

On the Downlink Capacity of Cellular CDMA and TDMA over Nondispersive Channels

Hikmet Sari⁽¹⁾, Heidi Steendam⁽²⁾, Marc Moeneclaey⁽²⁾

⁽¹⁾ Alcatel Radio Communications Division
5, rue Noel-Pons
92734 Nanterre Cedex, France
Hikmet.Sari@alcatel.fr

⁽²⁾ TELIN/DIGCOM
University of Ghent, St. Pietersnieuwstraat 41
B-9000 Gent, Belgium
Marc.Moeneclaey@telin.rug.ac.be

Abstract - We investigate the multiuser interference on the downlink channel (from base station to users) in cellular code-division multiple access (CDMA) and time-division multiple access (TDMA) systems. The results indicate that for the same total bandwidth occupancy and maximum number of users per cell, multiuser interference is approximately 10 dB higher in CDMA in fully-loaded cells. The implication of these results is that for the same total bandwidth and number of users per cell, TDMA gives significantly superior bit error rate (BER) performance, as confirmed by our numerical results on additive white Gaussian noise (AWGN) channels as well as on flat Rayleigh fading channels. Alternatively, CDMA needs approximately 10 times more bandwidth than TDMA to achieve the same signal-to-interference ratio (SIR) and bit error rate (BER) performance in fully-loaded cells.

1. INTRODUCTION

Code-division multiple access (CDMA) has become very popular over the past few years, not only in mobile and personal communications, but also in other applications including fixed wireless access and satellite systems. While it is undeniable that this multiple access technique has the virtue of freeing operators from frequency planning, the capacity of CDMA and its comparison to time-division multiple access (TDMA) has been a very controversial issue often dominated by commercial interests. In addition, comparisons are usually made between systems in which TDMA and CDMA are only one ingredient among many others. The consequence of this is that the capacity of CDMA is still not a well understood issue which needs further clarification.

The purpose of the present paper is to compare the downlink capacity of cellular CDMA and TDMA.

Before describing the CDMA scheme considered in this study, note that there are essentially two basic CDMA techniques: The first one directly derives from direct-sequence spread-spectrum (DS-SS) systems which were originally devised for military communication systems [1], [2]. DS-SS has two attractive features for those applications: The first feature is the low intercept probability which results from the fact that a DS-SS signal is virtually buried in background noise both in the time and in the frequency domains. The second feature is the robustness of DS-SS signals to intentional or unintentional jamming. It is obvious that the spreading sequence in these applications must be pseudo-random and difficult to replicate by an unauthorized user, because if such a user can replicate the spreading sequence, the whole process becomes useless. In the sequel, we will refer to CDMA systems which use pseudo-random (pseudo-noise, or PN) spreading sequences as pseudo-noise CDMA (PN-CDMA).

The second basic CDMA technique is also based on direct sequence spectral spreading, but uses a set of orthogonal sequences, e.g., Walsh-Hadamard (WH) sequences [3], instead of PN sequences. There is no randomness in this case, and the spreading sequence repeats itself from one symbol to the next. Furthermore, since the spreading sequences are orthogonal, there is no mutual interference between different user signals in this technique.

2. A BRIEF REVIEW OF TDMA AND CDMA

TDMA consists of sharing a data stream between different users by assigning to them different time slots. For simplicity, we will focus here on a simple TDMA scheme in which the data stream is formatted into frames of N time slots and each user gets one time slot per frame. That is, the channel

resources are equally shared between N different users. The total bandwidth of the transmitted signal in this scheme is N times the bandwidth which would be needed by a single user if this user were alone on the channel. Let R designate the symbol rate of each user, and W Hz the bandwidth required to transmit this symbol rate in single-user transmission. The TDMA scheme at hand thus transmits a symbol rate of NR baud using a bandwidth of NW Hz. Of course, the overhead needed for signal framing is neglected in this simple example.

An equivalent OCDMA scheme assigns a periodic sequence of length N to each user, where all sequences are mutually orthogonal. The period of the sequences coincides with the symbol interval, giving thus N chips per symbol. This process spreads the transmitted signal bandwidth by a factor of N . Since the number of orthogonal sequences of length N is exactly N , this OCDMA scheme can accommodate N users. In other words, exactly as in TDMA, OCDMA can accommodate N users when the available bandwidth is N times that required by one user, and this is achieved without any mutual interference.

