Second Order Sliding Mode Control of a 3-Dimensional Overhead Crane

Carlos Vazquez, Leonid Fridman and Joaquin Collado

Abstract—In this paper the 3-Dimensional Overhead Crane is the subject of study. The note includes the Euler-Lagrange equations of a 5 degrees of freedom, 5DOF, Overhead Crane and the design of a Second Order Sliding Mode Controller in order to ensure the precise load transfer during the load movement despite of model uncertainties and un-modeled dynamics. Moreover, the system is under-actuated and 5DOF should be controlled with 3 control actions. The stability analysis is developed based on Lyapunov second method ensuring the finite time convergence to the desired sliding output. Simulations and Experimental results are presented.

I. INTRODUCTION

The study of Overhead Crane systems has been considered of fundamental interest during the last decades. These kind of systems are widely used in several manufacturing and industrial activities for transportation of heavy loads and the impact of its performance is crucial, see [1]. Besides, Cranes are under actuated mechanical systems and the acceleration required for motion always induces undesirable payload oscillations. This undesirable swing motion is the principal cause of load damages and accidents.

The solution to this problem is not trivial and several attempts have been considered in literature. For the 2 DOF crane the optimal solution has been considered in [2]. In [3] a saturation control, considering constraints in the sates, was proposed. Other techniques which include delay in the measurement were developed in [4] and [5]. Sliding mode control techniques have been considered in [6]. Recently in [7] and adaptive tracking control was implemented.

In the work of [8], a new coordinate system for the 3-Dimensional Crane was introduced in order to avoid the singularity on the equilibrium point. Even thought, the majority of the work concerning the 3D Overhead Crane are based on small oscillations analysis, see [1], [8] and [9], new techniques that can handle the overall nonlinear model have been proposed, see [10] and [11]. However, perturbations are not considered in the analysis and only the case of constant rope length was considered in [11].

In order to compensate perturbations, Sliding Mode Control has been shown to be robust and easy to implement, see [12] and [13]. Basically, the design consists of two steps:

- The selection of a suitable sliding surface in the state space such that the system exhibits the desired behavior.
- The design of an appropriate sliding mode enforcement such that the trajectories of the system converge to the sliding surface in finite time.

The sliding mode enforcement requires control signals to commute at theoretically infinite frequency. Particularly, this is not realizable in the case of electromechanical systems and DC motors. In this case, Second Order Sliding Modes Control is a suitable technique which has been proven to be effective in chattering attenuation while at the same time preserving the sliding mode properties, see [14], [15] and [16]. This note presents a solution to the control problem of a 5DOF Overhead Crane System. First we adopt the coordinate definition suggested in [8] and obtain the Euler-Lagrange equations with the representation proposed in [17], for under actuated mechanical systems. Then we define a new nonlinear control law for the crane system based on Sliding Modes of Second Order. The stability analysis of the overall closed loop system which include three control inputs is achieved with the non-differentiable Lyapunov function approach, see [18] and [19]. With this method the convergence time to the desired surface is estimated.

The remainder of this note is organized as follows. First, the Statement of Problem is described in Section II. Section III presents the Control Design. Simulations are presented in section IV. In Section V, the obtained results are validated experimentally over a Laboratory IntecoTM 3D Crane. Finally in Section VI, the Conclusions are drawn for this study.

II. STATEMENT OF PROBLEM

Moving a suspended load along a pre-specified path is not an easy task since the system is under-actuated. This type of crane corresponds to a spherical pendulum attached to a moving support, the trolley. Figure 1 shows the coordinate system of the 3D crane, where $XYZ$ is the fixed coordinate system and $X_1Y_1Z_1$ is the trolley coordinate system which moves with the trolley. The origin of the trolley coordinate system is $(0, 0, 0)$ in the fixed coordinate system; $\theta_x$ and $\theta_y$ are the swing angles projected on the $X_1Z_1$ and $Y_1Z_1$ planes respectively, and $\beta$ is the swing angle measured from the $X_1Z_1$ plane. The trolley moves on the girder in the $Y_1$ direction (traverse) and the girder and $Y_1$ axis move in the $X_1$ direction (travel) direction. In this note, the load is considered as a point mass and the mass and stiffness of the rope are neglected.
The position of the load in the fixed coordinate system is given by

\[ \begin{align*}
x_m &= x + l \sin \theta \cos \beta, \\
y_m &= y + l \sin \beta, \\
z_m &= -z - l \cos \beta \cos \theta,
\end{align*} \]

