A New Precoder Design for Precoding-Based Blind Channel Estimation for MIMO-OFDM Systems

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Abstract—Semi-Blind/Blind channel estimation for multiple-input multiple-output (MIMO) orthogonal frequency-division multiplexing (OFDM) systems has received a lot of attention in recent years. A new linear precoder for blind channel estimation is proposed in which only a small number of subcarriers carry symbols that are linearly precoded to effect the inter-symbol correlation needed for the blind channel estimation scheme to function properly. The proposed precoding scheme leaves most of the subcarriers intact, thereby minimizing the number of symbols that have to be jointly estimated via either Maximum Likelihood (ML) or MMSE. An optimal precoder under the sparse structure is developed, which insures that the precoder matrix is well-conditioned to minimize any noise enhancement that may occur in the process of MMSE based joint symbol estimation.

I. INTRODUCTION

Semi-Blind/Blind channel estimation for OFDM systems is an active research area to boost spectral efficiency by reducing the number of pilot carriers needed for channel estimation [3], [4]. Gao et al proposed a blind channel estimation method for OFDM systems based on linear precoding [5], [6]. The linear precoding is applied to all of the information symbols to induce signal correlation across all of the carriers. The signal correlation is required for their blind channel identification scheme to function properly. Their blind scheme works even in the single-input single-output (SISO) case of only a single transmit and a single receive antenna. Their work has lead to several follow-up advances based on their so-called non-redundant precoder [7]–[10]. There are a number of issues associated with inducing correlation across all of the transmitted symbols. This dictates that all of the symbols have to be jointly estimated. The joint ML estimation becomes computationally prohibitive. Further, even if the minimum mean squared error (MMSE) based joint estimation is not computationally prohibitive due the large matrix equation that has to be solved, there is the additional issue of noise enhancement due to the conditioning of the precoder matrix.

In contrast, in the scheme proposed here, only a small number of subcarriers carry symbols that are linearly precoded to effect the inter-symbol correlation needed for the blind channel estimation. The proposed precoding scheme leaves most of the subcarriers intact, thereby minimizing the number of symbols that have to be jointly estimated via either ML or MMSE. An optimal precoder is developed to insure that the precoder matrix is well-conditioned to minimize any noise enhancement that may occur in the process of MMSE based joint symbol estimation. Again, joint symbol estimation is only required for the small number of symbols that are used for blind channel estimation. The vast majority of transmitted symbols may be estimated via ML independently.

A. Notation

We will make use of standard notational conventions. Vectors and matrices are written in boldface with matrices in capitals. All vectors are column vectors. For a matrix $A$, $A^\dagger$, $A^T$ and $A^H$ indicate the Moore-Penrose pseudo inverse, transpose and Hermitian transpose of $A$, respectively. Also, $A(I, J)$ denotes the submatrix composed of rows $I$ and columns $J$ of $A$ for sets $I$ and $J$, and $: \cdot$ denotes the entire set. For a vector $a$, we use $|a|$ for 2-norm $A \circ B$ and $\|A\|_F$ denote the Hadamard product and Frobenius norm for matrices, respectively. $I_n$ stands for the identity matrix of size $n \times n$.

II. SYSTEM MODEL

We consider the MIMO-OFDM system with $N_t$ transmit antennas and $N_r$ receive antennas employing $N$ subcarriers. The $i$-th data vector $x_k^{(i)} = [x_k^{(i)}(0), \ldots, x_k^{(i)}(N-1)]^T$ at the $k$-th transmit antenna is linearly precoded using the non-redundant precoding matrix $W_k$ for blind channel estimation. Each precoded data vector is OFDM modulated (normalized inverse discrete Fourier transform (IDFT)), then the cyclic prefix (CP) is added at the front of each block as shown in Fig. 1. Assume the CP is sufficient to mitigate the inter-symbol interference caused by channel time spread $h_{lk} = [h_{lk}(0), \ldots, h_{lk}(L-1)]^T$ from the $k$-th transmit antenna to the $l$-th receive antenna and the channel remains constant over an $N_s$ successive OFDM symbols. The CP is removed at the $l$-th receive antenna, then the $i$-th received signal is demodulated through inverse OFDM block (normalized DFT) as

