UNDERSTANDING TOA AND TDOA NETWORK CALIBRATION USING FAR FIELD APPROXIMATION AS INITIAL ESTIMATE

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Keywords: TOA, TDOA, calibration, sensor networks, self-localization

Abstract: This paper presents a study of the so called far field approximation to the problem of determining both the direction to a number of transmittors and the relative motion of a single antenna using relative distance measurements. The same problem is present in calibration of microphone and wifi-transmittor arrays. In the far field approximation we assume that the relative motion of the antenna is small in comparison to the distances to the base stations. The problem can be solved uniquely with at least three motions of the antenna and at least six real or virtual transmittors. The failure modes of the problem is determined to be (i) when the antenna motion is planar or (ii) when the transmittor directions lie on a cone. We also study to what extent the solution can be obtained in these degenerate configurations. The solution algorithm for the minimal case can be extended to the overdetermined case in a straightforward manner. We also implement and test algorithms for non-linear optimization of the residuals. In experiments we explore how sensitive the calibration is with respect to different degrees of far field approximations of the transmittors and with respect to noise in the data.

1 INTRODUCTION

Navigation covers a broad application area ranging from traditional needs in the terrestrial, aerial and naval transport sectors to personal objectives of finding your way to school if you are visually impaired, to the nearest fire exit in case of an emergency, or to specific goods in your local supermarket. Many potential applications are however presently hindered by performance limitations of existing positioning techniques and navigation systems.

Radio based positioning rely on either signal strength, direction of arrival (DOA) or time-based information such as time of arrival (TOA) or time differences of arrival (TDOA), or a combination thereof.

The identical mathematical problem occurs also in microphone arrays for audio sensing. Using multiple microphones it is possible to locate a particular sound-source and using beamforming to enhance sound quality of the speaker.

Although TOA and TDOA problems have been studied extensively in the literature in the form of localization of e.g. a sound source using a calibrated detector array, the problem of calibration of a sensor array using only measurement, i.e. the initialization problem for sensor network calibration, has received much less attention. One technique used for sensor network calibration is to manually measure the inter-distance between pairs of microphones and use multi-dimensional scaling to compute microphone locations, (Birchfield and Subramanya, 2005). Another option is to use GPS, (Niculescu and Nath, 2001), or to use additional transmittors (radio or audio), close to each receiver, (Elnahrawy et al., 2004; Raykar et al., 2005; Sallai et al., 2004). Sensor network calibration is treated in (Biswas and Thrun, 2004). In (Chen et al., 2002) it is shown how to estimate additional microphones, once an initial estimate of the position of some microphones are known. In (Thrun, 2005) the far field approximation is used to initialize the calibration of sensor networks. However the experiments and theory was only tested for the planar case and no study of the failure modes were given. Initialization of TOA networks has been studied in (Stewénius, 2005), where solutions to the minimal case of three transmittors and three receivers in the plane is given. The minimal case in 3D is determined to be four receivers and six transmittors for TOA, but this is not solved. Initialization of TDOA networks is studied in (Pollefeys and Nister, 2008), where solutions were give to two non-minimal cases of ten transmittors and five receivers, whereas the minimal solution for far field approximation in this paper are six transmittors and four receivers.

In this paper we study far field approximation as an initialization to the calibration problem. We use a similar factorization as (Thrun, 2005) but in three dimensions, and show that far field approximation is at least four measurement positions, i.e. three motions, and measurements to at least six real or virtual transmittors. In this paper we describe the failure modes of the algorithm and show what can be done when such configurations are present. We further propose two optimization strategies for more thorough calibration and evaluate them in regards to accuracy and convergence rate. Several test cases are simulated in which we validate far field approximation, accuracy of the proposed algorithms, optimization schemes and performance under noisy measurements.

