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Hypercomputational vs. Computational Complexity
A Challenge for Methodology of the Social Sciences

Motto One: *There are actually lots of threads that led to computer technology, which come from mathematical logic and from philosophical questions about the limits and the power of mathematics.* Greg Chaitin[1]

Motto Two: *Computer simulations are extremely useful in the social sciences. It provides a laboratory in which qualitative ideas about social and economic interactions can be tested. This brings a new dimension to the social sciences where 'explanations' abound, but are rarely subject to much experimental testing.* Richard J. Gaylor, Louis J. D'Andria[2]

1. The impact of logic and informatics on the current paradigm of the social sciences

1.1. The first term in the title of this essay, *hypercomputational*, requires elucidation as being quite a novelty (a bit shocking, perhaps) in the language of science. Fortunately, the term *computational* has a well-established meaning since Turing's seminal study of 1936. Fortunately, as well, it was the same Turing, in his work of 1938, who offered us a first hint toward the idea of the hypercomputational, the hint being involved in the concept of an *oracle* — a device (hypothetically postulated) to render values of uncomputable functions; such a rendering is now called hypercomputing.

Thus hypercomputational complexity is one which cannot be handled by algorithms, that is, computational devices. Nevertheless, it may be handled with other means. What means? Turing put forward his idea of oracle in order to make more precise the concept of intuition, specially as appearing in the context of Gödel's discovery of undecidable statements in mathematics. If such a statement is acknowledged as true without any proof (and even without a chance of being proved), the human faculty acting there is what one calls *intuition* or *insight*. The same faculty is busy in judging mathematical axioms as true. Turing [1938] attempted at formalizing this informal concept of intuition.

More on this subject is to be said later. Here it is enough to express the conjecture that mathematical intuition may deal with the uncomputable. However, as being mathematical, it remains in the realm of numbers, while numbers are capable of being computed; if not computed in the strict Turingian [1936] sense, then in a way called *hypercomputing* – as suggested recently by a circle of researchers led by Jack Copeland.

How should the uncomputable and the hypercomputable be related with each other? It is enough and is safe to tell that the former is included in the latter. That is to say, at least some uncomputable functions can be hypercomputed. To make this a plausible conjecture, let us suppose there are magnitudes in the world which are both continuous and uncomputable. Next, suppose

[1] "A Century of Controversy over the Foundations of Mathematics" in: C. Calude and G. Paun, *Finite versus Infinite*, Springer-Verlag, London 2000, pp. 75-100.

[2] From Introduction to *Simulating Society: A Mathematical Toolkit for Modeling Socioeconomic Behavior*, Springer Verlag 1998. See www.telospub.com/catalog/FINANCEECON/SimSoc.html

that such magnitudes can be correctly estimated by a cognitive apparatus (the senses, the brain) on the basis of analogue processing, that is, a kind of mirroring or copying, the copies being some states of the perceiving subject. As being true copies, they should involve magnitudes represented by continuous uncomputable numbers. Since they result from a process which operates on some magnitudes, they are somehow computed in an extended sense of the word. To avoid equivocation, it is reasonable that the difference between the computing in that broader sense and that in the strict (as in Turing [1936]) sense be rendered with a new term, such as hypercomputing.

The authors engaged in the information-processing approach to scientific research divide nowadays into the minority which claims the possibility of hypercomputing and the majority which rejects it. The name for the latter has been already coined, namely *computationism*. As to the former, no designation has been invented so far; it may seem misleading to use a term like "hypercomputationism" (as too similar in form to that so opposite in content), but no viable alternative is in view. To make the difference clearer, I emphasise it through adding the adjective "strict". Thus the opposition involved will be rendered as follows: **strict computationism** *versus* **hypercomputationism —** abbreviated as SC and HC, respectively.

1.2. Either point results in a specific methodological paradigm. Let us restrict discussion to methodology of social sciences. Computational methods enter a social science naturally in those parts in which phenomena can be handled in a quantitative way, as in economics, theory of social choice (votes are easily counted), and even in theories of social interactions as much as they are developed in a game-theoretical or similarily quantitative framework. At the same time, there is a strong trend from the very beginnings of sociology to shape it more on the image of historical investigations than that of natural sciences. And then enters the concept of understanding (German *Verstehen*) which appears in the technical term *understanding sociology*, going back to Max Weber.

There is an ambarassing multitude of possible understandings of the word "understanding", but one interpretation is specially encouraging. It consists in resorting to the concept of intuition as discussed in philosophy of mathematics, and exemplified with such mental acts as asserting axioms, or judging importance of a mathematical problem.

