Isosurfaces computation for approximating boundary surfaces within three-dimensional images

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Abstract. In the visualization of three-dimensional (3D) images, specific isosurfaces are usually extracted from 3D images and used to represent (approximate) boundary surfaces of certain structures within 3D images. In order to well approximate the boundary surfaces of these structures, it is important to determine a good isosurface for each boundary surface. An isosurface is said to be a good isosurface of a boundary surface if it can approximate the boundary surface with the smallest error under certain error measuring criteria. The mathematical model describing the approximation problem of a boundary surface by isosurfaces is constructed and studied. The method used to deduce good isosurfaces for the boundary surfaces within 3D discrete images is presented. The proposed method is illustrated by examples with different real 3D biomedical images. © 2007 SPIE and IS&T.

1 Introduction

Boundary surfaces of the structures within three-dimensional (3D) images are a class of important features for analyzing and understanding 3D images. Therefore, detecting and extracting boundary surfaces from 3D images has long been an important topic. Several classes of methods have been proposed to extract or approximate the boundary surfaces within 3D images. They include methods for reconstructing surface from contours, 2D deformable surface techniques, an algorithm for extracting polygonal boundary surfaces from 3D images, and an isosurface extraction method. In these methods, boundary surfaces are approximated by a number of simple surface patch—polygons, and eventually, polygonal surface models of boundary surfaces are generated. The isosurface extraction algorithm is mainly suitable for extracting those boundary surfaces that are located between such objects and backgrounds that have distinctive differences in gray values. Such boundary surfaces include the surface of bone structures within 3D CT images, the surface of machines within 3D industrial CT images, the surface of many anatomical structures within 3D medical images, etc. Although the isosurface extraction algorithm (the Marching-Cubes al-
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The isosurface extraction algorithm is widely studied and applied in medical visualization and computer graphics. With the isosurface extraction algorithm, a specific isosurface is extracted from 3D images and used to represent (approximate) the boundary surface of a structure within 3D images. However, in order to well approximate or extract a boundary surface, the following problem should be considered: Given a boundary surface contained within a 3D image, what is its suitable approximative isosurface and how can we deduce such a suitable isosurface from the discrete 3D image?

This is one of the basic problems in the visualization of 3D images. By solving this problem, it is possible to compute accurate isosurfaces to approximate boundary surfaces. This paper will tackle this problem. For the convenience of discussion, in this paper, the isosurface that can approximate one boundary surface with the smallest error under certain error measuring criteria is called the good isosurface of the boundary surface (subordinating to the given error measuring criteria). We assume that boundary surfaces within 3D discrete images could be well approximated by certain isosurfaces. Under this presumption, we study how to deduce or select good isosurfaces from a discrete 3D image so as to approximate well those boundary surfaces within the 3D image.

Recall that threshold selection techniques developed in two-dimensional (2D) image processing are usually used to segment 3D images. Their aim is to seek suitable thresholds to separate voxels of objects from voxels of background. In many cases, more than one good threshold exists in the sense of correctly separating voxels of object from those of background. For example, for a binary 3D image with the gray values 30 and 100, any value between 30 and 100 is a suitable threshold. In contrast to the threshold selection techniques, in this paper, the boundary surface of each structure within 3D images is treated as the continuous implicit surface, and we try to deduce a good isosurface from 3D images to approximate the continuous implicit boundary surface. The objectives of threshold selection and isosurface computation are different from each other. Therefore, many good thresholds might not be the isovalue of the good isosurface we try to compute. The conventional threshold selection techniques usually select thresholds based on the histogram of gray values of the whole 3D image or on the statistic analysis of gray values of the whole 3D image. However, the good isosurface of a given boundary surface mainly correlates to the boundary surface rather than to the gray values of those grid points far from the boundary surface. Thus, for a given boundary surface, it is better to deduce its good isosurface from its attribute values rather than from the gray values of the whole 3D images. This is our basic idea to design the algorithms in this paper.

Recently, in the visualization of 3D images, two important methods are proposed to select “significant” or “meaningful” isosurfaces from 3D images. Bajaj et al. proposed a method to compute and display such an isosurface that has the largest average gradient value and/or the largest area among all isosurfaces. Pekar et al. presented a computationally efficient method to compute and display such an isosurface that has the largest average gradient value among all isosurfaces. Usually, the significant or meaningful isosurface is related closely to the boundary surface of certain structures within 3D images. However, these methods cannot be used to compute multiple different suitable isosurfaces from 3D images for multiple boundary surfaces within the 3D images. Besides, there is no analytical mathematical analysis to demonstrate that the significant or meaningful isosurface is exactly the good isosurface of the continuous implicit boundary surface.

In this paper, we will directly study the approximation problem of boundary surfaces by suitable isosurfaces. We construct the mathematical model to describe the approximation problem of a boundary surface by a good isosurface. By solving the model, we derive the analytical expression of the isovalue of the good isosurface. Based on the analytical formula, we develop a strategy to compute such an isovalue from 3D discrete images. The proposed method is applied to many real-world biomedical images and real-world industrial images and is illustrated by examples with these 3D images. The proposed method can overcome some drawbacks existing in the isosurface selection methods and the conventional threshold selection techniques.