2 DETERMINING POSE

In the following treatment, we make no difference between real and virtual transmittors or base stations. Assume that the base station is stationary at position $b = \begin{pmatrix} b_x & b_y & b_z \end{pmatrix}$ and that the antenna is at position $z = \begin{pmatrix} z_x & z_y & z_z \end{pmatrix}$. By measuring the signal with known base band frequency one obtains a complex constant, whos phase depends on the distance $d = |b - z|$ between the antenna and the base station.

By tracking the phase during small relative motions of the antenna, it is feasible to determine the relative distance $d_{rel}(t) = d(t) + \tilde{C}$, where $\tilde{C}$ is an unknown constant for each base station. This is the so called TDOA setup. Furthermore if during measurements the relative motion is small in comparison with the distance d between the antenna z and the base station b it is reasonable to approximate the distance $d = |b-z| \approx |b-z_0| + (z-z_0)^T \mathbf{n} = z^T \mathbf{n} + \underbrace{(|b-z_0| - z_0^T \mathbf{n})}_{\bar{C}}$. Here z_0 is the initial position of the antenna and $\mathbf{n}$ is the direction from the base station towards the antenna, now assumed to be constant with unit length. By setting $C = \tilde{C} + \bar{C}$ one obtains the far field approximation

$$d_{rel}(\mathbf{n}, z) \approx z^T \mathbf{n} + C.$$

In this paper we are interested in the following far field time difference of arrival (FFTDOA) type problem that arise from this approximate relative distance measurement.

Problem 2.1. *Given measurements $D_{i,j}$, $i = 1, \ldots, m$ and $j = 1, \ldots, k$ from the antenna at m different positions to k base stations, determine both the positions $z_1, \ldots, z_m$ of the antenna during the relative motion and the directions $\mathbf{n}_1 \ldots \mathbf{n}_k$ from the base stations so that*

$$D_{i,j} = z_i^T \mathbf{n}_j - C_j,$$
$$||\mathbf{n}_j||_2 = 1$$

where C_j is a constant distance offset for each base station.

Lemma 2.1. *A problem with m measurements to k base stations with unknown constant C_j can without loss of generality be converted to a problem with $m-1$ measurements to k base stations with known constant.*

Proof. Note that because of the unknown constant C_j the problem does not change in character by modification $\bar{D}_{i,j} = D_{i,j} - K_j$. For simplicity we set $\bar{D}_{i,j} = D_{i,j} - D_{1,j}$. By also setting $z_1 = \begin{pmatrix} 0 & 0 & 0 \end{pmatrix}^T$, we get $C_j = 0$. This is equivalent to choosing the origin of the unknown coordinate system to the first point. □

For simplicity we will in the sequel assume that $C_j = 0$ and assume that the one measurement has already been used to resolve the ambiguity. Denote by D the matrix after removing that said point. This converts the FFTDOA problem into a FFTOA problem, i.e.

Problem 2.2. *Given measurements $D_{i,j}$, $i = 1, \ldots, m, j = 1, \ldots, k$ from the antenna at m different positions to k base stations, determine both the both the positions z_i of the antenna during the relative motion and the direction from the base stations $\mathbf{n}_j$ so that*

$$D_{i,j} = z_i^T \mathbf{n}_j.$$
$$||\mathbf{n}_j||_2 = 1$$

Lemma 2.2. *The matrix D with elements $D_{i,j}$ is of rank at most* 3.

Proof. The measurement equations are $D_{i,j} = z_i^T \mathbf{n}_j$. By setting

$$Z = \begin{pmatrix} z_1^T \\ z_2^T \\ \vdots \\ z_m^T \end{pmatrix}$$

and

$$N = \begin{pmatrix} n_1 & n_2 & \ldots & n_k \end{pmatrix}$$

we see that $D = ZN$. Both Z and N have at most rank 3, therfore the same holds for D. □

Assuming that k and m are large enough and assuming that the motion z_i and the base stations $\mathbf{n}_j$ are in general enough constellation the matrix D will have rank 3. If so it is possible to reconstruct both Z and N up to an unknown linear transformation. This can be done using singular value decomposition, $D = USV^T$.