The concept of mathematical intuition, in spite of its initial vagueness, has gained some preciseness owing to the theory of algorithmic complexity. After Gödel, Turing, Church and Post, nobody denies that there are acts of mathematical cognition which are not available for algorithms. Such acts, evidencing human creativity, are of two kinds. Some of them can do what algorithms cannot, the latter being too slow to solve a problem in a reasonable time (without a human prompt); thus human cleverness, when acting in a non-algorithmic manner, may succeed where a brute force fails. To account for such intuitions, it is not necessary to resort to the idea of hypercomputation. However, this may be necessary to account for human capability of finding axioms, including such ones as the famous Gödelian sentence.

Thus the social sciences owe much to mathematics, not only when taking models and algorithms from it, but also when facing the riddle of intuition. Once we follow the HC claim that mathematical axioms stem from intuition which may consist in hypercomputing, the same may be considered with respect to axiom-like propositions of social sciences (as, for instance, that concerning interaction between demand and supply, that stating an advantage of peace over war, etc).[3]

[3] Ludvig von Mises [1966], using the phrase *a priori* instead of *intuitive*, claims that numerous statements of social sciences are apriorical, exactly in the way mathematical axioms are. This view is convincing,

There is a price to be paid for such a help obtained from mathematics by social sciences. This requires endorsing a vision like that of Leibniz (preceded by Phytagoreans and Plato) according to which the whole reality is governed by numbers, as said in the Book of Wisdom: *omnia in numero disposuisti*. This favourite maxim of Leibniz was by him also expressed in the saying: *Cum Deus calculat fit mundus*, and this agreed with his juvenile (in the earliest dissertation) idea: *essentiae rerum sunt sicut numeri*.[4]

However, why not to perform a thought experiment and become a Leibnizian for a while? In this experiment, let us combine Leibniz's vision with the modern awareness of the computable, uncomputable and hypercomputable. Then a social scientist would deal with the world also (like the physical world) defined by numbers, and then the question would arise: whether in that world there are computable numbers alone, as believed by computationists (e.g. the physicist Ed Fredkin and the followers of his "digital philosophy"), or uncomputable numbers should be admitted as well, as believed by Penrose, Copeland and others?[5]

Even if there may seem something mysterious about such a philosophical framework, the strict-computationist alternative is mysterious as well. According to SC, what HC calls intuition also results from an algorithm, say Alg1. Nobody knows it, it has to be hidden somewhere in the interior of brain. At this point, one has to feel a touch of mystery. To wit, in those cases in which it has been proved about an algorithm that it cannot settle a question, SC has to assume, to avoid acknowledging intuition, that there exists a stronger algorithm (even if we know nothing of it) to perform the task too difficult for its predecessor. Only in such a way SC can avoid the verdicts of limitative theorems. Moreover Alg1 must be produced by something else, and that 'else' must be again an algorithm, say Alg2. And so on. As in the Catholic Church the principle "nulla salus extra Ecclesiam" (no salvation outside the Church), so in the SC epistemology there holds the principle "no cognition outside Algorithm".

Let us agree that SC and HC are equally justified as philosophical hypotheses which may be identified with what Popper calls *metaphysical research programmes*. Both have bright and dark sides. In fact, both provide a research paradigm which may prove fruitful, since in such a configuration negative results concerning any of them corroborate the other. Thus the best strategy is to carry out either programme, possibly, each by a different team. In what follows, the HC-vs-SC opposition will be exemplified with presenting some eminent representatives of either conjecture in social sciences.

1.3. Among the pioneers who cleverly acknowledged the complexity in social systems, such as free markets, or democratic societies, there was Friedrich Hayek (1899-1992). His insights can be appreciated against the contrastive background of the views of Oskar Lange (1904-1965). The latter has a merit of a different kind — that of comitting a fruitful instructive error. Lange believed

especially if axioms are construed as propositions of the kind called *meaning postulates* by Carnap [1958]; the occurrence of meaning postulates in empirical sciences is beyond any doubt.

[4] The quoted sayings mean, respectively. as follows. *You [God] arranged all the things with numbers. When God computes, the world is becoming. The essences of things are like numbers.*

[5] Leibniz himself was divided between these opposite options. As one who constructed arithmetical machines, and fancied logical machines to solve all possible problems, he was like a modern computationist. On the other hand, his metaphysics of infinity and continuity would make him close to the opposite camp (that Leibniz's internal split is discussed by Marciszewski [1996a, 1996b]).

that the computational complexity of computer systems (as having been attainable in the early sixties) can perfectly match the complexity of economic processes. It was that conviction on which he built his faith in the advantages of central planning over the *spontaneous order* (Hayek's expression) of free market, provided a relevant use of computers in the planning.

