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A Study on A New Method For Solving Interval Neutrosophic Linear Programming Problems

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Abstract

Neutrosophic set theory is a generalization of the intuitionistic fuzzy set which can be considered as a powerful tool to express the indeterminacy and inconsistent information that exist commonly in engineering applications and real meaningful science activities. In this paper an interval neutrosophic linear programming (INLP) model will be presented, where its parameters are represented by triangular interval neutrosophic numbers (TINNs) and call it INLP problem. Afterward, by using a ranking function we present a technique to convert the INLP problem into a crisp model and then solve it by standard methods.

Key words: Single Valued, Linear Programming Problem, Neutrosophic.
AMS classification: 90c05, 90c06, 90c90.

1 Introduction

A differential equation is a mathematical equation that relates some function with its derivatives. In applications, the functions generally represent physical quantities, the derivatives represent their rates of change, and the differential equation defines a relationship between the two. Because such relations are extremely common, differential equations play a prominent role in many disciplines including engineering, physics, economics, and biology. In pure mathematics, differential equations are studied from several different perspectives, mostly concerned with their solutions the set of functions that satisfy the equation. Only the simplest differential equations are solvable by explicit formulas; however, some properties of solutions of a given differential equation may be determined without finding their exact form. If a closed form expression for the solution is not available, the solution may be numerically approximated using computers. The theory of dynamical systems puts emphasis on qualitative analysis of systems described by differential equations, while many numerical methods have been developed to determine solutions with a given degree of accuracy.

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2 Preliminaries

This section recalls the necessary notions and definitions of NS theory on the real numbers line that can be used in this research.

Definition 2.1 A Neutrosophic Set (NS) N in a domain X (finite universe of objectives) can be represented by $T_N : X \to]0^-, 1^+[$, $I_N : X \to]0^-, 1^+[$ and $F_N : X \to]0^-, 1^+[$ such that $0^- \leq T_N(x) + I_N(x) + F_N(x) \leq 3^+ \ \forall x \in X$, where $T_N(x), I_N(x)$ and $F_N(x)$ denote the truth, indeterminacy, and falsity membership functions, respectively.

Definition 2.2 A single-valued neutrosophic set (SVNS) N in a domain X (finite universe of objectives) can be denoted as $N = \{x, T_N(x), I_N(x), F_N(x); x \in X\}$, where $T_N : X \to [0, 1]$, $I_N : X \to [0, 1]$ and $F_N : X \to [0, 1]$ are three maps in X that satisfy the condition $0 \leq T_N(x)+I_N(x)+F_N(x) \leq 3 \ \forall x \in X$. The numbers $T_N(x), I_N(x)$ and $F_N(x)$, are respectively the degrees of truth, indeterminacy and falsity membership of element x to N.

Definition 2.3 A neutrosophic number (NN) N is an extension of the fuzzy set on $\mathbb{R}$ such that the truth, indeterminacy and falsity membership functions could be defined as follows:

$$T_N(x) = \begin{cases} \frac{x-a^l}{a^m-a^l}, & a^l \leq x \leq a^m, \\ \frac{a^u-a^m}{a^u-x}, & a^m. \leq x \leq a^u, \\ 0, & \text{otherwise}, \end{cases}$$

$$I_N(x) = \begin{cases} \frac{a^m-x}{a^m-a^l}, & \delta a^l + (1-\delta)a^m \leq x \leq a^m, \\ \frac{x-a^m}{a^u-a^m}, & a^m \leq x \leq (1-\delta)a^m + \delta a^u, \\ \delta, & \text{otherwise}, \end{cases}$$

$$F_N(x) = \begin{cases} \frac{a^m-x}{a^m-a^l}, & a^l \leq x \leq a^m, \\ \frac{x-a^m}{a^u-a^m}, & a^m \leq x \leq a^u, \\ 1, & \text{otherwise}, \end{cases}$$

where $\delta \in (0, 1)$ is the maximum degree of indeterminacy and $a^l \leq a^m \leq a^u$.

The various functions of the single-valued neutrosophic number (SVNN) N are shown in Figure 1. Truth, indeterminacy andfalsity membership functions of N.

