REDOCED-COMPLEXITY MULTIPLE SYMBOL DIFFERENTIAL DETECTION FOR UWB COMMUNICATIONS

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ABSTRACT

In ultra-wideband (UWB) communications, the typical signal propagation through dense multipath fading offers potentially very large multipath diversity, but at the same time complicates receiver design as far as channel estimation and multipath energy capture are concerned. To strike a desired balance, we propose a multi-symbol differential detection framework that bypasses training or costly channel estimation by the use of autocorrelation principle. Furthermore, resorting properly to the Viterbi algorithm enables to attain an efficient performance versus affordable complexity tradeoff solution. Simulation results demonstrate that the proposed detection scheme is remarkably robust with respect to the effects of both noise and multiple access interference.

1. INTRODUCTION

UWB technology offers the potential of low-cost short-range high-speed wireless communications as required in wireless personal area networks (WPANs) [1]. The operating environments for such envisioned applications typically encounter, however, harsh multipath propagation phenomena, in which each transmitted pulse arrives at the receiver over hundreds of delayed echoes. Capturing the received symbol energy scattered over dense multipath reveals itself certainly as a very demanding task, especially in view of many stringent receiver constraints, first of all the affordable complexity. The well-known Rake receiver approach enables the capture of a significant level of energy [2], but at the cost of a large number of correlator-based fingers together with an intensive computational load in estimating the required gains and delays of the channel paths [3].

A viable alternative approach for efficient energy capture without requiring any prior channel estimation is given by the transmitted reference (TR) methods, as proposed originally in [4] and in its improved versions in [5]-[6]. The TR idea consists of transmitting per frame a reference pulse prior to each data pulse which is used as noisy template in a correlation receiver for data detection. This simple receiver does not require the costly task of channel estimation, but entails wastage of important communication resources, such as additional transmit power and decreased transmission rate due to the transmission of reference pulses. These drawbacks can be avoided by resorting to differential detectors (DD), as formerly outlined in [7] or more recently in [5] as differential transmitted reference (DTR) schemes. Here, a replica of the received waveform is used as a template for the current pulse to obtain side information about the propagation channel, thus retaining the simplicity of the TR approach but at the same time offering a SNR enhancement of about 3 dB.

The above developments suggest that the DD approach can be a promising solution for attaining low-complexity and energy-efficient receivers in comparison to the traditional Rake processing. It is thus motivated to search for other alternative differential schemes capable of avoiding channel estimation but at the same time providing enhanced performance, especially in severe multiple access scenarios. Along this line, this paper contributes to deriving a novel receiver structure based on multiple symbols differential detection (MSDD). The MSDD approach was pursued in [9] for differentially encoded M-ary phase-shift keying signals transmitted over flat fading channels to reach multiple receivers. Without explicit phase estimation and compensation, the error performance of this MSDD detector approaches that of maximal-ratio combining as the symbol block size increases. Conversely, our MSDD design targets point-to-point links with unknown frequency selective fading. Without the knowledge of the received pulse template due to the unknown channel, the solution proposed in [9] is not immediately applicable, whereas our derivation through a GLRT-based optimality approach, as suggested in [8], results in an auto-correlation-based receiver.

Likewise the aforementioned DTR solutions, the proposed MSDD receiver makes use of differential encoding performed symbol-by-symbol rather than frame-by-frame, and circumvents computationally intensive estimation of the multipath channel impulse response. Compared with the symbol-by-symbol GLRT-based receiver [8], the MSDD detector shows improved BER performance at the cost of increased computational complexity. Nevertheless, the level of desired performance can be accomplished through a judicious selection of the number of multiple symbols adopted in the detection step, enabling an efficient tradeoff between overall complexity and performance.

The rest of the paper will devote to the development and validation of the proposed MSDD scheme. Sections are organized to present the signal model, MSDD receiver design, reduced-complexity implementation of the MSDD based on the Viterbi Algorithm, simulation results and concluding summary. Also presented is an illustrative example comparing MSDD with its reduced-complexity version and the one-shot DD scheme.