However, the power of computing cannot be judged rightly without consulting mathematical logic from which issued the research in computational complexity (as recalled in Motto One). The latter is a tool in investigating complex systems, among which social systems display the uttermost complexity. When referring to mathematical logic, I mean those problems and results which are mainly due to Hilbert, Gödel, Turing and Post, to mention those most akin to computer science; close to them there are the achievements of Church, Tarski etc.

However, the limitative theorems that reveal the incompleteness of arithmetic and the undecidability of first-order logic were by many treated, up to some time, as little relevant to empirical research. People had believed that these theorems apply to some esoteric mathematics, and are not engaged in its applications, as modelling and digital simulation of social processes. Such an optimism – as to the chance of escaping computational limitations – is rooted in the following *Computability of the Empirical* assumption, which expresses the philosophical SC point.

[E-Comp] *All the relations holding in empirical reality* (1) *can be represented by computable functions and, moreover,* (2) *all of them can be calculated by algorithms which work with resources being within our reach.*

This claim is capable of being empirically tested and, possibly, falsified, at least in item 2. The core of Lange's contribution consists in stating such a falsifiable hypothesis; if falsified, it would confirm the contrary to it conjecture as defended by Hayek and the rest of the Austrian School. The claim E-Comp might seem plausible in the early sixties (the time of Lange's last publications), before the new science of computational complexity started to come into existence with contributions like Hartmanis and Stearns [1965]. Another instructive instance of disregarding complexity is found in the famous Club of Rome Report, based on computer simulations. A tone of such optimism sounds even in some recent views as that quoted in Motto Two (which can be endorsed, but only if supplied with due provisos).

This paper should (a) hint at some results which refute E-Comp(2), and (b) discuss chances of E-Comp(1). Such chances would challenged by a success of hipercomputation, that is, such performances in information processing which would exceed the possibilities of Turing machines.

In the title of this paper, let me recall, the complexity to be handled by an insight (oracle) is called hipercomputational, while that being, at least in principle, capable of algorithmic approach is called computational. The latter notion comprises those cases which are computationally tractable, and the cases of intractability, that is, those in which algorithms require such big resources of time, memory, etc, that are practically useless (hence the proviso "in principle").

To sum up. there is a vital reason for which philosophy and methodology of the social sciences should be interested in hipercomputational complexity. With this concept there appears an opportunity to build a bridge between recent developments in informatics and the traditional theory of intuition, involved in understanding sociology. The concept of intuition becomes methodologically justified when (i) defined as complementary to the concept of algorithm (in the sense of logical complement), and (ii) proved non-empty. The latter is shown by examples such as the intuitive accepting of the Gödelian sentence as well as any mathematical axioms.

What may social sciences gain from that strategy, that is, treating sociological understanding by analogy to non-algorythmic (hypercomputational) part of mathematical activity? Let us note the following. The most afflicting problem with understanding (or, intuition, etc) is its subjectivity and elusiveness. The problem is common, concerning the whole of knowledge, but only in the philosophy and methodology of mathematics it is being efficiently investigated; there are thorough discussions concerning methods of an objective justifying of axioms (e.g. Maddy [1966]). These discussions put axioms beyond the domain of the computable, hence in the conjectured domain of the hypercomputable. Once we agree that some assertions in social sciences are like axioms in mathematics, we may apply those mathematical considerations to a better understanding of "understanding" in social sciences (cp. footnote 3).

2. Degrees of complexity, poor awareness of them in the practice of social sciences

2.1. To account for the full meaning of SC-HC opposition, one should consider it in the context of degrees of complexity. This issue is discussed in more detail in Sections 3 and 4 (addressed to those less acquainted with basic notions of complexity theory). The present Section is to hint at a state of research in the social sciences which is far from being satisfactory from the methodological point of view. To prove such a diagnosis, which ought to encourage to repair the deficiency, some introductory remarks on complexity would do.

Mathematical logic led to the formalization of the notion of algorithm (Church, Markov, Post, Turing) as well as the understanding that certain problems are algoritmically unsolvable. As there appeared computing machines, and logicians with computer scientists started to inquire into practical capabilities and limitations of such devices, computational complexity theory emerged from the logical theory of algorithmic unsolvability. Church's and Turing's study in the *Entscheidungsproblem* have demonstrated that checking whether a sentence has a proof is algorithmically unsolvable. (cp. Sipser). This result is, so to speak, infinitary in the sense that infinite time (measured with the number of operations) may be needed in the search for solution. What the theory of complexity is concerned about can be seen as a finitary version of the *Entscheidungsproblem.* Now we do not ask whether an assertion has (any) proof but if it has a *short* algorithmic proof ("algorithmic" can be rendered by "formalized" in Hilbert's terminology).