$$
y_l^{(i)} = \sum_{k=1}^{M} \tilde{H}_{lk} W_k x_k^{(i)} + \tilde{n}_l^{(i)},$$

where $\tilde{H}_{lk}$ denotes the diagonal matrix with elements $\tilde{h}_{lk} = \tilde{F} h_{lk}$ and $\tilde{n}_l^{(i)}$ denotes a zero-mean uncorrelated Gaussian noise with covariance matrix $\sigma_n^2 I_N$. $\tilde{F}$ is defined by the first $L$ columns of the $N \times N$ normalized DFT matrix $F$. Here, the source covariance matrix is given by $E\{x_k^{(i)} x_k^{(i)H}\} = I_N$. 

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A. Blind estimation based on non-redundant precoding for OFDM systems

We briefly review the precoding-based blind channel estimation method for MIMO-OFDM systems which will provide useful background information for further understanding of the proposed method in the next section. The blind channel estimation technique is based on non-redundant precoding at transmitter and exploits the second-order statistics of the sample covariance with subspace decomposition. The covariance matrix of the received signal \( r_i \) at the l-th receive antenna is given by

\[
R_l := E(\tilde{y}_l \tilde{y}_l^H),
\]

\[
= \sum_{k=1}^{M} \tilde{H}_{lk} Q_k \tilde{H}_{lk}^H + \sigma_n^2 I_N,
\]

\[
= \left( \sum_{k=1}^{M} \tilde{h}_{lk} \tilde{h}_{lk}^* \right) \odot Q + \sigma_n^2 I_N,
\]

where \( Q_k := W_k W_k^H \) and \( Q_1 = \cdots = Q_M = Q \), referred to as the square precoder matrix. We will omit the time index \( (i) \) for notational simplicity whenever possible. Next, the elements of \( R_l \) are written by

\[
[R_l]_{i,j} = \sum_{k=1}^{M} \tilde{h}_{lk}(i) \tilde{h}_{lk}^*(j) [Q]_{i,j} + \sigma_n^2 \delta_{\{0\}}(i-j),
\]

where \( \delta_A(x) \) denotes the indicator function, i.e., \( \delta_A(x) = 1 \) if \( x \in A \) or zero otherwise. Prior works for blind channel estimation rely on (5), i.e., the channel parameters can be identifiable with given \( R_l \) and \( Q \) as

\[
[R_l]_{i,j} - \sigma_n^2 \delta_{\{0\}}(i-j) = \sum_{k=1}^{M} \tilde{h}_{lk}(i) \tilde{h}_{lk}^*(j),
\]

where \( i,j \in \{1, \ldots, N\} \). Here, the off-diagonal elements of \( R_l \) are used for blind channel estimation because the diagonal elements of one are contaminated by the additional unknown noise variance. For example, the method in [6] defines new variables for blind channel estimation as

\[
\mathbf{v}_j := \begin{bmatrix}
[R]_{1,j} & [R]_{2,j} & \cdots & [R]_{N,j}
\end{bmatrix}^T,
\]

\[
\tilde{F}_j := \begin{bmatrix}
F(1 : j - 1, :) \\ F(j + 1 : N, :)
\end{bmatrix}.
\]

Note that \( \mathbf{v}_j = \tilde{F}_j \sum_{k=1}^{M} \tilde{h}_{lk} \tilde{h}_{lk}^* \) is obtained from \( \tilde{h}_{lk} = \tilde{F} h_{lk} \) and (6). We then have

\[
\mathbf{J}^G := \tilde{F} \mathbf{v}_1 \cdots \tilde{F} \mathbf{v}_N \mathbf{F} = \sum_{k=1}^{M} \tilde{h}_{lk} \tilde{h}_{lk}^H.
\]

Now, the channel \( [h_{11}, \ldots, h_{MM}] \) is identifiable from the eigenvalue decomposition (EVD) of \( \mathbf{J}^G \) with a \( M \times M \) unitary ambiguity matrix which is inherent in the blind estimation methods as [11], [12]

\[
\mathbf{H}_l := [h_{11}, \ldots, \tilde{h}_{MM}] = \mathbf{U}(:, 1 : M) \Sigma(1 : M, 1 : M)^{1/2},
\]

where \( \mathbf{J}^G = \mathbf{U} \Sigma \mathbf{U}^H \) by the EVD.