Even with noisy measurements, the closest rank 3 approximation in the L_2 norm can be found using the first 3 columns of U and V. By setting $\tilde{Z} = U_3$ and $\tilde{N} = S_3 V_3^T$ we get all possible solutions by $N = A\tilde{N}$, with A a general full rank 3×3 matrix. Changing A corresponds to rotating, affinely stretching and possibly mirroring the coordinate system. The true reconstruction also fulfills $\mathbf{n}_j^T \mathbf{n}_j = 1$, which gives constraints on A of type

$$\mathbf{n}_j^T A^T A \mathbf{n}_j = 1,$$

which after substitution $B = A^T A$ becomes linear

$$\mathbf{n}_j^T B \mathbf{n}_j = 1$$

in the unknown elements of B. Since symmetric 3×3 matrices have 6 degrees of freedom we need at least 6 base stations to determine the matrix uniquely. Once B has been determined A can be determined by Cholesky factorization. This gives the transformation A up to an unknown rotation and possible mirroring of the coordinate system. We summarize the above in the following theorem.

Theorem 2.1. *The minimal case for reconstructing m positions z_i and k orientations $\mathbf{n}_j$ from relative distance measurements $D_{i,j}$ as formulated in Problem 2.2 is $m = 4$ and $k = 6$.*

Accordingly, we have the following algorithm for the minimal case of the problem:

Algorithm 2.1.
Given the measurement matrix D of size 4×6.

1. *Set $\bar{D}_{i,j} = D_{i,j} - D_{1,j}$*
2. *Remove the first row of $\bar{D}$*
3. *Calculate a singular value decomposition $\bar{D} = USV^T$.*
4. *Set $\tilde{Z}$ to first 3 columns of U and $\tilde{N}$ to first three columns of SV^T.*
5. *Solve for the six unknowns in the symmetric matrix B using the 6 linear constraints $\tilde{n}_j^T B \tilde{n}_j = 1$.*
6. *Calculate A by Cholesky factorization of B, so that $A^T A = B$.*
7. *Transform motion according to $Z = \tilde{Z} A^{-1}$ and structure according to $N = A\tilde{N}$.*

Note that using minimal information $m = 4$ and $k = 6$ results in estimates that fulfill the measurements exactly (up to machine precision) even if the measurements are disturbed by noise.

2.1 Failure modes of the algorithm

It is interesting and enlightening to know the failure modes of the algorithm. This is captured by the following theorem.

Theorem 2.2. *The minimal case for reconstructing m orientations $\mathbf{n}_j$ and k positions z_i from relative distance measurements $D_{i,j}$ as formulated in Problem 2.2 is for $m = 4$ and $k = 6$. As long as the orientations $\mathbf{n}_j$ do not lie on a common quadratic cone $\mathbf{n}_j^T \Omega \mathbf{n}_j = 0$ and the measurement positions z_i do not lie on a plane, there will not be more than one solution to the problem of determining both structure $\mathbf{n}_j$ and motion z_i up to an unknown translation, orientation and reflection of the coordinate system.*

Proof. The algorithm can fail if the measurement matrix D has rank 2 or lower. This could e.g. happen if either all measurement positions z_i lie in a plane or if all directions $\mathbf{n}_j$ lie in a plane (or both). The algorithm can also fail if there are two solutions to the matrix B in $\mathbf{n}_j^T B \mathbf{n}_j = 1$. But then the difference $\Omega = B_1 - B$ of these two solutions is a three by three matrix for which

$$\mathbf{n}_j^T \Omega \mathbf{n}_j = 0,$$

which in turn implies that the directions $\mathbf{n}_j$ lie on a common conic as represented by the matrix Ω. □

Yet another type of failure mode of the algorithm is if the data is corrupted by noise or far field approximation is not vaild, so that the matrix B obtained is not positive definite. Then the algorithms fails because there is no Cholesky factorization of B into $A^T A$. If B is unique, there are no real solution to the problem in this case.

