Optimal Robust Control of Underactuated Manipulators via Actuation Redundancy

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In this article we focus on post-failure control of a mechanical manipulator from the point of view of optimal control, and present a novel method for controlling the positions of the failed, passive joints in an optimal way. It used the so-called coupling index as an optimization criterion to minimize the energy spent by the underactuated manipulator. Although the optimization is performed locally, the results indicate the validity and feasibility of the proposed theory.

1. INTRODUCTION

The utilization of manipulators for applications in hazardous or hard-to-reach environments have led to a corresponding increase on the study of fault tolerant control methods for these mechanisms. The manipulators working in these kind of environments turns its repair difficult and costly, therefore, these mechanisms require sophisticated fault tolerant control methodologies. Thus, the research in post-failure control of manipulators has fundamental importance in such cases. Fault tolerant manipulator control has traditionally been studied as a combination of separate tasks:

(i) hardware redundancy,1–3
(ii) manipulator design and trajectory planning for fault tolerance,4–6
Manipulator fault detection has been combined with post-failure control algorithms in a unified method in an article by Shin. There, the authors present a hybrid systems-based framework consisting of three basic units that guarantee task completion in the presence of any number of failed joints. The first unit is a fault detection and isolation scheme which continuously monitors the manipulator to detect and identify a joint failure. The second unit is responsible for control reconfiguration. The third unit is composed of control algorithms appropriate for each control mode, based on input from the control reconfiguration unit.

In the current article we focus on the control algorithm unit, and more specifically at the problem of controlling the position of a failed joint to any desired set-point in an optimal way when actuation redundancy is available. This is possible, for example, when one joint of a three-joint manipulator fails, and the position of the failed joint is controlled by the remaining two joints. Our solution to this problem is based on local redundancy resolution, extensively studied in the context of inverse kinematics. Our main contribution is the utilization of the so-called coupling index as an optimization criterion to minimize the energy spent by the manipulator during the motion of the failed joints.

Despite the large number of works recently published on the subject of underactuated manipulators, covering its properties and control methods, to the authors’ knowledge there have been no contributions to the optimization of the control for such mechanisms.

This paper is organized as follows: in Section 1 we present the introduction and motivation of the work. In Section 2 we present the system description and the dynamic equations of the manipulator, including the partitions of these equations in order to study the underactuated behavior of the system. In Section 3 we review the coupling index, which measures the coupling between the active and passive joints of the underactuated manipulator. In Section 4 we discuss the basic ideas behind the use of redundancy for the control of the passive joints and develop the optimization technique. Section 5 presents the design of the robust controllers utilized. The results of the procedure for experiments with a planar 3 degree-of-freedom manipulator are shown in Section 6. Finally, the conclusion of the work is presented in Section 7.

2. SYSTEM DESCRIPTION AND DYNAMIC MODELING

The system considered in this work consists basically of a serial robot manipulator with rigid links and fixed base. The joints and links are numbered from 1 to \( n \), with joint 1 and link 1 being the closest to the base. We use \( q \) to represent the robot’s \( n \times 1 \) joint vector, and \( \tau \) to represent its \( n \times 1 \) torque vector. The dynamic parameters of link \( i \), namely, its mass and inertia, are represented by \( m_i \) and \( I_i \), respectively. The kinematic parameters, length and location of the center of mass are represented by \( l_i \) and \( l_{ci} \), respectively.

The dynamic equations of the manipulator calculated by Lagrange approach, according to ref. 29, are given by

\[
\tau = M(q)\ddot{q} + b(q, \dot{q}),
\]

where \( M(q) \) is called the inertia matrix, \( n \times n \), symmetric, positive definite. \( b(q, \dot{q}) \) represents the \( n \times 1 \) vector of Coriolis, centrifugal, gravitational, and frictional torques.

If \( n_a \) joints of this manipulator lose actuation due to failures, and the remaining \( n_u \) joints continue to operate normally, where \( n_u > n_a \), our problem is to find a control method that brings the \( n_u \) passive joints to a desired position in an optimal way, and then control the \( n_a \) active joints to their desired positions.

To solve this problem we make the following assumptions, valid for most manipulators:

- (i) only free-swinging joint failures are considered;
- (ii) all joints are equipped with brakes and encoders;
- (iii) joint failures occur one at a time.