The short word "short" is crucial for defining degrees of complexity of a problem. *The longer is a shortest algorithmic proof needed for solution, the more complex is the problem whose solution is to be proved.* There is a method of distinguishing practically important intervals in such a scale, called *complexity classes*; an ordering of them yields degrees of complexity.

Now, as to hypercomputational complexity, it is beyond that scale, in the sense that only finite procedures are taken into account. However, in another sense it can be viewed as being on the top (in that sense in which we speak that an infinite number is greater than any finite number). It is the latter sense to be referred to in the further discussion.

Those problems whose complexity is beyond any capabilities of computation are called *undecidable*. Those whose complexity would require inaccesible resources of time, memory, etc. for their algorithmic solution are called *intractable*.

When a research in social phenomena involves algorithms, it ought to be accompanied by the awareness of that conceptual framework, worked out in logic and computer science. In particular, a researcher should be aware of limitations of algorithm; only then he will be able to give up projects being unrealistic, and in the case of hard ones find measures to make his project more feasible. Here are some questions to be considered with such awareness.

— [1] Whether the algorithms needed to model and simulate social phenomena happen to have complexity which would make the issue involved an undecidable or intractable problem?
— [2] If so, are there any methods to transform the problem into being both tractable and duly approximating the answer required?
— [3] If so, these methods should be presented.
— [4] Are there in any social theory assertions not being justified by any algorithm?

The reply to [4] is obvious both for mathematical and for empirical sciences. In a deductive mathematical theory such assertions are axioms, while in an empirical theory no algorithms are necessary to obtain observational statement and meaning postulates (the latter in the sense given by Carnap [1956]). In spite of such obviousness, question [4] should be stated to create opportunity for the next question, to wit:
— [5] What is the basis of accepting a statement in a theory if the acceptance is not substantiated by any algorithm?

When questions [1], [2] and [3] are put to physicists, one obtains clear answers accompanied by a list of examples of the problems being undecidable or intractable or else those having only approximate solutions. Such a state of affairs is nicely exemplified by Stephen Wolfram's paper *Undecidability and Intractability in Theoretical Physics* [1985].[6] When exemplifying undecidable and intractable problems, he takes advantage of the term *reducibility* which can be also rendered by *compressibility* as used in the theory of algorithmic information (Chaitin, Kolmogoroff). The lack of this property causes that an algorithm simulating the process in question has to reproduce it step by step, in an explicit simulation, without any possibility of shortening.

Irreducible computations may turn intractable because of the lack of time or space (memory). Also the undecidability appears as an usual phenomenon, as exemplified by undecidable propositions about the behaviour of the cellular automaton (CA). The occurrence of such undecidable propositions may be viewed as a consequence of computational irreducibility.

Undecidability is found in physics in many areas what can by seen if physical systems are viewed as universal computers. There are many physical systems in which it is known to be possible to construct universal computers. "Apart from those modeled by CA — writes Wolfram [1985] — some examples are electric circuits, hard-sphere gases with obstructions, and networks of chemical reactions. The evolution of these systems is in general computationally irreducible, and so suffers from undecidable and intractable problems. [...] It is the thesis of this paper that such problems are in fact common. Certainly there are many systems whose properties are in practice studied only by explicit simulation or exhaustive search: Few computational shortcuts (often stated in terms of invariant quantities) are known."

2.2. It is worth while to compare a physicist's awareness of such limitations with the belief in unlimited power of computation characteristic of some social scientists. As if social phenomena had not been enormously more complex from those in the physical world. This criticism is directed against some projects enjoying an enormous prestige and influence. There is a lot of expert

[6] Wolfram is widely known owing to his works on cellular automata, collected in [1994] and his being the author of software called "Mathematica". His monumental book [2002] claiming that cellular automata constitute an adequate model of physical world has become a scientific bestseller of the year.

studies which bring important limitative results concerning the use of algorithms in social sciences (referred to in the next Sections), but these have not even a fraction of the fame of those rather pretentious projects.[7]

Here are some examples of treating very complex problems as if they were easily tractable in an algorithmic way.

Example 1. • Strong AI (Artificial Intelligence). This is a project demanding to a highest degree, as it aims at a perfect simulation of the most complex entity in the whole Nature, namely the human brain. The accomplishing of that project would be of great consequence for social sciences since intelligent artificial agents could be organized into artficial societies (AS) whose behaviour would be fully predictable (as rsulting from the algorithms known to the constructors). This branch of computer science happens to be called *multi-agent simulation*. Not only AS is assisted by AI, the reverse holds too, since intellectual development of artificial agents depends on social interactions in AS, and such a feedback has to result in a monstrous complexity. In spite of that, one hardly encounters messages (from the researchers involved) which would concern about either undecidability or intractability of the problems being addressed.