2.2 Analysis of failure modes

If the rank of the matrix D is 2, this could be because the points z_i lie on a plane or that $\mathbf{n}_j$ lie on a plane.

In this case of coplanar z_i it is still possible to estimate the planar coordinates $Z = U_2 A$ and $N = A S_2 V_2^T$ up to an unknown 2×2 matrix A representing a choice of affine coordinate system. Here we do get inequality constraints that

$$\left| A \begin{pmatrix} n_{j,x} \\ n_{j,y} \end{pmatrix} \right| \leq 1.$$

Each such A is a potential solution. It is possible to extend with a third coordinate in the normal direction according to

$$n_{j,z} = \pm\sqrt{1 - n_{j,x}^2 - n_{j,y}^2}.$$

Another possibility is that the directions $\mathbf{n}_j$ lie on a plane. In this case it is possible to reconstruct two of the coordinates for both the positions z_i and the directions $\mathbf{n}_j$. Since the normals are assumed to lie in a plane, we can exploit the equality constraints $\mathbf{n}_j^T A^T A \mathbf{n}_j = 1$ similar to the rank 3 case. In this particular case we only need three directions $\mathbf{n}_j$, i.e. the minimal case is for $m = 3$ and $k = 3$. This gives the full reconstruction of both points and directions up to an unknown choice of Euclidean coordinate system and unknown choice of z-coordinate for the points z_i.

If the rank is 1, this could be because the directions are parallel. In this case. Similar to the discussions above we can obtain one of the coordinates of the positions z_i, but this is trivial since the measurements $D_{i,j}$ are such coordinates by definition.

If the rank is 1 because the points lie on a line, we obtain a one-parameter family of reconstructions based on $Z = U_1 a$ and $N = aS_1V_1^T$, where a is an unknown constant that has to fulfill $a \leq 1/l$, where $l = max_j |S_1 V_{1,j}|$. For each such a it is possible to extend the directions $\mathbf{n}_j$ so that they have length one, but there are several such choices.

2.3 Overdetermined Cases

When more measurements are available than the minimal case discussed in the previous section, we need to solve an overdetermined system in least-square sense or with robust error measures e.g. L_1-norm. Here we focus on the following least-square formulation for the pose problem:

Problem 2.3. *Given measurements $D_{i,j}$, $i = 1,\ldots,m$ and $j = 1,\ldots,k$ from the antenna at m different positions to k base stations, determine both the relative motion of the antenna $\mathbf{z}_i$ and the direction to the base stations $\mathbf{n}_j$ so that*

$$\begin{array}{ll} \min_{Z,N} & ||D - Z^T N||^2_{Frob} \\ s.t. & ||\mathbf{n}_j||_2 = 1,\ j = 1,\ldots,k. \end{array} \tag{1}$$

where $||.||_{Frob}$ denotes the Frobenius norm.

For the over-determined cases, that is $m > 4$ and $k \geq 6$ or $m \geq 4$ and $k > 6$, it is possible to modify Algorithm 2.1 to obtain an efficient but not necessarily optimal algorithm that finds a reconstruction that fits the data quite good using the following three modifications (i) the best rank 3 approximation can still be found in step 4-5 using the singular value decomposition, (ii) the estimate of B in step 6 can be performed in a least squares sense and (iii) re-normalize the columns of N to length 1. This results in a reconstruction that differs from the measurements, but both steps are relatively robust to noise. The problem of B not being positive semi-definite can be attacked by non-linear optimization. Here we try to optimize A so that $\sum_{j=1}^{k}(\mathbf{n}_j^T A^T A \mathbf{n}_j - 1)^2$ is minimized. This can be achieved e.g. by initializing with $A = I$ and then using non-linear optimization of the error function.