We now analyze PN-CDMA. Let p_{kl} designate the portion of the k th user's PN sequence that is used during the l th symbol period. Since the p_{kl} sequences are uncorrelated PN sequences of length N , we have

$$E(p_{kl}p_{kl'}) = N\delta_{ll'} \quad (1)$$

where δ denotes the Kronecker delta which takes the value of 1 for $l = l'$ and of 0 for $l \neq l'$. Therefore, the correlator preceding the decision circuit at the receiver gives a useful signal value of N (times the transmitted symbol), while the average interference value from any other user is 0. The mean-squared value of this interference is

$$E\left\{\left(p_{k'l}p_{kl}\right)^2\right\} = N. \quad (2)$$

The signal-to-interference ratio (SIR) is therefore $N^2/N = N$, or equivalently the interference power normalized by the useful signal power is $1/N$. Interference from different users adds up, and the normalized interference power becomes n/N when the number of users is n . If we assume that the number of active users is N , then each user will get interference from the other $N-1$ users, and the total normalized interference power will be $(N-1)/N$,

which indicates that for large N , the interference power is as large as the useful signal power. Obviously, no operation is possible at such a high interference level even with the best error correction codes, and this means that PN-CDMA accommodates a much smaller number of users than TDMA and OCDMA. If the interference power is to be limited to for example 20 % of the signal power, then only $N/5$ users can be accommodated, and this represents a capacity decrease by a factor of 5 with respect to TDMA and OCDMA. The capacity of PN-CDMA is therefore not a fixed number. It depends on the interference level that one is prepared to tolerate. To summarize the single-cell case, while both TDMA and OCDMA can accommodate N users without any mutual interference on a channel whose bandwidth is N times that required by a single user, PN-CDMA leads to multiuser interference as soon as there are two users on the channel, and the interference power is proportional to the number of users. As a result, the number of users that can be accommodated is substantially lower than N if the performance is to be kept at an acceptable level.

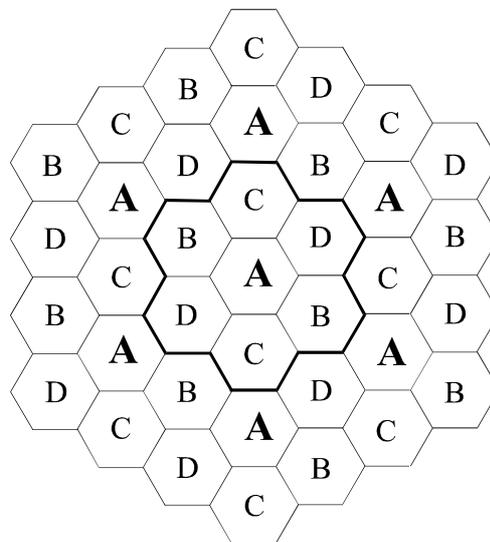


Fig. 1 : Cell structure with reuse factor of 4.

3. FREQUENCY REUSE AND MULTIUSER INTERFERENCE

In the previous section, we analyzed the capacity of CDMA in a single cell. In many real-world applications, communication systems operate in a multicell environment, and the basic question is the achievable frequency reuse factor among cells. We will consider here a hexagonal cell pattern

commonly used in mobile radio systems. In TDMA, two adjacent cells can not use the same carrier frequency, and therefore the smallest number of frequencies is 3. If the SIR with this frequency reuse factor is not sufficient, a frequency reuse factor of 4 or 7 can be used instead. Such frequency reuse patterns are commonly found in state-of-the-art textbooks on digital mobile radio systems [4]. In what follows, we will assume a frequency reuse factor of 4, following the cell pattern of Fig. 1. Each cell in this figure is labeled with a letter which indicates the frequency (or group of frequencies) used. If we consider an A-cell, the 6 cells surrounding it are of type B, C, D, B, C, and D, respectively. If R designates the cell radius (i.e., the radius of the largest circle that fits inside the hexagonal cell), then the distance between two base stations using the same frequency is 4R. Furthermore, each cell has 6 nearest neighbors which employ the same frequency.

A. TDMA

Let us now examine the downstream (from base station to users) interference problem in TDMA in this environment. Interference is obviously limited to cells that employ the same carrier frequency, i.e., an A-cell gets interference only from other A-cells, a B-cell from other B-cells, and so forth. If the clocks of different cells are mutually synchronized, then the interference will be limited to users that share common time slots. In other words, there is a one-to-one correspondence between interfering users in that case. However, it is quite unlikely that clocks of different cells will be synchronized in practice. In the presence of clock frequency offset, a given user will sequentially interfere with all other users of the interfering cells, the interference resembling a periodic impulse noise in unloaded cells. In the sequel, we assume that all base stations transmit the same signal level.

Now we investigate the multiuser interference assuming that the propagation path loss is proportional to the fourth power of the distance. To do this, we focus on a user located on the cell boundary in the direction of one of the nearest cells which employ the same frequency. Such a user is at a distance R from its home base station and at a distance 3R from the nearest interfering base station. Assuming that there are N active users in that cell, i.e., the cell is full, the normalized interference power from this particular cell for the

user of interest equals $1/9^2$. Considering the other 5 nearest cells with the same frequency assignment, the total interference level is easily shown to be

$$I_d = \frac{1}{9^2} + \frac{2}{13^2} + \frac{2}{21^2} + \frac{1}{25^2}. \quad (3)$$

The denominators on the righthand side of (3) are the fourth powers of the normalized distances of the interfering base stations to the user considered. Neglecting interference from cells at a larger distance, the SIR for the downlink is $SIR_d = 33$, or $SIR_d = 15.2$ dB in the dB scale.