Since the method for blind channel estimation requires that all off-diagonal elements of \( Q \) be non-zero to construct \( \mathbf{v}_j \) in (7), prior methods consider the following square precoder matrix for \( p \neq 0 \) [6], [7], [10], [13]:

\[
Q_p = (1-p)I_N + p \mathbf{e} \mathbf{e}^T, \quad -1 < p < 1,
\]

where \( \mathbf{e} \) is the column vector composed of all ones. Note that the dense precoder matrix \( \mathbf{W} \) incurred by a fully dense square precoder matrix \( Q_p \) precodes all subcarriers for blind channel estimation. This limits the primary advantage of MIMO-OFDM systems which provides an \( M \times M \) MIMO flat-fading channel so that channel equalization and the MIMO encoding/decoding algorithms can be applied independently to each subcarrier [14, Ch. 5].

B. Blind channel estimation based on sparse linear precoder

To reduce the number of subcarriers that are linearly precoded for blind channel estimation, we propose the method that uses only a mall number of samples of \( R_l \). We will consider a \( T \times T \) submatrix of \( R_l \) for \( T \geq L \). In order to construct the submatrix, we define the index sets of selected rows and columns such that

\[
\mathcal{I} = \{i_1, \ldots, i_T\} \subset \{1, \ldots, N\},
\]

\[
\mathcal{J} = \{j_1, \ldots, j_T\} \subset \{1, \ldots, N\},
\]

\[
\mathcal{I} \cap \mathcal{J} = \emptyset.
\]

The condition (14) is required to exclude the diagonal elements of \( R_l \). From (6), the proposed square precoder matrix \( Q \) needs to have non-zero off-diagonal elements only at \( \mathcal{I} \times \mathcal{J} \) and also \( \mathcal{J} \times \mathcal{I} \) due to its symmetry, as illustrated in (34). For \( i_s, j_t \in \mathcal{I} \) and \( j_t \in \mathcal{J} \), let us define

\[
\mathbf{v}^t_{j_t} := [R]_{i_s,j_t}/[Q]_{i_s,j_t}, \ldots, [R]_{i_T,j_t}/[Q]_{i_T,j_t}
\]

\[
= \tilde{F}(\mathcal{I}, :) \left( \sum_{k=1}^{M} \tilde{h}_{lk} \tilde{h}_{lk}^* \right)
\]

\[
= \tilde{F}(\mathcal{I}, :) \left( \sum_{k=1}^{M} \tilde{h}_{lk} \tilde{h}_{lk}^* \right),
\]

where \( \tilde{F}(\mathcal{I}, :) \in \mathbb{C}^{T \times L} \) is composed of rows \( \mathcal{I} \) of the skinny DFT matrix \( \mathbf{F} \). Since \( \tilde{F}(\mathcal{I}, :) = \tilde{F}(\mathcal{J}, :) = \tilde{F}(\mathcal{I}, :)^H = \sum_{k=1}^{M} \tilde{h}_{lk} \tilde{h}_{lk}^* \), we obtain

\[
\mathbf{J}_{\mathcal{I},\mathcal{J}} := \tilde{F}(\mathcal{I}, :)^T [\mathbf{v}_1^T, \ldots, \mathbf{v}_T^T] (\tilde{F}(\mathcal{J}, :)^H)^H = \sum_{k=1}^{M} \tilde{h}_{lk} \tilde{h}_{lk}^*.
\]

Similar to (10), the channel can be estimated from \( \mathbf{J}_{\mathcal{I},\mathcal{J}} \) [1], [2].