where \( x, y, l, \theta_x \) and \( \beta \) are defined as the generalized coordinates to describe the motion. The kinetic energy of the crane and its load, \( T \), and the potential energy of the load, \( P \), are given by

\[ \begin{align*}
T &= \frac{1}{2} m (\dot{x}_m^2 + \dot{y}_m^2 + \dot{z}_m^2) + \frac{1}{2} m_c (\dot{x}_c^2 + \dot{y}_c^2) + \frac{1}{2} m_r (\dot{x}_r^2) \\
P &= mgl(1 - \cos \theta \cos \beta)
\end{align*} \]

where \( g \) is the gravity, \( m_c \) is the trolley mass, \( m \) is the load mass, \( m_r \) is the girder mass and \( l \) is the rope length.

**Remark 1**: In the rest of the paper, the following notation will be used: \( \cos \theta_x = c_{\theta_x} \) and \( \sin \theta_x = s_{\theta_x} \).

The equations of motion of the crane system, obtained by the Lagrange’s equations associated with the generalized coordinates \( x, y, l, \theta_x \) and \( \beta \) respectively, are given by:

\[ \begin{align*}
M_{11}\ddot{q}_1 + M_{12}\ddot{q}_2 + h_1 + \phi_1 &= 0 \\
M_{21}\ddot{q}_1 + M_{22}\ddot{q}_2 + h_2 + \phi_2 &= \tau
\end{align*} \]

where

\[ \begin{align*}
q_1 &= \begin{bmatrix} \theta_x, \beta \end{bmatrix}^T, \\
q_2 &= \begin{bmatrix} x, y, l \end{bmatrix}^T, \\
\phi_2 &= [0, 0, -mgc_{\beta}\theta_x]^T, \\
\phi_1 &= \begin{bmatrix} mlcg_{\beta}s_{\theta_x}, mlcg_{\beta}s_{\beta} \end{bmatrix}^T, \\
h_1 &= C_{11}\ddot{q}_1 + C_{12}\ddot{q}_2, \\
h_2 &= C_{21}\ddot{q}_1 + C_{22}\ddot{q}_2 \\
\tau &= \begin{bmatrix} f_x, f_y, f_3 \end{bmatrix}^T
\end{align*} \]

The corresponding matrices are the next ones:

\[ \begin{align*}
M_{11} &= \begin{bmatrix} ml^2c_2^2 & 0 & 0 \\
0 & ml^2 & 0 \\
0 & 0 & ml^2
\end{bmatrix}, \\
M_{21} &= \begin{bmatrix} ml^2c_{\beta} & 0 & 0 \\
0 & ml_{\beta} & 0 \\
0 & 0 & ml_{\beta}
\end{bmatrix}, \\
M_{12} &= \begin{bmatrix} mlc_{\beta} & 0 & 0 \\
0 & mlc_{\beta} & 0 \\
0 & 0 & mlc_{\beta}
\end{bmatrix}, \\
M_{22} &= \begin{bmatrix} m_{\beta} & 0 & m_{\beta}s_{\beta}c_{\beta} \\
0 & m_{\beta} & 0 \\
m_{\beta}s_{\beta}c_{\beta} & m_{\beta}s_{\beta}c_{\beta} & m_{\beta}
\end{bmatrix}
\]
where $\sigma = [\sigma_x, \sigma_y, \sigma_l]^T$, $\bar{u} = [\bar{u}_x, \bar{u}_y, \bar{u}_l]^T$ and $d_0 = [-\dot{x}_{ref}, -\dot{y}_{ref}, -\dot{l}_{ref}]^T$. We may rewrite system (8) like,
\[ \ddot{\sigma}_\alpha = \bar{u}_\alpha - \dot{\sigma}_{ref}, \quad \text{for } \alpha = x, y, l \] (9)

With linear feedback, the variable $\sigma$, at best, approaches zero in infinite time; with an appropriate nonlinear feedback the variable $\sigma$ can reach zero in finite time. For this purpose we propose the control law:
\[ \bar{u}_\alpha = u_{0\alpha} + u_{1\alpha} \] (10)

where $u_{0\alpha}$ represents the linear part of the controller,
\[ u_{0\alpha} = -k_{p\alpha}\sigma_l - k_{d\alpha}\dot{\sigma}_\alpha \] (11)

and $u_{1\alpha}$ is the twisting algorithm,
\[ u_{1\alpha} = -r_{1\alpha}\text{sign}(\sigma_\alpha) - r_{2\alpha}\text{sign}(\dot{\sigma}_\alpha) \] (12)

for $\alpha = x, y, l$. In the next section we study stability of system (9)-(10) by Lyapunov second method, see [22]. In the analysis we will use an almost everywhere differentiable Lyapunov function, see [18] and [19].

