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Next-to-leading order mass effects in QCD Compton process of polarized DIS

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Abstract. The method originally developed for the exact calculations in QED theory is applied for the calculation of NLO effects in QCD Compton processes. QCD corrections to the structure functions and sum rules are obtained. Different interpretations of the NLO effects due to finite quark mass are discussed.

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1 Introduction

A lot of articles dedicated to NLO QCD corrections to polarized DIS have been published. Calculations in most of them are performed in an assumption that the quark mass is equal to zero (see [1, 2, 3, 4, 5, 6, 7] and review [8]). However there are some works when authors estimate a finite quark mass effect for both unpolarized [9] and polarized structure functions [10, 11, 12, 13, 14, 15, 16]. The main difference of these two approaches is the method of tending the quark mass to zero and the procedure of integration of the squared matrix element over the phase space of emitted gluon. In the massless approsch a fermion mass is equal to zero *before* the integration while in the massive one this quantity goes to zero *after* the integration and survives only in LO terms.

Notice that QCD and QED radiative corrections (RC) having different origins possess some common features on the one-loop level. If our consideration is restricted to so-called QCD Compton process then both corrections should be described by the identical sets of Feynman graphs. The transition from the strong radiative effects to the electromagnetic ones could be performed by the following replacement:

$$\frac{4}{3}\alpha_s \rightarrow e_q^2 \alpha_{QED}. \tag{1}$$

In the present paper we investigate QCD Compton process for polarized nucleon target by the method developed for exact calculations [17, 18, 19, 20, 21, 22] in QED theory for massive polarized fermions but never used for QCD.

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We estimate the influence of the finite quark mass effects on the value of the polarized sum rules and discuss some different interpretations of the first moment of the polarized structure function g_1.

The present paper is organized in the following way. The method of calculation is described in Section 2. The explicite expressions for the QCD-improved structure functions as well as their first moments can be found in Section 3. Then some conclusions are made in Section 4. We also discuss there different points of view how the NLO finite quark mass effects influence the first moment of g_1. Details of the integration procedure are presented in Appendix.

2 Method of Calculation.

The cross section of polarized lepton-nucleon DIS

$$l(k_1,\xi) + N(p,\eta) \rightarrow l(k_2) + X \tag{2}$$

on the level of the one-photon exchange can be presented as a convolution of the well-known lepton tensor:

$$L_{\mu\nu} = \frac{1}{2} Sp\, \gamma_\mu (\hat{k}_1 + m)(1+\gamma_5\hat{\xi})\gamma_\nu(\hat{k}_2 + m), \tag{3}$$

and the hadronic one:

$$\begin{aligned} W_{\mu\nu} = \quad & -g_{\mu\nu}F_1 + \frac{p_\mu p_\nu}{pq}F_2 \\ & + \frac{iM}{pq}\epsilon_{\mu\nu\lambda\sigma}q^\lambda(\eta^\sigma g_1 + (\eta^\sigma - \frac{\eta q}{pq}p^\sigma)g_2). \end{aligned} \tag{4}$$

Here p and η are an initial 4-momentum of the target and its polarization vector respectively and q is a 4-momentum of the virtual photon ($p^2 = M^2$, $q^2 = -Q^2$).

On the Born level in the frame of the naive parton model the structure functions contributed to the hadronic tensor $W^0_{\mu\nu}$ have the following form:

$$\begin{aligned} F_1^0(x) = \quad & \frac{1}{2}\sum_q e_q^2 f_q(x), \\ F_2^0(x) = \quad & x\sum_q e_q^2 f_q(x), \\ g_1^0(x) = \quad & \frac{1}{2}\sum_q e_q^2 \Delta f_q(x), \\ g_2^0(x) = \quad & 0. \end{aligned} \tag{5}$$

Here $f_q(x)$ and $\Delta f_q(x)$ are unpolarized and polarized parton distributions respectively[1], and $x = Q^2/2pq$ is the standard Bjorken variable.

[1]Here and later the independence of the polarized parton distribution on the polarization vector is assumed. See discussion in [5].

