf{S}^{\top}$ has K non-zero eigenvalues and $N - K$ zero eigenvalues corresponding to the orthogonal complement of $\mathbf{S}$ or equivalently, corresponding to the noise space. These eigenvalues are not relevant since $\mathbf{r}_s$ has no components in null $\left(\mathbf{S}^{\top}\right)$ so the corresponding elements of $\mathbf{v}_{i,1}$ are zero. It then follows that $\mathbf{\Phi}$ has $N - K$ eigenvalues of one that are irrelevant for convergence. The K relevant eigenvalues[5] of $\mathbf{\Phi}$ are the same as the eigenvalues of $\mathbf{I} - \left(\mathbf{I} + \mathbf{L}\right)^{\perp 1}\mathbf{S}^{\top}\mathbf{S} = -\mathbf{M}$. Thus, all the eigenvalues have modulus less than one.

It is possible to show that rank $(()\,\mathbf{\Phi}) < N$ so $\mathbf{\Phi}$ has a zero eigenvalue. It is not possible in general to analytically conclude anything further about the K relevant eigenvalues with respect to the geometric and algebraic multiplicities and their relation to the individual Jordan blocks. We therefore resort to simulations in order to gain some useful insight. For all experiments conducted, all non-zero eigenvalues were simple, corresponding to Jordan blocks of order 1. Only the zero eigenvalue was observed to cause $\mathbf{\Phi}$ to occasionally become defective. Fortunately the elements of $v_{i,1}$ corresponding to the zero eigenvalues have obviously no impact on the convergence rate,

[5] If $\mathbf{S}$ is of rank $L < K$ then we have $N - L$ eigenvalues of one that are still all irrelevant. The rank defficiency indicates that $K - L$ spreading codes are linearly dependent and therefore have to be treated as one user.

so we only need to consider the remaining $K - m_a(0)$ non-zero eigenvalues. We can thus express (38) as $\mathbf{v}_{i,1} = \mathbf{\Lambda}^{i-1}\mathbf{v}_{1,1}$ where

$$v_{i,1}(n) = \lambda_n^{i-1} v_{1,1}(n), \quad n \in \mathcal{I}, \tag{42}$$

and $\mathcal{I}$ is the set of indices of cardinality $(K - m_a(0))$, corresponding to the elements of $\mathbf{v}_{i,1}$ relevant for convergence. A similar expression for $\mathbf{e}_{i,1}$ is obtained from (40) and (42)

$$e_{i,1}(j) = \sum_{n \in \mathcal{I}} x_n(j) \lambda_n^{i-1} v_{1,1}(n), \quad j = 1, 2, ..., N. \tag{43}$$

Letting

$$|\lambda_n| = \exp\left(-\frac{1}{\tau(n)}\right) \Rightarrow \tau(n) = \frac{-1}{\ln(|\lambda_n|)}, \tag{44}$$

we have

$$\frac{v_{i,1}(n)}{v_{1,1}(n)} = |\lambda_n|^{i-1} \exp(j(i-1)\phi(n)) = \exp\left(\frac{i-1}{\tau(n)}\right) \exp(j(i-1)\phi(n)), \tag{45}$$

where $\exp(j(i-1)\phi(n))$ is the phase of the complex eigenvalue λ_n. It follows that the absolute value of each element of $\mathbf{v}_{i,1}$ decreases exponentially with each additional stage with a time constant of $\tau(n)$. Eqn. (43) shows that each element in the residual received vector $\mathbf{e}_{i,1}$ converges as a weighted sum of exponentials. The time $\tau(n)$ required for each term to reach $1/e$ of its initial value is given by Eqn. (44). However, the overall time constant τ, defined as the time required for the summation term in Eqn. (43) to decay to $1/e$ of its initial value, cannot be expressed in a simple closed form. The slowest rate of convergence is attained when $x_n(j)v_{1,1}(n) = 0$ for all n except for the mode corresponding to the largest eigenvalue $\lambda_{\max}$. The fastest rate of convergence is attained when only $x_n(j)v_{1,1}(n) \neq 0$ for the n corresponding to the smallest eigenvalue $\lambda_{\min}$.

Accordingly, the overall time constant τ for any element in the residual received vector is bounded by

$$\frac{-1}{\ln(|\lambda_{\min}|)} \leq \tau \leq \frac{-1}{\ln(|\lambda_{\max}|)}. \tag{46}$$

For a large eigenvalue spread, the convergence rate is limited by the largest eigenvalue or the slowest rate of convergence. In general all modes of convergence are the same for all users since $\mathbf{\Phi}_k$ for all k are similar, and thus all users experience similar convergence characteristics, i.e., they have the same convergence rate. This is true for any set of received power levels as these levels only influence the initial estimates, and not the convergence rate. In contrast, user ordering does influence convergence rate as the eigenvalues of $\mathbf{\Phi}$ change. The changes are however quite small and the benefits of power ordering in terms of improvements of the initial estimates are usually more significant.