With these assumptions, we believe that the work described here is applicable in a variety of scenarios. The ones we have in mind are manipulators operating in harsh or hard-to-reach environments, such as in space or deep-sea, where actuator repair is too costly or impossible.
Whenever an actuator fails, we analyze the dynamics of the resulting underactuated manipulator by partitioning Eq. (1) in components, corresponding to the active and passive joints, as

\[
\begin{bmatrix}
\tau_a \\
0
\end{bmatrix} = 
\begin{bmatrix}
M_{aa}(q) & M_{au}(q) \\
M_{ua}(q) & M_{uu}(q)
\end{bmatrix} 
\begin{bmatrix}
\dot{q}_a \\
\dot{\tilde{q}}_u
\end{bmatrix} + 
\begin{bmatrix}
b_a(q, \dot{q}) \\
b_u(q, \dot{q})
\end{bmatrix},
\]

(2)

where the subscripts \(a\) and \(u\) denote quantities related to the active and unlocked passive joints, respectively.

In order to bring all joints to a desired position, thus bringing the end-effector to a desired Cartesian configuration, we have to drive the joints in two distinct phases. First, the passive joints are driven to their set-points via their dynamic coupling with the active ones. Each passive joint is locked as it reaches its set-point. After all passive joints reach their desired positions and are locked, the second phase takes place, namely, control of the active joints to their set-points. One must note that, in ref. 16, when \(n_a > n_u\) a subset of the active joints can be controlled along with the passive ones in the first phase. For comparison purposes in the results section, we shall denote this by nonredundant control strategy. When only the passive joints are controlled in the first phase, the strategy is denoted redundant control strategy.

Clearly, the problem addressed in this article benefits from this redundancy. Here we propose a method for optimally controlling the position of the passive joint in the first phase.

3. COUPLING INDEX

The positions of an underactuated manipulator's passive joints cannot be directly controlled because these joints are not equipped with actuators. The passive joints, however, are dynamically coupled to the active joints because of the presence of nonzero off-diagonal elements in the inertia matrix. To be able to efficiently utilize underactuated manipulators for manipulation operations such as object placement or inspection, one must quantify the dynamic coupling to measure and to control the motion of the passive joints. In this section, we review our previous work to determine the acceleration of the unlocked passive joints due to the torques of the active ones.30

Factoring \(\ddot{q}_a\) in the first line of (2) we obtain

\[
\ddot{q}_a = M_{aa}^{-1}(q) (\tau_a - M_{aa}(q) \ddot{q}_a - b_a(q, \dot{q})).
\]

(3)

Substituting this expression on the second line of Eq. (2), the following relationship between the acceleration of the passive joints and the torques applied at the active ones is obtained [for the sake of convenience the dependency on \(q\) and \(\dot{q}\) of matrices \(M(.,.)\) and \(b(.,.)\) will be omitted]:

\[
\ddot{\tilde{q}}_u = -W_{uu} M_{aa}^{-1} \tau_a + W_{uu} (M_{aa}^{-1} b_a - b_u),
\]

(4)

where the matrix \(W_{uu}\) is positive definite, because it is equal to the lower diagonal block of the inverse of the inertia matrix.

Rewriting Eq. (4), the following relationship between the accelerations of the passive joints and the torques of the active joints can be stated:

\[
\ddot{\tilde{q}}_u = -W_{uu} M_{aa}^{-1} \tau_a = W_{aa} \tau_a,
\]

(6)

where

\[
\ddot{\tilde{q}}_u = \ddot{q}_u - W_{uu} (M_{aa}^{-1} b_a - b_u).
\]

(7)

The vector \(\ddot{q}_u\) can be considered as a virtual acceleration of the unlocked passive joints, generated by the active torques and the nonlinear torques due to velocity and gravitational effects. We defined in our previous works the following torque to acceleration coupling index (henceforth referred to as coupling index for the sake of simplicity):

\[
\rho_s(q) = \prod_{i=1}^{n_a} \sigma_i(W_{aa}),
\]

(8)

where \(\sigma_i\) are the singular values of \(W_{aa}\).

The torque coupling index provides a local measure of how well active joint torques are transmitted to the unlocked passive joints, because the elements of \(W_{aa}\) are functions of the manipulator’s current position \(q\). This index will be utilized as an optimization measure for the optimal control of the underactuated manipulator’s passive joints, to minimize the torques of the actuators and consequently the energy consumption of the process.
4. REDUNDANCY AND OPTIMAL CONTROL OF MANIPULATORS WITH PASSIVE JOINTS

Redundancy has been incorporated in a large variety of systems that require high reliability, such as airplanes, spaceships, and military systems. For example, in the case of a spaceship, to improve the reliability of the computational system, many computers make the same calculations simultaneously.