The structure of the paper is as follows. Section 2 describes the mathematical theory on which the algorithms in this paper are based. In Sec. 3, we propose a strategy to compute or estimate the isovalue of a good isosurface from 3D discrete images. In Sec. 4, some examples with many real 3D images are displayed that illustrate the proposed method. In Sec. 5, we compare the proposed method to the related methods. Conclusions are drawn in Sec. 6.

2 Mathematical Model

In this section, we explain the mathematical theory on which the algorithms in this paper are based. It is known that, in the visualization of 3D images, each 3D discrete image can be treated as the discrete sampling of a 3D continuous function [represented by $f(x,y,z)$] at grid points of a 3D regular grid, as shown in Fig. 1. Correspondingly, boundary surfaces of the structures within the 3D image are continuous implicit surfaces contained within the continuous sampling region of the 3D image. Here, eight adjacent grid points form a cube, and all such cubes constitute the continuous sampling region of the 3D image. In 3D images, different structures usually correspond to different gray intensities. Therefore, their boundary surfaces belong to step-like boundary surfaces and can be described as specific continuous zero-crossing surfaces with high gradient values. Mathematically, continuous implicit step-like boundary surfaces within $f(x,y,z)$ can be represented as follows:

$$f(x,y,z)$$
reduce noise in 3D images and to provide a multiscale representation within the 3D image. 20 Therefore, generally, 3D edge points that are detected from 3D grid points by 3D gradient threshold selection. 20, 23 In Eq. (1), we smooth the Gaussian function $G(x, y, z, \sigma)$ with the scale $\sigma$ and represent the Laplacian function and gradient magnitude function of $g(x, y, z)$, respectively. $T$ is a predetermined gradient threshold, which could be selected by using the same method as is used in the edge detection for the gradient threshold selection. 20, 21 In Eq. (1), we smooth $f(x, y, z)$ with the Gaussian function $G(x, y, z, \sigma)$ so as to reduce noise in 3D images and to provide a multiscale frame for computing derivatives from discrete 3D images. 31 Sonka et al. pointed out that this will inevitably fail to detect the points satisfying $\nabla^2 g(x, y, z)=0$ from 3D grid points of 3D discrete images. 20 Therefore, generally, 3D edge points that are detected from 3D grid points by 3D edge detection techniques 22, 24, 25 do not belong to the continuous implicit boundary surfaces. The continuous implicit boundary surfaces are continuous surfaces located between adjacent grid points. In this research, we try to deduce good isosurfaces from a discrete 3D image to approximate or represent those continuous implicit boundary surfaces within the 3D image.

Suppose that $S(x, y, z)$ represents a given continuous implicit boundary surface contained within the continuous 3D image $g(x, y, z)$ and $I(k)$ represents an isosurface of $g(x, y, z)$ with the isovalue $k$, defined as $I(k) = \{(x, y, z): g(x, y, z) = k\}$. In what follows, based on two error-measuring criteria, we introduce two good isosurfaces for $S(x, y, z)$.

$$\begin{cases} \nabla^2 g(x, y, z) = 0, \\ \|\nabla g(x, y, z)\| \geq T, \end{cases}$$

where $g(x, y, z) = f(x, y, z) * G(x, y, z, \sigma)$ represents the convolution between $f(x, y, z)$ and the Gaussian function $G(x, y, z, \sigma)$ with the scale $\sigma$ and represent the Laplacian function and gradient magnitude function of $g(x, y, z)$, respectively. $T$ is a predetermined gradient threshold, which could be selected by using the same method as is used in the edge detection for the gradient threshold selection. 20, 21 In Eq. (1), we smooth $f(x, y, z)$ with the Gaussian function $G(x, y, z, \sigma)$ so as to reduce noise in 3D images and to provide a multiscale frame for computing derivatives from discrete 3D images. 31 Sonka et al. pointed out that this will inevitably fail to detect the points satisfying $\nabla^2 g(x, y, z) = 0$ from 3D grid points of 3D discrete images. 20 Therefore, generally, 3D edge points that are detected from 3D grid points by 3D edge detection techniques 22, 24, 25 do not belong to the continuous implicit boundary surfaces. The continuous implicit boundary surfaces are continuous surfaces located between adjacent grid points. In this research, we try to deduce good isosurfaces from a discrete 3D image to approximate or represent those continuous implicit boundary surfaces within the 3D image.

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### 2.1 Graylevel-Based Good Isosurface of the Boundary Surface

The square error between gray values of $S(x, y, z)$ and the gray value (isovalue) of $I(r)$ can be described by $\int_{S(x, y, z)} [g(x, y, z) - r]^2 dS$, which represents the surface integral of the function $[g(x, y, z) - r]^2$ over the boundary surface $S(x, y, z)$.