For theoretical convergence, $\mathbf{e}_{i,1} \to \mathbf{0}$ for $i \to \infty$. In practice however, convergence can be declared when $\mathbf{e}_{i,1}$ is sufficiently small for some finite i. Defining an arbitrary $\epsilon < 1$ we will

consider convergence to have occured when

$$\|\mathbf{e}_{i,1}\|^2 \leq \|\mathbf{r}\|^2 \epsilon. \tag{47}$$

We will denote this behaviour as ϵ-convergence. Based on Eqns. (40) and (43)

$$\|\mathbf{e}_{i,1}\|^2 = \left(\sum_{n\in\mathcal{I}} \mathbf{x}_n \lambda_n^{i-1} v_{1,1}(n)\right)^{\mathrm{H}} \left(\sum_{j\in\mathcal{I}} \mathbf{x}_j \lambda_j^{i-1} v_{1,1}(j)\right). \tag{48}$$

A condition for ϵ-convergence can be expressed as a function of the overall time constant τ. However, in order to relate the rate of ϵ-convergence to the number of stages, it is more convenient to consider the individual time constants $\tau(n)$. An upper bound on $\|\mathbf{e}_{i,1}\|^2$ is obtained by letting $\lambda_n = \lambda_{\max}$ for all n, i.e.,

$$\begin{aligned} \|\mathbf{e}_{i,1}\|^2 &\leq |\lambda_{\max}|^{2(i-1)} \left(\sum_{n\in\mathcal{I}} \mathbf{x}_n v_{1,1}(n)\right)^{\mathrm{H}} \left(\sum_{j\in\mathcal{I}} \mathbf{x}_j v_{1,1}(j)\right) \\ &= |\lambda_{\max}|^{2(i-1)} \|\mathbf{r}\|^2. \end{aligned} \tag{49}$$

Comparing (47) and (49) we can conclude that ϵ-convergence is obtained if

$$|\lambda_{\max}|^{2(i_\epsilon - 1)} \leq \epsilon \tag{50}$$

$$\Rightarrow i_\epsilon \geq \frac{\log \epsilon}{2 \log |\lambda_{\max}|} + 1. \tag{51}$$

In general, a heavily loaded system has several eigenvalues close to one, while lightly loaded systems have smaller eigenvalues. A heavily loaded system therefore converges quite slowly as compared to a lightly loaded system. For example, if $|\lambda_{\max}| = 0.95$ and $\epsilon = 0.01$, $i_\epsilon = 46$ as compared to the case of $|\lambda_{\max}| = 0.5$ with a corresponding $i_\epsilon = 5$.

In a short code system the above analysis leads to the same results for all bits transmitted. When long codes are used, the involved spreading codes and hence, all the involved matrices change for each bit interval. In such systems we must therefore consider the averaged behaviour over all segments of length N.

C. *MSE Considerations*

Assuming perfect cancellation for all users after i stages, then $\mathbf{y}_i = \mathbf{Cd}$ and

$$\mathbf{e}_{i,k} = \mathbf{r} - \mathbf{S}\mathbf{y}_i = \mathbf{r} - \mathbf{SCd} = \mathbf{n}. \tag{52}$$

For the linear SIC however, $\mathbf{e}_{i,k} \to \mathbf{0}$ for $i \to \infty$. At intermediate stages where the residual received vector gets close to estimating the noise vector correctly, it is therefore possible that the linear SIC performs better in terms of BER than at convergence. Due to the principle of alternating projections controlling the linear SIC, perfect cancellation is rarely, if ever, achieved. It is therefore likely that the best estimates of $c_k d_k$ are found at different stages for different users. To estimate the number of stages required for minimising the BER for each user, we consider

the mean squared error,

$$J_k(i) = \mathrm{E}\left\{|y_{i,k} - c_k d_k|^2\right\} = \sigma^2 \|\mathbf{g}_{i,k}\|^2 + c_k^2 + \|\mathbf{C}\mathbf{S}^\top \mathbf{g}_{i,k}\|^2 - 2c_k^2 \mathbf{s}_k^\top \mathbf{g}_{i,k}. \tag{53}$$

If we impose the constraint that all users pass through the same number of stages before the final decision, the overall MSE is considered instead,

$$\begin{aligned} J(i) &= \mathrm{E}\left\{\|\mathbf{y}_i - \mathbf{C}\mathbf{d}\|^2\right\} = \sum_{k=1}^{K} J_k(i) \\ &= \mathrm{Tr}(\sigma^2 \mathbf{G}_i \mathbf{G}_i^\top) + \mathrm{Tr}(\mathbf{C}^2 + \mathbf{C}\mathbf{S}^\top \mathbf{G}_i \mathbf{G}_i^\top \mathbf{S}\mathbf{C} - 2\mathbf{C}^2 \mathbf{S}^\top \mathbf{G}_i) \qquad (54) \\ &= \mathrm{Tr}\left(\sigma^2 \mathbf{R}^{\perp 1} + \sigma^2 \mathbf{R}^{\perp 1}\left(\left(\mathbf{I} - (-\mathbf{M})^i\right)^\top \left(\mathbf{I} - (-\mathbf{M})^i\right) - \mathbf{I}\right)\right) + \mathrm{Tr}\left(\mathbf{C}^2 \left(\mathbf{M}^i\right)^\top \mathbf{M}^i\right) \qquad (55) \\ &= J_{\mathrm{dec}} + J_{\mathrm{ex}}(i), \end{aligned}$$

where Tr denotes the trace operator. In (55) the first term corresponds to the MSE level of the decorrelator, while the remaining terms represent the excess MSE $J_{\mathrm{ex}}(i)$. Comparing (54) and (55), then the two trace terms of (54) correspond directly to the two trace terms of (55). We can also relate (53) to (54) and hence to (55). There is a one-to-one correspondence of diagonal element k of each of the matrix terms in (54) to the similar terms in (53). Due to the asymmetry of $\mathbf{M}$ it is not possible to simplify the MSE expressions any further.