A. Stability Analysis

Setting $\tilde{\sigma}_1 = \sigma_x$, $\tilde{\sigma}_2 = \sigma_y$ and $\bar{\sigma} = [\tilde{\sigma}_1, \tilde{\sigma}_2]^T$. We can rewrite system (9)-(10) in state space form:
\[ \begin{align*}
\ddot{\bar{\sigma}}_1 &= \tilde{\sigma}_2 \\
\ddot{\bar{\sigma}}_2 &= -k_{p\alpha}\sigma_1 - k_{d\alpha}\dot{\sigma}_2 + u_{1\alpha} + \bar{d}
\end{align*} \] (13)

where $k_{p\alpha} > 0$, $k_{d\alpha} > 0$, $u_{1\alpha}$ is the twisting controller and $\bar{d} = -\dot{\sigma}_{ref}$ is the perturbation which is bounded, i.e. $|\bar{d}| \leq \bar{r}$.

Theorem 1: With the twisting algorithm,
\[ u_{1\alpha} = -r_{1\alpha}\text{sign}(\sigma_\alpha) - r_{2\alpha}\text{sign}(\dot{\sigma}_\alpha) \] (14)

and considering $r_{1\alpha} > r_{2\alpha} + \bar{r}$ and $r_{2\alpha} > \bar{r}$; also $k_{p\alpha} > \frac{1}{\bar{r}}$, $k_{d\alpha} > \frac{1}{\gamma_0}$ with $\gamma_0 > 0$. Then system (13) is exponentially stable.

Proof: Consider the next Lyapunov function:
\[ V_0(\bar{\sigma}) = \frac{1}{2} \bar{\sigma}^T P \bar{\sigma} + r_{1\alpha}\gamma_0|\sigma_1| \] (15)

with $P = \begin{bmatrix} \gamma_0 k_{p\alpha} + k_{d\alpha} & 1 \\ 1 & \gamma_0 \end{bmatrix}$. This Lyapunov function is differentiable almost everywhere. With $\gamma > 0$, $k_{1\alpha} > \frac{1}{\gamma_0}$ and $k_{2\alpha} > 0$ we have $P > 0$. Now defining,
\[ V_{01} = \frac{1}{2} \lambda_{min}[P]|\bar{\sigma}|^2, \quad V_{02} = \frac{1}{2} \lambda_{max}[P]|\bar{\sigma}|^2 + r_{1\alpha}\gamma_0|\sigma_1| \]

which implies: $V_{01} \leq V_0 \leq V_{02}$.

Remark 2: $\lambda_{max(min)}$ represents the operation of taking largest (smallest) eigenvalue.

Now $V_0$ is given by,
\[ V_0 \leq -\bar{\sigma}^T Q \bar{\sigma} - r_{min}|\bar{\sigma}| \] (16)

with $Q = \begin{bmatrix} k_{p\alpha} & 0 \\ 0 & \gamma_0 k_{d\alpha} - 1 \end{bmatrix}$ and $r_{min} = \min\{r_{1\alpha} - r_{2\alpha} - \bar{r}, \gamma_0(r_{2\alpha} - \bar{r})\} > 0$. If $k_{d\alpha} > \frac{1}{\gamma_0}$ we have $Q > 0$. Moreover, $V_0 \leq -\lambda_{min}[Q]|\bar{\sigma}|^2 - r_{min}||\bar{\sigma}||$. ■

B. Finite Time Convergence

Theorem 2: If conditions of theorem 1 are satisfied together with:
\[ (\gamma_0 - c)(\gamma_0 k_{p\alpha} + k_{d\alpha}) - 1 > 0 \quad \text{and} \quad \lambda_{1}^2 \gamma_0^2 - \frac{c^2}{4} > 0 \]

$\gamma_0 > 1$, $c > 0$, $\gamma_0 > 0$ constants and $\gamma_0 - c > 1$

\[ r_{1\alpha} > r_{2\alpha} > \bar{r} + \frac{k_{d\alpha}}{\gamma_0} \quad \text{and} \quad \lambda_0 > (1 + \frac{k_{d\alpha}}{\gamma_0}) \]

