Asymptotically Optimal Subcarrier Matching and Power Allocation for Cognitive Relays With Power and Interference Constraints

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Abstract—In this paper, the problem of resources allocation in decode-and-forward relayed OFDM based cognitive system is considered. The subcarrier matching and power allocation are optimized jointly under the individual power constraints in source and relay so that the sum rate is maximized while the interference induced to the primary system is kept below a pre-specified interference temperature limit. Therefore, the dual decomposition technique is adopted to obtain an asymptotically optimal solution. Moreover, a suboptimal scheme is presented to get rid of the high computational complexity of the optimal scheme. The suboptimal algorithm solves the problem in two phases. In the first phase, the subcarrier powers are fixed according to the interference that can be induced to the primary system as well as the available power budgets. The subcarriers are matched in the second phase based on the fixed powers and the channel qualities. Selected numerical examples are provided to demonstrate the performance of the different schemes.

I. INTRODUCTION

The available spectrum is divided into several frequency bands which are allocated traditionally to a specific user or service provider exclusively in order to be protected from any interference. Since most of current frequency bands have been already allocated, it will be very hard to find vacant bands for the emerging wireless systems or services. Moreover, recent measurements show that the spectrum utilization varies from 15% to 85% depending on time, frequency and geographical location [1]. These observations motivate the development of cognitive radio (CR) communications whereby the secondary users (SUs) are allowed to access the unused radio spectrum (spectrum holes) originally allocated to the primary users (PUs). In this way, CR will greatly improve the spectrum utilization without major changes to the existing primary systems. Using one of the spectrum sensing techniques [2], the SUs need to periodically monitor the radio spectrum in order to detect in time and frequency the PUs transmission. The interference introduced to the primary system due to CR transmission should be below a predefined value called the interference temperature limit. Multicarrier communication systems have been considered as an appropriate candidate for CR systems [3].

The system performance and the spectrum utilization can be improved by combining the cognitive radio with the cooperative communication techniques in which the relays (R) is used to assist the source (S) to destination (D) transmission. An overview of the cooperative communications in cognitive scenario has been presented in [4], [5].

Uncounted research work has been done on the resources allocation in non-cognitive relay systems (see e.g. [6]–[8] and references therein). The methods used in the non-cognitive relays are not solving efficiently the resource allocation problem in cognitive relays because the different resources should be distributed adequately so that the interference introduced to the primary system isn’t harmful. Jia et al. proposed in [5] a centralized heuristic algorithm to select the most profitable pair of nodes and to allocate the different channels based on the availability of the spectrum. The interference to the primary system was not considered. Lijing et al. presented in [9] a joint relay selection and power allocation algorithm where the cognitive relay system is prevented from inducing severe interference to the primary system by limiting its maximum transmission power. In [10], the authors proposed an algorithm to select the best transmit way between the network nodes. The algorithm can select direct, dual or diversity transmission based on the available spectrum as well as the maximum allowable transmission powers. The algorithms presented in [9] and [10] are dealing with the point-to-point systems. In [11], a power allocation algorithm in decode and forward OFDM based CR system has been proposed under the interference constraint only. By assuming a perfect subcarrier matching in the S-R and R-D links, the authors treated the optimization problem in S and R individually. The algorithm performance degrades significantly if the relay has to forward the receiving message on the same subcarrier, i.e. there is no subcarrier matching.

In this work, an asymptotically optimal joint subcarrier matching and power allocation algorithm in decode and forward relayed OFDM based cognitive system is proposed. The objective is to maximize the total throughput of the cognitive system under separate power constraints. The interference introduced to the primary system should be under the pre-specified interference temperature limit. The main contribution of this paper is as follows: 1) Applying the decode and
forward relaying strategy, the problem is formulated so that
the dual decomposition technique can be applied to find the
powers and subcarrier pairs jointly. 2) An efficient subopti-
mal scheme is presented to reduce the high computational
complexity of the optimal solution. The proposed suboptimal
scheme performs the resource allocation in two phases. The
subcarriers powers are firstly fixed and the different pairs are
determined afterwards depending on the induced interference,
fixed powers and channels quality.