For a received signal only perturbed by additive white Gaussian noise, minimising the MSE corresponds to minimising the BER [16]. For systems with a sufficient number of active users such that the multiple access interference adequately resembles AWGN, it is therefore expected that the MSE can accurately estimate the required number of stages for minimising the BER. For fewer active users, where the MAI is not behaving like AWGN, the MSE will only provide a reasonable indication of the required number of stages. This is demonstrated in the following section through numerical examples.

IV. Numerical Results

The numerical examples considered in this section are based on a symbol-synchronous system with perfect power control[6]. We focus on a $K = 8$ and a $K = 16$ system with $N = 32$. Typical sets of randomly selected short codes are first used for both cases, where $\lambda_{\max} = 0.4207$ for $K = 8$ and $\lambda_{\max} = 0.7333$ for $K = 16$. In the examples using short codes, all results are obtained by direct evaluation of Eqns. (27), (47), (50) and (54). The use of long codes is modelled by selecting a random set of spreading codes for each bit interval. The performance curves have been obtained by Monte Carlo averaging of (27) and (54) over the distribution of the random codes.

In Figure 3, we demonstrate the convergence behaviour of the scheme using short codes as quantified by (47). The plot shows $\|\mathbf{e}_{i,k}\|^2$ normalised with respect to $\|\mathbf{r}\|^2$ as a function of the number of stages, where 3 of the $K = 8$ and $K = 16$ users are considered. A noiseless case

[6] Unequal powers can easily be introduced. The characteristic behaviour is however more pronounced, assuming perfect power control.

with a fixed data sequence is depicted since neither noise nor data influence (47) as it describes a relative relationship. The same behaviour is observed for all data sequences and all SNR. It is clear that all users converge at the same rate, which is not to say that they converge to the same performance. That depends on the initial starting point determined by $\mathbf{s}_k^\top \mathbf{r}$, i.e., the correlation characteristics of the codes. As discussed previously, it is clear that the system load significantly influences the convergence rate. The convergence rate for $K = 8$ is more than twice as fast as for $K = 16$. The ϵ-bounds based on (50) are included in the figure. For these cases, the bounds are generally within 4 stages of the actual stage where ϵ-convergence is achieved.

The BER and MSE performances as functions of the number of stages for the same typical short-code system as above with $K = 8$ are shown in Figure 4 for an E_b/N_0 of 4 dB and 12 dB, respectively. Three users, each with different convergence behaviour at 4 dB have been selected. Based on ϵ-convergence at $\epsilon = 10^{-2}$, we should consider up to 4 stages as indicated by Figure 3. For user $k = 6$, both the BER and the MSE reach a minimum before converging to the decorrelator performance. This is the most common behaviour for any user of any code set. A small fraction of users do not reach a minimum or the minimum is achieved for $i = 1$, i.e., they converge monotonically either from above or below as exemplified by users $k = 3$ and $k = 8$ in Figure 4. When a user converges from below, the performance consistently degrades towards convergence. The averaged behaviour is also included in the figure. The averaged performance is usually dominated by users like $k = 6$.

Comparing the BER and the MSE behaviour, we observe that for users $k = 3$ and $k = 8$, minima are achieved for the same number of stages. For user $k = 6$ and for the averaged performance, the MSE suggests 2 stages instead of 3 which actually leads to the true minimum BER. The differences both in BER and MSE level for $i = 2$ and $i = 3$ are however quite small. For an E_b/N_0 of 12 dB, all users converge from above and there is a good agreement between the number of stages achieving minima for the BER and the MSE, respectively. The MSE, which is easier to determine than the BER, therefore gives a good indication as to where the minimum BER is obtained. In a few cases, the number of stages required to minimise the MSE and the BER do not agree. The corresponding BER differences are small however. For increasing E_b/N_0, the number of stages required for achieving the minimum BER eventually coincides with the number of stages required for ϵ-convergence as demonstrated for $E_b/N_0 = 12$ dB. This is due to the fact that $\mathbf{e}_{i,k} \to \mathbf{n}$ for perfect cancellation and $\mathbf{e}_{i,k} \to \mathbf{0}$ at convergence. For high noise levels, better cancellation is therefore achieved on average at a number of stages prior to convergence, while for lower noise levels where $\sigma^2 \to 0$, the two criteria are virtually identical.