The one-loop lowest-order RC to the hadronic current consists of two parts with contributions calculated in different ways:

$$W_{\mu\nu}^{1-loop} = W_{\mu\nu}^{V} + W_{\mu\nu}^{R}. \tag{6}$$

The first part of the one-loop RC to the hadronic current comes from the gluon exchange and it will be described later. The second one corresponds to the gluon emission and requires the integration over its phase space:

$$\begin{aligned} W_{\mu\nu}^{R} &= \frac{\alpha_s}{12\pi^2}\sum_q e_q^2 \int \frac{d^3k}{k_0}\frac{1}{(p_{2q}-p_{1q})^2} \\ &\quad \times [w_{\mu\nu}^{Ru} f_q(x/z) + w_{\mu\nu}^{Rp}\Delta f_q(x/z)], \end{aligned} \tag{7}$$

where

$$\begin{aligned} w_{\mu\nu}^{Ru} &= Sp\, \Gamma_{\mu\alpha}(\hat{p}_{1q}+m_q)\bar{\Gamma}_{\alpha\nu}(\hat{p}_{2q}+m_q), \\ w_{\mu\nu}^{Rp} &= Sp\, \Gamma_{\mu\alpha}(\hat{p}_{1q}+m_q)\gamma_5\hat{\eta}\bar{\Gamma}_{\alpha\nu}(\hat{p}_{2q}+m_q), \end{aligned} \tag{8}$$

and p_{1q} (p_{2q}) is an initial (final) 4-momentum of the quark ($p_{1q}^2 = p_{2q}^2 = m_q^2$).

Quantities $\Gamma_{\mu\alpha}$ and $\bar{\Gamma}_{\alpha\nu}$ are defined as

$$\begin{aligned} \Gamma_{\mu\alpha} &= 2\Omega_q^{\alpha}\gamma_\mu - \frac{\gamma_\mu\hat{k}\gamma_\alpha}{v_q} - \frac{\gamma_\alpha\hat{k}\gamma_\mu}{u_q}, \\ \bar{\Gamma}_{\alpha\nu} &= 2\Omega_q^{\alpha}\gamma_\nu - \frac{\gamma_\alpha\hat{k}\gamma_\nu}{v_q} - \frac{\gamma_\nu\hat{k}\gamma_\alpha}{u_q}, \end{aligned} \tag{9}$$

where

$$\Omega_q = \frac{p_{1q}}{v_q} - \frac{p_{2q}}{u_q}, \tag{10}$$

and $v_q = 2kp_{1q}$, $u_q = 2kp_{2q}$. A variable $z = Q^2/(Q^2+u_q)$ is a standard one used for the description of QCD effects.

Within the requirements of the naive parton model in (7) we assume that the quark mass depends on the variable z as

$$m_q = x/zM \tag{11}$$

for bremsstrahlung process and

$$m_0 = m_q(z=1) = xM \tag{12}$$

for non-radiative one. Notice that the factor before M in these formulae coincides with the argument of the corresponding parton distribution.

Both contributions in right hand side in (6) include the infrared divergences to be carefully considered for canceling. Similar to QED we use an identity:

$$W_{\mu\nu}^{R} = W_{\mu\nu}^{R} - W_{\mu\nu}^{IR} + W_{\mu\nu}^{IR} = W_{\mu\nu}^{F} + W_{\mu\nu}^{IR}. \tag{13}$$

Here an infrared part can be written in the following way:

$$W_{\mu\nu}^{IR} = \frac{4}{3}\frac{\alpha_s}{\pi}\sum_q \delta^{IR} W_{\mu\nu}^{0q}, \tag{14}$$

where $W^{0q}_{\mu\nu}$ is a contribution of q-quark to the hadronic tensor on the Born level and δ^{IR} should be decomposed in a sum of a soft and hard parts:

$$\delta^{IR} = \frac{1}{\pi}\int\frac{d^3k}{k_0}F^{IR} = \frac{1}{\pi}\int\frac{d^3k}{k_0}F^{IR}\theta(\epsilon - k_0) + \frac{1}{\pi}\int\frac{d^3k}{k_0}F^{IR}\theta(k_0 - \epsilon) = \delta^{IR}_{soft} + \delta^{IR}_{hard}. \tag{15}$$

Here k_0 is the energy of the emitted gluon, ϵ is an infinitesimal parameter of the separation and

$$F^{IR} = \frac{(Q^2 + u_q)}{(Q^2 + u_q - v_q)}\left(\frac{Q^2 + u_q}{u_q v_q} - \frac{m_q^2}{u_q^2} - \frac{m_q^2}{v_q^2}\right). \tag{16}$$

Notice that there is some arbitrariness in the definition of F^{IR}: the asymptotic expression ($k \to 0$) is fixed. We choose it like (16) to write the QCD-improved structure functions (see formulae (27)) in the simplest form. Recall that the quark mass in (16) is defined by (11).