(17)

where $\lambda_0 = 2\lambda_{min}[P] \cdot \min\{\lambda_{min}[Q], r_{min}\}$. Then point $\bar{\sigma} = [0, 0]^T$ of system (13) is globally exact finite time stable. Moreover, (18) is a strict Lyapunov function assuring the robustness and convergence properties of the twisting controller. An upper bound, $T_{\text{reach}}$, of the convergence time of a trajectory starting at $\bar{\sigma}(0)$ can be estimated by:
\[ T_{\text{reach}} \leq \frac{2}{\lambda_2} V_0^{\frac{1}{2}}(\bar{\sigma}(0)) \]

where:
\[ \lambda_1 = \lambda_0 - \gamma(1 + \frac{k_{d\alpha}}{\gamma_0}) \quad \text{and} \quad \bar{c}_1 = \frac{3}{2} \bar{c}_0 + 1 \]

$\lambda_2 = \min\{\lambda_1, \gamma(r_{1\alpha} - r_{2\alpha} - \bar{r} + \frac{k_{d\alpha}}{\gamma_0})\}$

$\tau_0 = \max\{\frac{1}{2}\lambda_{max}[P]^2, r_{1\alpha}\gamma_0\lambda_{max}[P_0], r_{1\alpha}\gamma_0\lambda_{max}[P_\xi]\}$

Proof: Consider the Lyapunov function:
\[ V_1(\bar{\sigma}) = V_0^2 + \gamma_1|\sigma_1|^2|\dot{\sigma}_{\gamma}| \] (18)

with $V_0$ as in (15) and $\gamma_1$ constant. $V_1$ can be rewritten in the next form:
\[ V_1 = \frac{1}{4}(\bar{\sigma}^T P \bar{\sigma})^2 + r_{1\alpha}\gamma_0|\sigma_1|(|\bar{\sigma}|^2 P_0 \bar{\sigma} + \zeta^T P_\xi \zeta) \] (19)

where $\zeta = [\bar{\sigma}_{\gamma}^2 \text{sign}(\sigma_1), \sigma_2^2]^T$, $P$ as in (15) and
\[ P_0 = \begin{bmatrix} \gamma_0 k_{p\alpha} + k_{d\alpha} & 1 \\ 1 & \gamma_0 - c \end{bmatrix}, \quad P_\xi = \begin{bmatrix} \frac{r_{1\alpha}^2 \gamma_0^2}{2} & \gamma_1 \frac{3}{2} \\ \frac{3}{2} & c \end{bmatrix}, \quad c \] is a constant to be chosen.

1) Definite Positive: First we will show that the function (18) is definite positive and decrescent. Because of theorem 1 $P > 0$. In order to get $P_0 > 0$ and $P_\xi > 0$ we need to satisfy:
\[ \begin{align*}
(\gamma_0 - c)(\gamma_0 k_{p\alpha} + k_{d\alpha}) - 1 > 0 \\
\frac{r_{1\alpha}^2 \gamma_0^2}{2} - \frac{c^2}{4} > 0 \\
c > 0
\end{align*} \]

Setting,
\[ V_{10} = \frac{1}{4} \lambda_{min}[P]|\bar{\sigma}|^4 + r_{1\alpha}\gamma_0|\sigma_1|(|\bar{\sigma}|^2 + \lambda_{min}[P_\xi]|\zeta|^2) \]

\[ V_{11} = \frac{1}{4} \lambda_{max}[P]|\bar{\sigma}|^4 + r_{1\alpha}\gamma_0|\sigma_1|(|\bar{\sigma}|^2 + \lambda_{max}[P_\xi]|\zeta|^2) \]

It implies,
\[ V_{10} \leq V_1 \leq V_{11} \] (20)

Moreover,
\[ V_1 \leq \tau_0(|\bar{\sigma}|^4 + |\sigma_1|(|\bar{\sigma}|^2 + |\zeta|^2)) \] (21)

which implies,
\[ V_1 \leq \frac{3}{2} \tau_0(|\bar{\sigma}|^4 + |\sigma_1|^2 + |\sigma_2|^4) \] (22)
where,
\[
\sigma_0 = \max\left\{ \frac{1}{4} (\lambda_{\max}[P])^2, r_{1a}\gamma_0\lambda_{\max}[P_0], r_{1a}\gamma_0\lambda_{\max}[P_c] \right\}
\]

Now we have,
\[
\frac{1}{\|\vec{\sigma}\|^2}(V_1 - \frac{3}{2}\sigma_0(\|\vec{\sigma}\|^4 + |\sigma_1|^2)) \leq \frac{|\sigma_2|^4}{\|\vec{\sigma}\|^4} \leq 1 \tag{23}
\]

It follows \(V_1 \leq \sigma_1(\|\vec{\sigma}\|^4 + |\sigma_1|^2)\) where \(\sigma_1 = \frac{3}{2}\sigma_0 + 1\).