Such sparse design for \( Q \) can leverage the system performance by minimizing the number of subcarriers that are linearly precoded for blind channel estimation. The most uncoded subcarriers can benefit from data rate enhancing MIMO precoding [15] and ML symbol detection. This issue will be discussed in details in Section IV.
III. OPTIMAL SPARSE PRECODER DESIGN

In this section, we provide an optimal precoder design under the sparsity introduced in Section II-B. Since the exact $R_t$ is not available in practice, the performance of the blind channel estimation depends on the design $Q$ when we use an imperfect sample covariance matrix $\hat{R}_t = \frac{1}{N} \sum_{n=1}^{N} x_i^{(n)} x_i^{(n)\dagger}$. We first define the difference between the sample covariance and the exact covariance matrix as

$$\Delta R_t := \hat{R}_t - R_t,$$

then the difference between $\nu_{j_1}'$ and $\nu_{j_1}$ is naturally given by

$$\Delta \nu_{j_1}' = \frac{\|\Delta R_t\|_{1,1,1} \cdot |\{Q\}|_{1,1,1} \cdot \|\Delta R_t\|_{1,1,1}}{|\hat{R}_t|_{1,1,1} \cdot |\{Q\}|_{1,1,1} \cdot |\hat{R}_t|_{1,1,1}} T,$$

where $i_s \in \mathcal{I}$ and $j_1 \in \mathcal{J}$. The estimation error of $J_{\mathcal{I}, \mathcal{J}}$ is given by

$$\Delta J := \hat{J}_{\mathcal{I}, \mathcal{J}} - J_{\mathcal{I}, \mathcal{J}} = \tilde{F}(\mathcal{I}, :)^\dagger [\Delta \nu_{j_1}' \cdots \Delta \nu_{j_T}'] (\tilde{F}(\mathcal{J}, :)^\dagger)^H,$$

where $\hat{J}_{\mathcal{I}, \mathcal{J}}$ and $J_{\mathcal{I}, \mathcal{J}}$ are computed from $\hat{R}_t$ and $R_t$, respectively. Since the channel is completely estimated from the exact $J_{\mathcal{I}, \mathcal{J}}$ in (17), minimizing the difference $\Delta J$ leads to better estimates of the channel. Thus, we choose a criterion for blind channel estimation to minimize the mean square error (MSE) $E\{\|\Delta J\|_F^2\}$. Next, we consider required constraints for the optimal design $Q$.

(C.1) $Q$ is positive semi-definite.

(C.2) The diagonal elements of $Q$ are ones.

(C.3) $Q$ has non-zero off-diagonal elements only at $\mathcal{I} \times \mathcal{J}$ and $\mathcal{J} \times \mathcal{I}$, where $|\mathcal{I}| = |\mathcal{J}| = T \geq L$, $\mathcal{I} \cap \mathcal{J} = \emptyset$, (I and $\mathcal{J}$ are equi-spaced.)

Condition 1 is required for the uniqueness of the precoder $\mathbb{W} = Q^{\frac{1}{2}}$ [11]. Condition 2 yields that there is no power boosting for each subcarrier. Condition 3 is the considered sparsity constraint discussed in the previous section. Since the optimization does not render a tractable solution, we restrict Condition 1 further as follows.

(C.1') $Q$ is diagonally dominant, i.e., $|Q|_{i,i} \geq \sum_{j \neq i} |Q|_{i,j}$.

Note that the relaxed condition (C.1') implies (C.1), then the optimization is formulated as

$$\min_Q E\{\|\Delta J\|_F^2\}$$

s.t. (C.1'), (C.2) and (C.3).

To minimize the perturbation effects in (21), $\tilde{F}(\mathcal{I}, :)$ and $\tilde{F}(\mathcal{J}, :)$ should be well conditioned. Hence, we consider equi-spaced index sets $\mathcal{I}$ and $\mathcal{J}$ where the minimum condition number is achieved assuming $T$ divides $N$ exactly. We further assume that $T = 2^m$ for some positive integer $m$ for practical purposes. The following theorem provides a property of the optimal square precoder matrix.