In Figure 5, similar results for the case of $K = 16$ are presented and similar behaviour is observed. In this case, we expect ϵ-convergence for $\epsilon = 10^{-2}$ at $i = 8$ stages. User $k = 1$ reaches an obvious minimum, while user $k = 5$ is practically converging from above. User $k = 16$ is in this case converging from above as opposed to user $k = 8$ in the previous example. The MSE however, still has a minimum for $i = 1$, indicating convergence from below. The resulting BER differences are however, quite small. As for the previous case, we observe a diminishing difference in the number of stages required for convergence and for minimising the BER as the E_b/N_0 increases.

The trends discussed above are typically observed for short code systems. The examples have

been chosen to demonstrate this typical behaviour. In Figure 6 and 7, we show the performance for a $K = 8$ long code system with perfect power control. The convergence behaviour of (47) and (50) is illustrated in Figure 6 for users $k = 2, 4, 6, 8$. For random codes, we observe that the size of the residual errors are user-ordered at each stage. Intuitively, this make sense as the last user supposedly benefits the most from successive cancellation. For practical ϵ-values, the bound of (50) is within 2-3 stages of the actual number of stages required for ϵ-convergence. The bound however, gets looser for decreasing ϵ-values.

In Figure 7, the BER and the MSE for the long code example are presented. The performance of users $k = 2, 6, 8$ and the averaged performance are included. The performance of user $k = 4$ is virtually identical to the averaged performance. For random codes, all users of course converge to the same performance. At an E_b/N_0 of 12 dB, all users have a minimum for both BER and MSE. The minima, however, are only marginally smaller than the convergence level. At an E_b/N_0 of 4 dB, all users also achieve a minimum for the BER. For the MSE, all but user $k = 8$ reaches a minimum. User $k = 8$ converges from below for MSE. In this case, the MSE is therefore a poor indicator of the BER performance for user $k = 8$. For the other users, the discrepancies between the BER achieved for a minimum MSE and the minimum BER are small.

Based on our simulation study, a rule of thumb for the number of stages required for achieving a BER close to, or better than the corresponding decorrelator performance for either short codes or long codes, is obtained by considering the number of stages required for achieving ϵ-convergence for $\epsilon = 10^{-2}$. For the 3 cases considered here, that corresponds to $i = 4$, $i = 8$ and $i = 4$, respectively, which gives close to the minimum achievable BER.

V. Concluding Remarks

In this paper we have developed a matrix-algebraic approach to linear successive interference cancellation. Based on this approach, it is realised that both single- and multi-stage linear SIC schemes correspond to a one-shot linear matrix filtering. It follows that the bit error rate can be determined analytically as for the conventional detector. The description also allows for quantitative statements concerning convergence in general and convergence rate in particular. Additionally, the concept of ϵ-convergence is introduced as a tool for determining the number of stages necessary for practical convergence.

It is observed that the BER does not generally decrease monotonically for all users as the number of stages increase. The MSE, which is easier to determine than the BER, is used to give an indication of the corresponding BER performance. The ϵ-convergence can also be used as a performance indicator. As a rule of thumb, ϵ-convergence for $\epsilon = 10^{-2}$ is suggested for estimating the required number of stages for achieving close to, or better than, the corresponding decorrelator performance.

In general, only a few stages are required to approach, or supersede, the decorrelator performance. This makes linear successive interference cancellation a very low-complexity implementation alternative for powerful linear detectors. The simplicity of the interference cancellation unit makes it particularly suitable for dedicated hardware implementation as opposed to the decorrelator.

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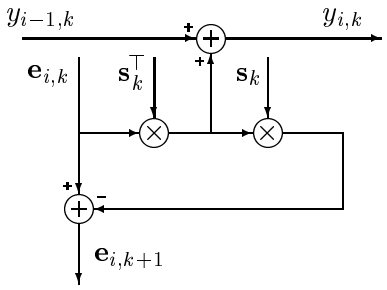

Figure 1: Linear successive interference cancellation unit.

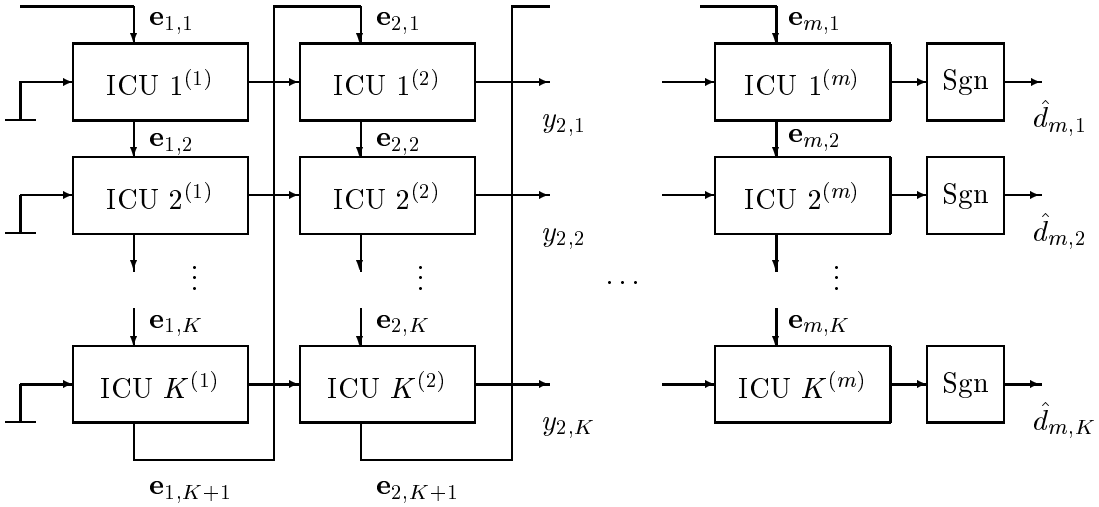

Figure 2: Multi-stage linear successive interference cancellation structure.