In this paper, the isosurface whose gray value (i.e., its isovalue) approximates the gray values of $S(x, y, z)$ with the least (square) error is called the graylevel-based good isosurface of $S(x, y, z)$. In many cases, the graylevel-based good isosurface of boundary surfaces can well separate voxels belonging to an object from voxels belonging to the background and therefore can be applied in the segmentation of 3D images. 11-14 The isovalue of the graylevel-based good isosurface of $S(x, y, z)$ is the solution of the following optimization problem:

$$\min_r \int_{S(x, y, z)} [g(x, y, z) - r]^2 dS, \quad r \in (0, \infty).$$

Let $F(r) = \int_{S(x, y, z)} [g(x, y, z) - r]^2 dS$. The solution of Eq. (2) satisfies $F'(r) = 0$. Thus, it is easy to see that the solution of the optimization problem (2) is as follows:

$$r = \left( \frac{\int_{S(x, y, z)} g(x, y, z) dS}{\int_{S(x, y, z)} dS} \right)^{1/2}.$$  

Equation (3) indicates that the mean of gray values of $S(x, y, z)$ determines the isovalue of the graylevel-based good isosurface of $S(x, y, z)$.

### 2.2 Distance-Based Good Isosurface of the Boundary Surface

Suppose that the point $P \in S(x, y, z)$. Let $\text{dist}[P, I(r)]$ and $\text{dist}[S(x, y, z), I(r)]$ be defined as follows, respectively:

$$\text{dist}[P, I(r)] = \min \{\|q - P\|: q \in I(r)\},$$

$$\text{dist}[S(x, y, z), I(r)] = \left\{ \begin{array}{l} \int_{S(x, y, z)} \text{dist}[P, I(r)] dS, \\ \int_{S(x, y, z)} dS \end{array} \right\},$$

where $\|q - P\|$ represents the Euclidean norm of the vector $q - P$ [i.e., $\|q - P\| = (x_1^2 + y_1^2 + z_1^2)^{1/2}$ for each point $x = (x_1, y_1, z_1)$], and $\int_{S(x, y, z)} \text{dist}[P, I(r)] dS$ represents the surface integral of the function $\text{dist}[P, I(r)]$ over the boundary surface $S(x, y, z)$ [note that $P \in S(x, y, z)$]. Then $\text{dist}[P, I(r)]$ represents the distance from $P$ to $I(r)$, and $\text{dist}[S(x, y, z), I(r)]$ represents the average value of all distances $\{\text{dist}[P, I(r)]: P \in S(x, y, z)\}$. Here, $\text{dist}[S(x, y, z), I(r)]$ actually reflects the average distance from the points of $S(x, y, z)$ to $I(r)$. Thus, it can be used to describe the approximation error between $S(x, y, z)$ and $I(r)$. In this paper, the isosurface $I(r)$ is said to be able to approximate the boundary surface $S(x, y, z)$ with a small distance error if $\text{dist}[S(x, y, z), I(r)]$ has a small value. The isosurface that
can approximate $S(x,y,z)$ with the least distance error is called the distance-based good isosurface of $S(x,y,z)$. The distance-based good isosurface of boundary surfaces can be used in the registration of 3D images, where 3D images or boundary surfaces are matched based on geometrical features of boundary surfaces and the geometrical features can be computed from the distance-based good isosurfaces of boundary surfaces.\textsuperscript{26,27} The isovalue of the distance-based good isosurface of $S(x,y,z)$ is the solution of the following optimization problem:

$$
\min_r \left\{ \int_{S(x,y,z)} \text{disf}[p,I(r)]dS \right\}, \quad r \in (0, \infty).
$$

(4)

Based on the discussion in the appendix, the solution of the optimization problem (4) is, approximately, as follows:

$$
r = \left[ \frac{1}{\int_{S(x,y,z)} \|\nabla g(x,y,z)\|g(x,y,z)dS} \right] \left[ \frac{1}{\int_{S(x,y,z)} \|\nabla g(x,y,z)\|dS} \right].
$$

(5)

Equation (5) shows that the isovalue of the distance-based good isosurface of $S(x,y,z)$ actually is the weighted average value of the gray values of $S(x,y,z)$. Here, for each gray value $g(x,y,z)$, its weight is $1/\|\nabla g(x,y,z)\|^2$.

Equations (3) and (5) reveal that isovalue of two good isosurfaces of a boundary surface are uniquely determined by the attribute values of the boundary surface. In this paper, we mainly consider the computation of isovalue of the graylevel-based good isosurface and the distance-based good isosurface for the boundary surface within 3D discrete images. The difference between these two good isosurfaces will be explained in Sec. 5 in detail.

3 Computation

In this section, based on Eqs. (3) and (5), we design algorithms to deduce good isosurfaces for the boundary surfaces within discrete 3D images. Equations (3) and (5) show that the isovalue of good isosurfaces of $S(x,y,z)$ are the average value or weighted average value of gray values of $S(x,y,z)$. Generally, we do not know $S(x,y,z)$. However, according to the statistical property of the average value, if we could compute many discrete samplings of $S(x,y,z)$ [here, the discrete samplings include gray values and gradient values of discrete sampling points of $S(x,y,z)$] from 3D discrete images, we can well deduce isovalue for good isosurfaces of $S(x,y,z)$ based on these computed discrete samplings. On the basis of this fact, a computational strategy is proposed to compute isovalue from discrete 3D images and consists of the following two steps:

1. Compute discrete samplings of boundary surfaces from 3D discrete images. Here, the discrete samplings refer to discrete sampling points of boundary surfaces as well as gray values and gradient values of these sampling points.

2. Deduce isovalue of good isosurfaces based on these discrete samplings.

In the following sections we will explain each step in detail.