Lemma 1:

$$E\{[\Delta \nu_{j_1}' \cdots \Delta \nu_{j_T}']^H (\tilde{F}(\mathcal{I}, :)^\dagger \tilde{F}(\mathcal{J}, :)^\dagger)^H [\Delta \nu_{j_1}' \cdots \Delta \nu_{j_T}']\} = C \begin{bmatrix} \sum_{s=1}^{T} |Q|_{i_s,j_s}^{-1} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \sum_{s=1}^{T} |Q|_{i_s,j_s}^{-1} \end{bmatrix},$$

where $C = \frac{NL}{T \sum_{i,s,j} |Q|_{i_s,j_s}^{-1}} (\frac{1}{N} \sigma_n^2 + \sigma_n^2)$.

Proof: See [16].

Theorem 1: Under constraints (C.1'), (C.2) and (C.3), $E\{\|\Delta J\|_F^2\}$ is minimized if the absolute values of off-diagonal elements of $Q$ at $\mathcal{I} \times \mathcal{J}$ are identical with the value of $1/T$, under the assumption that the channel tap coefficients in time domain are zero-mean independent and identically distributed (i.i.d.) proper complex Gaussian random variables with variance $\sigma_n^2$, i.e., i.i.d. Rayleigh-faded.

Proof: $E\{\|\Delta J\|_F^2\}$ in (22) is given by

$$E\{\|\Delta J\|_F^2\} = E\{\|\tilde{F}(\mathcal{I}, :)^\dagger [\Delta \nu_{j_1}' \cdots \Delta \nu_{j_T}'] (\tilde{F}(\mathcal{J}, :)^\dagger)^H\}^2,$$

where the expectation is over noise and channel distributions. (25) is by $\|A\|_F^2 = tr(A^H A)$ and (26) holds by $tr(\mathcal{A} \mathcal{B} \mathcal{C}) = tr(\mathcal{B} \mathcal{C} \mathcal{A})$. Using Lemma 1 and the fact that $[\tilde{F}(\mathcal{I}, :)^\dagger \tilde{F}(\mathcal{J}, :)^\dagger]^H \mathcal{I} = |\mathcal{I}| \times |\mathcal{J}|$, for the equi-spaced index set $\mathcal{I}$ and $\mathcal{J}$, we reduce (27) to

$$E\{\|\Delta J\|_F^2\} = C \frac{NL}{T^2} \sum_{i_s,j_s \in \mathcal{I} \times \mathcal{J}} \frac{1}{|Q|_{i_s,j_s}^{-1}}.$$

The optimal design problem (22) under (C.1'), (C.2) and (C.3) can be redescibed as

$$\min_Q C \frac{NL}{T^2} \sum_{i_s,j_s \in \mathcal{I} \times \mathcal{J}} \frac{1}{|Q|_{i_s,j_s}^{-1}}$$

s.t. (C.1'), (C.2) and (C.3).

The column-wise sum constraint $\sum_{i_s \in \mathcal{I}} |Q|_{i_s,j_s}^{-1} \leq 1$ is because each column vector of submatrix $Q(\mathcal{I}, \mathcal{J})$ is translated into a row vector due to the symmetric structure of $Q$. (See (34).) Since the cost is an inverse of square of $|Q|_{i_s,j_s}$, the minimum occurs when the constraint is satisfied with equality, i.e.,

$$\sum_{j_s \in \mathcal{J}} |Q|_{i_s,j_s}^{-1} = 1$$

for $i_s$ and $\sum_{i_s \in \mathcal{I}} |Q|_{i_s,j_s}^{-1} = 1$ for $j_s$.