Clearly, we lose any guarantee on the optimality of the solution when we enforce the constraints as in step (iii). However, the solution can serve as a good initialization for subsequent optimization algorithms we present in this section. We discuss how to use alternating optimization and Levenberg-Marquardt algorithm (LMA) to obtain better solution. The first algorithm starts with an initial feasible solution for Z and N, and then it alternates between optimizing Z given N and vice versa. The latter is essentially a method combining Gauss-Newton algorithm and a gradient descent that improve the solution locally. For both methods, we need to treat the constraints on the direction vectors properly to ensure convergence.

2.3.1 Alternating Optimization

In order to find the local minima of Problem 2.3, we can use a coordinate descent scheme. Specifically, we would like to iteratively optimize the cost function in Problem 2.3 with respect to Z given N, and then find the optimal feasible N with fixed Z. If we initialize N such that it satisfies the norm constraints, we can easily see that the alternating procedure is converging (Algorithm 2.2).

Algorithm 2.2.
Given the measurement matrix D with $m > 4$ and $k \geq 6$ or $m \geq 4$ and $k > 6$,

1. *Construct $\bar{D}$ and initialize Z and N as in Algorithm 2.1*
2. *Fix N , find optimal Z*
3. *Fix Z, solve the constrained minimization for each $\mathbf{n}_j$, $j = 1, ..., k$*
4. *Repeat (2) and (3) until convergence or predefined number of iterations is reached*

To enable the alternating optimization, we need to solve two separate optimization problems. The first one is to find the optimal Z given N. This is the classic least squares problem and is known to be convex and can be solved efficiently. On the other hand, solving for optimal $\mathbf{n}_j$ given Z is not always convex due to the additional constraints on the $\mathbf{n}_j$'s. In this case, we seek the local minima for each $\mathbf{n}_j$ as a constrained minimization problem. We solve the small constrained problems (3 variables each) independently with interior point method. Alternatively, we can solve the constrained optimization as solving polynomial equations. This can be related to the fact that for a given Z, level sets of the cost function with respect to $\mathbf{n}_j$ are surfaces of a ellipsoid in $\mathbb{R}^3$ (the centers are in this case the solution from singular value decomposition). The norm 1 constraints on $\mathbf{n}_j$ geometrically means that the feasible solutions lie on the unit sphere centered at origin. Therefore, the optimal solution of $\mathbf{n}_j$ is one of the points that the ellipsoid is tangent to the unit sphere, which can be found by solving polynomial equations. While there could exist multiple solutions, we can choose the one with minimum euclidean distances to the center of the ellipsoid. Unlike interior point solver, we always find the global optimum. However, in practice, we found that in the alternating procedure, interior point method and polynomial solving give similar performance.

2.3.2 Levenberg-Marquardt Algorithm

It is well-known that alternating optimization as a coordinate descent scheme converges slowly in practice. Alternatively, we can solve the minimization problem by iteratively finding the best descent direction for N and Z simultaneously. The difficulty here is again the constraints on the direction vectors $\mathbf{n}_j$. The key idea here is to re-parameterize the orientation vectors. Given a direction vector $\mathbf{n}$ having unit length, any direction vectors can be represented by $\mathbf{n} \cdot exp(S)$, where S is a 3×3 skew-symmetric matrix. This is due to the fact that the exponential map of any such matrix is a rotation matrix. In this case, if we use the (current) orientation $\mathbf{n}$ as axis direction, any local change of the orientation on the sphere can be easily parameterized via the exponential map. Therefore, the gradient of D_{ij} with respect to $\mathbf{n}_j$ can be expressed without any constraints. We can then construct the Jacobian for the Levenberg-Marquardt algorithm to compute the optimal descent direction. In the following, we use $\mathbf{y}$ to denote the vector formed by stacking variables in Z and N, $\bar{\mathbf{d}}$ is the vectorized version of $\bar{D}$ based on the ordering of g.