The rest of this paper is organized as follows. Section II
gives the system model while the problem is formulated
and the optimal power allocation is derived in Section III.
The sub-optimal scheme is presented in Section IV. Selected
numerical results are discussed in Section V. Finally, Section
VI concludes the paper.

II. SYSTEM MODEL

In this paper, an OFDM-based relay CR will be considered. As
shown in Fig. 1, the CR relay system coexists with the
primary system in the same geographical location. Due to
the existence of an obstacle or a large distance, there is no
direct link between S and D so that S tries to communicate
with D through R. The CR system’s frequency spectrum is
divided into \( N \) subcarriers each having a \( \Delta f \) bandwidth. It is
assumed that the CR system can use the inactive PU bands
provided that the total interference introduced to the PU band
does not exceed the maximum interference power that can be
tolerated by PU, \( I_{th} \). The relay is assumed to be half-duplex,
thus receiving and transmitting in two different time slots. In
the first time slot, S transmits to R over the \( j^{th} \) subcarrier while
in the second time slot R decodes the message, re-encodes it
and then forwards it to D over the \( k^{th} \) subcarrier which may
not be the same as \( j \) and they form the subcarrier pair \( (j,k) \).
The maximum total transmission powers that can be used in
S and R are \( P_S \) and \( P_R \) respectively.

Assume that \( \Phi_i \) is the power spectrum density (PSD) of the
\( i^{th} \) subcarrier in S or R. The expression of the PSD depends
on the used multicarrier technique. If an OFDM based CR is
assumed, the PSD of the \( i^{th} \) subcarrier can be written as [12]

\[
\Phi_i(f) = P_i T_s \left( \frac{\sin \pi f T_s}{\pi f T_s} \right)^2
\]

where \( P_i \) is the total transmit power emitted by the \( i^{th} \) sub-
carrier and \( T_s \) is the symbol duration. The mutual interference
introduced by the \( i^{th} \) subcarrier to PU, \( I_i (d_i, P_i) \), is the
integration of the PSD of the \( i^{th} \) subcarrier across the PU
band, \( B \), and can be expressed as [12]

\[
I_i (d_i, P_i) = \int_{d_i - B/2}^{d_i + B/2} G_i \Phi_i(f) \, df = P_i \Omega^i
\]

where \( d_i \) is the spectral distance between the \( i^{th} \) subcarrier
and the PU band, \( G_i \) denotes the channel gain between the \( i^{th} \)
subcarrier and the PU band while \( \Omega^i \) denotes the interference
factor of the \( i^{th} \) subcarrier to the PU band.

By the same way, the interference power introduced by PU
signal into the band of the \( j^{th} \) subcarrier is [12]

\[
J_j = \int_{d_j - \Delta f/2}^{d_j + \Delta f/2} Y_j T (e^{j\omega}) \, d\omega
\]

where \( T (e^{j\omega}) \) is the power spectrum density of PU signal and
\( Y_j \) is the channel gain between the \( i^{th} \) subcarrier and the
PU signal.

III. PROBLEM FORMULATION AND OPTIMAL SOLUTION

The transmission rate of the \( j^{th} \) subcarrier in the source
coupled with \( k^{th} \) subcarrier in the relay, \( R(j,k) \), can be
evaluated as follows

\[
R(j,k) = \frac{1}{2} \min \left\{ \log_2 \left( 1 + \frac{P_{jk} H_{jk}^2}{\sigma^2} \right), \log_2 \left( 1 + \frac{P_{jk} H_{jk}^2}{\sigma^2} \right) \right\}
\]

where \( P_{jk}^{SR}(P_{jk}^{RD}) \) is the power transmitted over the \( j^{th}(k^{th}) \)
subcarrier in the S-R(R-D) link while \( H_{jk}^{SR}(H_{jk}^{RD}) \) is
the \( j^{th}(k^{th}) \) subcarrier fading gain over S-R(R-D) link.