2. SIGNAL MODEL

In UWB impulse radio signaling, each symbol is transmitted over $N_f$ frames with one pulse $g(t)$. The symbol, frame, pulse durations are denoted as $T_s$, $T_f$ and $T_p$ respectively, satisfying $T_s = N_f T_f$ and $T_f \gg T_p$. Multiple access can be enabled by using pseudo-random time hopping (TH)
codes \(c_j \in [0, N_c - 1]\), which shift each pulse time position at multiples of the chip period \(T_c\), with \(N_c T_c < T_f\). We consider here differential encoding, in which the independent information-bearing symbols \(a_i \in \{\pm 1\}\) are differentially encoded into channel symbols \(b_i \in \{\pm 1\}\) through \(b_i = a_i a_{i-1}\). The transmitted signal is given by

\[
x(t) = \sum_i b_ig_s(t - iT_s),
\]

where \(g_s(t)\) is the transmitted symbol-long waveform that can be written as \(g_s(t) = \sum_{j=0}^{N_f-1} g(t - jT_f - c_j T_c)\).

The propagating multipath channel is slow fading and has \(h(t) = \sum_{l=0}^{L-1} \alpha_l \delta(t - \tau_l)\) as impulse response, where \(L\) is the total number of paths, each with gain \(\alpha_l\) and delay \(\tau_l\). The received pulse \(p(t) = \sum_{l=0}^{L-1} \alpha_l g(t - \tau_l)\) is the convolution of \(g(t)\) with \(h(t)\), resulting in a widened pulse width \(T_p = \tau_{L-1} + T_c\). Thus, the received symbol-level waveform \(p_s(t)\), given by the convolution of \(g_s(t)\) and \(h(t)\), takes on the form

\[
p_s(t) = \sum_{j=0}^{N_f-1} p(t - jT_f - c_j T_c),
\]

and accordingly, the received signal can be expressed by

\[
y(t) = \sum_i b_ip_s(t - iT_s) + w(t),
\]

where the additive noise \(w(t)\) stands for the contribution of both Gaussian thermal noise and multiple access interference (MAI).

### 3. MULTI-SYMBOL DIFFERENTIAL DETECTION

The objective of this section is to derive a multi-symbol differential detector (MSDD) that recovers \(M\) consecutive information symbols \(a = [a_1, a_2, \ldots, a_M]^T\) based on the received signal \(y(t)\) in the interval \(0 \leq t \leq (M+1)T_s\).

The following assumptions are adopted: (i) accurate symbol timing has been acquired, (ii) the data block size \((M + 1)T_s\) is set to be smaller than channel coherence time, (iii) the frame period \(T_f\) satisfies \(T_f \geq T_p + N_c T_c\) to prevent inter-frame interference (IFI), and (iv) the channel impulse response \(h(t)\) is unknown and will not be explicitly estimated during detection with the aim of reducing the receiver complexity. It is worth observing that the last constraint (together with the fact that the UWB fading channel is highly frequency selective) sets our detection problem apart from the existing MSDD for flat fading channels proposed in [9], and additionally, makes it necessary to develop a new autocorrelation-based MSDD specifically tailored for UWB systems.

According to the differential encoding rule, the channel symbol \(b_i\) can be expressed as \(b_i = b_0 \prod_{k=1}^i a_k\), \(i > 0\). Using (2), the received signal in (3) can therefore be put in the alternative form

\[
y(t) = \sum_{i=0}^{M} \prod_{k=1}^i a_k \sum_{j=0}^{N_f-1} q(t - jT_f - c_j T_c - iT_s) + w(t),
\]

where \(q(t) \triangleq b_0 p(t)\) of width \(T_p\) contains both the unknown channel parameters and the initial channel symbol \(b_0\). Given that \(q(t)\) is not known, to detect \(a\) we will follow the GLRT approach [8], which amounts to finding the maximum of the log-likelihood metric

\[
\Lambda[y(t) | \hat{a}, \hat{q}(t)] = \int_0^{(M+1)T_f} y(t) \hat{s}(t) dt - \int_0^{(M+1)T_f} \hat{s}^2(t) dt
\]

with respect to \(\hat{a}\) and all the finite-energy functions \(\hat{q}(t)\) with support \([0, T_p]\), where we define as

\[
\hat{s}(t) = \sum_{i=0}^{M} \prod_{k=0}^{i} \hat{a}_k (t + iT_s),
\]

a possible realization of the signal component in (4) corresponding to \(\hat{a}\) and \(\hat{q}(t)\).