3.1 Computation of Discrete Samplings of Boundary Surfaces from 3D Discrete Images

As shown in Fig. 1, boundary surfaces within each 3D image are continuous implicit surfaces contained within the continuous sampling region of the 3D image. Thus, they will divide the set of all cubes into two categories: the set of edge-cubes, referring to the cubes that are intersected by boundary surfaces, and the set of cubes that are not intersected by any boundary surface. The boundary surfaces are contained within the set of all edge-cubes. Therefore, we can compute a discrete sampling of boundary surfaces from the 3D discrete image by following these three steps:

1. Compute gradient values and Laplacian function values for all grid points of the 3D discrete image. Those methods used in 3D edge detection techniques for estimating gradient values and Laplacian function values can be applied.\textsuperscript{24,25}

2. Detect edge-cubes from the 3D discrete image. Each cube has twelve edges, and each edge-cube contains at least three edges intersected by the boundary surface. Based on this fact, we proposed a method to detect edge-cubes from the 3D discrete image in Refs. 6 and 28. In this method, we first mark those edges intersected by the boundary surface. Based on whether one cube contains at least three edges intersected by the boundary surface, we can then detect edge-cubes from the 3D image.

3. In each edge-cube, compute the points of intersection between boundary surfaces and twelve edges of the edge-cube, and compute gray values and gradient values of these intersecting points. For the point of intersection between a boundary surface and an edge, its position (or gray value and gradient value) can be computed by linearly interpolating the positions (or gray values and gradient values) of two vertices of the edge. See Refs. 28 and 29 for a detailed description.

Consequently, the set of points of intersection between boundary surfaces and all edge-cubes constitutes the discrete sampling points of boundary surfaces. Due to noise, a very small number of edge-cubes might not be detected. However, usually those lost edge-cubes contribute very little, and hence missing them does not harm the validity of those computed discrete sampling points. Discrete sampling points of boundary surfaces are obtained by linearly interpolating the adjacent grid points and are different from 3D edge points detected by various 3D edge detecting techniques.

3.2 Deducing Isovalues from Discrete Samplings of Boundary Surfaces

Suppose that the discrete samplings of boundary surfaces computed from the 3D discrete image are as follows:
We try to deduce isovalues of good isosurfaces from $\Omega$. Based on whether several boundary surfaces are contained within the 3D image, we will discuss the computation of isovalues in two different cases, respectively.

First, assume that in the 3D image, only one boundary surface exists. Then based on Eqs. (3) and (5), the isovalue of the graylevel-based good isosurface and the isovalue of the distance-based good isosurface can be directly estimated as follows, respectively:

$$
\Omega = \{(x_i, y_i, z_i, m_i, n_i)| m_i = g(x_i, y_i, z_i), n_i = \| \nabla g(x_i, y_i, z_i) \|, i = 1, 2, \ldots, n \}.
$$

Here, $r_1$ is the mean of gray values of all discrete sampling points of the boundary surface, and $r_2$ is the weighted average value of gray values of all discrete sampling points of the boundary surface.

Second, assume that several boundary surfaces are contained within the 3D image. In this case, we need to compute the good isosurface for each boundary surface. Since discrete samplings computed from the 3D image belong to multiple different boundary surfaces and generally it is difficult to separate discrete samplings belonging to different boundary surfaces from each other, we cannot deduce the isovalue of the distance-based good isosurface from $\Omega$ for each boundary surface. Here we consider only the computation of multiple graylevel-based good isosurfaces from $\Omega$.

It is known that if one boundary surface could be well approximated by a specific isosurface, then gray levels of points lying on the boundary surface usually cluster together around their mean. Thus, when several boundary surfaces contained within the 3D image all can be well approximated by different isosurfaces respectively, we can observe the following facts:

- Gray values of discrete sampling points of each boundary surface will cluster together and manifest themselves as one of the main clusters in the histogram of DSPBS. Here, histogram of DSPBS is an abbreviation of “histogram of gray values of discrete sampling points of the boundary surfaces computed from the 3D image.”

- Usually, in the histogram of DSPBS, except for several main clusters corresponding to different boundary surfaces, there still exist many other small clusters or small peaks induced by noise or small details. In Fig. 2, such a typical example is shown, where a histogram of DSPBS containing three different objects is displayed. In the histogram, three main clusters exist, and each one corresponds to a boundary surface. In addition, some small peaks exist as well.

- Discrete sampling points induced by noise and small details usually occupy only a very small percentage in

| Table 1 The isovalue of graylevel-based good isosurface (GBGI) and the isovalue of distance-based good isosurface (DBGI) computed from six 3D images [a binary 3D image containing a solid cube (Cube), a binary 3D image containing a solid sphere (Sphere), a 3D CT image of the dry skull (Skull), a 3D CT image of the head (Head bone), a 3D electronic microscopic image of the dendrite (Dendrite), and a 3D MRI image of a pumpkin (Pumpkin)]. |
|-----------------|--------|--------|-----------|----------|--------|--------|
| Isovalue of GBGI | 57.06  | 57.19  | 72.63     | 133.81   | 104.9  | 47.4   |
| Isovalue of DBGI | 56.52  | 57.05  | 70.19     | 104.95   | 104.4  | 46.4   |
total discrete sampling points computed from the 3D image. Thus, in the histogram of DSPBS, although noise or small objects might affect slightly the shape of each main cluster, the position of the peak of each main cluster is seldom changed or its change is very small. Even if there are some noise and small details in the 3D image, the positions of peaks of main clusters in the histogram are comparatively stable.