Closed link mechanisms can present actuation redundancy. These types of mechanisms generally have actuated and nonactuated joints. The number of actuated joints is usually equal to the dimension of the output variables. Additional actuators in some of the nonactuated joints produce mechanisms with actuation redundancy.

There are two approaches for utilizing redundancy in optimization: local optimal control and global optimal control. Although the local approach, that instantaneously determines the utilization of redundancy based on current information, has low computational cost, it lacks the guarantee of global optimality. Thus, this approach is better for real time applications. The global optimal control is better for off-line trajectory planning for tasks that require strict optimality, such as obstacle avoidance in complex work spaces.

Our objective at this point is to control the positions of the passive joints through the dynamic coupling with the active ones, taking advantage of the actuation redundancy to minimize a determined criterion of interest. Thus, we will have a redundant system with $n_a$ active joints controlling $n_u$ passive joints.

We start by observing the second order nonholonomic equation in the second line of Eq. (2):

$$M_{uu} \ddot{q}_u + M_{ua} \ddot{q}_a + b_u = 0.$$

As we are considering the case $n_a > n_u$, this equation represents a system of $n_a$ equations, and the matrix $M_{uu}$ has dimension $n_a \times n_a$.

Because matrix $M_{ua}$ is not square, factoring $\ddot{q}_a$ we have the following equation:

$$\ddot{q}_a = -M_{ua}^\# (M_{uu} \ddot{q}_u + b_u) + (I - M_{ua}^\# M_{ua}) z,$$  \hspace{1cm} (9)

where $I$ is the identity matrix of order $n_a$ and $z$ is an arbitrary vector. Equation (9) has infinite solutions, among which we can choose the one with minimum norm for $\ddot{q}_a$ (when $z$ is the zero vector) or one that minimizes a predetermined criterion.

The main idea in the development of the general solution of Eq. (9) derives intuitively from the fact that it is possible to add to the solution of the second line of Eq. (2) any vector consistent with the restrictions imposed to the system. Ligeois\textsuperscript{31} proves this affirmation through a kinematic approach.

Choosing $z$ as the gradient of a potential function, $P(q)$,

$$z = -k \left( \frac{\partial P(q)}{\partial q_a} \right)^T,$$  \hspace{1cm} (10)

where $k$ is a positive constant that represents the step of the gradient, then this component of Eq. (9) forces $P(q)$ to decrease during the manipulator trajectory.\textsuperscript{23}

Thus, defining the potential function proportional to a criterion of interest, the minimization of the potential function will imply the minimization of the criterion chosen. We investigate the use of the negative of the coupling index as the potential function:

$$P(q) = -\rho_{\tau} = -\prod_{i=1}^{n_u} \sigma_i(W_{ui}).$$  \hspace{1cm} (11)

The rationale behind this choice is as follows. As seen before, the coupling index measures how coupled are the active and passive joints. The bigger the index, the smaller is the torque that must be applied in the active joints to control the passive joints. In other words, as the magnitude of $\rho_{\tau}$ increases, less torque is necessary in the actuators to produce the same movement and, therefore, the energy consumption is smaller.

To achieve the objective of controlling the passive joints while the torque is minimized we substitute Eqs. (11) and (10) in (9), obtaining

$$\ddot{q}_a = -M_{ua}^\# (M_{uu} \ddot{q}_u + b_u) + (I - M_{ua}^\# M_{ua})(-k) \times \left( \frac{\partial P(q)}{\partial q_a} \right)^T.$$  \hspace{1cm} (12)

Substituting Eq. (12) in the first line of Eq. (2) we obtain a new open-loop relationship between $\ddot{q}_u$ and $\tau_u$:...
\[ \tau_a = (M_{aa} - M_{aa}M_{uu}^T M_{uu}) \ddot{q}_d - M_{aa}M_{uu}^T b_u + b_a + M_{aa} (I - M_{aa}^T M_{aa}) (-k) \left( \frac{\partial P(q)}{\partial q_a} \right)^T. \]  

(13)

This redundancy is explicitly represented in Eq. (13) by the term

\[ M_{aa} (I - M_{aa}^T M_{aa}) (-k) \left( \frac{\partial P(q)}{\partial q_a} \right)^T. \]  

(14)

Thus, we can define an auxiliary control signal \( u \), that applied in

\[ \tau_a = (M_{aa} - M_{aa}M_{uu}^T M_{uu}) u - M_{aa}M_{uu}^T b_u + b_a + M_{aa} (I - M_{aa}^T M_{aa}) (-k) \left( \frac{\partial P(q)}{\partial q_a} \right)^T, \]

(15)

gives the torque vector, \( \tau_a \), necessary to control the passive joints and at the same time minimize \( P(q) \).