Algorithm 2.3.
Given the measurement matrix D (over-determined), initialize $\mathbf{y}$ *and construct* $\bar{\mathbf{d}}$ *as in Algorithm 2.1,*

1. *Compute the Jacobian of* $\bar{\mathbf{d}}$ *with respect to* $\mathbf{y}$, $J = (\frac{\partial \bar{D}_{11}}{\partial \mathbf{y}}, \ldots, \frac{\partial \bar{D}_{ij}}{\partial \mathbf{y}}, \ldots, \frac{\partial \bar{D}_{(m-1)k}}{\partial \mathbf{y}})$
2. *Calculate* $\Delta\mathbf{y} = (J^T J + \lambda \cdot diag(J^T J))^{-1} J^T \Delta\bar{\mathbf{d}}$, *where* $\Delta\bar{\mathbf{d}}$ *is the residue and* λ *a damping factor.*
3. $\mathbf{y} = \mathbf{y} + \Delta\mathbf{y}$
4. *repeat (1),(2) and (3) until convergence or predefined number of iterations is reached*

3 EXPERIMENTAL VALIDATION

In this section, we present comprehensive experimental results for simulated data. We focus on the performance of the minimal solver, verification of the far field approximation as well as the comparisons between solvers for overdetermined cases.

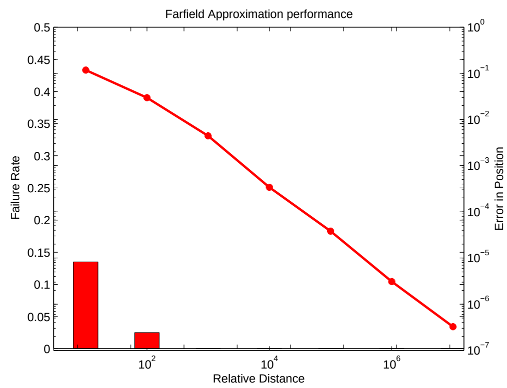

Figure 1: Performance on minimal case solver. Bars show failure rate (left y-axis) and curve shows the norm in estimated position as a function of distance (right y-axis). Note that bar height has linear scale.

3.1 Minimal Solver Accuracy

The numerical performance of the algorithm was evaluated by generating problems where the far field approximation is true and not degenerate. In essence this constitutes generating directions $\mathbf{n}_j$ and relative distance measurements $D_{i,j}$ and culling cases where the three largest singular values of the measurement matrix aren't above a threshold or the directions lie on a conic. The error is then evaluated as the average norm-difference of the estimated reciever positions. The reciever positions were selected as the corners of a tetrahedron with arc-length one. The average error of 10000 such tests was $6.8 \cdot 10^{-15}$, close to machine epsilon.

3.2 Far Field Approximation Accuracy

3.2.1 Minimal Case

To evaluate the performance of the assumption that senders can be viewed as having a single common direction to receivers, data was generated using 3D positions for both senders and receivers at different relative distances in-between receivers and senders to receivers. The constellation of receivers is again the tetrahedron and senders are randomly placed on a sphere surrounding it. A graph showing the error, as defined in section 3.1, as a function of radius of the sphere (that is relative distance), as well as the failure rate of the solver is shown in Figure 1. A failure constitutes a case in which the B matrix in algorithm 2.1 is not positive definite. As can be seen this is infrequent

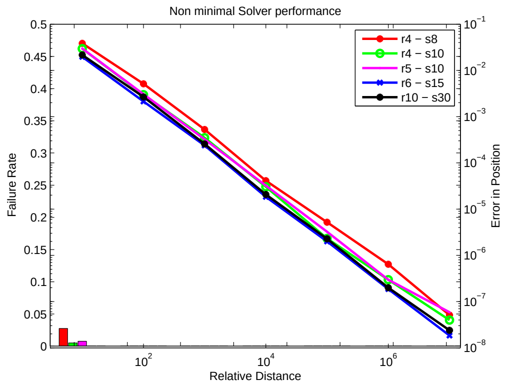

Figure 2: Performance on non-minimal cases. Bars show failure rates, line is error as a function of distance in loglog scale. Size of test-cases are noted in figure with rx-sy denoting x recievers and y senders. This plot is best viewed in color. Note the scale difference to the graph in Figure 1.

even at small relative distances in when one would not expect a far field approximation to work. As can be expected the approximation gets better when the relative distance increases.