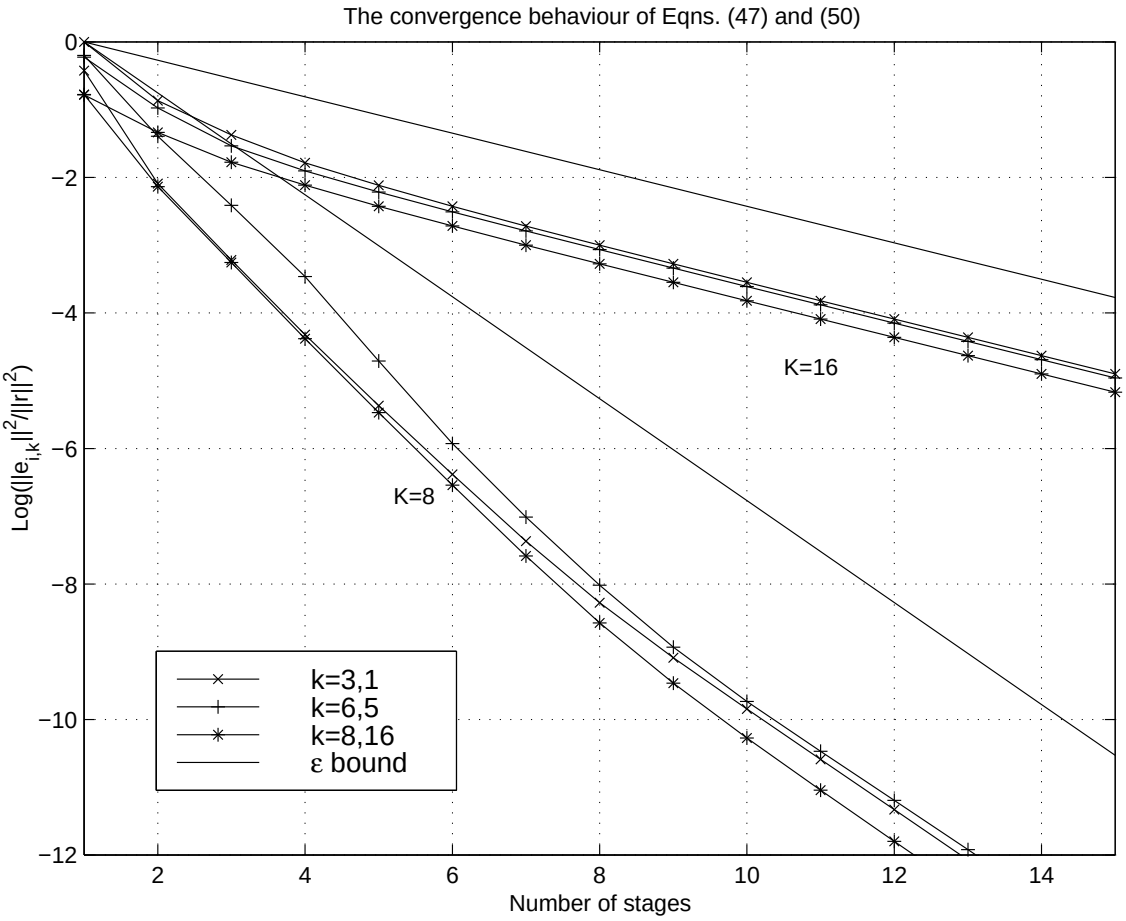

Figure 3: The normalised length of the residual error vector as a function of the number of stages for typical short code systems with $K = 8$ and $K = 16$, $N = 32$. Users $k = 3, 6, 8$ and $k = 1, 5, 16$, respectively, are considered for the two cases. Also included are the ϵ-bounds obtained from (50).