The way of calculation of typical integrals in the soft and hard parts of δ^{IR} is widely discussed in [21, 22]. After explicit integration the sum of the soft

$$\delta^{IR}_{soft} = 2(l_0 - 1)\left(P^{IR} + \log\frac{2\epsilon}{\mu}\right) - l_0^2 + l_0 + 1 - \pi^2/6 \tag{17}$$

and the hard parts

$$\delta^{IR}_{hard} = 2(l_0 - 1)\log\frac{m_0}{2\epsilon} + \frac{3}{2}l_0^2 + l_0 l_v - \frac{1}{2}l_v^2 - l_0 - l_v - \pi^2/6 \tag{18}$$

gives the result independent on the parameter ϵ. Here

$$l_0 = \log\frac{Q^2}{m_0^2}, \qquad l_v = \log\frac{1-x}{x}. \tag{19}$$

The pole term which corresponds to the infrared divergence is contained in

$$P^{IR} = \frac{1}{n-4} + \frac{1}{2}\gamma_E + \log\frac{1}{2\sqrt{\pi}}. \tag{20}$$

The arbitrary parameter μ has a dimension of a mass and γ_E is the Euler constant.

To extract some information about QCD contributions to the polarized structure functions g_1 and g_2 an integration in $W^F_{\mu\nu}$ over the gluon phase space should be performed without any assumptions about the polarization vector η. So the technique of tensor integration has to be applied in this case. All exact expressions for tensor, vector and scalar integrations can be found in the appendix. The ultrarelativistic limit $m_q \to 0$ can be performed in a straightforward way, with the exception of two terms: $f(x)/\tau$, $u_q f(x/z)/\tau^2$. The first one including $1/\tau$ appears only in the substracting parts of $W^F_{\mu\nu}$. Since the corresponding parton distributions do not depend on u_q it can be integrated over u_q analytically.

The second one u_q/τ^2 comes from $W^R_{\nu\mu}$. The ultrarelativistic limit in $W^R_{\nu\mu}$ can be applied after the transformation

$$\int du_q \frac{u_q}{\tau^2} f(x/z) = \int du_q \frac{f(x/z) - f(x)}{u_q} + f(x) \int du_q \frac{u_q}{\tau^2} \tag{21}$$

and the analytical integration of the last term. Here f is any (either unpolarized or polarized) parton distribution. The first term in the right hand side of (21) gives the contribution to $W^F_{\mu\nu}$. The second one together with the result coming from the integration $1/\tau$ over u_q gives an additional factorized correction δ^m:

$$\delta^m = -\frac{1}{4}(3l_0 + 3l_v + 1). \tag{22}$$

An explicit expression for the gluon exchange contributions reads:

$$W^V_{\mu\nu} = \frac{4}{3}\frac{\alpha_s}{\pi} \sum_q \delta^V W^{0q}_{\mu\nu} + W^{AMM}_{\mu\nu}, \tag{23}$$

where

$$\delta^V = -2\left(P^{IR} + \log\frac{m_0}{\mu}\right)(l_0 - 1) - \frac{1}{2}l_0^2 + \frac{3}{2}l_0 - 2 + \frac{\pi^2}{6} \tag{24}$$

is a vertex correction part factorizing before $W^{0q}_{\mu\nu}$, and

$$W^{AMM}_{\mu\nu} = \frac{\alpha_s}{6\pi} l_0 \sum_q \frac{iM}{pq} \epsilon_{\mu\nu\lambda\sigma} q^\lambda \left[\eta^\sigma - \frac{\eta q}{pq} p^\sigma\right] \sum_q \Delta f_q(x) \tag{25}$$

is the quark anomalous magnetic moment.