\[
\sigma^2_{(j,k)} = \sigma^2_{AWGN} + J_{(j,k)} \]

where \( \sigma^2_{AWGN} \) is the variance of the additive white Gaussian noise (AWGN) and \( J_{(j,k)} \) is
the interference introduced by the PU signal into the \( j^{th}(k^{th}) \)
subcarrier which is evaluated using (3) and can be modeled as
AWGN as described in [3]. To make the analysis more clear
and without loss of generality, the noise variance is assumed
to be constant for all subcarriers, i.e. \( \sigma^2 = \sigma^2_{AWGN} = \sigma^2 \).

Our objective is to maximize the CR system throughput by
optimizing the subcarrier pairing and distribute the available
power budgets in S and R between the subcarrier pairs so that
the instantaneous interference introduced to the primary sys-
tem is below the maximum limit. Therefore, the optimization

![Fig. 1. Cooperative relay cognitive radio network.](image-url)
function of the power at $S$ as follows
\[ P^j_{SR} > 0, P^0_{RD} > 0, t_{j,k} \]
subject to
\[ \sum_{j=1}^{N} \sum_{k=1}^{N} t_{j,k} R(j,k) \]
Therefore, the power allocated at $R$ can be expressed as function of the power at $S$ as follows
\[ P^k_{RD} = \frac{P^j_{SR} H^j_{SR}}{H^k_{RD}} \]
Hence, the optimization problem in (5) can be re-written as follows
\[ \max_{P^j_{SR} > 0,t_{j,k}} \sum_{j=1}^{N} \sum_{k=1}^{N} t_{j,k} R(j,k) \]
subject to
\[ \sum_{j=1}^{N} P^j_{SR} \leq P_S \]
\[ \sum_{j=1}^{N} \sum_{k=1}^{N} t_{j,k} \frac{P^j_{SR} H^j_{SR}}{H^k_{RD}} \leq P_R \]
\[ \sum_{j=1}^{N} \sum_{k=1}^{N} t_{j,k} \Omega_k \leq I_{th} \]
Finding the optimization variables $P^j_{SR}$ and $t_{j,k}$ in (8) is a mixed binary integer programming problem. There are $N!$ subcarrier matching possibilities and hence the complexity is prohibitive for large number of subcarriers. The problem in (8) is satisfying the time sharing condition described in [14] and hence, the duality gap of the problem is negligible as the number of subcarrier is sufficiently large regardless of the convexity of the problem. The solution obtained by the dual method is asymptotically optimal [14].

The dual problem associated with the primal problem (8) can be written as
\[ \min_{\beta \geq 0, \gamma \geq 0, \lambda \geq 0, \mu \geq 0} g(\beta, \gamma, \lambda, \mu) \]
\[ \beta \geq 0, \gamma \geq 0, \lambda \geq 0, \mu \geq 0 \]
\[ g(\beta, \gamma, \lambda, \mu) \triangleq \max_{P^j_{SR} > 0, t_{j,k}} \sum_{j=1}^{N} \sum_{k=1}^{N} t_{j,k} R(j,k) \]
subject to
\[ \sum_{j=1}^{N} t_{j,k} \leq 1, \forall j; \sum_{j=1}^{N} t_{j,k} \leq 1, \forall k \]
where $N$ denotes the total number of subcarriers while $I_{th}$ is the interference threshold prescribed by PU. $P_S$ and $P_R$ are the available power budget in $S$ and $R$ respectively. $\Omega^j_{SP}$ and $\Omega^k_{RP}$ are the $j^{th} (k^{th})$ subcarrier interference factor to the PU band from $S$ and $R$ respectively. The subcarrier pairing constraint ensures that each subcarrier in $S$ is paired with only one subcarrier in $R$ where $t_{j,k} \in \{0,1\}$ is the subcarrier pairing indicator, i.e. $t_{j,k} = 1$ if the $j^{th}$ subcarrier in $S$ is paired with the $k^{th}$ in $R$, and zero otherwise. $S$ will be in charge of the resources allocation where all the instantaneous fading gains are assumed to be perfectly known. Note that the channel gains between the CR system parts ($S$, $R$ and $D$) can be obtained practically by the classical channel estimation techniques while the channel gains between the CR system and the PU can be obtained by estimating the received signal power from the primary terminal when it transmits, under the assumptions of pre-knowledge on the primary transmit power levels and the channel reciprocity [13].