Thus, the optimization problem is now reduced to (29) and (31). The constraint (31) requires both row-wise and column-wise sums of the absolute values of elements of submatrix $Q(\mathcal{I}, \mathcal{J})$ to be one. Here, we relax this constraint so that the total sum of absolute values of elements of $Q(\mathcal{I}, \mathcal{J})$ is $T$.
Then, the relaxed optimization is given by

$$\min_{Q} \frac{CNL}{T^2} \sum_{i_s,j_t \in J} 1 \left\|Q_{i_s,j_t}\right\|^2 \quad \text{s.t.} \sum_{i_s,j_t \in J} \left\|Q_{i_s,j_t}\right\| = T. \tag{32}$$

Let \( q := \left[ \left\|Q_{i_s,j_t}\right\|, \left|Q_{i_s,j_t}\right|, \ldots, \left|Q_{i_s,j_t}\right| \right]^T \). The objective function in (32) is continuously differentiable and satisfy Schur’s condition [17],

and therefore the objective function in (32) is a Schur-convex function of \( q \). It is known that a Schur-convex function with \( l_1 \)-norm constraint on \( q \) is minimized when all elements of \( q \) are equal [17], i.e., \( q = \frac{T}{n}e \). This solution also satisfies the constraint (31), and thus it is the optimal solution of the original problem (29) and (30).

**Corollary 1:** With the optimal solution, \( \left\|Q_{i_s,j_t}\right\| = 1/T \) for all \( i_s \) and \( j_t \), the minimum MSE \( E\{\|\Delta J\|^2\} \) is given by

$$E\{\|\Delta J\|^2\} = C^{NL} T^2 \sum_{i} \left( \frac{LM}{N^2} \right)^2 \left( \frac{\sigma_i^2 + \sigma_n^2}{\lambda_2} \right)^2,$$

independent of \( T \).

Since the channel estimation performance is independent of the number of samples \( T \), the use of the minimum \( T \) is preferred to guarantee the sparse design of \( Q \), i.e., simply we set \( T = L \). From Theorem 1, we now use the same absolute value of \( \rho/T \) \((0 < \rho \leq 1)\) for all elements of \( Q(I, J) \) and \( Q(J, I) \) for blind channel estimation, and \( Q \) is diagonally dominant for all such \( \rho \). Note that the resulting design from Theorem 1 is obtained under relaxed constraints, thus the loss of the cost function \( E\{\|\Delta J\|^2\} \) incurred by the relaxation will be discussed in [16].

### A. Sign determination of the precoder square matrix

In this section, we consider the problem of sign determination for \( Q(I, J) \). The signs for elements of \( Q(I, J) \) are irrelevant to the channel estimation performance since there are only square terms of \( \left\|Q_{i_s,j_t}\right\| \) in (32), whereas each sign determination of \( Q(I, J) \) can change the condition number of \( Q \). Since we require the inversion of \( W \) for data decoding, the condition number of \( W \) defined by the square root of that of \( Q \) affects the data decoding performance. From Condition 3, we have \( T^2 \) degree of freedom in sign determination because \( Q \) only have non-zero off-diagonal elements at \( I \times J \) where \( I \times J \) with its symmetry. To see how some factors affect the condition number, consider the following example of \( Q \) with \( N = 8, T = 2, I = \{4, 8\} \) and \( J = \{3, 7\} \):

\[
Q = I_N + \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}.
\]

Let us define \( Q' := Q - I_N \), then we introduce a sub-block matrix including non-zero elements of \( Q' \) as

\[
Q_{mn} := \begin{bmatrix}
0 & [Q']_{mn}^{n,m} \\
0 & 0 \\
\end{bmatrix}, \quad \text{for } m, n = 1, \ldots, T \tag{35}
\]

then we have a \( 2T \times 2T \) sub-matrix as

\[
\tilde{Q}' := \begin{bmatrix}
Q'_{11} & \cdots & Q'_{1T} \\
\vdots & \ddots & \vdots \\
Q'_{T1} & \cdots & Q'_{TT} \\
\end{bmatrix} \tag{36}
\]

For example, \( Q' \) of (34) is given by

\[
\tilde{Q}' := \begin{bmatrix}
0 & q_{i_1,j_1} & 0 & q_{i_1,j_2} \\
0 & q_{i_2,j_1} & 0 & q_{i_2,j_2} \\
\end{bmatrix} \tag{37}
\]