The infrared terms P^{IR}, parameters μ, ϵ and the squared logarithms containing mass singularities are canceled in the sum

$$\begin{aligned} \delta^{IR} + \delta^m + \delta^V = & \quad \frac{1}{2}\left(2l_0 l_v - l_v^2 + \frac{3}{2}l_0 - \frac{7}{2}l_v - \frac{5}{2} - \frac{\pi^2}{3}\right) \\ = & \quad \frac{1}{2}\delta_q \end{aligned} \tag{26}$$

that is identical to (C.17) from [23].

Since the result of the analytical integration has the same tensor structure as the usual hadronic tensor in polarized DIS, the coefficients before the corresponding tensor structures (like $g_{\mu\nu}$, $p_\mu p_\nu$ ...) in the sum $W^V_{\mu\nu} + W^R_{\mu\nu}$ can be interpreted as one-loop QCD contributions to the corresponding structure functions.

3 QCD-improved structure functions

In the last section we obtained one-loop correction to the hadronic tensor and structure functions. The explicit result for them is:

$$F_1(x, Q^2) = \frac{1}{2x}[F_2(x, Q^2) - F_L(x, Q^2)],$$

$$
\begin{aligned}
F_2(x,Q^2) = \quad & x\sum_q e_q^2 f_q(x,Q^2),\\
F_L(x,Q^2) = \quad & \frac{4\alpha_s}{3\pi} x \sum_q e_q^2 \int\limits_x^1 dz f_q(x/z),\\
g_1(x,Q^2) = \quad & \frac{1}{2}\sum_q e_q^2 \Delta f_q(x,Q^2),\\
g_2(x,Q^2) = \quad & \frac{\alpha_s}{6\pi}\sum_q e_q^2 \Big\{(1-2l_q-\log(1-x))\Delta f_q(x)\\
& + \int\limits_x^1 dz \Big[(4l_q - 4\log z(1-z) - 12\\
& - \frac{1}{(1-z)})\Delta f_q(x/z) + \frac{\Delta f_q(x)}{(1-z)}\Big]\Big\}. \qquad (27)
\end{aligned}
$$

The Q^2-dependent unpolarized and polarized parton distributions are defined as

$$
\begin{aligned}
f_q(x,Q^2) = \quad & (1+\frac{2\alpha_s}{3\pi}\delta_q)f_q(x) + \frac{2\alpha_s}{3\pi}\int\limits_x^1 \frac{dz}{z}\Big[\Big(\frac{1+z^2}{1-z}\\
& \times(l_q - \log z(1-z)) - \frac{7}{2}\frac{1}{1-z} + 3z + 4\Big) f_q(\frac{x}{z})\\
& - \frac{2}{1-z}\left(l_0 + \log\frac{z}{1-z} - \frac{7}{4}\right) f_q(x)\Big],
\end{aligned}
$$

$$
\begin{aligned}
\Delta f_q(x,Q^2) = \quad & (1+\frac{2\alpha_s}{3\pi}\delta_q)\Delta f_q(x) + \frac{2\alpha_s}{3\pi}\int\limits_x^1 \frac{dz}{z}\Big[(\frac{1+z^2}{1-z}\\
& \times(l_q - \log z(1-z)) - \frac{7}{2}\frac{1}{1-z} + 4z + 1\Big)\Delta f_q(\frac{x}{z})\\
& - \frac{2}{1-z}\left(l_0 + \log\frac{z}{1-z} - \frac{7}{4}\right) \Delta f_q(x)\Big], \qquad (28)
\end{aligned}
$$

where $l_q = \log Q^2/m_q^2$ and δ_q is defined by (26).

Notice that our formulae (27,28) are in an agreement with ones obtained earlier. The unpolarized structure functions coincide with (2.24,2.49) from [9], $g_1(x,Q^2)$ corresponds to the expression (13) in [10]. At last $g_2(x,Q^2)$ can be compared with the sum (18) and (19) from [11].