The control signal \( u \) can be obtained from a variety of control techniques that were designed for the experimental manipulator. Among these techniques we will introduce in the next section controllers designed by computed torque, \( H_2 \), and \( H_\infty \) theories.

5. ROBUST CONTROLLERS

As is usual in the robotics literature, the nonlinear system (15) can be controlled with the computed torque method, where \( u \) is given by

\[ u = \ddot{q}_d + K_p (\dot{q}_d - \dot{q}) + K_v (q_d - q), \]

(16)

where \( q_d, \dot{q}_d \) and \( \ddot{q}_d \) are the desired positions, velocities and accelerations of the passive joints, respectively; and \( K_p \) and \( K_v \) are diagonal matrices \( n_u \times n_u \), in which each element of the diagonal is a positive scalar gain.

The block diagram of this method is shown in Figure 1.

Here \( M_{est} \) and \( b_{est} \) are estimated models of the inertia matrix, \( M \), and of the noninertial torque vectors, \( b \), of the real robot. The closed loop system equation is

\[ \ddot{\epsilon} + K_p \dot{\epsilon} + K_v \epsilon = M_{est}^{-1} [(M - M_{est}) \ddot{q}_u + (b - b_{est}) + d_{ext}]. \]

(17)

where \( \epsilon \) is the error between the desired and actual passive joints positions.

In a real manipulator, external disturbances such as friction, torque variation of the actuators, and disturbances due to loads are commonplace. If the summation of these disturbances is defined as \( d_{ext} \) and added to Eq. (17), we have

\[ \ddot{\epsilon} + K_p \dot{\epsilon} + K_v \epsilon = M_{est}^{-1} [(M - M_{est}) \ddot{q}_u + (b - b_{est}) + d_{ext}]. \]

(18)

If we have perfect knowledge of all the parameters of the robot and there is no external disturbance, then the right-hand side of Eq. (18) becomes zero and the computed torque method is capable of providing excellent performance for all the robot configurations. However, modeling uncertainties and external disturbances will cause errors and will degrade its performance.

To overcome this deficiency we combine a robust controller (represented by the block \( K \) in Figure 2) in addition to weighting functions which reflect the performance objectives of the system. The control law, therefore, consists of an internal loop of feedback linearization and an external robust control loop. The control techniques that will be implemented in this work are based on \( H_2 \) and \( H_\infty \) theory. Here we provide only an outline of the derivation. Further details can be found in ref. 32.

In Figure 2 the portion limited by the dotted line, which is the portion relative to the PD controller, corresponds to the diagram of Figure 1 and allows us to formulate the linear robust controller. In this method, the portion relative to the computed torque precompensates the dynamics of the robot and the \( H_2 \) / \( H_\infty \) controllers postcompensate the residual errors that were not removed by the computed torque method.

The derivation of the robust controller is based on the diagram shown in Figure 3. The plant \( G \) corresponds to the portion of Figure 2 limited by the dot-
There is a representation of the nonstructured uncertainties in the input of the plant, representing possible errors of the actuators of the manipulator, neglected dynamics of high frequency, and uncertain zeros on the right half plane. $W_e(s)$, $W_u(s)$ and $W_d(s)$ are weighting functions described below.

The weighting function $W_e(s)$ reflects the performance requisites of the closed loop transfer function shape of the system. This is a diagonal matrix with dimension consistent with the number of degrees of freedom of the system, with the elements of the diagonal given by

$$W_e(i,i) = \frac{s/M_s + \omega_b}{s + \omega_b e}.$$ \(19\)

The choice of $W_u(s)$ is similar to $W_e(s)$. The matrix $W_u(s)$ is diagonal and its components are given by:

$$W_u(i,i) = \frac{s + \omega_{bc} / M_u}{\varepsilon_1 s + \omega_{bc}}.$$ \(21\)

This function should yield

$$\|T(s)W_u(s)\|_{\infty} < 1,$$ \(22\)

which means that the highest singular value of the complementary sensitivity function $T(s) = K(s)G(s) \times (I + K(s)G(s))^{-1}$ should be smaller than the least singular value of $W_u^{-1}(s)$ in the frequency range of interest. The weighting function $W_d(s)$ is selected as the identity matrix.