Let \( \lambda_i \) be the eigenvalue of \( \tilde{Q}' \) such that \( \lambda_1 \leq \cdots \leq \lambda_{2T} \). From the fact that the trace of \( Q' \) is zero in (37), we know that \( Q' \) has some positive and some negative eigenvalues, i.e., \( \lambda_i \leq 0 \) and \( 0 < \lambda_{i+1} \) for some \( i \). Note that \( Q' \) includes all non-zero eigenvalues of \( \tilde{Q}' \) and the eigenvalue of \( Q' \) is the sum of one and an eigenvalue of \( \tilde{Q}' \) as shown in (34). This yields that the smallest eigenvalue of \( Q' \) is \( 1 + \lambda_1 \) and the largest eigenvalue of \( Q \) is \( 1 + \lambda_{2T} \).

We now design an optimization problem for sign determination which minimizes the condition number of \( Q \) under some constraints:

\[
\min_{\{\lambda_1, \ldots, \lambda_{2T}\}} \frac{1 + \lambda_{2T}}{1 + \lambda_1} \tag{38}
\]

s.t. \( \text{tr}(Q') = \sum_i \lambda_i = 0 \), \( \text{tr}(Q' H^H) = \sum_i \lambda_i^2 = 2 \rho^2 \),

\[-1 \leq \lambda_1 \leq \cdots \leq \lambda_{2T} \tag{39}\]

where the second constraint is from Theorem 1, i.e., \( |q_{i_s,j_t}| = \rho/T \).

**Theorem 2:** Under the assumption that all \( \left\|Q_{i_s,j_t}\right\| \) are equal, the minimum condition number of \( Q \) is achieved when \( Q(I, J) \) is a scaled version of a unitary matrix.

**Proof:** See [16].

Theorem 2 provides a simple solution for sign determination, i.e., a unitary matrix with an equal absolute value for all its elements is given by Fast Fourier Transform (FFT) matrix or Hadamard matrix. Since Hadamard matrix \( H_T \in \mathbb{R}^{T \times T} \) requires easy operation of sign change, we design \( Q(I, J) = \overline{H_T} \) with \( 0 < \rho \leq 1 \). In this case, \( Q' \) is given by

\[
\tilde{Q}' = \rho T \overline{H_T} \otimes I_2, \tag{40}
\]

where \( \Pi \in \mathbb{R}^{T \times 2T} \) is some permutation matrix such that \( \Pi = \Pi_T \) and \( \otimes \) denotes the Kronecker product. Since \( \overline{Q}' \) is orthonormal and symmetric, its eigenvalues have modulus one, i.e., \( \pm 1 \) [18]. Then the eigenvalues of \( \tilde{Q}' \) comprise of \( \{\rho/\sqrt{T}, \rho/\sqrt{T}\} \) with the same multiplicity \( T \), and thus the condition number of \( Q \) is given by

\[
\chi(Q) = \frac{1 + \rho/\sqrt{T}}{1 - \rho/\sqrt{T}}. \tag{41}
\]
Note that we can change \( \rho \) up to \( \sqrt{T} \) for the positive semi-definiteness of \( Q \) from (41), while \( Q \) is diagonally dominant for \( 0 < \rho \leq 1 \). Increasing \( \rho \) toward \( \sqrt{T} \) can improve the channel estimation performance by reducing the objective function in (32) further. However, an increase of \( \rho \) entails an ill-conditioned square precoder \( Q \) as the denominator in (32) goes to zero. The numerical results also validate such a trade-off in \( \rho \).

IV. LINEAR PRECODING FOR BLIND ESTIMATION TO MULTIPLE USER CASE

To perform the blind channel estimation for MIMO-OFDM systems, we consider an equal absolute value \( \frac{\rho}{\sqrt{T}} \) for all elements of \( Q(I, J) \), \( 0 < \rho < \sqrt{T} \), and \( [Q]_{i,i} = 1 \). We now discuss the way to design a sparse precoder \( W \) resulting from the sparse square precoder matrix \( Q \) instead of computing \( W = Q^\dagger \) directly. That is, we first collect the non-zero off-diagonal elements of \( Q \) into a \( 2T \times 2T \) matrix, then obtain the square root of the \( 2T \times 2T \) matrix by the EVD. Next, we re-embed the square root of a matrix into the precoder matrix.

