Computational Aspects of Algebra, Geometry and Combinatorics

Friday, August 8, 1:00 p.m. - 5:15 p.m., Hilton Portland, Plaza Level, Pavilion West

This session will highlight recent advances in mathematics inspired by experimental and computational aspects of research. The talks will be in areas of combinatorics and probability related to algebra and geometry. This is a highly active area of research, which often lends itself to interesting talks accessible to a wide audience.

Sara Billey, University of Washington
Benjamin Young, University of Oregon

The Combinatorics of CAT(0) Cubical Complexes and Robotic Motion Planning

1:00 p.m. - 1:30 p.m.

A cubical complex is CAT(0) if it has global non-positive curvature; informally, "all its triangles are thin". These complexes play an important role in pure mathematics (group theory) and in applications (phylogenetics, robot motion planning, etc.). In particular, as Abrams and Ghrist observed, when one studies the possible states of a discrete robot, one often finds that they naturally form a CAT(0) cube complex. Gromov gave a remarkable topological/combinatorial characterization of CAT(0) cube complexes. We give an alternative, purely combinatorial description of them, allowing a number of applications. In particular, for many robots, we can use these tools to find the fastest way to move from one position to another one. The talk will describe joint work with Tia Baker, Megan Owen, Seth Sullivant, and Rika Yatchak. It will require no previous knowledge of the subject.

Federico Ardila, San Francisco State University

A Borsuk-Ulam Equivalent that Directly Implies Sperner’s Lemma

1:45 p.m. - 2:15 p.m.

Sperner's Lemma is an elegant combinatorial analog to Brower's Fixed Point Theorem and both are equivalent to the KKM Lemma. Additionally, these three cozy theorems are implied by another triple of related results: Tucker's Lemma, the Borsuk-Ulam Theorem, and the LSB Theorem. In each of these triples, the first result is combinatorial, the second topological, and the third is a set covering result. Although direct proofs have been shown between the topological and set covering pairs of these theorems, a direct link showing Tucker's Lemma implies Sperner's Lemma was missing. We show that another combinatorial result, Fan's N+1 Lemma, is also equivalent to the Borsuk-Ulam Theorem and directly implies Sperner's Lemma. Consequently, we might think of Fan's N+1 Lemma as a more natural combinatorial analogue to the Borsuk-Ulam Theorem. Joint work with Francis Edward Su.

Kathryn Nyman, Willamette University
The Combinatorics of Fully Packed Loops and Razumov-Stroganov Conjectures
2:30 p.m. - 3:00 p.m.

Fully packed loops are certain interesting arrangements of lines on an NxN square lattice that form connections between different points along the boundary. In 2001, physicists Razumov and Stroganov discovered using computer experimentation that these objects encode the answer to a seemingly unrelated question on the probability distribution of a certain random walk on handshake patterns, which are ways for an even number of people standing around a table to shake hands without different hands having to cross over each other. This empirical discovery was finally given a beautiful proof in 2010 by Cantini and Sportiello. I will explore the story of this remarkable result and tell about some mysterious variations of the same problem that remain poorly understood.

Dan Romik, University of California, Davis

Parking Functions and Tree Inversions
3:15 p.m. - 3:45 p.m.

We describe a bijection between the parking functions and the spanning trees of a labeled graph which relates the degree of a parking function with the number of inversions of its associated spanning tree. The special case of the complete graph solves a problem posed by Richard Stanley. This is joint work with Qiaoyu Yang and Kuai Yu.

David Perkinson, Reed College

Expanding Hall-Littlewood Polynomials into Schur Functions
4:00 p.m. - 4:30 p.m.

The Hall-Littlewood polynomials and the Schur functions are important in the fields of symmetric functions, representation theory, and gauge theory, amongst others. The Hall-Littlewood polynomials have a simple expansion in terms of the Fundamental quasisymmetric functions. By thinking of each term of these expansions as the node of a graph - called a dual equivalence graph - we demonstrate how to associate each component with a single Schur function, providing a new expansion of Hall-Littlewood polynomials in terms of Schur functions. We will also point out other topics in algebraic combinatorics where graph structures may lead the way to simpler transitions between bases.

Austin Roberts, University of Washington

Self-Organizing Cellular Automata
Cellular automata display an extraordinary range of behavior, ranging from very simple to apparently chaotic, with many cases in between. Perhaps the most interesting rules are those that yield multiple behavior types from different initial conditions - this is common even for one-dimensional rules started from finitely-supported seeds. If a rule yields chaos from some initial condition, it is tempting to conclude by analogy with the second law of thermodynamics that chaos should be prevalent from almost all initial conditions. For a certain natural class of rules, we prove that the opposite holds: typical (i.e. random) initial seeds self-organize into predictable (but non-trivial) evolution, while exceptional seeds generate more complicated behavior, including apparent chaos.

Deterministic evolution from random initial conditions is a combination that strikes fear into the hearts of probabilists - some randomness in the evolution typically makes analysis much easier. However, we were able to side-step this issue by taking advantage of simple linear algebra together with fractal-like properties of certain cellular automata.

See [http://research.microsoft.com/~holroyd/ca/webxor.png](http://research.microsoft.com/~holroyd/ca/webxor.png) for a picture. No prior knowledge assumed. Joint work with Janko Gravner.

**Alexander E. Holroyd, Microsoft Research**