Example 2. • Central socialist planning with the help of computers, in the current literature called *socialist calculation*, as mentioned above, was defended by Oskar Lange in his polemics with von Mises and Hayek. Their objections hinted at the enormous complexity of economic and other social phenomena, too big to be processed by the brains of planners. In the early sixties, Lange replied that what had been impossible before the inventing of computers, became viable and easy with their help. Even now this idea is defended by some leftist authors (e.g. Cottrell and Cockshott [1993]). Such a debate is likely to bring a conclusion, provided that Lange's followers will supply us with a realistic model of economy. Owing to that lucky feature of economic phenomena that they cen be measured, it should be possible to estimate the order of magnitude of input data to be taken into account. Moreover, there are proposals as to mathematicsl models of the demand-supply equilibrium, economic development, etc. as created by Pareto, Lange and other eminent authors. The equations forming the model provide suitable algorithms; these considered with the magnitude of input data should give us an idea of computational complexity to be handled by computers employed by planners (provided that decidability is granted).

Example 3. • The Club of Rome Report of 1972 entitled *Limits to Growth* predicted a world-wide economic and ecological disaster after the end of the 20th century. That divination was backed not only by a respected group of intellectuals but also by the authority of computer science as simulations were carried out by MIT experts with the best then available machines. It was based on a simulation model, a mathematical representation of the main variables and their dynamic interactions known as the WORLD III model. "The forms of exhaustion predicted in the various scenarios simulated in the model start to emerge in the early twenty-first century, as the world population grows to a peak of 10 billion, per capita food production drops to a mere 15-25 percent of 1970 levels, pollution has risen tenfold, and the most important resources, such as oil and gas, have become depleted. Because of the so-called exponential character of growth and depletion, half-hearted or one-sided measures are of little avail. A drastic program of technological improvement such as energy conservation, for example, achieving 50 percent savings in 20 years against a background of, say, 2 percent growth in consumption, postpones the date of depletion by a mere 3 years." (See Van Dieren [1995, Introduction]).

When coming into the 21st century, one can easily judge the reliability of the message reported in Example 3. Let it be just recalled that after the collapse of socialism the food production in most post-socialist countries so dramatically rose that those being EU candidates are obliged to artificially cut production; otherwise they would be too competitive in the EU market.

The success of a model much depends on intelligent simplifications. Among the simplifications made in the Report there was the total omitting of the factors of scientific research and technologi-

[7] Negligencies of social scientists happen to be repaired by logicians and computer scientists who watch what is going on in social sciences and comment on it from a logico-methodological point of view.

cal invention (there is no need to comment if that was an intelligent simplifying). Obviously, such factors cannot be grasped in central economic planning. Even if the Laplacean demon revealed what is to be going on in the heads of future discoverers, the unimaginable complexity of each brain separately and still greater of their world-wide interactions would unavoidably hamper any computer-based predictions.

On the other hand, an intuitive understanding as expressed in axiom-like maxims, e.g. "the more economic freedom, the more economic information" may prove more reliable and more useful than results of algorithmic procedures. Do such understandings result from some hypercomputational processes in our brains? This is an open question. They may be due to some algorithms which would be by far more efficient than "classical" algorithms based on logic of predicates, or probability theory, or else game theory. In the moment, having no way to settle this question, we should be glad for our ability to see the problem that by no means is trivial, and forms a considerable progress. To grasp it, one needs some messages which will be discussed in the next Section.

3. Game-theoretical models and the concept of rationality

3.1. As an example of modelling in social sciences, let us consider the mathematical game theory, going back to von Neumann and Morgenstern [1944], which supplies social scientists with a standard model of rational human interactions. A game which became a standard paradigm of game theory, comprising a very large class of social processes, is called "prisoner's dilemma" because of the following story to exemplify the problem (cp. pespmc1.vub.ac.be/PRISDIL.html).

> Two criminals arrested under the suspicion of having committed a crime together. To obtain a sufficient proof in order to have them convicted, they are by the police isolated from each other, and offered a deal: the one who offers evidence against the other one will be freed. If none of them accepts the offer, they prove cooperating against the police, and both will get only a small punishment because of lack of proof. Thus they both gain. However, if one of them betrays the other one, by confessing to the police, the defector gains more: he will be freed, while the other, remaining silent, will receive the full punishment (as one who did not help the police). If both betray, both will be punished, but less severely than if they had refused to talk. The dilemma resides in the fact that each prisoner has a choice between only two options, but cannot make a good decision without knowing what the other one will do.