From Jensen’s inequality, we now can write,
\[
\frac{V_1}{\sigma_1} \leq (\|\vec{\sigma}\|^4 + |\sigma_1|^2) \leq (\|\vec{\sigma}\|^3 + |\sigma_1|^\frac{3}{2})^\frac{4}{3} \tag{24}
\]

2) Computing \(\dot{V}_1\): Computing the derivative of (18) we have,
\[
\dot{V}_1 = 2\dot{V}_0\dot{V}_0 + \frac{3}{2} \gamma_1 |\sigma_1|^2 \sigma_2^2 + \gamma_1 |\sigma_1|^\frac{3}{2} \sigma_2 \text{sign}(\sigma_1) \tag{25}
\]

It implies,
\[
\dot{V}_1 \leq -\lambda_0(\|\vec{\sigma}\|^4 + \|\vec{\sigma}\|^3) + \frac{3}{2}\gamma_1 |\sigma_1|^2 \sigma_2^2 - k_{p\alpha} |\sigma_1|^\frac{3}{2} - \gamma_1 \sigma_1 |\sigma_1|^\frac{3}{2} + k_{d\alpha}\gamma_1 |\sigma_1|^\frac{3}{2} |\sigma_2 | \tag{26}
\]

where \(\lambda_0 = 2\lambda_{\min}[P]\cdot\min\{\lambda_{\min}[Q], r_{\min}\}\). Now taking into account the Young’s inequality, we obtain:
\[
|\sigma_1|^\frac{3}{2} \sigma_2^2 \leq \frac{1}{2} |\sigma_1|^\frac{3}{2} + \frac{3}{2} |\sigma_2|^3 \tag{27}
\]

It implies,
\[
\dot{V}_1 \leq -\lambda_1(\|\vec{\sigma}\|^4 + \|\vec{\sigma}\|^3) - \gamma_1 (k_{p\alpha} - \frac{k_{d\alpha}}{6}) |\sigma_1|^\frac{3}{2} - \gamma_1 (r_{1a} - r_{2a} - \tilde{r}) |\sigma_1|^\frac{3}{2} \tag{28}
\]

where \(\lambda_1 = \lambda_0 - 2(1 + \frac{k_{d\alpha}}{6}) > 0\). Now we have,
\[
\dot{V}_1 \leq -\lambda_2(\|\vec{\sigma}\|^3 + |\sigma_1|^\frac{3}{2}) \tag{29}
\]

where \(\lambda_2 = \min\{\lambda_1, \gamma_1 (r_{1a} - r_{2a} - \tilde{r} - \frac{k_{d\alpha}}{6})\}\). Finally,
\[
\dot{V}_1 \leq -(|\vec{\sigma}|^3 + |\sigma_1|^\frac{3}{2}) \tag{30}
\]

Finally using (24) we obtain,
\[
\dot{V}_1 \leq -\frac{\lambda_2}{\sigma_1} V_1^\frac{4}{3} \tag{31}
\]

### IV. SIMULATION

Simulations were performed in Matlab-Simulink with system (1) and the control law (7)-(10). We consider that the trolley can be controlled along the straight line from a starting position \((x_0, y_0, z)\) to a final desired position \((x_f, y_f, z)\). We distinguish two phases, the "traveling phase", including the whole travel of the payload toward the final location, i.e. \(t_0 < t \leq t_f\), and the "arrival phase", i.e. \(t > t_f\). The reference for the car position is given by,
\[
\alpha_{\text{ref}}(t) = \begin{cases} 
    a_0 + a_1 t^3 + a_2 t^4 + a_3 t^5 & t \leq t_f \\
    a_f & t > t_f
\end{cases}
\]

where \(\alpha = x, y\).