3.2.2 Initialization for Overdetermined Cases

As described in section 2.3 algorithm 2.1 can with some modifications be used on overdetermined cases without guarantees on optimality of the solution. In these situations the solutions serves as an initial guess of some other optimization method. The additional information should however give some numerical stability and it is interesting to evaluate the algorithm for initial guess estimates in overdetermined cases. To do this the synthetic dataset is augmented with additional randomly placed senders and receivers. The four first receivers are again the tetrahedron and the rest are randomly uniformly distributed within the unit cube. Senders are again placed on a sphere around the receivers. Results for different problem sizes are shown in Figure 2. One immediately notices that the failure ratio drops, in many cases to zero. One can also see that adding more data will (in general) result in smaller errors, for the cases shown here up to one order of magnitude smaller than a min case.

3.3 Overdetermined Cases

We also investigate the performance of the two schemes for over-determined cases. In all experiments below, we initialize both the alternating optimization and LMA based on the minimal solver modified for over-determined case. The simulated data is of a true far field approximation with gaussian white noise, i.e. measurements are simulated as

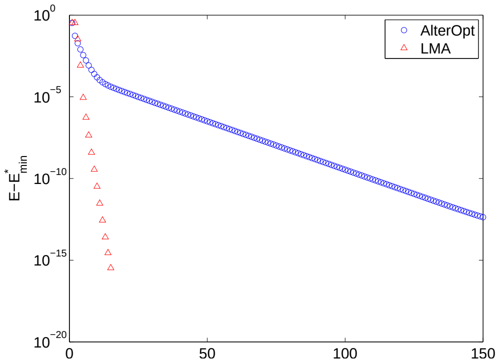

Figure 3: Convergence of alternating optimization and LMA on simulated TDOA measurements with gaussian white noise ($\sigma = 0.1$). Here $m = 10$ and $k = 10$.

$D_{i,j} = \mathbf{z}_i \mathbf{n}_j + \varepsilon_{i,j}$ where $\varepsilon_{i,j} \in N(0, \sigma)$ i.i.d. In Figure 3, we can see that alternating optimization and LMA all decrease the reconstruction errors compared to the minimal solver. On the other hand, from figure 3, LMA converges much faster than alternating scheme (20 vs. 150) and obtains relatively lower reconstruction errors. This verifies the superiority of LMA over coordinate descent. This observation is consistent over different m and k as well as a variety of noise levels. Note that here for all the experiments, we set the damping factor λ to 1.

It it also of interest to view the complete system when the measurements $D_{i,j}$ does not fulfill the far field approximation and when disturbed by noise. The relative distances of the simulated senders and receivers are set to 10^2 for a mediocre far field approximation and 10^7 for a good far field approximation. TDOA measurements $D_{i,j}$ are then simulated, perturbed with gaussian white noise. Figure 4 shows the results. The pictures show that the initialization method is fairly good, but in many cases the LMA brings down the position error. The system is also fairly robust to noise.

4 CONCLUSIONS

In this paper we study the far field approximation to the calibration of TDOA and TOA sensor networks. The far field approximation of the problem results in a factorization algorithm with constraints. The failure modes of the algorithm is studied and particular emphasis is made on what can be said when any of these failure conditions are met. The experimental validation gives a strong indication that a far field approximation is a feasible approach both for getting direct estimates as well as initial estimates for other solvers. Even considering that there are cases when the algo-