From (4), the maximum capacity over a given subcarrier pair $(j,k)$ can be achieved when
\[ P^j_{SR} H^j_{SR} = P^k_{RD} H^k_{RD} \]
Therefore, the power allocated at $R$ can be expressed as function of the power at $S$ as follows
\[ P^k_{RD} = \frac{P^j_{SR} H^j_{SR}}{H^k_{RD}} \]

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Therefore, the power allocated at $R$ can be expressed as function of the power at $S$ as follows
\[ P^k_{RD} = \frac{P^j_{SR} H^j_{SR}}{H^k_{RD}} \]
Hence, the optimization problem in (5) can be re-written as follows
\[ \max_{P^j_{SR} > 0, t_{j,k}} \sum_{j=1}^{N} \sum_{k=1}^{N} \frac{1}{2} t_{j,k} \log \left( 1 + \frac{P^j_{SR} H^j_{SR}}{\sigma^2} \right) \]
subject to
\[ \sum_{j=1}^{N} \sum_{k=1}^{N} t_{j,k} \leq t(j,k) \]
\[ \sum_{k=1}^{N} \sum_{j=1}^{N} t_{j,k} \leq I_{th} \]
\[ \sum_{j=1}^{N} \sum_{k=1}^{N} t_{j,k} \leq I_{th}, \forall j \]
where
\[
D\left(P_{SR}^j, k\right) = \frac{1}{2} \log \left(1 + \frac{P_{SR}^j H_{SR}^j}{\sigma^2} \right) - \beta P_{SR}^j - \gamma P_{SR}^j H_{RD}^j \Omega_j - \mu P_{SR}^j H_{RD}^j \Omega_k
\]

Hence, the optimal power allocation and subcarrier pairing can be found in three steps as follows:

A. Optimal Power Allocation for a Given Subcarrier Pair

We can start by assuming any initial values for the different dual variables and also assuming that \((j, k)\) is a valid subcarrier pair and is already matched. Hence, (12) is decomposed into \(N\) independent subcarrier allocation subproblems. The optimal power allocation can be determined by solving the following subproblem

\[
\max_{P_{SR}^j} D\left(P_{SR}^j, k\right) \quad \text{s.t.} \quad P_{SR}^j \geq 0
\]

for every \((j, k)\) pair. Solving (14) for the optimal power we can find

\[
P_{SR}^{\ast,j} = \left[\frac{1}{\beta + \gamma \Omega_j + \lambda \Omega_j + \mu \Omega_k} - \frac{\sigma^2}{H_{SR}^j} \right]^+ \tag{15}
\]

where \([x]^+ = \max(0, x)\).

B. Optimal Subcarrier Matching

Substituting the optimal power allocation expression in (15) into (12) to eliminate the power variable, we obtain the corresponding dual function

\[
g(\beta, \gamma, \lambda, \mu) \triangleq \max_{t_{j,k}} \sum_{j=1}^{N} \sum_{k=1}^{N} t_{j,k} D\left(P_{SR}^{\ast,j}, k\right) + \beta P_S + \gamma P_S + I_{th}(\lambda + \mu) \tag{16}
\]

s.t. \(\sum_{j=1}^{N} t_{j,k} \leq 1, \forall k; \sum_{k=1}^{N} t_{j,k} \leq 1, \forall j\).