Remark 3: For the car reference position \(\alpha_{\text{ref}}(t)\), based on the crane dimensions, we choose: \(t_0 = 0, t_f = 6, \alpha_r(t_0) = 0.1\) and \(\alpha_{\text{ref}}(t_f) = 0.7\); additionally, in order to satisfy the next equalities \(\alpha_r(t_0) = \alpha_{\text{ref}}(t_0) = 0\) and \(\alpha_r(t_f) = \alpha_{\text{ref}}(t_f) = 0\), we obtain: \(a_0 = 0.1, a_1 = 0.2778, a_2 = -0.006944\) and \(a_3 = 0.0046296\).

It is also assumed that the load is hoisted up or down between the minimum and the maximum rope length, \(\ell_1\) and \(\ell_2\), respectively, in order to avoid all possible known obstacles in the workspace.

The reference for the rope length is given by,
\[
l_{\text{ref}} = \begin{cases} 
    \frac{(t + \frac{\ell_1}{2})(1 + \cos \omega_2 t)}{\ell}, & 0 < \omega_2 t \leq \ell_a \\
    \frac{\ell}{(\frac{t + \ell_1}{2})(1 - \epsilon)}, & t_a < \omega_2 t \leq \ell_b \\
    \frac{(t + \frac{\ell_1}{2})(1 + \cos(\omega_2 t - \ell_b - \pi))}{\ell}, & \ell_b < \omega_2 t \leq \ell_f
\end{cases}
\]

Fig. 2: Simulation: trolley and rope length position (meters), payload angles (degrees) and control inputs (kg m/s²).

Based on the dimensions of the Laboratory 3D Intecotm crane, we consider \(\ell_1 = 1.05, \ell_2 = 1.45, \epsilon = (\ell_2 - \ell_1)/(\ell_1 + \ell_2)\) and \(\omega_2 = 0.22\omega_0\).

Remark 4: The system parameters considered in this note are: \(m = 1\) kg, \(m_c = 0.6\) kg and \(m_r = 1\) kg.

The parameters of design are:
- \(K = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix}
- r_{1x} = 2, r_{2x} = 1, k_{px} = 15, k_{dx} = 3
- r_{1y} = 2, r_{2y} = 1, k_{py} = 15, k_{dy} = 3
- r_{1\theta} = 2, r_{2\theta} = 1, k_{p\theta} = 15, k_{d\theta} = 3
- \gamma_0 = 1.5, \gamma_1 = 1.1, c = 0.5 and \text{dr} = 0.1
- With these parameters we have: \(T_{\text{reach}} \leq 4.22V_1^\frac{1}{2}(\sigma(0))\)

The initial condition considered in simulation was \(x(0) = y(0) = 0\) m, \(l(0) = 1.45\) m, \(\theta_x(0) = 5^\circ\), \(\beta(0) = 2^\circ\), \(\tilde{\theta}(0) = 5.8^\circ/s\), \(\tilde{\beta}(0) = 0\), \(\tilde{x}(0) = \tilde{y}(0) = \tilde{l}(0) = 0\). With this initial condition we are introducing an initial swing on
the payload. Figure 2 show the obtained results. From figure we can observe a good performance even in the presence of a initial payload swing.

V. EXPERIMENT

Experiments were performed on the laboratory crane. This laboratory crane has the values of \((x, \theta_x, y, \theta_y, l)\) available for measurement and they are interfaced to a personal computer through an Inteco™ Data Acquisition Board. The \(\beta\) angle is obtained by the relation \(\beta = \arctan(\cos \theta_x \sin \theta_y)\). The control algorithms are implemented on a Matlab/Simulink™ enviroment. The derivatives have been estimated by using online differentiators based on second order sliding modes, see [23].

As well as in simulation an initial payload swing is intentionally introduced on the experiment. Figure 3 shows the effectiveness of the proposed controller and we can note how well the results presented in fig 3 resembles the corresponding simulation in fig 2.

VI. CONCLUSIONS

In this paper, the 5 Degrees of Freedom Overhead Crane System is analyzed. The study includes the design of a Second Order Sliding Mode Controller, the stability analysis and the estimation of the time convergence to the desired sliding output via Lyapunov second method. The proposed controller can realize trajectory tracking of the system actuated degrees keeping at the same time the attenuation of the payload oscillations. Experimental results, over an Inteco™ 3D Crane, show the effectiveness of the approach even in the presence of an initial payload swing. The case of Ship Mounted Crane is considered for future work.

REFERENCES