Let us calculate the difference between QCD-improved parton distributions found in both of the discussed schemes:

$$
\begin{aligned}
\frac{3\pi}{4\alpha_s}(f_q^{m_q=0}(x,Q^2) - f_q^{m_q\neq 0}(x,Q^2)) = \Big[(P^{IR}\\
+ \log\frac{m_0}{\mu})(\frac{3}{2}+2l_v) - l_v^2 - l_v + 1\Big] f_q(x)
\end{aligned}
$$

$$+\int_x^1 \frac{dz}{z}\Big[\Big(\frac{1+z^2}{1-z}(P^{IR}+\log\frac{m_q}{\mu}+\log(1-z))$$
$$+\frac{1}{1-z}-\frac{1}{2}z-\frac{1}{2}\Big)f_q(\frac{x}{z})$$
$$-\frac{2}{1-z}\Big(P^{IR}+\log\frac{m_0}{\mu}+\log\frac{1-z}{z}+\frac{1}{2}\Big)f_q(x)\Big],$$
$$\frac{3\pi}{4\alpha_s}(\Delta f_q^{m_q=0}(x,Q^2)-\Delta f_q^{m_q\neq 0}(x,Q^2))=\Big[(P^{IR}$$
$$+\log\frac{m_0}{\mu})(\frac{3}{2}+2l_v)-l_v^2-l_v+1\Big]\Delta f_q(x)$$
$$+\int_x^1 \frac{dz}{z}\Big[\Big(\frac{1+z^2}{1-z}(P^{IR}+\log\frac{m_q}{\mu}+\log(1-z))$$
$$+\frac{1}{1-z}-\frac{3}{2}z+\frac{1}{2}\Big)\Delta f_q(\frac{x}{z})$$
$$-\frac{2}{1-z}\Big(P^{IR}+\log\frac{m_0}{\mu}+\log\frac{1-z}{z}+\frac{1}{2}\Big)\Delta f_q(x)\Big]. \qquad (29)$$

It can be seen from (29) that the partonic distributions within the two schemes have the same structure and all leading logarithmic terms are canceled after the following replacement:

$$\log m_{q,0}\to\log\mu-P^{IR}. \qquad (30)$$

The standard QCD results include infinite terms P^{IR} (see ref.[6]) that corresponds to quark mass singularity terms in our formulae. These divergent and scheme dependent terms in the classical results are rejected by the additional procedure of the renormalization of the parton distributions (see ref.[8] for details). Contrary, our formulae (28) being dependent on $\log m_{q,0}$ do not require any renormalization. The main origin of differences of the non-leading logarithmic terms in these formulae comes from the calculation procedure itself since the integration over the emission gluon phase space and tending of the quark mass to zero are not commutative with each other.

In order to find one-loop QCD contributions to the sum rules it is necessary to integrate (27) over the scaling variable x. After an identical recombination like

$$\int_0^1 dx\int_x^1 dzR(z)f_q(x/z)=\int_0^1 dzzR(z)\int_0^1 dxf_q(x) \qquad (31)$$

an explicit integration over z gives

$$\int_0^1 dxF_1(x,Q^2)=\Big(1-\frac{2}{3}\frac{\alpha_s}{\pi}\Big)\int_0^1 dxF_1^0(x),$$
$$\int_0^1 \frac{dx}{x}F_2(x,Q^2)=\int_0^1 \frac{dx}{x}F_2^0(x),$$

$$\int_0^1 dx g_1(x,Q^2) = \Big(1 - \frac{5}{3}\frac{\alpha_s}{\pi}\Big)\int_0^1 dx g_1^0(x),$$
$$\int_0^1 dx g_2(x,Q^2) = 0, \qquad (32)$$

where the structure functions with index "0" are defined by (5).

Notice that QCD RC to the first moment to the unpolarized structure functions as well as g_2 have identical values for both massive and massless quark approaches. At the same time these two methods give the different results for the QCD corrections to the first moment of g_1.

Till present time the calculations have been performed within the naive parton model when the quark mass is defined as in (11, 12). Another possibility is to consider it as a constant like it was assumed in [24]. An additional factor can depend only on z and x. So the result for the structure functions would differ only in an argument of logarithm containing the quark mass. An additional contribution to the sum rules can appear after integration over x and z. However it could be shown by the explicit calculations that this contribution is equal to zero. Indeed an additional integrand has DGLAP-like structure, so this cancellation is consequence of DGLAP equations. Thus the final results for sum rules in these two cases are identical.

4 Discussion and Conclusion

In this paper we applied the approach traditionally used in QED and electroweak theory for calculation of QCD correction to the DIS structure functions and sum rules. Since the quark was considered massive within this approach, it allowed us to estimate the finite quark mass effects at the NLO level. LO correction contributes to the DIS structure functions but vanishes for the sum rules, so this mass effects are important just for the sum rules.