"This simple game-theoretic model seems to capture in miniature something of the tensions between individual acquisitiveness and the goals of collective cooperation. That is of course precisely why it has become a major focus of modelling within theoretical sociology, theoretical biology, and economics. [...] It is no simplification to say that our strongest and simplest models of the evolution of biological and sociological cooperation–and in that respect our strongest and simplest models of important aspects of ourselves as biological and social organisms–are written in terms of the Iterated Prisoner's Dilemma." (see www.sunysb.edu/philosophy/faculty/pgrim/SPATIALP.HTM).

Originally the game was considered for two persons playing one-shot version of the game (i.e., without iterations), but from a logical point of view (e.g. the point of decidability) the thing becomes interesting in iterated many-player games. At the same time, iterated games are in focus of a theory of social evolution as tending towards more cooperative behaviour. This is why they are worth to be studied. In a non-iterated game the most advantageous behaviour consists in acting selfish, that is, with the loss of the other player in order to maximalize ones own gain. In an iterated game it is cooperation what proves more advantageous. The reason is that the increasing experience reduces uncertainty as to the partner's strategy; at the same time, they get opportunity to learn the advantages which in long term result from cooperation. Thus the prisoner's dilemma yields a model of social evolution.

In studying evolution, a very efficient tool is provided by the theory of cellular automata (CA) (created by von Neumann, together with Stanisław Ulam). Individual cells represent agents interacting with their neighbours (i.e. the surrounding cells) according some fixed rules, and changing their states (from a definite set of states) owing to interactions. As cells are rendered, e.g., by squares at a blackboard, each cell has eight neighbours. If the interaction rules are those involved in the prisoner's dilemma, then there are nine players. The number of strategies may exceed two (the cooperative and the competitive one).

Each player in such a display competes with each of its neighbors in an iterated prisoner's dilemma and totals its scores from those competitions. A player surveys its neighbours. If no neighbour has a higher local score, the player retains its original strategy. If it has neighbours with higher scores, it converts to the strategy of its most successful neighbour. The result is a model in which success is in all cases calculated against local competitors.

In the course of game, some strategies become more frequent than other ones, and in this sense they start to dominate; this evolution consists of changes of configurations in the two-dimensional field. In some cases, it is possible to forecast the direction of evolution. Is it possible in each case? Is there an algorithm which in each case would tell us about the final result of evolution? That is: which strategy would preserve its domination? When translating the issue into a concrete example, one may ask whether a peace and alliance between some former enemies will last for ever, or is to turn again into old animosity?

Grim [1997] who examined the case of dilemma as reported above replies with the following conclusion. "There is no general algorithm [...] which will in each case tell us whether or not a given configuration of Prisoner's Dilemma strategies embedded in a uniform background will result in progressive conquest. Despite the fact that it is one of the simplest models available for basic elements of biological and social interaction, *the Spatialized Prisoner's Dilemma proves formally undecidable in the classical Gödelian sense."* (Italics mine – WM.)

Thus, as to the example of competitors who become cooperative allies, when taking into account the complexity of real social situations, one has reasons to believe that such a case, so involved, is no more solvable than the relatively simple case investigated by Grim. That is to say, one cannot hope that any algorithm would settle the question, while one may hope that a clever politician would do. Analogously, no arithmetical algorithm does recognize the Gödelian sentence as true, but a clever mathematician does. Does this result form a process of understanding which in mathematical terms would mean a hypercomputing? Obviously, this is not a question to be settled at once, but (let me say it again) the dawning awareness of this question is a real achievement in exploring the limits of the human mind.

3.2. The prisoner's dilemmma forms an opportune framework to consider the notion of rationality which since the time of Max Weber (at least) is vital for social sciences. Among the authors discussing the dilemma, there are two different uses of "rational". To put the difference in a nutshell, let it be roughly expressed with the following identities:

R1: rational = efficiently acting for self-interest;
R2: rational = able to correctly perform every computation.

Interpretation R2 may illuminate, by analogy or metaphor, the Weberian sense as found in the study of the Protestant ethics and capitalism, since the capitalistic rationality is related to precise calculations.

Let us turn to explaining both R1 and R2 in more detail. The former is defined as follows (see *Principia Cybernetica Web*).

> "The problem with the prisoner's dilemma is that if both decision-makers were purely rational, they would never cooperate. Indeed, rational decision-making means that you make the decision which is best for you whatever the other actor chooses." (pespmc1.vub.ac.be/PRISDIL.html)

This use of the term "rational". fairly frequent in the game-theoretical literature, should be treated as elliptic.[8] When fully articulated, the term should be replaced by the following:

> instrumentally rational (*zweckrational* – Weber's term) with respect to the goal being defined as the decision-maker's gain, irrespective of possible losses which his choice might bring to other beings.