Therefore, the following realization represents the augmented plant $G$ and will be used for the computation of the controllers:

$$G = \begin{bmatrix}
0 & I & 0 & 0 & 0 & 0 & 0 \\
-K_p & -K_v & 0 & 0 & I & 0 & I \\
0 & 0 & A_{W_u\Delta} & 0 & 0 & B_{W_u\Delta} \\
B_{W_e} & 0 & 0 & A_{W_e} & 0 & B_{W_e} \\
0 & 0 & C_{W_u\Delta} & 0 & 0 & D_{W_u\Delta} \\
D_{W_e} & 0 & 0 & C_{W_e} & 0 & D_{W_e} \\
I & 0 & 0 & 0 & I & 0
\end{bmatrix},$$ \(23\)

where $(A_{W_u\Delta}, B_{W_u\Delta}, C_{W_u\Delta}, D_{W_u\Delta})$ is a realization in
state space for the weighting function \( W_u(s) \) and \((A_{W}, B_{W}, C_{W}, D_{W})\) for the weighting function \( W_s(s)\).

The \( H_{\infty} \) solution involves the following Hamiltonian matrices:

\[
H_{\infty} := \begin{bmatrix}
A & \gamma^{-2} B_{1} B_{1}^* - B_{2} B_{2}^* \\
-C_{1}^* C_{1} & -A^*
\end{bmatrix},
\]

and

\[
J_{\infty} := \begin{bmatrix}
A^* & \gamma^{-2} C_{1}^* C_{1} - C_{2}^* C_{2} \\
-B_{1} B_{1}^* & -A
\end{bmatrix}.
\]

The controller is given by

\[
K_{sub}(s) = \begin{bmatrix}
\hat{A}_{\infty} & Z_{\infty} L_{\infty} \\
F_{\infty} & 0
\end{bmatrix},
\]

where

\[
\hat{A}_{\infty} = A + \gamma^{-2} B_{1} B_{1}^* X_{\infty} + B_{2} F_{\infty} + Z_{\infty} L_{\infty} C_{2},
\]

\[
F_{\infty} = B_{2}^T X_{\infty},
\]

\[
L_{\infty} = -Y_{\infty} C_{2}^*,
\]

\[
Z_{\infty} = (I - \gamma^{-2} X_{\infty} Y_{\infty})^{-1}.
\]

For the \( H_2 \) we have the following Hamiltonian matrices:

\[
X_{2} := \text{Ric} \begin{bmatrix}
A & -B_{2} B_{2}^T \\
-C_{1}^* C_{1} & A^T
\end{bmatrix},
\]

and

\[
Y_{2} := \text{Ric} \begin{bmatrix}
A^T & -C_{2}^* C_{2} \\
-B_{1} B_{1}^* & -A
\end{bmatrix},
\]

where \( \text{Ric} \) represents the solution of the algebraic Riccati equation associated with this Hamiltonian.

The optimal \( H_2 \) controller is

\[
K_{opt}(s) = \begin{bmatrix}
\hat{A}_{2} & L_{2} \\
F_{2} & 0
\end{bmatrix},
\]

where

\[
\hat{A}_{2} = A + B_{2} F_{2} + L_{2} C_{2},
\]

\[
F_{2} = B_{2}^T X_{2},
\]

\[
L_{2} = -Y_{2} C_{2}^T.
\]

6. RESULTS

To validate the optimal control strategy proposed in this article, we use an experimental planar three-link manipulator UARM II, designed and built by H. Benjamin Brown and Randal Casciola from Pittsburgh, PA (Figure 4). In this manipulator, all three joints are equipped with an actuator, a brake, and an encoder.

The choice of the negative of the coupling index, \(- \rho_{\tau}\), as the potential function, \( P(q) \), for the optimal control strategy provides energy minimization in the long run operation of the manipulator, rather than on every individual PTP (point-to-point) motion. Due to the complexity in obtaining an analytical proof, we show that for a large variety of PTP motions the technique is successful.

In the sequel we show three examples that illustrate our claim. The first example is a simulation of a large variety of motions of the manipulator. The second and third examples show experimental results using the actual manipulator UARM II.

In each example we compare the energy spent during the control of the passive joints and the energy spent during the entire motion, both for our method (redundant strategy) and the only other one available in the literature (nonredundant strategy). The coupling index, \( \rho_{\tau} \), and its gradient were calculated symbolically via the mathematical software package.
Maple, and the literal expressions were implemented in MATLAB, which calculates the gradient at each position of the manipulator. Despite their complexity, the gradient expressions take only a small part of the sampling period adopted for the manipulator control.

Example 1: Consider the manipulator in configuration AAP, i.e., joints 1 and 2 active and joint 3 passive. Suppose the three-link robot is mounted on a space structure, and that it has to move in a PTP fashion to inspect bolts located at the set of Cartesian points shown in Figure 5, in the order shown.