We found that there is no any additional effect for the first moment of the unpolarized structure functions as well as for g_2. However there is some non-zero correction to the first moment of g_1 and as a result to the Ellis-Jaffe and Bjorken sum rules. A value of the correction is in agreement with the results of refs.[10, 13], obtained by different methods. We confirm also the statement of [10] that the classical value of correction to the first moment of g_1 is reproduced if we take into account the leading term of the polarization vector $\eta = p_{1q}/m_q$. However contrary to the paper we would interpret the result with (-5/3) in eqs.(32) as physical one. In our calculation we took the proton (quark) polarization vector in a general form. Using of the exact representation of the vector [18] (k_1 is an initial lepton momentum and $k_1^2 = m_l^2$):

$$\eta = \frac{1}{\sqrt{(k_1 p_{1q})^2 - m_l^2 m_q^2}}\Big[\frac{k_1 p_{1q}}{m_q} p_{1q} - m_q k_1\Big] \qquad (33)$$

leads to exactly the same result (with -5/3) for the first moment of g_1. The second term giving the additional contribution (-2/3) cannot be neglected within the approximation under consideration. It can be easyly understood from pure calculation of QED and electroweak corrections to the lepton current in DIS process [20, 25], where the lepton polarization vector looks like (33), and this additional contribution was extracted to separate non-vanishing contribution. The result with (-5/3) was obtained under similar assumptions in the report [12]. The additional correction was discussed both within OPE (operator product expansion) and within improved quark-parton model in paper [13]. It was shown in this paper that there is no contradiction between this result and classical correction obtained for massless QCD, if we carefully take into account the finite quark mass effects for coefficient function and matrix element. As it was reviewed in recent paper [15] the renormalization of the axial-vector current completely suppresses the additional contribution obtained within massive approach (see section 4 of the paper).

We note that Burkhard-Cottingham sum rule is held withing taking into account mass effects at the NLO level. The target mass correction of the next order ($\sim m_q^2/Q^2$) was analysed and was found negative in the paper [14]. However, in the case of conserved currents (see [16]) the polarized structure function g_2 with the target mass correction obeys not only Burkhard-Cottingham sum rule but Wandzura-Wilczek relation too.

Estimating the diagrams with one gluon radiation we have also result for QED radiative correction to hadronic current. The calculation within the quark parton model shows that QED correction is not so large, however there are at least two arguments to be taken into account. First, the DIS structure functions are defined in the one-photon exchange approximation as an objects including only strong interaction. It means that transferring from the cross section to the structure functions we neglect these QED effects, so their contributions have to be included to systematic error of corresponding measurements. Second reason is that there exist some measurements where QED correction is important. One of the examples is calculation of α_s by comparing theoretical and experimental values for the Bjorken sum rule [26]. In this case α_s is suggested to be extracted from QCD corrections to the Bjorken sum rule, which is a series over α_s, and at least five terms of the expansion are known. Simple estimation shows that QED contribution is comparable with fourth (or even third) term of the expansion.

There are several papers devoted to calculation of QED and electroweak corrections to the hadronic current [24, 25, 27]. As usual methods similar to ours are used for that. Positive moment here is the keeping quark mass nonzero. From the other side there are some effects which are not considered within QED calculations: taking into account the confiment effect in integration of soft region, consideration of the quarks as non-free particles. Thus the methods (see for example [12]) developed for careful treatment of the QCD effects should be applied for the analysis of photon emission from the hadronic current. However, probably the best way to take into account the photon radiative correction to the hadronic current is further generalization of OPE technique to include QED effects.

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Appendix

The gluon emission phase space on the parton level can be written in a covariant form:

$$\frac{d^3k}{k_0} = \frac{(Q^2 + u_q - v_q)}{2(Q^2 + u_q)} \frac{du_q dv_q dz_1}{\sqrt{-\Delta(k_1, k_2, p_{1q}, p_{2q})}}. \tag{A.1}$$

Here $v_q = 2kp_{1q}$, $u_q = 2kp_{2q}$, $z_1 = 2kk_1$, $\Delta(k_1, k_2, p_{1q}, p_{2q})$ is Gram determinant, $\lambda_q = (Q^2 + u_q)^2 + 4m_q^2 Q^2$ and p_{1q} (p_{2q}) is an initial (final) quark momentum.