Considering that \(\hat{q}(t)\) has support \([0, T_p]\) and \(T_f \geq T_p + N_c T_c\), it is possible to show that

\[
\int_0^{(M+1)T_f} y(t) \hat{s}(t) dt = \int_0^{T_p} \hat{q}(t) \left[ \sum_{i=0}^M \prod_{k=0}^i \hat{a}_k (t + iT_s) \right] dt
\]

where

\[
z(t) \triangleq \sum_{j=0}^{N_f-1} y(t + jT_f + c_j T_c),
\]

and

\[
\int_0^{(M+1)T_f} \hat{s}^2(t) dt = (M + 1)N_f \int_0^{T_p} \hat{q}^2(t) dt.
\]

Substituting (7) and (9) into (5) yields

\[
\Lambda[y(t) | \hat{a}, \hat{q}(t)] =
\]

\[
2 \int_0^{T_p} \hat{q}(t) \left[ \sum_{i=0}^M \prod_{k=0}^i \hat{a}_k (t + iT_s) \right] dt - (M + 1)N_f \int_0^{T_p} \hat{q}^2(t) dt.
\]

and therefore, the GLRT-based decision strategy works as

\[
\hat{a} = \arg \max_{\hat{a}} \left\{ \max_{\hat{q}(t)} \{ \Lambda[y(t) | \hat{a}, \hat{q}(t)] \} \right\}.
\]

In order to solve (11), we will first keep \(\hat{a}\) fixed and compute the inner term \(\Gamma[y(t) | \hat{a}] = \max_{\hat{q}(t)} \{ \Lambda[y(t) | \hat{a}, \hat{q}(t)] \}\). To this end, one can resort to variational techniques: i) impose \(\hat{q}(t) = q_0(t) + \lambda \epsilon(t)\), where \(q_0(t)\) is the optimum solution to be solved for and \(\epsilon(t)\) is a generic real function with support in \([0, T_p]\); ii) take the first-order derivative of \(\Lambda[y(t) | \hat{a}, \hat{q}(t)]\) with respect to \(\lambda\) and set it to zero. In doing so, we obtain

\[
q_0(t) = \frac{1}{(M + 1)N_f} \sum_{i=0}^M \prod_{k=0}^i \hat{a}_k (t + iT_s),
\]

and consequently,

\[
\Gamma[y(t) | \hat{a}] = \frac{1}{(M + 1)N_f} \int_0^{T_p} \left[ \sum_{i=0}^M \prod_{k=0}^i \hat{a}_k (t + iT_s) \right]^2 dt.
\]

According to (11), the information symbols will be detected through evaluating the maximum of \(\Gamma[y(t) | \hat{a}]\) in (13)
with respect to $\tilde{a}$. The fact that the information symbols take values in $\{\pm 1\}$ enables a further rearrangement of (13) as

$$
\Gamma[y(t) | \tilde{a}] = \sum_{i=1}^{M} \sum_{l=0}^{M-i} \prod_{k=1}^{i} \tilde{a}_{k+i} Z_{l+i} = \sum_{i=1}^{M} \prod_{k=1}^{i} \tilde{a}_{k+i} Z_{l+i} \quad (14)
$$

where we define

$$
Z_{a, j} = \frac{1}{(M+1)N_f} \int_{0}^{T_p} z(t + iT_i) z(t + jT_i) dt. \quad (15)
$$

All in all, the proposed decision rule can be written as

$$
\tilde{a} = \arg \max_{\tilde{a}} \{ \Gamma[y(t) | \tilde{a}] \}, \quad (16)
$$

where $\Gamma[y(t) | \tilde{a}]$ is given by (14).

Two remarks about (14)-(16) are now in order. From (15) it comes out that the coefficients $Z_{a, j}$ do not require the estimation of channel parameters in that they are derived by correlating segments of the waveform $z(t)$ which in turn is solely constructed from the received signal $y(t)$ in (4). Hence, the MSDD boils down to an autocorrelation-based receiver that, even in the presence of an unknown multipath channel, works only by means of the sequence $Z_{l+i}$, $1 \leq i \leq M$, $0 \leq l \leq i - 1$. The second remark concerns the computational complexity that goes up exponentially in the number $M$ of symbols to be detected. This prompts our investigation on reduced-complexity alternatives, which we will detail next.