By defining the \(N \times N\) profit matrix \(D = [D\left(P_{SR}^{\ast,j}, k\right)]\) for every \((j, k)\) pair, the objective in (16) can be maximized by picking elements from the matrix \(D\) such that the sum of profits is as large as possible. This is a linear assignment problem that can be solved efficiently by the Hungarian method with a complexity of \(O(N^3)\) [15].

C. Solving the Dual Problem

The subgradient method can be used to solve the dual problem with guaranteed convergence. After finding the optimal solution, i.e. \(P_{SR}^{\ast,j}\) and \(t_{j,k}^{\ast}\), of the dual function at a given dual point \((\beta, \gamma, \lambda, \mu)\), the dual variables at the \((i + 1)^{th}\) iteration are updated as

\[
\beta^{(i+1)} = \beta^{(i)} - \delta^{(i)} \left(P_S - \sum_{j} P_{SR}^{(j)}\right) \tag{17}
\]

\[
\gamma^{(i+1)} = \gamma^{(i)} - \delta^{(i)} \left(P_R - \sum_{k} P_{RD}^{(k)}\right) \tag{17}
\]

\[
\lambda^{(i+1)} = \lambda^{(i)} - \delta^{(i)} \left(I_{th} - \sum_{j} P_{SR}^{(j)}\right) \tag{17}
\]

\[
\mu^{(i+1)} = \mu^{(i)} - \delta^{(i)} \left(I_{th} - \sum_{k} P_{RD}^{(k)}\right) \tag{17}
\]

where \(\delta^{(i)}\) is the step size that can be updated according to the nonsummable diminishing step size policy. With the updated values of the dual variables, the optimal power allocation and subcarrier matching are evaluated again. The iterations are repeated until convergence.

IV. SUBOPTIMAL ALGORITHM

Suppose that the complexity of the dual variables update in (17) is polynomial in the number of dual variables, i.e. \(4^o\), and the number of iterations required to converge is \(M\). Then, by including the computational complexity of the Hungarian method, the asymptotically optimal solution derived in the previous section has a complexity of \(O(4^oMN^3)\) which may not be efficient with high number of subcarriers [14]. In order to solve the problem efficiently, we propose in this section a suboptimal algorithm by which the resource allocation problem is solved in two phases. In the first phase, the powers are fixed to the different subcarriers while the different subcarrier pairs are determined afterwards in the second phase. The detailed description of the proposed suboptimal scheme follows.

A. Fixing the Subcarrier Powers

Assume that the interference induced to the primary system is divided uniformly on the subcarriers, i.e. every subcarrier is able to induce interference to PU equal to \(\frac{1}{N}\). Therefore, form (2), the maximum power that can be allocated to the \(j^{th}\) subcarrier in \(S(R)\) is

\[
P_{SR,\text{max}}^j (P_{RD,\text{max}}^j) = \frac{I_{th}}{N \Omega_{SP}(\Omega_{RP})} \tag{18}
\]

Similarly, the power constraints can be distributed uniformly on the different subcarriers to get

\[
P_{SR,\text{uni}}^j (P_{RD,\text{uni}}^j) = \frac{P_S(P_R)}{N} \tag{19}
\]

and hence, the allocated power to the \(j^{th}\) subcarrier is

\[
P_{SR}^j (P_{RD}^j) = \min\left(P_{SR,\text{max}}^j (P_{RD,\text{max}}^j), P_{SR,\text{uni}}^j (P_{RD,\text{uni}}^j)\right) \tag{20}
\]
B. Matching the Subcarriers

The optimal subcarrier pairing strategy in non-cognitive OFDM based system is achieved by ordering the subcarriers in S and R according to their signal to noise ratio (SNR) and pair the subcarriers with same order together. This strategy is not optimal in CR system due to the existence of the interference constraints. The already fixed powers in (20) is considering the interference and the power constraints, therefore, the channel qualities should be considered also in order to achieve a good subcarrier matching criteria. Hence, the subcarriers in S and R are ordered according to the product of the powers found using (20) by the channel gains, i.e. $P_{SR}(P_{SR})H_{SP}(H_{RP})$. Then, every subcarrier in S will be matched with the subcarrier with the same order in R. The sub-optimal algorithm has the sorting computational complexity of $O(N \log N)$.