Since the gluon is radiated from the quark legs it is clear that the matrix element squared for this process does not depend on z_1. Therefore an analytical integration of the right part of (A.1) over z_1 can be performed

$$\int\limits_{z_1^{min}}^{z_1^{max}} \frac{dz_1}{2\sqrt{-\Delta(k_1, k_2, p_{1q}, p_{2q})}} = \frac{\pi}{\sqrt{\lambda_q}}, \tag{A.2}$$

where the limits $z_1^{max/min}$ are defined as the solutions of the equation

$$\Delta(k_1, k_2, p_{1q}, p_{2q}) = 0. \tag{A.3}$$

The following expression

$$J[A] = \int\limits_{v_q^{min}}^{v_q^{max}} dv_q A \tag{A.4}$$

for the integration over v_q will be used in this appendix. Explicit expressions for the limits of integration which can be obtained as the solution of the equation $z_1^{max} = z_1^{min}$ read:

$$v_q^{max/min} = \frac{u_q(Q^2 + u_q + 2m_q^2 \pm \sqrt{\lambda_q})}{2\tau}, \tag{A.5}$$

where $\tau = u_q + m_q^2$.

Tensor and vector integrals can be reduced to scalar ones using the following expressions:

$$\begin{aligned} J\Big[k_\mu k_\nu A\Big] = &\quad \frac{g_{\mu\nu}}{2\lambda_q} J\Big[T_1 A\Big] + \frac{x^2 p_\mu p_\nu}{\lambda_q^2} J\Big[T_2 A\Big] \\ &+ \frac{x(p_\mu q_\nu + q_\mu p_\nu)}{\lambda_q^2} J\Big[T_3 A\Big] + \frac{q_\mu q_\nu}{\lambda_q^2} J\Big[T_4 A\Big], \\ J\Big[k_\mu A\Big] = &\quad \frac{x p_\mu}{\lambda_q} J\Big[V_2 A\Big] + \frac{q_\mu}{\lambda_q} J\Big[V_2 A\Big]. \end{aligned} \tag{A.6}$$

The coefficients T_i and V_i are defined as the solution of the systems of equations obtained by the convolutions (A.6) with p_{1q}, q and $g_{\mu\nu}$:

$$\begin{aligned}
T_1 = \quad & (u_q - v_q)^2 m_q^2 + u_q v_q (v_q - u_q - Q^2), \\
T_2 = \quad & ((u_q + v_q)^2 + 2u_q v_q) Q^4 + 2(u_q + 2v_q) Q^2 u_q \\
& + (u_q^2 - 2m_q^2 Q^2)(u_q - v_q)^2, \\
T_3 = \quad & (4u_q - v_q) Q^2 u_q v_q + (2u_q + v_q) Q^3 v_q \\
& - m_q^2 (u_q - v_q)(3u_q(Q^2 + u_q - v_q) + Q^2 v_q), \\
T_4 = \quad & (Q^2 + u_q)^2 v_q^2 + 6(u_q - v_q)^2 m_q^4 \\
& - 2m_q^2 v_q [(Q^2 + u_q)(3u_q - 2v_q) - u_q v_q], \\
V_1 = \quad & Q^2(u_q + v_q) + u_q(u_q - v_q), \\
V_2 = \quad & v_q(u_q + Q^2) + 2m_q^2(v_q - u_q).
\end{aligned} \tag{A.7}$$

The list of scalar integrals over variable v_q in NLO approximation reads as

$$\begin{aligned}
J[\frac{1}{v_q^2}] = \quad & \frac{\sqrt{\lambda_q}}{m_q^2 u_q}, \\
J[\frac{1}{v_q}] = \quad & \log \frac{(Q^2 + u_q + 2m_q^2 + \sqrt{\lambda_q})^2}{4m_q^2 \tau}, \\
J[1] = \quad & \frac{u_q \sqrt{\lambda_q}}{\tau}, \\
J[v_q] = \quad & \frac{u_q^2 (Q^2 + u_q + 2m_q^2) \sqrt{\lambda_q}}{2\tau^2},
\end{aligned} \tag{A.8}$$

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