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Definition 2.4 The addition and subtraction operations between two SVNNs such as $N = [(a^l, a^m, a^u); \alpha_N,\ \delta_N, \beta_N]$ and $M = [(b^l, b^m, b^u); \alpha_M,\ \delta_M, \beta_M]$ could be defined as:

$$N + M = [(a^l + b^l, a^m + b^m, a^u + b^u); \alpha_N \wedge \alpha_M,\ \delta_N \vee \delta_M, \beta_N \vee \beta_M],$$
$$N - M = [(a^l - b^l, a^m - b^m, a^u - b^u); \alpha_N \wedge \alpha_M,\ \delta_N \vee \delta_M, \beta_N \vee \beta_M],$$

furthermore, the scalar multiplication is defined as:

$$kN = \begin{cases} \left[(ka^T, ka^m, ka^u); \alpha_N, \delta_N, \beta_N\right], k > 0, \\ \left[(ka^u, ka^m, ka^T); \alpha_N, \delta_N, \beta_N\right], k < 0. \end{cases}$$

Definition 2.5 Let N and M are two NNs The ranking orders of these two numbers will be as:

- If $L(N) > L(M)$ then N is bigger than M,
- If $L(N) < L(M)$ then N is smaller than M,
- If $L(N) = L(M)$ then N is equal to M.

Definition 2.6 Let X be a space of discourse, an interval neutrosophic set (INS) N through X taking the form $N = \{x, T_N(x), I_N(x), F_N(x); x \in X\}$ where $T_N(x), I_N(x), F_N(x) \subseteq [0, 1]$ and $0 \leq \text{Sup } T_N(x) + \text{Sup } I_N(x) + \text{Sup } F_N(x) \leq 3$ for all $x \in X$. $T_N(x), I_N(x)$ and $F_N(x)$ represent truth membership, indeterminacy membership, and falsity membership of x to N, respectively.

Definition 2.7 An interval neutrosophic number (INN) N is an extended version of the fuzzy set on $\mathbb{R}$ whose the truth, indeterminacy and falsity membership functions are given as follows:

$$T_N^L(x) = \begin{cases} \frac{x - a^T + h_N(a^T - x)}{a^m - a^T}, & a^T \leq x \leq a^m, \\ \frac{a^u - x + h_N(x - a^u)}{a^u - a^m}, & a^m \leq x \leq a^u, \\ 0, \text{ otherwise}, \end{cases}$$
$$T_N^U(x) = \begin{cases} \frac{x - a^l + h_N(a^m - x)}{a^m - a^l}, & a^l \leq x \leq a^m, \\ \frac{a^u - x + h_N(x - a^m)}{a^u - a^m}, & a^m \leq x \leq a^u, \\ h_N, \text{ otherwise}, \end{cases}$$

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where $T_N(x) = [T_N^L(x), T_N^U(x)]$,

$$I_N^L(x) = \begin{cases} \frac{a^m - x + h_N(x-a^m)}{a^m - a^l}, & \delta a^l + (1-\delta)a^m \le x \le a^m, \\ \frac{x - a^m + h_N(a^m - x)}{a^u - a^m}, & a^m \le x \le (1-\delta)a^m + \delta a^u, \\ \delta, \text{ otherwise}, \end{cases}$$

$$I_N^U(x) = \begin{cases} \frac{a^m - x + h_N(x-a^l)}{a^m - a^l}, & \delta a^l + (1-\delta)a^m \le x \le a^m, \\ \frac{x - a^m + h_N(a^u - x)}{a^u - a^m}, & a^m \le x \le (1-\delta)a^m + \delta a^u, \\ 1-\delta, \text{ otherwise}, \end{cases}$$

where $I_N(x) = [I_N^L(x), I_N^U(x)]$,

$$F_N^L(x) = \begin{cases} \frac{a^m - x + h_N(x-a^m)}{a^m - a^l}, & a^l \le x \le a^m \\ \frac{x - a^m + h_N(a^m - x)}{a^u - a^m}, & a^m \le x \le a^u \\ 1 - h_N, & \text{otherwise}, \end{cases}$$

$$F_N^U(x) = \begin{cases} \frac{a^m - x + h_N(x-a^l)}{a^m - a^T}, & a^T \le x \le a^m, \\ \frac{x - a^m + h_N(a^u - x)}{a^u - a^m}, & a^m \le x \le a^u, \\ 1, \text{ otherwise}, \end{cases}$$

where $F_N(x) = [F_N^L(x), F_N^U(x)]$ and $h_N = T_N^U(x) - T_N^L(x)$ such that $\delta \in (0,1)$ and $h_N \le \delta$.