4. VITERBI ALGORITHM FOR MSDD

The reduced-complexity implementation of the MSDD we will focus on is based on the well-known Viterbi Algorithm (VA) as it stands for an efficient performance versus complexity tradeoff solution. However, a first look at our problem makes us conclude that VA is not immediately applicable, because the MSDD formulation (14) cannot be represented by a fixed number of states over stages (indexed by $i$). To overcome this obstacle, we remove some summands in (14) and approximate $\Gamma[y(t) | \tilde{a}]$ as

$$
\tilde{\Gamma}[y(t) | \tilde{a}] = \sum_{i=1}^{L} \sum_{l=0}^{M-i} \prod_{k=1}^{i} \tilde{a}_{k+i} Z_{l+i} \quad (17)
$$

where $L(M)$ is a design parameter reflecting the desired performance-complexity tradeoff.

Let $\tilde{a}^{(L-1)}_{i-1} = (\tilde{a}_{i-L+1}, \tilde{a}_{i-L+2}, \ldots, \tilde{a}_{i-1})$ define, for binary signaling, a total number of $2^{L-1}$ trellis states. Hence, the objective function (17) can be rewritten as

$$
\tilde{\Gamma}[y(t) | \tilde{a}] = \sum_{i=1}^{M} \lambda_{i}(\tilde{a}^{(L-1)}_{i-1}, \tilde{a}_i) \quad (18)
$$

where

$$
\lambda_{i}(\tilde{a}^{(L-1)}_{i-1}, \tilde{a}_i) \triangleq \sum_{l=0}^{M-i} \prod_{k=1}^{i} \tilde{a}_{k+i} Z_{l+i} \quad (19)
$$

indicates the branch metric in a trellis diagram representation, whereas in accordance with (18)-(19), the accumulated metric $J_i(\cdot)$ at the $i$-th trellis stage can be denoted with respect to $\tilde{a}_i := (\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_i)$ as

$$
J_i(\tilde{a}_i) = J_{i-1}(\tilde{a}_{i-1}) + \lambda_i(\tilde{a}^{(L-1)}_{i-1}, \tilde{a}_i). \quad (20)
$$

Having defined the branch metric (19) and accumulated metric (20), the VA can be readily applied to solve for the optimal sequence $\tilde{a}$.

5. AN ILLUSTRATIVE EXAMPLE

This section gives an example to illustrate our proposed reduced-complexity VA-based MSDD (VA-MSDD) as compared with the conventional symbol-by-symbol DD and the full-complexity MSDD. A possible mechanization of the VA procedure will be illustrated in detail as well.

In MSDD, the objective function (14) can be expressed as

$$
\Gamma[y(t) | \tilde{a}] = Z_{0,1} \tilde{a}_1 + Z_{1,2} \tilde{a}_2 + Z_{0,2} \tilde{a}_1 \tilde{a}_2 + Z_{2,3} \tilde{a}_3 + Z_{1,3} \tilde{a}_1 \tilde{a}_2 \tilde{a}_3 + Z_{3,4} \tilde{a}_4 + Z_{2,4} \tilde{a}_2 \tilde{a}_3 \tilde{a}_4 + Z_{0,4} \tilde{a}_1 \tilde{a}_2 \tilde{a}_3 \tilde{a}_4 + \ldots \ldots \quad (21)
$$

For the special case $M = 1$, the above metric is reduced to

$$
\Gamma[y(t) | \tilde{a}] = \tilde{a}_1 Z_{0,1}, \quad (22)
$$

which turns the decision (16) into $\tilde{a}_1 = \text{sgn}\{Z_{0,1}\}$. This is nothing but the conventional one-shot DD, that evidently does not take into account any cross-terms for detection over more than one symbol.