Figure 6 shows the coupling index, $r$, as a function of the angles of joints 2 and 3 (from 0 to $2\pi$, in radians) for the configuration considered. The objective of our methodology is to take advantage of the actuation redundancy to position the passive joint while keeping the values of the coupling index high during most of the process. As mentioned in the previous section, the consequence of this is the decrease in the efforts of the active joints to position the passive joints, and, consequently, the decrease in the energy consumption of the system.

To compare the energy consumption of each strategy, we compute the sum of the absolute value of the torques applied at the active joints during the motion of the manipulator, i.e.,

$$E = N \frac{T_f}{\Delta t} \sum_{N=0}^{N-T_f/\Delta t} \tau_1(N\Delta t) + \sum_{N=0}^{N-T_f/\Delta t} \tau_2(N\Delta t). \quad (37)$$

Since the actuators of the UARM II are armature controlled DC motors, for whom torque is directly proportional to current, $E$ provides a fairly accurate measure of the electric power spent during the motion.

We simulate the motion of the manipulator between each pair of Cartesian points $i$ and $i+1$, and measure the amount of energy spent during each PTP motion, according to Eq. (37). In each motion, we performed a search for the best value of the gradient step, $k$, comparing the total energy computed for each $k$.

Table I presents the energy values. For each strategy we present the values of energy spent during the positioning of the passive joint, ($E_{NR_1}$ and $E_{R_1}$), and positioning of all the joints, ($E_{NR_2}$ and $E_{R_2}$). Due to the symmetry of the points of this set, we present only the values relative to the points 1–10; the data repeats.

<table>
<thead>
<tr>
<th>Points</th>
<th>$E_{NR_1}$</th>
<th>$E_{NR_2}$</th>
<th>$E_{R_1}$</th>
<th>$E_{R_2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1→2</td>
<td>1.07</td>
<td>2.18</td>
<td>0.56</td>
<td>1.62</td>
</tr>
<tr>
<td>2→3</td>
<td>1.16</td>
<td>2.38</td>
<td>0.65</td>
<td>1.79</td>
</tr>
<tr>
<td>3→4</td>
<td>1.34</td>
<td>2.71</td>
<td>0.85</td>
<td>1.93</td>
</tr>
<tr>
<td>4→5</td>
<td>1.60</td>
<td>3.15</td>
<td>1.06</td>
<td>2.14</td>
</tr>
<tr>
<td>5→6</td>
<td>12.08</td>
<td>13.89</td>
<td>6.91</td>
<td>7.58</td>
</tr>
<tr>
<td>6→7</td>
<td>1.65</td>
<td>2.97</td>
<td>1.02</td>
<td>2.35</td>
</tr>
<tr>
<td>7→8</td>
<td>1.39</td>
<td>2.56</td>
<td>0.82</td>
<td>1.90</td>
</tr>
<tr>
<td>8→9</td>
<td>1.19</td>
<td>2.28</td>
<td>0.64</td>
<td>1.18</td>
</tr>
<tr>
<td>9→10</td>
<td>1.08</td>
<td>2.15</td>
<td>0.50</td>
<td>1.07</td>
</tr>
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<td>(Σ×2)</td>
<td>45.12</td>
<td>68.54</td>
<td>25.86</td>
<td>43.12</td>
</tr>
</tbody>
</table>

Figure 5. Set of points to be reached by the manipulator.

Figure 6. Torque coupling index—configuration AAP.
itself for the other points. We also show in the last line of the table the energy spent for the entire motion, i.e., the manipulator moving from point 1 and returning to this position making a stop in each point. We compute the total amount of energy as the sum of the values exhibited multiplied by 2.

For all the PTP motions of this example, the redundant strategy proposed resulted in minimization of energy, both for the positioning of the passive joint and for all the joints of the manipulator. A peculiarity of this simulation is the high value of energy spent between points 5 and 6, despite the fact that these points are relatively close in Cartesian space. This occurred because of the choice of the joint angles configurations for these points. In the next example we make a more detailed analysis of this motion in particular.

We can notice a significant economy of energy in all the cases. Considering the entire motion and the positioning of all the joints (passive and active) this economy reaches 37%. For the most critical case (from points 5 to 6) the advantages of this methodology are even greater, as we can see comparing the absolute values of energy.