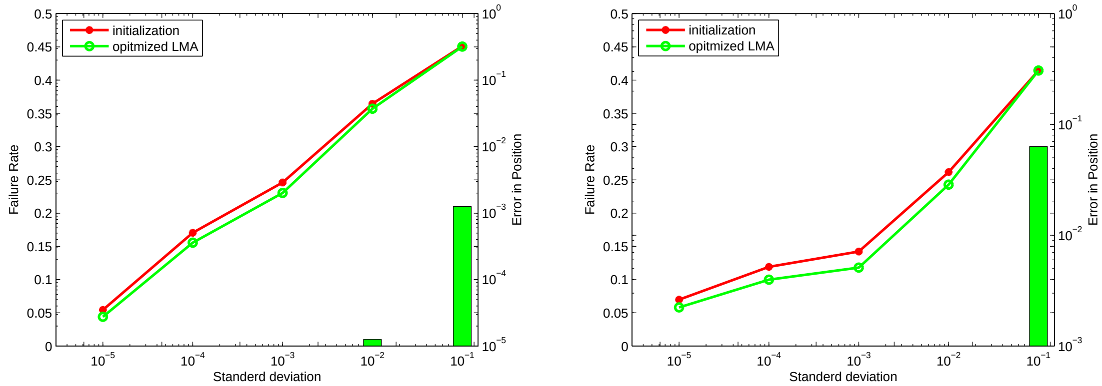

Figure 4: Performance on non-minimal cases with simulated TDOA measurements with gaussian white noise. The mean error in position of the receivers are plotted against the noise standard deviation. Here m=5 and k=10, and the relative distance to receivers and senders are 10^7 (left) and 10^2 (right). Failure rates for the initialization are also shown for completeness.

rithm fails, obtained solutions are good even at small relative distances. This validation is done on 3D problems and confirms findings in (Thrun, 2005) where evaluation was done in 2D.

Further we analyze two optimization schemes and what difficulties may arise when employing them. Both of these schemes are experimentally evaluated and confirmed to successfully optimize the initial guess on a problem fulfilling the far field assumptions, although at quite different convergence rates. The faster of the two is also employed on cases when senders are given true locations and measurements are subject to noise with good results.

It would be interesting in future work to study to what extent it can be shown that the local optimum obtained to the problem can be proven to be global optimum. To integrate the solvers with robust norms is also worth studying to handle situations with outliers. It would also be interesting to verify the algorithms on real measured data and investigate the possibilities of using our algorithms in a RANSAC approach to remove potential outliers that may occur in real life settings.

ACKNOWLEDGEMENTS

The research leading to these results has received funding from the Swedish strategic research projects ELLIIT and ESSENSE, the Swedish research council project Polynomial Equations and the Swedish Strategic Foundation projects ENGROSS and Wearable Visual Systems.

REFERENCES

Birchfield, S. T. and Subramanya, A. (2005). Microphone array position calibration by basis-point classical multidimensional scaling. *IEEE transactions on Speech and Audio Processing*, 13(5).

Biswas, R. and Thrun, S. (2004). A passive approach to sensor network localization. In *IROS 2004*.

Chen, J. C., Hudson, R. E., and Yao, K. (2002). Maximum likelihood source localization and unknown sensor location estimation for wideband signals in the near-field. *IEEE transactions on Signal Processing*, 50.

Elnahrawy, E., Li, X., and Martin, R. (2004). The limits of localization using signal strength. In *SECON-04*.

Niculescu, D. and Nath, B. (2001). Ad hoc positioning system (aps). In *GLOBECOM-01*.

Pollefeys, M. and Nister, D. (2008). Direct computation of sound and microphone locations from time-difference-of-arrival data. In *Proc. of International Conference on Acoustics, Speech and Signal Processing*.

Raykar, V. C., Kozintsev, I. V., and Lienhart, R. (2005). Position calibration of microphones and loudspeakers in distributed computing platforms. *IEEE transactions on Speech and Audio Processing*, 13(1).

Sallai, J., Balogh, G., Maroti, M., and Ledeczi, A. (2004). Acoustic ranging in resource-constrained sensor networks. In *eCOTS-04*.

Stewénius, H. (2005). *Gröbner Basis Methods for Minimal Problems in Computer Vision*. PhD thesis, Lund University.

Thrun, S. (2005). Affine structure from sound. In *Proceedings of Conference on Neural Information Processing Systems (NIPS)*, Cambridge, MA. MIT Press.