Recall that, in this paper, the 3D image is smoothed by a Gaussian function. Thus, without loss of generality, we can assume that, if one boundary surface could be well approximated by a specific isosurface, then the distribution of gray values of the boundary surface follows the Gaussian distribution. Consequently, the mean of gray values of the boundary surface is just the gray level at the peak of the main cluster formed by the gray values of the boundary surface. Based on this analysis, the means of gray values of different boundary surfaces actually can be computed by detecting directly peaks of main clusters from the histogram of DSPBS. Here, peaks can be detected from the histogram by using the interactive method or by using the automatic peak detection method developed in 2D image processing. Thus, in the histogram of DSPBS, gray values at peaks of main clusters determine isovalue of graylevel-based good surfaces of different boundary surfaces. Although we assume the Gaussian distribution, we have not found the inexact matching of the real world to the Gaussian ideal to limit the application of our technique.

4 Experimental Results

We first give a qualitative analysis on the effect of \( T \) on isovalue of two good isosurfaces. In 3D images, usually boundary surfaces have high gradient values but other regions have very low gradient values. Thus, there is a wide range in which any value can be selected as a suitable gradient threshold for separating boundary surfaces from 3D images. This implies that, when different gradient thresholds are selected in the wide range, discrete samplings of boundary surfaces computed from 3D images have little change. Correspondingly, the isovalue of two good isosurfaces have little change as well. These facts are illustrated in Fig. 3. In Fig. 3, we show the changing trend of isovalues of two good isosurfaces when the gradient threshold \( T \) is changed. Here, isovalue of two good isosurfaces are computed from one 3D CT image of the skull. In Fig. 3, the solid line \( l \) represents the changing trend of the isovalue of the graylevel-based good isosurface, and the dashed line \( L \) represents the changing trend of the isovalue of the distance-based optimal isosurface. We can observe that when the gradient threshold \( T \) is selected in the wide range from 600 to 3800, isovalue of two good isosurfaces have little change. Specifically, the isovalue of the graylevel-based good isosurface is much more robust with respect to the change of gradient threshold \( T \). The reason that the computed isovales have certain robust properties is because these isovales are deduced by computing the average value or weighted average value of the gray values of discrete sampling points of the boundary surfaces. For the same reason, the computed isovales have certain robust properties with respect to Gaussian noise.

Subsequently, we present some experimental results to illustrate the proposed method. Three cases are considered, respectively. First, we deduce the good isosurface from those 3D images in which only one boundary surface exists. Six 3D images are considered, respectively: a binary 3D image that contains a solid cube and has gray values 20 and 100, a binary 3D image that contains a solid sphere and has gray values 20 and 100, a 3D CT image of a dry skull, a 3D CT image of a head, a 3D electron microscope image of a dendrite, and a 3D MRI image of a pumpkin. Although the 3D CT image of the head contains two boundary sur-

Fig. 4 Discrete sampling points of boundary surfaces computed from six 3D images.

Fig. 5 Histograms of gray levels of six 3D images (above), and histograms of gray values of discrete sampling points shown in Fig. 4 (below).

Fig. 6 Graylevel-based good isosurfaces (above) and distance-based good isosurfaces (below) extracted from the same four 3D images (CT image of a dry skull, CT image of a head, electron microscope image of a dendrite, and MRI image of a pumpkin).

Fig. 7 Graylevel-based good isosurfaces extracted from three 3D industrial CT images.
faces (skin surface and bone surface), by selecting a higher gradient threshold \( T \) in Eq. (1), we could extract discrete sampling points of only bone surface from the 3D image. The proposed method is used to deduce both graylevel-based good isosurface and distance-based good isosurface from these 3D images. Discrete sampling points of the boundary surface computed from the six 3D images are shown in Fig. 4. In Fig. 5, above are histograms of gray values of the six 3D images, and below are histograms of gray values of the discrete sampling points shown in Fig. 4.

It can be observed that the histogram of DSPBS contains one main cluster, and the main cluster distributes in a narrow region. Thus, based on Eq. (7), isovalues of two good isosurfaces could be computed from the six 3D images, respectively. These computed isovalues are shown in the Table 1. In Fig. 6, above are graylevel-based good isosurfaces computed from four 3D images (3D CT image of a dry skull, 3D CT image of a head, 3D electron microscope image of a dendrite, and 3D MRI image of a pumpkin), and below are the corresponding distance-based good isosurfaces computed from the same four 3D images. In Fig. 7, graylevel-based good isosurfaces extracted from three 3D industrial CT images are displayed, respectively. Table 1 shows that, sometimes, for the same boundary surface, a large difference exists between the isovalues of its two good isosurfaces. From the histograms shown in Fig. 5 (above), we can see that there are four 3D images in which voxels belonging to the object of interest occupy only a very small percentage of the whole 3D image. Consequently, voxels of the object of interest cannot be “recognized” obviously from the histogram of gray levels of each 3D image. In these cases, conventional threshold selection techniques, which are based on the histogram of the whole image, cannot work. However, based on the proposed method, discrete sampling points of the boundary surface of the object of interest within these 3D images could manifest as a main cluster in the histograms [see the histograms shown in Fig. 5 (below)], and therefore the isovalue of the good isosurface is comparatively easy to compute.