Example 2: Considering the most critical motion of the previous example, we configure the actual UARM II as APA, i.e., with joints 1 and 3 active and joint 2 passive. The PTP motion to be performed corresponds to the manipulator starting at \( q = [50^\circ; -40^\circ; -40^\circ] \) and reaching \( q^d = [-50^\circ; 40^\circ; 40^\circ] \) as shown in Figure 7. Here the robust controller used is based on \( H_\infty \) theory. Two strategies were tested: non-redundant control and redundant control with optimization for \( k = 0.0005 \) in Eq. (10).

Figure 8 shows the coupling index, \( \rho_T \), as a function of the angles of joints 2 and 3 (from 0 to \( 2\pi \), in radians) for the configuration considered. Figure 9 presents the same index, as a function of time, for both control strategies of the passive joint. The dotted line represents the values for the nonredundant strategy and the continuous line represents the values for the redundant strategy.

Figures 10–12 present the graphics of the positions, velocities and torques of all joints, including not only the control of the passive joints but the control of the active joints as well. Figures 13–15 are the graphics related to the redundant optimized strategy.

The results are shown in Table II, where \( E_1 \) represents the energy spent during phase 1 (control of the passive joints) and \( E_2 \) represents the energy spent during the entire motion (phases 1 and 2).

As mentioned in the previous sections, the efforts
of the active joints to position the passive joints decrease, and, consequently, the energy consumption of the system decreases. Compared to the nonredundant strategy, this method provided 35.2% of energy saving in the passive joint control phase and 25.5% of energy saving in the whole process.

Example 3: We still utilize the manipulator UARM II with configuration APA, but the controller used is $H_2$. The initial angular positions of the PTP motion are $q = [0^\circ; 0^\circ; 0^\circ]$ and final positions are $q^d = [-30^\circ; -20^\circ; -10^\circ]$. The value of $k$ in the redundant strategy is 0.01.

The graphics of positions, velocities and torques for the joints using the nonredundant strategy are presented in Figures 16–18. The graphics related to redundant strategy are presented in Figures 19–21. Computing the energy consumption of the motion during phase 1 and for the entire motion we have the results of Table III.

Again, the method proposed presents the least energy expenditure. The economy of energy for the redundant strategy compared to nonredundant strategy is 66.6%. Considering the entire motion this economy is 11.1%.

7. CONCLUSION

As manipulators start to operate in space, in deep sea, and in hazardous environments, fault tolerant control
becomes ever more fundamental. The result presented in this article allows one to drive all joints of a manipulator with both active and passive joints to a desired configuration, with the added bonus of energy consumption minimization during the phase when the passive joints are controlled. Extensions of this work that the authors are dealing with include control of passive joints not equipped with brakes, or whose encoders have failed along with their actuators.

The optimization method proposed is valid locally. The results obtained suggest that for most of the cases the torque and the energy consumed by the system are minimized during the trajectory, no matter whether the passive joint or all the joints are being controlled. This happens despite the fact that we utilize redundancy without focusing global effects. Our future work will focus on global optimization, based on optimal control theory.

### Appendix: Equations of Inertia Matrix and Parameters

The inertia matrix $M$ and the noninertial torque vector $b$ are computed according to Section II resulting in the expressions given below:

<table>
<thead>
<tr>
<th>Strategy</th>
<th>$E_1$ (Nm)</th>
<th>$E_2$ (Nm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nonredundant</td>
<td>18.72</td>
<td>32.33</td>
</tr>
<tr>
<td>Redundant ($k=0.0005$)</td>
<td>12.13</td>
<td>24.07</td>
</tr>
</tbody>
</table>
Figure 18. Joint torques—nonredundant strategy.

\[
M = \begin{bmatrix}
M_{11} & M_{12} & M_{13} \\
M_{21} & M_{22} & M_{23} \\
M_{31} & M_{32} & M_{33}
\end{bmatrix},
\]

\[
M_{11} = I_1 + I_2 + I_3 + m_1 l_1^2 + m_2 (l_1^2 + l_2^2 + 2 l_1 l_2 \cos(\theta_2)) + m_3 (l_1^2 + l_2^2 + 2 l_1 l_2 \cos(\theta_2) + 2 l_1 l_3 \cos(\theta_2) + \theta_3) + 2 l_2 l_3 \cos(\theta_3),
\]

\[
M_{12} = l_2 + I_3 + m_2 (l_2^2 + 2 l_1 l_2 \cos(\theta_2)) + m_3 (l_2^2 + l_3^2 + l_1 l_2 \cos(\theta_2) + l_1 l_3 \cos(\theta_2 + \theta_3) + 2 l_2 l_3 \cos(\theta_3)),
\]

\[
M_{13} = I_3 + m_3 (l_3^2 + l_1 l_3 \cos(\theta_2 + \theta_3) + l_2 l_3 \cos(\theta_3)),
\]

\[
M_{21} = M_{12},
\]

\[
M_{22} = I_2 + I_3 + m_2 (l_2^2 + l_3^2 + 2 l_2 l_3 \cos(\theta_2)) + m_3 (l_2^2 + l_3^2 + 2 l_2 l_3 \cos(\theta_2)),
\]

\[
M_{23} = M_{22}.
\]

Figure 19. Joint positions—redundant strategy.