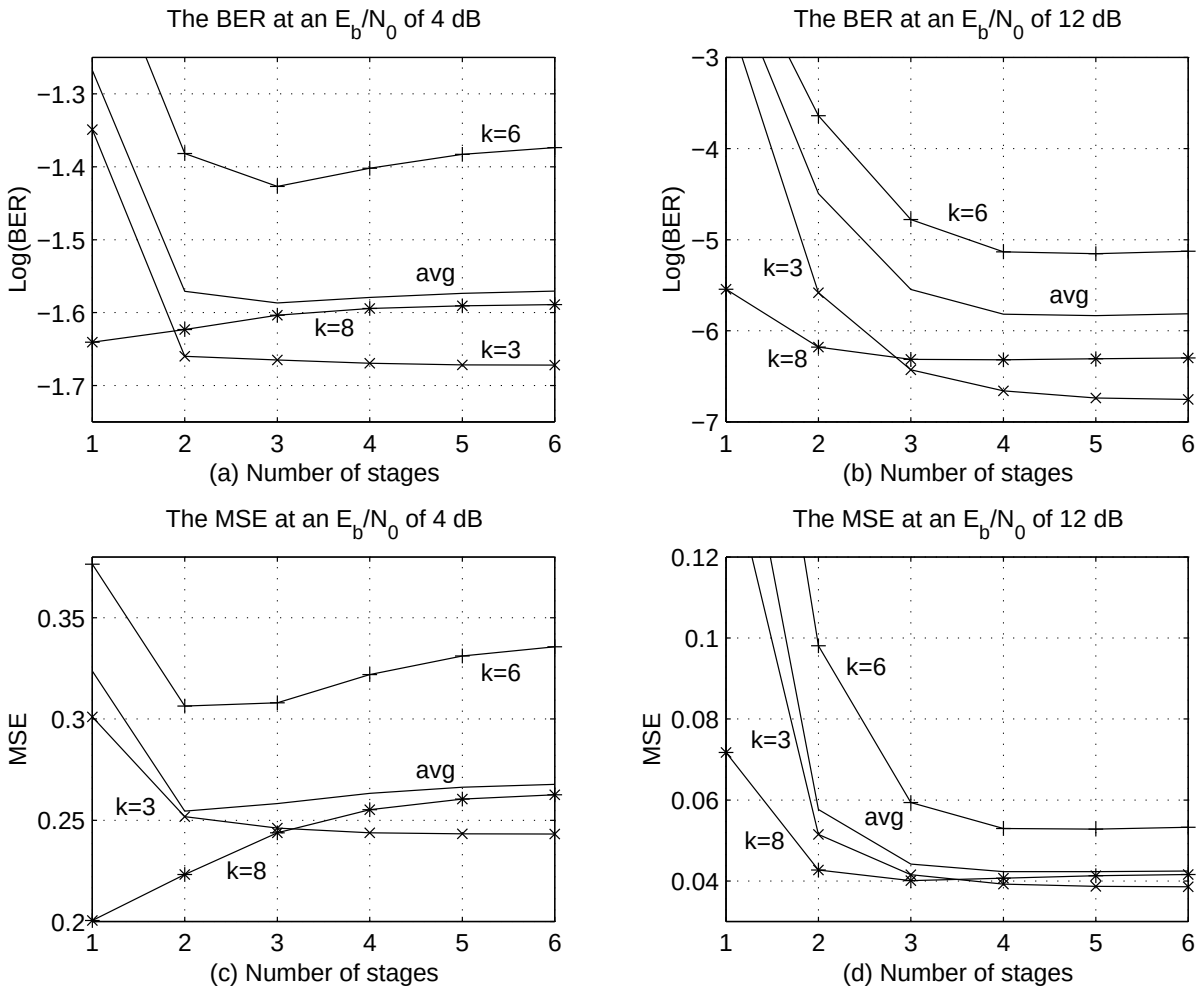

Figure 4: The BER and MSE performance as a function of the number of stages for a typical short code systems with $K = 8$, $N = 32$. Users $k = 3, 6, 8$ are considered for an E_b/N_0 of 4 and 12 dB. Also included is the averaged performance.