This interpretation of the term "rational" does not imply that treat as equivalent these two philosophical maxims: *homo est animal rationale* and *homo homini lupus est*. Rationality does not consist in mere selfishness but in the ability to find out means for any goals, while in economics one's own profit is the goal considered.

The other concept of rationality in game theory, here referred to as R2, is related not to motivation but, most generally speaking, to information. With the unattainable perfect rationality there is contrasted the *bounded rationality*, that is, one which suffers some informational limitations, as incomplete information, absent-mindedness, limited foresight, limited reasoning capabilities, too small memory, etc.

This concept proves very fruitful for game theory as it allows to solve mamy problems concerning optimal strategies. For instance, A. Neyman [1985] (who much contributed to the notion of bounded rationality) stated that if the size of memory (measured in the number of states in finite automata implementing strategy as players) is contained in the interval $[n^{1/k}, n^k]$, where n is the number of rounds, and $k > 1$, then cooperation is a profitable stategy for both players. Another example: there is a proof that cooperation is more advantageous than competition when the number of rounds is not known to players; this kind of uncertainty makes competition more risky (cp. Papadimitriou and Yannakakis [1991]).

In the final Section which follows, which is to lead to the point stated in the title of this paper, it is the second game-theoretical concept of rationality which will be taken into account. We shall see how it contributes to the notion of intelligence.

4. Intelligence as rationality with inventiveness

4.1. The challenge to be met by the methodology of social sciences when faced with the theory of computational complexity may be rendered in the following reasoning.

(i) Computer simulation of social processes is possible then and only then if there is an algorithm for explanations and predictions concerning the process in question. (ii) Any theoretical

[8] Such an elliptic use is manifest in statements like the following. "Indeed, social actors are not merely agents following rules in a strict way or pure rationalists maximizing a value. They also try to realize their social relationships and cultural forms." (Gomolińska [1999, p. 96). Take we this wording literally (instead of elliptically), then realizing social relationshios and behaving in a cultural way should be qualified as irrational.

explanations and predictions of social processes require taking into account intelligent behaviour of the actors involved. (iii) Hence, computer simulation of social processes requires an algorithm to simulate intelligent behaviour of the actors involved.

Thus the challenge will be met if one offers a definition of intelligence which would tell whether every intelligent behaviour could be simulated with an algorithm having a reasonable complexity. This is to mean that the answer in the affirmative will be substantiated if (a) an algorithm does exist and, moreover, (b) it enjoys a required feasibility; that is, the size of resources needed (as time and memory) is not as great as, say, exponential with respect to the size of input data. Correspondingly, the answer in the negative will be substantiated if either ($non-a$) no algorithm is attainable or, if it is attainable, ($non-b$) its computational complexity exceeds the limits of feasibility (tractability).

In what follows, I am to argue that the $non-a$ situation is sometimes the case. The argument has to consider a theory of intelligence. A necessary feature of intelligence is one discussed in the previous Section, to wit rationality. This feature is defined by a set of criteria. Depending on a set of accepted criteria, one obtains one from among various notions of rationality, and in consequence of that choice, one from among various notions of intelligence. Such a multiplicity is conspicuous in the case of bounded rationality, since differently defined bounds result in different concepts (as to a perfect rationality, it would, presumably, be unique).

For instance, one may postulate that no rational being is ready to assert in one sentence that both A and non-A is true; but if a person asserts A in Sunday, and B in Monday, the behavioural process including both is not necessarily non-rational. However, we would not be so tolerant with respect to an entity supposed to be perfectly rational, that is, without any limits (like an omniscient God). Another example: we would hardly regard as rational a bahaviour in which one does not grasp the validity of modus ponens reasoning. However, we would abstain from such a verdict in the case of a very long and involved reasoning; or in the case of calculating expected utility which would require instant multiplication of, say, some 15-digit numbers.

Thus we obtain a generic concept (i.e., a necessary condition) for defining intelligence which yields the following proper inclusion: *every intelligent behaviour is rational*, that is, conforming to certain criteria of rationality. In this sense, if the criteria are suitably chosen, the behaviour of a machine can achieve a high level of rationality. Is it sufficient to call it intelligent? This depends on what is required more.

4.2. In search for another feature of intelligence, to complete the one discussed above, let us consider a connection between intelligence and life. Intelligence as the problem-solving ability is necessary to survive and to develop in a desired direction. Both survival and development require efficient problem-solving.