Definition 2.8 An INS $N = [(a^l, a^m, a^u); [\alpha_N^l, \ \alpha_N^u], [\delta_N^l, \ \delta_N^u], [\beta_N^l, \ \beta_N^u]]$ will be reduced to the NS if $\alpha_N^l = \alpha_N^u$, $\delta_N^l = \delta_N^u$ and $\beta_N^l = \beta_N^u$.

Definition 2.9 Let $N = [(a^l, a^m, a^u); [\alpha_N^l, \alpha_N^u], [\delta_N^l, \ \delta_N^u], [\beta_N^l, \ \beta_N^u]]$ and $M = [(b^l, b^m, b^u); [\alpha_M^l, \ \alpha_M^u], \ [\delta_M^l, \delta_M^u], [\beta_M^l, \ \beta_M^u]]$ are two INNs. The addition and subtraction operations for these two INNs are defined as follows:

$$\begin{aligned} N + M =& [(a^l + b^l, a^m + b^m, a^u + b^u); [\alpha_N^l + \alpha_M^l - \alpha_N^l\alpha_M^l, \alpha_N^u + \alpha_M^u - \alpha_N^u\alpha_M^u], \\ & [\delta_N^l\delta_M^l, \delta_N^u\delta_M^u], [\beta_N^l\beta_M^l, \beta_N^u\beta_M^u]], \\ N - M =& [(a^l - b^l, a^m - b^m, a^u - b^u); [\alpha_N^l + \alpha_M^l - \alpha_N^l\alpha_M^l, \alpha_N^u + \alpha_M^u - \alpha_N^u\alpha_M^u], \\ & [\delta_N^l\delta_M^l, \delta_N^u\delta_M^u], [\beta_N^l\beta_M^l, \beta_N^u\beta_M^u]]. \end{aligned}$$

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3 Proposed Interval Neutrosophic Linear Programming Method

In this section, by using a new ranking function for interval neutrosophic numbers we suggest a new method for solving INLP problems. The basis of our work will be presented as follows:

Step 1. Insert the INLP problem with triangular interval neutrosophic numbers.

Step 2. By using the following method convert the INLP problem to the crisp model. In order to compare any two triangular INNs based on the proposed ranking function, let $N = [(a^l, a^m, a^u); [\alpha_N^l, \alpha_N^u], [\delta_N^l, \delta_N^u], [\beta_N^l, \beta_N^u]]$ be a symmetric interval neutrosophic number, where $[\alpha_N^l, \alpha_N^u], [\delta_N^l, \delta_N^u]$ and $[\beta_N^l, \beta_N^u]$ are respectively the truth, indeterminacy, and falsity membership degrees of N. Also a^l, a^m and a^u are respectively the lower, median, and upper bounds for N. The ranking function for the interval neutrosophic number N will be defined as follows:

$$L(N) = \frac{1}{4}[a^l + a^u + 2a^m] + (\overline{\alpha}_N - \overline{\delta}_N - \overline{\beta}_N), \tag{1}$$

where $\overline{\alpha}_N = \dfrac{\alpha_N^l + \alpha_N^u}{2}$, $\overline{\delta}_N = \dfrac{\delta_N^l + \delta_N^u}{2}$ and $\overline{\beta}_N = \dfrac{\beta_N^l + \beta_N^u}{2}$.

Moreover, we have : $N \geq \tilde{0}$ if $\dfrac{a^l + a^u + 2a^m}{4} \geq 0$.

Step 3. By applying the previous ranking function, convert each triangular INN to a crisp number. This leads to convert the INLP problem to the crisp model.

Step 4. Solve the crisp model using the standard simplex method to achieve the optimal solution.

4 Conclusion

In this paper, by considering an LP problem based on INNs we have presented a new linear programming model. In this model in view of considering the truthiness, indeterminacy, and falsity degrees we can cover all aspects of real daily life circumstances. It should be noticed there is no necessity the values of these degrees be crisp values. In this respect, we proposed a ranking function that is capable of converting every triangular interval neutrosophic number to its equivalent crisp value. Subsequently, every INLP problem could be converted to the crisp model where can be solved by standard methods easily. The proposed model indicates more simplicity applicability and more efficiency in comparison with other existing models.

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