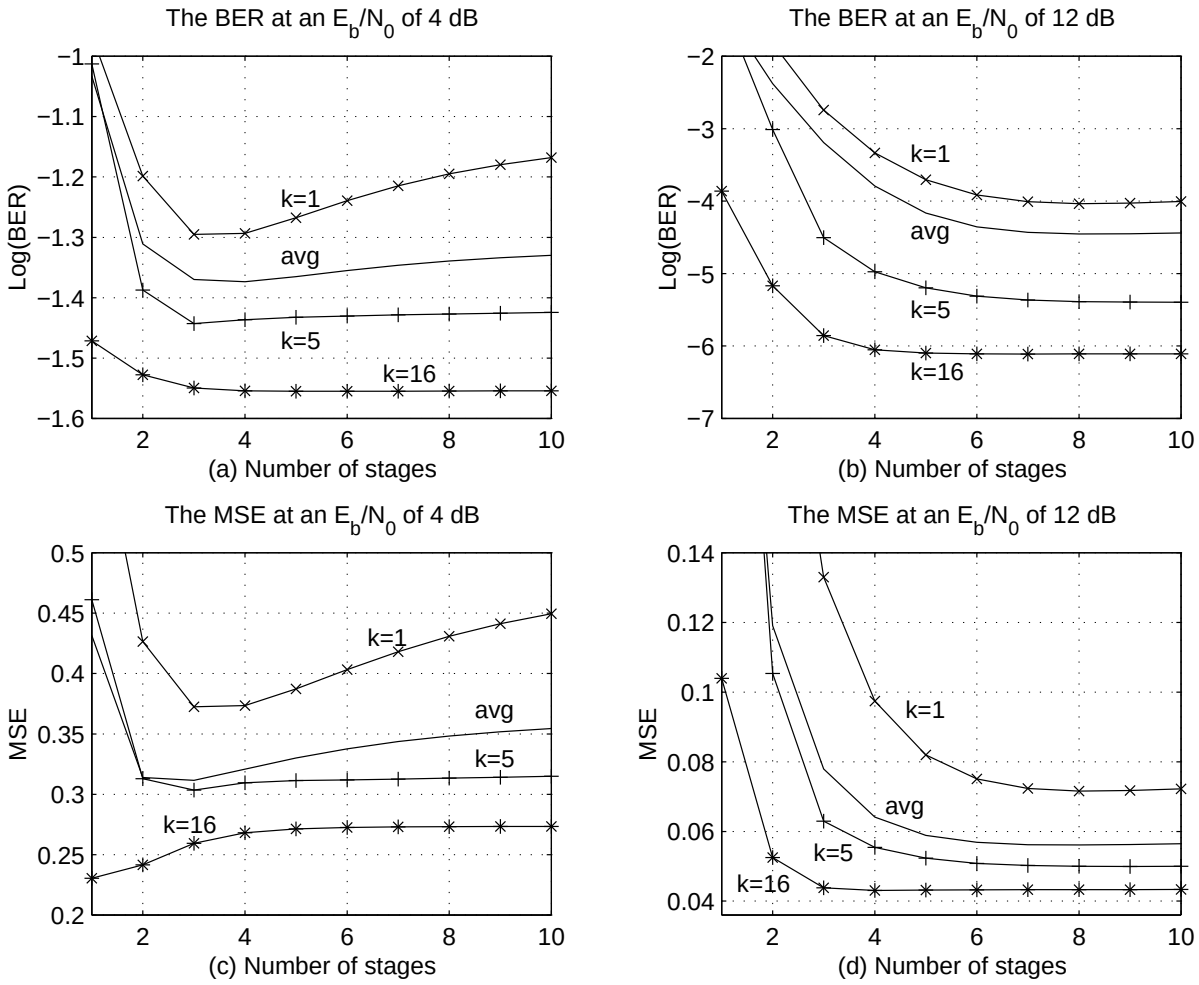

Figure 5: The BER and MSE performance as a function of the number of stages for a typical short code systems with $K = 16$, $N = 32$. Users $k = 1, 5, 16$ are considered for an E_b/N_0 of 4 and 12 dB. Also included is the averaged performance.