In such a context, it is easy to observe that living in a continously changing environment requires reacting to many unexpected situations. In such situations any once possessed *routine* does not suffice. Instead, one needs what is called *inventiveness*, that is, the capability of making *innovations*. The opposition of routine and innovation is crucial for this discussion. However, before we focus on it, let us look at the phenomenon of innovation in the broader context of evolutionary processes; such processes form a great part of what social scientists try to render with computer simulations.[9]

[9] In the following discussion of evolutionary processes an extensive use is made of George Modelski's (1996) paper "Evolutionary Paradigm for Global Politics". In some places I quote his statements in a form

The accumulation of countless innovations leads to systems as intricate as market economies or democratic states. This course of events is embedded into an all-embracing process of social evolution driven by four basic mechanisms. To describe such a process (with hinting at these driving mechanisms) and its phases, we may start observation from any point of time whatever.

(0) At any point there is a set of strategies (or policies) that persist, that is, successfully reproduce themselves. This means the transmission of a program, or code, or set of rules, to the next generation of strategies. Such a persistence is accounted for by the basic inertia of all social systems. It is easy to notice its counterpart in biological dimension, when strategies amount to well-tested ways of behaviour to serve survival and development of a species.

(1) While some of these strategies are reproduced in a routine fashion, by copying, others will undergo change, e.g. chance mutation in the biological domain, or will be proposed as reforms by policy makers in response to certain problems. These are the sources of variation that introduce *innovation* into the set of strategies.

(2) Here a moral from the prisoner's dilemma proves to be in order. Innovation may disturb an established equilibrium of strategies; the ways of behaviour being safe so far become more risky because of the weakened orientation of agents in the altered environment. The actors may choose strategies either of conflict or of cooperation. In a long term, the latter proves more profitable for all the actors involved, they become the focus of effective alliances, and so the society ever better appreciates advantages of *cooperation*. It learns to coooperate. Such a course of events is more probable in free societies being, moreover, more advanced civilizationally. With others this requires a longer time and costs more, but eventually a cooperative strategy is likely to win everywhere; for, as Adam Smith put it, when accounting for what prompts humanity to save, there is the ever-present "desire for bettering our condition".

(3) However, an equilibrium so regained after the innovation had disturbed the previuos one, does not mean a total peace and security. There is another factor, namely constant competition which is necessary for any development either in biological (the Darvinian Struggle for life) or in social dimension (economic competition, political elections). This feature of evolution is being summarized with the concept of *selection*. One should agree with Darvin that selection is a crucial factor for progress in evolution.

(4) As in every process of learning, the success of selected strategies amounts to *reinforcement* (that is reward, combined with punishment for non-selection). The so reinforced revised strategies are then diffused, via mechanisms of amplification, and transmitted via a system of inheritance, in successive generations of strategies (compare item 0).

4.3. It has been asserted above that two properties constitute intelligence: rationality (in certain defined bounds) and inventiveness. The argument is simply derived from the concept of intelligence as the ability of efficient solving problems. Obviously, problems are either expected or unexpected. Those expected are tackled through a routine which in most perfect form becomes an algorithm, and this is the domain of rationality (as conceived by computer scientists, not necesarily

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similar to his original statements; however, I do not use quotations marks since adjustments to this text must have been made, thus departing from the literal wording.

by philosophers). Those unexpected must be handled through innovations, and this is the domain of inventiveness. It involves having new, even strange, ideas, and also the ability to put questions, to feel astonished, to discern what is important and what is not. There is no job for algorithms in such situations.

It follows, then, that intelligence consists of these two factors, each of them being necessary; taken together they form the sufficient condition for a behaviour to be intelligent. The rationality factor is predictable, the inventiveness factor is not. Now, to continue the argument concerning chances of the perfect computer simulation of social processes, we should address the question of how is predictability related to computability. Certainly, whatever is computable is predictable; there is no need to bother whether the reverse holds. For our argument this assertion is sufficient since it is equivalent with the assertion that non-predictability implies non-computability. Hence the inventiveness factor, as being not predictable, is thereby not computable, hence not subject to digital simulation.

In the evolutionary framework as sketched above, inventiveness plays the main role in the phase 1. It constitutes that part of evolutionary process which cannot have any computable mathematical model and any algorithm to be used in a digital simulation. This is not to mean that the method of simulation is useless in social research. On the contrary, the greater is the area of unpredictable, the more we should try to have a precise, certain, and as vast as possible knowledge about what can be known. This is what increases the chance of clever guesses.

Suppose that some uncomputable phenomena may be hypercomputable, that is, accessible for a non-algorithmic skill, like with an inventive researcher who finds out new axioms and new methods of reasoning. Suppose, that the skill grows when the researcher wins more knowledge from computer simulations. Furthermore, consider that successful innovations in the world of algorithms are due to creative powers of intuition. Then the success of a cognitive enterprise requires a close cooperation between the algorithmic and the intuitive thinking. And then the challenge for methodology of social sciences consists in the arranging of a close aliance between these two powers in dealing with complexity of the social world.

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