Next, we deduce the graylevel-based good isosurfaces from the 3D images in which several boundary surfaces are contained. In this case, we need to compute a good isosurface for each boundary surface. Three 3D images are considered: a 3D CT image of a child’s head, a 3D CT image of a leg, and a 3D CT image of a foot. Each image contains at least two anatomical structures. In Fig. 8, discrete sampling points of boundary surfaces computed from three 3D images are shown. Each set of discrete sampling points belongs to at least two boundary surfaces. In Fig. 9, above are the histograms of gray levels of three different 3D images, and below are the histograms of DSPBS. We can see that the histograms of gray levels of each 3D image show a broad valley or very unequal peaks in shape. In particular, the bone in each image occupies only a very small percentage of the whole 3D image, and therefore voxels of the bone cannot be recognized from the histogram of gray levels of the whole 3D image. For such 3D images, classical threshold selection techniques that are based on the histogram of the whole image cannot work. However, in the histograms of DSPBS, each boundary surface corresponds to a main cluster. Therefore, it is comparatively easy to determine the graylevel-based good isosurface for each boundary surface. Multiple graylevel-based good isosurfaces extracted from three 3D images are shown in Figs. 10–12, respectively.

Last, we consider 3D images that contain several boundary surfaces, but only partial boundary surfaces could be
well approximated by specific isosurfaces. We try to compute graylevel-based good isosurfaces for the partial boundary surfaces. To the best of our knowledge, the conventional threshold selection techniques and the existing isosurface selection techniques are difficult to use to accomplish such a task. Based on the analysis in Sec. 3.2, we know that if one boundary surface could be well approximated by a specific isosurface, then gray values of discrete sampling points of the boundary surface will cluster together and manifest as one main cluster in the histogram. Thus, in the histogram of DSPBS, by detecting peaks of main clusters distributed in a narrow region, we can still determine the isovalues of graylevel-based good isosurfaces for the partial boundary surfaces. Consider a 3D MRI image of a head and a 3D MRI image of an orange. Each 3D image contains more than one boundary surface. In the former 3D image, the surface of skin could be well approximated by certain isosurfaces, but surfaces of other soft tissues cannot be well approximated by any isosurface. In the latter 3D image, the pericarp of an orange could be well approximated by certain isosurfaces, but the other boundary surfaces contained cannot be well approximated by any isosurface. We note that in the histograms of DSPBS shown in Fig. 13(a) and 13(b), there is a main cluster distributed in a narrow region in each histogram. By detecting the peak from the first cluster of the histogram shown in Fig. 13(a) and detecting the peak from the main cluster of the histogram shown in Fig. 13(b), we can obtain the isovalues of the graylevel-based good isosurfaces for the surface of skin and the pericarp of orange. These graylevel-based good isosurfaces are shown in Fig. 13(c) and 13(d), respectively.

5 Comparison

We first compare the graylevel-based good isosurface and the distance-based good isosurface. These are derived based on different optimization criteria. If gray values or gradient values of discrete sampling points of one boundary surface have comparatively homogeneous values, then the isovalues of two different good isosurfaces are nearly equal. However, in other cases, for the same boundary surface, a great difference might exist between isovalues of two different good isosurfaces. For example, Table 1 shows that isovalues of two different good isosurfaces, which are computed from the 3D CT image of a head (8-bits gray image) are 133.81 and 104.95, respectively. In an 8-bit gray image, this is a significant difference. Meanwhile, the obvious difference can be observed from the extracted graylevel-based good isosurface and the distance-based good isosurface, which are shown in Figure 6(d) (see the eye socket and neck bone of the two good isosurfaces). Figure 6(d) shows that many holes appearing on the graylevel-based good isosurface are filled in by the distance-based good isosurface. Consider another example, in Fig. 14, where two different good isosurfaces extracted from the same 3D image shown in Fig. 15 are displayed, respectively. The 3D image shown in Fig. 15 contains a boundary surface with varying gray values and varying gradient values. Figure 14 shows that on the extracted graylevel-based good isosurface [Fig. 14(a)], the distortion exists in two local regions encircled by two circles. However, on the extracted distance-based good isosurface [Fig. 14(b)], such distortion is greatly improved. Here, we give an explanation of the phenomena in Fig. 6(d) and Fig. 14. Because of noise and the high gradient threshold , in the proposed method, some edge-cubes (and therefore some discrete sampling points with small gradient values) might not be detected. Thus, the discrete sampling points of boundary surfaces with small gradient values have less contribution to the deduced isovalues of the graylevel-based good isosurface. Consequently, the corresponding graylevel-based good isosurface will exhibit some distortions or will have some small holes in the local region where gradient values are low. Conversely, when computing the isovalues of one distance-based good isosurface, each discrete sampling point is assigned a weight . Therefore, the discrete sampling points with small gradient values will have a larger contribution to the isovalues. Consequently, the corresponding distance-

Fig. 12 Two graylevel-based good isosurfaces extracted from a 3D CT image of a foot. (a) Graylevel-based good isosurface of the skin surface. (b), (c) Graylevel-based good isosurface of the foot bone (different views).