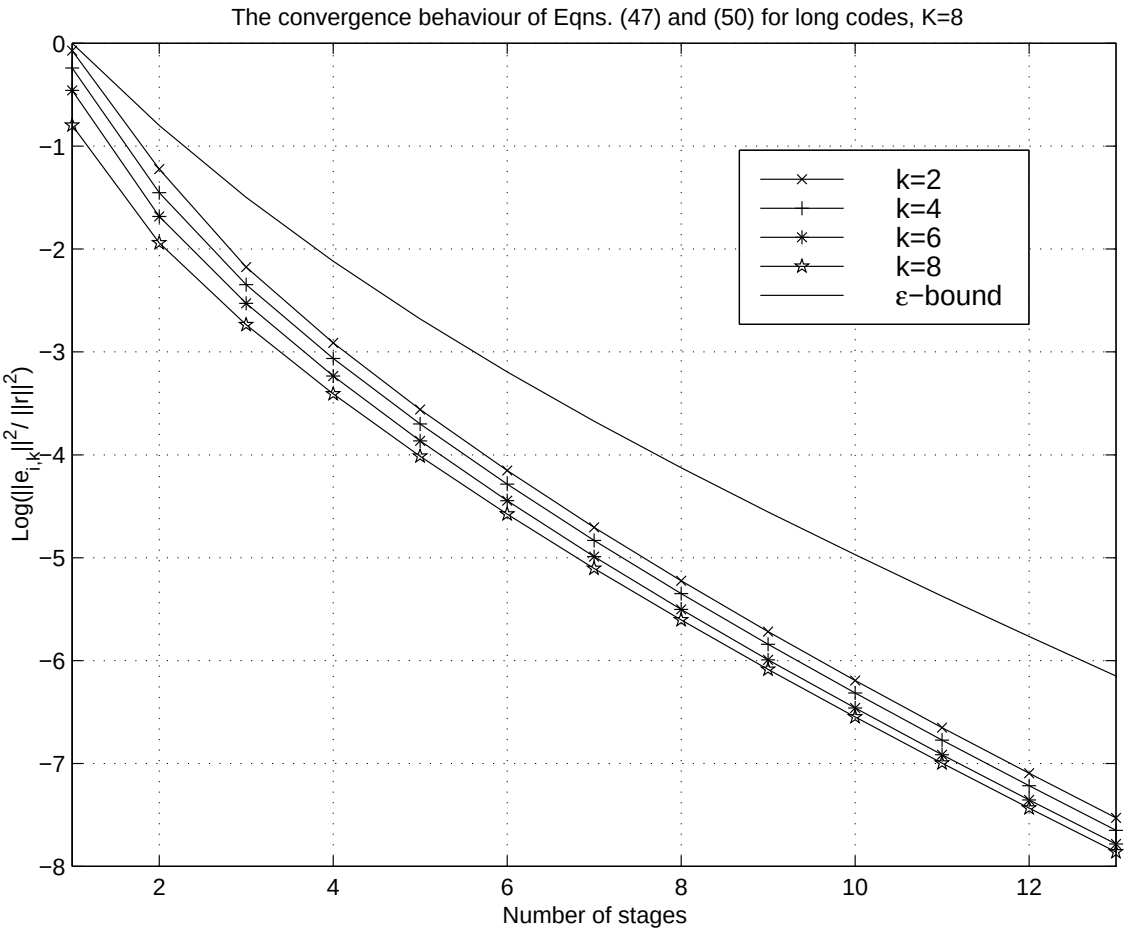

Figure 6: The normalised length of the residual error vector as a function of the number of stages for a long code example with $K = 8$, $N = 32$. Users $k = 2, 4, 6, 8$ are considered and the ϵ-bound obtained from (50) is included.

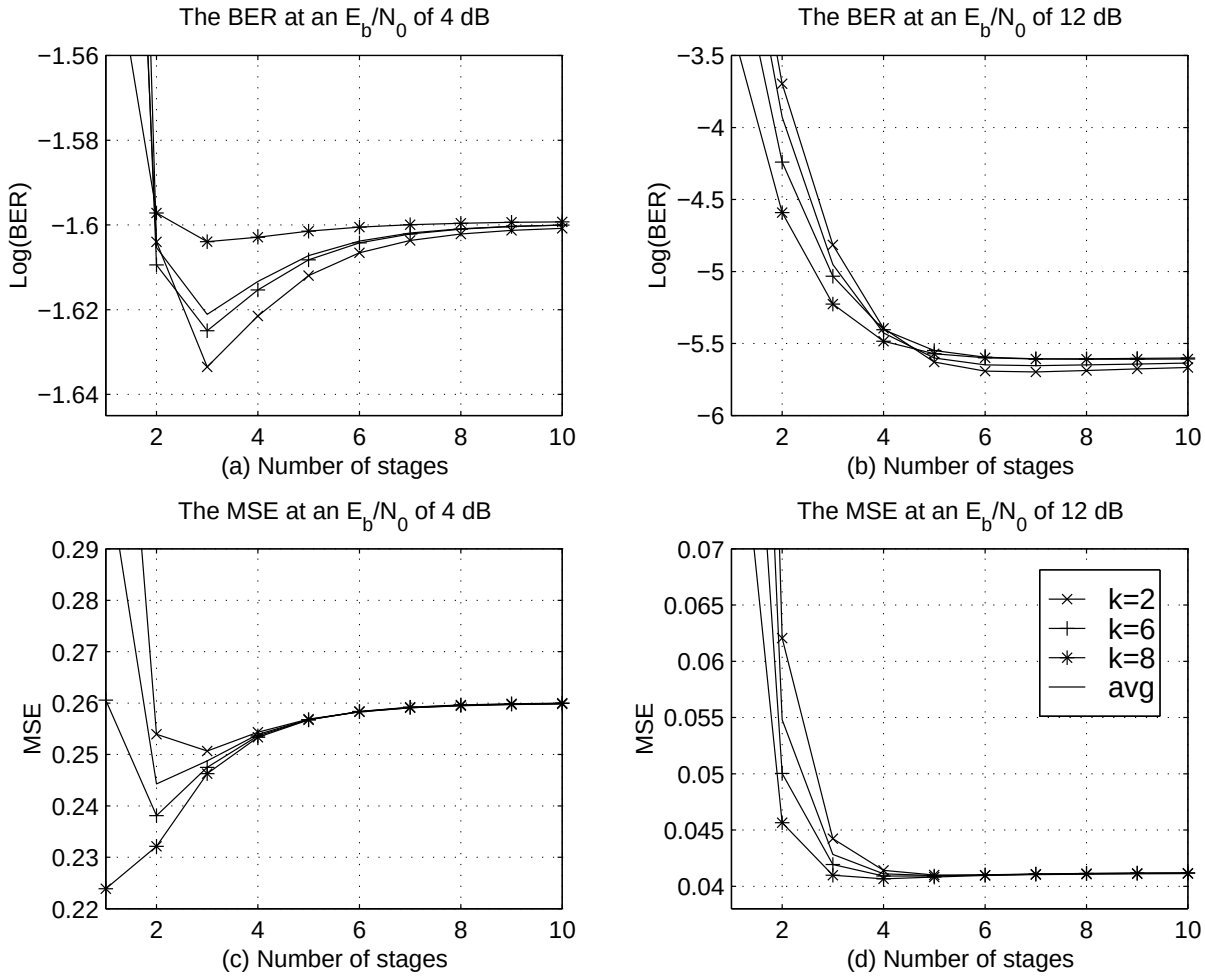

Figure 7: The BER and MSE performance as a function of the number of stages for a long code system with $K = 8$, $N = 32$. Users $k = 2, 6, 8$ are considered for an E_b/N_0 of 4 and 12 dB. Also included is the averaged performance which is virtually identical to the performance of user $k = 4$.