Concerning the VA-MSDD, we set $L = 3$ for the sake of illustration. The approximate objective function in (17) reads as

$$
\tilde{\Gamma}[y(t) | \tilde{a}] = Z_{0,1} \tilde{a}_1 + Z_{1,2} \tilde{a}_2 + Z_{0,2} \tilde{a}_1 \tilde{a}_2 + Z_{2,3} \tilde{a}_3 + Z_{1,3} \tilde{a}_1 \tilde{a}_2 \tilde{a}_3 + Z_{3,4} \tilde{a}_4 + Z_{2,4} \tilde{a}_2 \tilde{a}_3 \tilde{a}_4 + Z_{1,4} \tilde{a}_1 \tilde{a}_2 \tilde{a}_3 \tilde{a}_4 + \ldots \ldots \quad (23)
$$

Compared with the lattice structure (21) that exhausts all cross-correlation combinations, the metric in (23) trim off some addends to keep only those cross-terms with a memory length no greater than $L = 3$ symbols, thereby resulting in an approximation to the original GLRT solution. Depending on $L$, VA-MSDD will trade off between full-complexity MSDD and conventional DD.

Next, follows the VA implementation of (23). We take as example binary signaling and again $L = 3$, which specialize the trellis states $\tilde{a}^{(2)}_{i-1} := (\tilde{a}_{i-2}, \tilde{a}_{i-1})$ to be

$$
S_0 = \{0, 0\}; \quad S_1 = \{0, 1\}; \quad S_2 = \{1, 0\}; \quad S_3 = \{1, 1\}.
$$
Accordingly, the trellis transitions are given by

\[
S_0 \rightarrow \begin{cases} 
S_0 & \text{for } \bar{a}_i = 0 \\
S_1 & \text{for } \bar{a}_i = 1 \\
S_2 & \text{for } \bar{a}_i = 0 \\
S_3 & \text{for } \bar{a}_i = 1
\end{cases}
\] (24)

Therefore, starting from \( S_0 \), we obtain

Stage 0: \( J_0 = 0 \);
Stage 1: \( \lambda_1(\bar{a}_1) = Z_0,1 \bar{a}_1 \);
Stage 2: \( \lambda_2(\bar{a}_1, \bar{a}_2) = Z_2,2 \bar{a}_2 + Z_0,2 \bar{a}_1 \bar{a}_2 \);
Stage 3: \( \lambda_3((\bar{a}_1, \bar{a}_2), \bar{a}_3) = Z_3,3 \bar{a}_3 + Z_1,3 \bar{a}_1 \bar{a}_3 + Z_0,3 \bar{a}_1 \bar{a}_2 \bar{a}_3 \);

\[
J_3(\bar{a}_3) = J_2(\bar{a}_2) + \lambda_3(\bar{a}_3, \bar{a}_3)
\]

\[
= Z_0,1 \bar{a}_1 + Z_1,2 \bar{a}_2 + Z_0,2 \bar{a}_1 \bar{a}_2 + Z_2,3 \bar{a}_3 + Z_3,3 \bar{a}_1 \bar{a}_3 + Z_0,3 \bar{a}_1 \bar{a}_2 \bar{a}_3;
\]

\[
\text{......}
\]

The above steps proceed until reaching Stage \( M \) (\( i = M \)), which gives \( J_M(\bar{a}_M) = \Gamma [\bar{y}(t)|\bar{a}] \). Note that the number of states in each stage is given by \( 2^{L-1} \), while the overall complexity grows linearly in \( M \), on the order of \( (M \cdot 2^{L-1}) \), \( L < M \).