Fig. 13 (a), (b) Histograms of gray values of discrete sampling points of boundary surfaces computed from two 3D images. (c), (d) Graylevel-based good isosurfaces of partial boundary surfaces within these two 3D images.
based good isosurface can partially decrease distortions and small holes in the local region where gradient values are low. Generally, it is difficult to deduce multiple distance-based good isosurfaces from a 3D image for multiple boundary surfaces within the image. In addition, Eq. (7) show that the distance-based good isosurface is more sensitive to noise and to the change of gradient threshold $T$.

Subsequently, we compare the proposed method with the conventional threshold selection techniques, which select thresholds based on the histogram of gray values of the whole image, and with the existing isosurface selection techniques. Researchers in Refs. 17 and 18 studied the automatic detection of meaningful isosurfaces so as to produce informative visualizations of 3D images. In Ref. 17, a user interface has been developed. This allows interactive determination of optimal isovalues by computing certain characteristics (called the contour spectrum, including contour length, contour area, gradient integral, etc.) of the corresponding isosurfaces at a selected isovalue. Based on the metrics evaluated over the range of possible isovalues, the user can readily decide which isovalue to use. Reference 18 is based on similar principles as Ref. 17, but it uses a completely different method to compute isosurface spectra. In Ref. 18, the intensity transitions in 3D images are detected as maxima in cumulative Laplacian-weighted gray-value histograms. These intensity transitions correspond to the thresholds that segment parts of the data volume with large contour surfaces and/or large gradient values along the contour surfaces. Since in Ref. 18, only one pass through the data volume is required to compute the histogram, it is a computationally very efficient method.

For the 3D image without any prior knowledge, it is important to judge whether boundary surfaces within the 3D image are suitable to be represented (approximated) by certain isosurfaces. Threshold selection techniques and isosurface selection techniques usually cannot provide such judgment. However, by observing whether gray values of discrete sampling points of a boundary surface manifest as one main cluster distributed in a narrow region, the proposed method is usually able to judge whether the boundary surface is suitable to be approximated by a specific isosurface. Generally, gray values of discrete sampling points of boundary surfaces within uneven 3D images distribute in a wide region (see Fig. 9 of Ref. 28). Thus, boundary surfaces within uneven 3D images cannot be well approximated by any isosurfaces.

Some 3D images might contain several boundary surfaces, but only partial boundary surfaces could be well approximated by specific isosurfaces. In this case, we need to deduce good isosurfaces from the 3D images for the partial boundary surfaces. We note that some threshold selection techniques might be able to select multiple thresholds from 3D images. But threshold selection techniques usually cannot be used to select thresholds for the partial objects. Generally, isosurface selection techniques cannot be used to select multiple good isosurfaces for multiple boundary surfaces. They also cannot be used to select one or multiple good isosurfaces from 3D images for the partial boundary surfaces. However, as shown by the experimental results in Fig. 13, the proposed method can be used to select good isosurfaces for multiple boundary surfaces and for the partial boundary surfaces.

The aim of threshold selection techniques is to seek suitable thresholds to separate voxels of objects from voxels of background. But the purpose of the proposed method is to select the good isosurface to well approximate the continuous implicit boundary surface. Usually, there is more than one good threshold that could separate correctly object voxels from background voxels. However, most good thresholds are not the isovalues of good isosurfaces we are trying

![Fig. 14 The graylevel-based good isosurface (a) and the distance-based optimal isosurface (b) extracted from the same 3D image shown in Fig. 15.](image)

![Fig. 15 2D slices of the 3D image containing a boundary surface with varying gray values and varying gradient values.](image)
to compute. Conventional threshold selection techniques select thresholds mainly based on the histogram of gray levels of 3D images. Figures 5 and 9 show that histograms of 3D images used in this paper have broad valleys or unequal peaks. In addition, usually some objects of interest occupy only a very small percentage in the whole 3D image. Consequently, these objects of interest cannot be recognized from the histogram of gray levels of the 3D image. For 3D images with such histograms, threshold selection techniques usually cannot effectively and correctly select thresholds from them. However, the proposed method deduces the isovalue from the histogram of DSPBS. In Figs. 5 and 9, we can see that in the histogram of DSPBS, each boundary surface corresponds to a main cluster, and therefore it is easy to determine isovalues for it. Thus, the proposed method can overcome some drawbacks of threshold selection techniques. In fact, the good isosurface of a given boundary surface mainly correlates to the boundary surface rather than to the gray values of those grid points far from the boundary surface. Thus, for a given boundary surface, it is better to deduce its good isosurface from its attribute values than from the gray values of the whole 3D image.