V. Simulations Results

The simulations are performed under the scenario given in Fig.1. An OFDM system of $N = 64$ subcarriers is assumed. The values of $T_s$, $\Delta f$, and $\sigma^2$ are assumed to be $4\mu$s seconds, 0.3125 MHz and $10^{-6}$ respectively. The channel gains are outcomes of independent Rayleigh distributed random variables with mean equal to 1. All the results have been averaged over 1000 iterations. For the purpose of performance comparison, the following algorithms are considered:

1) **Optimal**: the problem is solved using the dual decomposition technique presented in Sec. III.

2) **Suboptimal**: the problem is solved using the proposed method presented in Sec. IV.

3) **Fix+Hungarian**: the powers are fixed as in Sec. IV-A, while the subcarriers are matched according to the Hungarian method as in (16).

4) **SNR+optimal**: the subcarriers are matched according to their SNR values while the powers are evaluated by solving (8) with the known $t_{j,k}$.

5) **Without matching**: the data transmitted by S over a given subcarrier in the first time slot will be forwarded by R over the same subcarrier in the second time slot. The powers are evaluated by solving (8) with $t_{j,k} = 1$ for every $j = k$ and zero otherwise.

Fig. 2 depicts the achieved capacity of the optimal and suboptimal schemes vs. the interference threshold. The solid lines plots the case when $P_S = P_R = 1$ mWatts while the dashed ones when $P_S = P_R = 100$ mWatts. The achieved capacity is compared with that when only one of interference or power constraint is applied. The interference (power) only performance forms an upper bound for that with both constraints. To that end, the performance of the optimal solution under both constraints has three different regions. Considering the case of $P_S = P_R = 1$ mWatts, the three region could be explained as follows

1) If $I_{th} \geq -40$ dBm : the performance is equal to that of the interference only case. The limited effect of the power constraints comes from the small value of the allowed interference since only a small quantity of the available power can induce the maximum allowed interference.

2) If $I_{th} \geq -10$ dBm : the performance is equal to that of the power only. The system in this region performs like a non-cognitive one since the available power budgets cannot induce the maximum allowed interference threshold.

3) If $-40 < I_{th} \leq -10$ dBm : in this region both the power and interference constraints are affecting the optimization problem. The optimal solution performs close to the upper bound formed by the interference (power) only curves.

The same observations can be applied on the case of $P_S = P_R = 100$ mWatts but with different ranges of the regions.

Fig. 3 shows the achieved capacity of the different algorithms vs. the interference threshold. One can notes that the CR system capacity increases with the interference threshold as the CR system become able to use more power on the different subcarriers. Additionally, the throughput increases - as expected- with the increase of the available power budgets. However, the increment in the throughput by changing the available power form 1 mWatts to 100 mWatts is very small in the low interference region since both systems use approximately the same amount of power to induce the maximum allowed interference to the PU. Moreover, the suboptimal algorithm with low computational complexity has a near optimal performance. The Fix+Hungarian algorithm has more computational complexity than the suboptimal algorithm and its performance lies in between the optimal and the suboptimal algorithms. In the low interference thresholds region, the SNR-based matching criteria applied in the non-cognitive system has limited performance in comparison with optimal because it doesn’t take the interference to the primary system into account. Furthermore, the gap between the optimal algorithm and the SNR+optimal algorithm is decreased with the interference threshold as the system behaves closer to the
The proposed suboptimal algorithm performs the resource allocation in two phases. The subcarrier powers are fixed in the first phase while the different pairs are determined in the second phase. It shown that the suboptimal algorithm has a near optimal performance with much less complexity and its observed that the proposed scheme outperforms the SNR-based used in the non-cognitive systems. We are currently working on the extension of the proposed system by considering more interference constraints as well as considering multiple relay nodes.

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