6. SIMULATIONS RESULTS

Computer simulations have been conducted to verify the effectiveness of the MSDD detector together with the suboptimal version VA-MSDD. In all tests, each active user transmits a burst of \( M \) information-bearing symbols and experiences a multipath propagation channel that is generated randomly according to the model in [10]. Specifically, the multipath components arrive in clusters with independent double-sided Rayleigh distributed amplitude having mean square value exponentially decaying with the cluster delay as well as with the ray within the cluster, with decay factors \( \Gamma = 30 \) ns and \( \gamma = 5 \) ns, respectively. The clusters and the rays within each cluster have Poisson distributed arrival times with arrival rates \( \lambda = 0.5 \) ns\(^{-1} \) and \( \lambda = 2 \) ns\(^{-1} \), respectively. The monocyte \( g(t) \) is selected as the second derivative of a Gaussian function with normalized unit energy and pulse width \( T_p = 1.0 \) ns. The frame and chip intervals are \( T_f = 100 \) ns and \( T_c = 1.0 \) ns, respectively, while the frame repetition factor is \( N_r = 25 \). Users’ time-hopping codes are randomly picked up in the interval \([0, N_c - 1]\) with \( N_c = 91 \). When MAI is concerned, the time origins of the desired user and \( N_u - 1 \) interfering users are chosen randomly over the symbol interval \([0, N_r/T_f]\) to reproduce an asynchronous access to the channel. The timing offset of the desired user is assumed to be known and the modulation format is binary PAM. Further, the receive filter is pass-band over the \( \pm 10 \) dB bandwidth of the monocyte, whereas the thermal noise is white, with a two-sided power spectral density \( N_0/2 \).

The following UWB receivers are considered: i) the full- and low-complexity MSDD, wherein the symbol decisions are obtained through searching exhaustively for the maximum of the metrics (14) and (17), respectively, over the \( 2^M \) possible sequences; ii) reduced-complexity VA-MSDD with memory length \( L \); iii) symbol-by-symbol DD receiver corresponding to \( M = 1 \); iv) ideal Rake (IRake) receiver under perfect channel state information (CSI) as performance benchmark.

Test A (MSDD for various memory length \( L \)): Fig. 1 compares the MSDD receivers for full-complexity and low-complexity with \( L = 3, 4, 6, 8, 10 \) configurations in a single-user system (\( N_u = 1 \)) scenario with \( M = 20 \). The MSDD outperforms DD by nearly 5 dB at BER = \( 10^{-4} \) and is less than 4 dB apart from the ideal Rake that requires formidable channel estimation. The low-complexity MSDD degrades its performance with respect to MSDD due to the approximations involved in the objective metric, but shows at least a gain of 4 dB over the DD for \( L = 6 \).

Test B (VA-MSDD for various memory length \( L \)): Focusing on VA-MSDD for its affordable complexity, Fig. 2 depicts the BER performance for \( N_u = 1, M = 50 \) and \( L = 3, 4, 6, 8, 10 \). The data block size \( M \) is set to be as large to render MSDD impractical in computation, while the complexity of VA-MSDD is still manageable. It is observed that the BER performance improves as \( L \) increases. Indeed, the VA-MSDD with \( L = 10 \) is 2 dB better than the less complex \( L = 3 \) case, shows an overall 5 dB gain over the DD scheme, and is only 4 dB apart from the ideal Rake.

Test C (VA-MSDD for various data size \( M \)): In Fig. 3, the memory length of the VA-MSDD is fixed at \( L = 6 \) with \( N_u = 1 \), while the data block size is chosen as \( M = 50, 200, 500 \). All \( M \) values yield very close BER curves. This indicates that the MSDD performance is constrained by the memory length \( L \) imposed by the complexity concern, regardless of \( M \). Indeed, even though increasing \( M \) in MSDD potentially may enable stronger averaging over the noise, the memory \( L \) in VA-MSDD limits the number of cross-terms in the objective metric, thus limiting the noise averaging effect. Therefore, unless faster algorithms can be devised for MSDD, setting a very large number for \( M \) will result in diminishing return in performance under practical complexity constraints.

Test D (VA-MSDD in the presence of MAI): BER per-
The V A-MSDD exhibits remarkable robustness with respect to the DD scheme with $N_u$. Decoding schemes, given that the information data symbols undergo some form of channel coding.

The MSDD framework appears to fit well with the turbo principle, and thus can be applied in iterative joint detection and decoding schemes, given that the information data symbols undergo some form of channel coding.

7. SUMMARY

In this paper, we have derived a multi-symbol differential detector based on the GLRT optimality criterion. The proposed detector achieves good performance without requiring training in the demanding multiple access scenario, and shows affordable complexity by both avoiding channel estimation and applying the reduced-complexity Viterbi algorithm. The MSDD framework appears to fit well with the turbo principle, and thus can be applied in iterative joint detection and decoding schemes, given that the information data symbols undergo some form of channel coding.

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