Note that \( Q' \) is a \( 2T \times 2T \) sub-matrix of \( Q \) including all its non-zero off-diagonal elements as in (36) and \( I_{2T} + Q' \) is positive semi-definite from Theorem 2. Define the square root of \( I_{2T} + Q' \) and its partitions as

\[
W = U\Sigma^\dagger = \begin{bmatrix}
W_{11} & \cdots & W_{1T} \\
\vdots & \ddots & \vdots \\
W_{T1} & \cdots & W_{TT}
\end{bmatrix},
\]

(42)

where \( I_{2T} + Q' = \bar{U}\Sigma\bar{U}^H \) by the EVD and \( W_{mn} \) is a \( 2 \times 2 \) partitioned matrix. Then, the precoder matrix \( W \) is defined by embedding \( W \) into the precoder matrix as

\[
W = \begin{cases}
\begin{bmatrix} W_{jm, jn} & W_{jm, jn} \\
W_{jm, jn} & W_{jm, jn} \end{bmatrix} = W_{mn} & \text{for } i_m \in I, j_n \in J, \\
\bar{W}_{ij} = \delta_{ij}(i - j) & \text{for } i, j \notin I \cup J.
\end{cases}
\]

(43)

Here, \( W \) obtained in (43) only mixes \( 2T \) subcarriers at \( I \cup J \) while leaving \( N - 2T \) subcarriers intact. For the subcarrier \( n \notin I \cup J \), we have a conventional \( M \times M \) MIMO system by collecting all received signals:

\[
\begin{bmatrix}
\hat{y}_1(n) \\
\vdots \\
\hat{y}_M(n)
\end{bmatrix} = \begin{bmatrix}
\hat{h}_{11}(n) & \cdots & \hat{h}_{1M}(n) \\
\vdots & \ddots & \vdots \\
\hat{h}_{M1}(n) & \cdots & \hat{h}_{MM}(n)
\end{bmatrix} \begin{bmatrix}
x_1(n) \\
\vdots \\
x_M(n)
\end{bmatrix} + \begin{bmatrix}
\tilde{n}_1(n) \\
\vdots \\
\tilde{n}_M(n)
\end{bmatrix}
\]

(44)

Therefore, such \( N - 2T \) subcarriers can be applied to data rate enhancing MIMO precoding based on the channel state information at the transmitter [15], and decoded using ML symbol detection with low complexity.\(^2\) For subcarriers \( n \in I \cup J \), the channel part is equalized with sub-optimal detection such as MMSE filtering.

V. NUMERICAL RESULTS

In this section, we provide some numerical results to evaluate the performance of the proposed method. We consider

\(^2\)When the precoder \( W \) is dense \([6],[7],[9]\), ML decoding suffers from high computational complexity. Thus, it requires sub-optimal detection (e.g., MMSE filtering) or additional coding schemes.
The proposed precoder linearly precodes only a small number of subcarriers, equalized by the inverse of the well-conditioned precoding matrix. Note that the proposed precoder is superior to the other precoders in the BER performance since the precoded channels are equalized by the inverse of the well-conditioned precoding matrix. Fig. 3 shows that the non-redundant precoding-based blind method has the trade-off between the performance of channel estimation and data decoding given by (29) and (41) w.r.t. $\rho$ when $Q_{i,j} = \rho/T$.

VI. CONCLUSIONS

We have proposed a new precoder design for blind channel estimation for MIMO-OFDM systems based on a sparse structure. The proposed precoder linearly precodes only a small number of subcarriers, blind channel estimation and is well conditioned for MMSE based joint symbol estimation. The proposed method can employ the data rate enhancing MIMO precoding for most of the subcarriers and exploit the optimal symbol detection with low complexity. Up to now, we have considered the block-wise operation for the proposed algorithm, but the algorithm can also be implemented in a symbol-by-symbol manner using sliding-window type sample covariance estimation techniques to track the slow fading channel.

REFERENCES