Isosurface selection techniques are mainly used to select the isosurface with the largest average gradient value from a 3D image. Their main drawback is that they cannot select multiple suitable isosurfaces from a 3D image for multiple boundary surfaces within the 3D image. In addition, although the selected significant isosurface is related closely to the boundary surface of certain structures within 3D images, a subtle difference exists between the significant isosurface and the boundary surface. Consider a binary 3D image used in the preceding experimental results. The binary 3D image has gray values 20 and 100 and contains a solid cube. It is known that in discrete 3D images, the gradient value of the point located between two adjacent grid points is computed by linearly interpolating the gradient values of these two grid points. This implies that the isosurface with the largest average gradient value passes through many grid points with gray values either 20 or 100. However, Sonka et al. have pointed out that this will inevitably fail to detect the points satisfying \( \nabla^2 g(x,y,z)=0 \) from 3D grid points. Thus, the isosurface with the largest average gradient value is not the boundary surface. Based on the proposed method, we show that isovalues of good isosurfaces are positive numbers near 60 (see Table 1); these are more reasonable isovalues than 20 and 100. Meanwhile, we can see that any value between 20 and 100 is a good threshold for segmenting correctly these two 3D binary images. The proposed method needs to compute positions, gray values, gradient values, and Laplacian values for discrete sampling points of boundary surfaces and for grid points. In addition, it needs a lot of memory to store these values. Therefore, usually the proposed method is more complex than the algorithm of Pekar et al.

6 Conclusion

This paper addresses the issue of the computation of good isosurfaces for well approximating boundary surfaces within 3D images. The mathematical model describing the approximation problem of a boundary surface by good isosurfaces is constructed. The method used to deduce good isosurfaces from 3D discrete images is presented. Based on these obtained results, we not only explain which isosurface is more suitable for approximating the boundary surface within a 3D image, but also provide a method to compute such isosurfaces from 3D discrete images. The proposed method has been applied to 3D biomedical images.

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Appendix

In 3D images, each boundary point \( P \in S(x,y,z) \) has a high gradient value. Thus, gray values will have a sharp change at the places close around \( P \). This indicates that isosurfaces corresponding to these sharply changed gray values all are very close to the point \( P \). Let \( I(r) \) represent one such isosurface. Let

\[ P_1 = \min_{Q \in I(r)} \| Q - P \|, \]

namely, \( P_1 \) represents such a point that lies on the isosurface \( I(r) \) and is the closest point to \( P \). Since \( I(r) \) is very close to \( P \) and the gradient vector \( \nabla g(P) \) is the steepest descent direction of \( g(x,y,z) \) at the point \( P \), we can assume that \( P_1 \) can be represented approximately as follows:

\[ P_1 = P + t_0 \cdot \nabla g(P). \]

In other words, \( P_1 \) is located in the direction of \( \nabla g(P) \). Here, \( t_0 \) is a real number with a small absolute value. This fact implies that the distance from a point \( P \) to the isosurface \( I(r) \) has the following expression:

\[ \text{dist}[P,I(r)] = \min_{Q \in I(r)} \| Q - P \| = \| P - P_1 \| = t_0 \cdot \| \nabla g(P) \|. \]

Denote \( F(t) = g(P) + t \cdot \nabla g(P), t \in (-\infty, +\infty) \). Then \( F(t) \) is a one-dimensional continuous function, and it has the following Taylor expansion at \( t = 0 \):

\[ F(t) = F(0) + F'(0)t + o(t^2). \]

Here, \( F'(0) = \nabla g(P) \cdot \nabla g(P) = \| \nabla g(P) \|^2 \). This implies that \nabla g(P) \|^2 + o(t_0^2). Thus, we have

\[ \text{dist}[P,I(r)] = |t_0| \cdot \| \nabla g(P) \| = \frac{|r - g(P)|}{\| \nabla g(P) \|} \cdot o(t_0^2). \]

Since \( t_0 \) is a real number with a small absolute value, without loss of generality, we can assume approximately that
\[ \text{dist}(P, I(r)) = \left\| r - g(P) \right\| / \| \nabla g(P) \|. \]

Thus, the distance error between \( S(x, y, z) \) and \( I(r) \) can be represented as follows:

\[ \text{dist}(S(x, y, z), I(r)) = \left\{ \left[ \int_{S(x, y, z)} \frac{r - g(x, y, z)}{\| \nabla g(x, y, z) \|} \, dS \right]^2 + \left[ \int_{S(x, y, z)} \frac{g(x, y, z)}{\| \nabla g(x, y, z) \|} \, dS \right]^2 \right\}^{1/2}. \]

Here, \( \int_{S(x, y, z)} |r - g(x, y, z)| / \| \nabla g(x, y, z) \| \, dS \) represents the surface integral of the distance function \( |r - g(x, y, z)| / \| \nabla g(x, y, z) \| \) over the boundary surface \( S(x, y, z) \). This implies that the isovale of the distance-based optimal isosurface of \( S(x, y, z) \) can be determined approximately by solving the following optimization problem:

\[ \min_r \left\{ \left[ \int_{S(x, y, z)} \frac{r - g(x, y, z)}{\| \nabla g(x, y, z) \|} \, dS \right]^2 + \left[ \int_{S(x, y, z)} \frac{g(x, y, z)}{\| \nabla g(x, y, z) \|} \, dS \right]^2 \right\}^{1/2}, \quad r \in (0, \infty), \quad (8) \]

Let \( F(r) = \int_{S(x, y, z)} \left[ r - g(x, y, z) \right] / \| \nabla g(x, y, z) \| \, dS \). The solution of Eq. (8) satisfies \( F'(r) = 0 \). It is easy to see that the solution of the optimization problem (8) is as follows:

\[ r = \frac{1}{\left[ \int_{S(x, y, z)} \| \nabla g(x, y, z) \| \, dS \right]^2}. \quad (9) \]

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