Table III. Values of energy expenditure for each control strategy.

<table>
<thead>
<tr>
<th>Strategy</th>
<th>(E_1) (Nm)</th>
<th>(E_2) (Nm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nonredundant</td>
<td>8.96</td>
<td>31.55</td>
</tr>
<tr>
<td>Redundant ((k=0.01))</td>
<td>2.99</td>
<td>28.35</td>
</tr>
</tbody>
</table>

Figure 20. Joint velocities—redundant strategy.

Figure 21. Joint torques—redundant strategy.


\[ M_{23} = I_3 + m_3(l_c^2 + I_2 l_c \cos(\theta_2)), \]
\[ M_{31} = M_{13}, \]
\[ M_{32} = M_{23}, \]

\[ b_1 = \left( -m_2 l_1 l_c \sin(\theta_2) + m_3 (-l_1 l_2 \sin(\theta_2) - l_1 l_3 \sin(\theta_2 + \theta_3)) \left( \frac{l_2}{2} + 2 \dot{\theta}_2 \dot{\theta}_2 \right) + m_3 (-l_1 l_3 \sin(\theta_2 + \theta_3) - l_2 l_3 \sin(\theta_3)) \right) \]
\[ \times \left( \frac{l_2^2}{2} + 2 \dot{\theta}_1 \dot{\theta}_1 \right) \]
\[ + l_3 \cos(\theta_1 + \theta_2 + \theta_3), \]

\[ b_2 = m_2 l_1 l_c \sin(\theta_2) + m_3 (l_1 l_2 \sin(\theta_2) + l_1 l_3 \sin(\theta_2 + \theta_3)) \frac{l_2^2}{2} - m_3 l_1 l_3 \sin(\theta_1) \left( \frac{l_2^2}{2} + 2 \dot{\theta}_1 \dot{\theta}_1 \right) - m_3 g (l_2 l_c \cos(\theta_1 + \theta_2)) \]
\[ + m_2 g (l_1 l_2 \cos(\theta_1 + \theta_2)) + m_3 g (l_1 l_2 \cos(\theta_1 + \theta_2)) + l_3 \cos(\theta_1 + \theta_2 + \theta_3), \]

\[ b_3 = m_3 (l_1 l_3 \sin(\theta_2 + \theta_3) + l_2 l_3 \sin(\theta_3)) \frac{l_2^2}{2} + m_3 l_2 l_3 \sin(\theta_2) \left( \frac{l_2^2}{2} + 2 \dot{\theta}_2 \dot{\theta}_2 \right) + m_3 g (l_3 \cos(\theta_1 + \theta_2 + \theta_3)). \]

\[ (A2) \]

**Table IV.** Parameters of UARM II.

<table>
<thead>
<tr>
<th>( i )</th>
<th>( m_i ) (kg)</th>
<th>( l_i ) (kgm²)</th>
<th>( l_i ) (m)</th>
<th>( l_{ci} ) (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.850</td>
<td>0.0075</td>
<td>0.203</td>
<td>0.096</td>
</tr>
<tr>
<td>2</td>
<td>0.850</td>
<td>0.0075</td>
<td>0.203</td>
<td>0.096</td>
</tr>
<tr>
<td>3</td>
<td>0.625</td>
<td>0.0060</td>
<td>0.203</td>
<td>0.077</td>
</tr>
</tbody>
</table>

The parameters of the experimental manipulator are presented in Table IV.

Graphics of Example 1, with manipulator in configuration APA, starting at \( q = (50°; -40°; -40°) \) and reaching \( q^d = (-50°; 40°; 40°) \). The robust controller used is based on \( H_\infty \) theory.

Graphics of Example 2, with manipulator in configuration APA, starting at \( q = (0°; 0°; 0°) \) and reaching \( q^d = (-30°; -20°; -10°) \). The controller used is based on \( H_2 \) theory.

**REFERENCES**

13. H. Schneider and P.M. Frank, Observer based supervision and fault detection in robots using nonlinear and