Note that the foregoing analysis holds when all the interfering cells are full. Otherwise, the downlink SIR of 15.2 dB will be valid only when the time slot assigned to the user considered is also simultaneously used in all of the 6 nearest cells that employ the same frequency.

B. Two-Layer CDMA

In order to avoid the large intracell interference that occurs in a pure PN-CDMA system, we consider a two-layer CDMA which consists of an intracell OCDMA and an intercell PN-CDMA. The transmitted signal is first spread through multiplication by a WH sequence with N chips per symbol. Then, a PN sequence is overlaid either without further spreading (i.e., with the same chip rate) or with further spreading by a factor of 2 or 4. All users of a cell employ the same PN sequence, and the N orthogonal sequences are reused in each cell. This kind of multiple access is used in IS-95 [4] and in current proposals for the future universal mobile telecommunications systems (UMTS) [5]. In this scheme, there is no interference between different users in the same cell, but a given user interferes with all users of all other cells. If the chip rates of the orthogonal and PN sequences are the same, the mutual interference between two users is $1/N$ (attenuated by a factor which is function of the user location). If the number of PN chips per WH chip is 4, then the mutual interference is $1/4N$, and in terms of bandwidth occupancy the resulting system is equivalent to the TDMA system with a frequency reuse factor of 4 considered above. The basic question is whether this system can accommodate as many users as does TDMA and, if so, at what performance level.

In what follows, we will evaluate the total downlink interference level in two-layer CDMA assuming that the PN sequence has 4 chips per WH

chip (so that the bandwidth is the same as for TDMA with a reuse factor of 4). We take into account only the interference terms $I_{d,1}$, $I_{d,2}$, and $I_{d,3}$, coming from the 6 nearest cells, the 6 second-nearest cells, and the 6 third-nearest cells, respectively. When the number of users per cell and the number of chips per symbol are equal to n and $4N$, respectively, we obtain

$$\begin{aligned} I_{d,1} &= \frac{n}{4N} \left(1 + \frac{2}{3^2} + \frac{2}{7^2} + \frac{1}{9^2} \right) \\ I_{d,2} &= \frac{n}{4N} \left(\frac{2}{7^2} + \frac{2}{13^2} + \frac{2}{19^2} \right) \\ I_{d,3} &= \frac{n}{4N} \left(\frac{1}{9^2} + \frac{2}{13^2} + \frac{2}{21^2} + \frac{1}{25^2} \right) \end{aligned} \quad (4)$$

which yields $SIR = S/(I_{d,1}+I_{d,2}+I_{d,3}) = 0.733(4N/n)$. For $n = N$, we obtain $SIR = 4.7$ dB. This is 10.5 dB worse than the downlink SIR in the equivalent TDMA scheme.

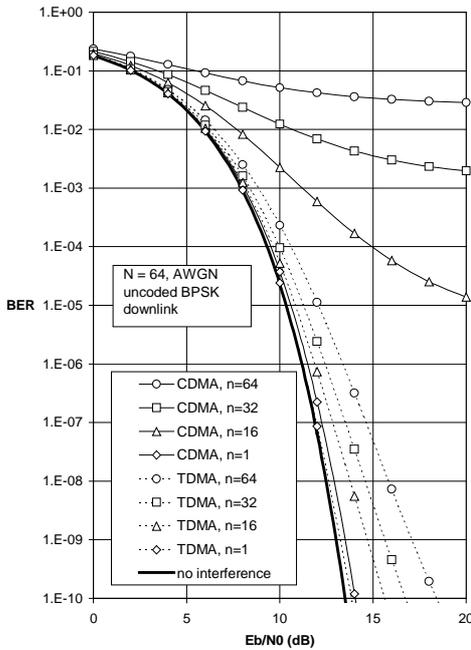


Fig. 2 : Downlink BER performance for uncoded transmission over AWGN channel

4. PERFORMANCE ANALYSIS

In this section, we will report some numerical results comparing bit error rate (BER) performance of cellular TDMA and CDMA with and without coding, and both at low and high user densities in different cells. The TDMA system has a frequency reuse factor of 4 and N users per cell each having the same bit rate as the users in the CDMA scheme to which it is compared. The CDMA system has $4N$ chips per symbol, and N orthogonal sequences are

allocated to each cell, i.e., the maximum number of users per cell is N . The TDMA and CDMA systems considered obviously require the same total bandwidth. For these two systems, numerical results are reported in Figs. 2-4 assuming binary phase-shift keying (BPSK) modulation and $N = 64$.

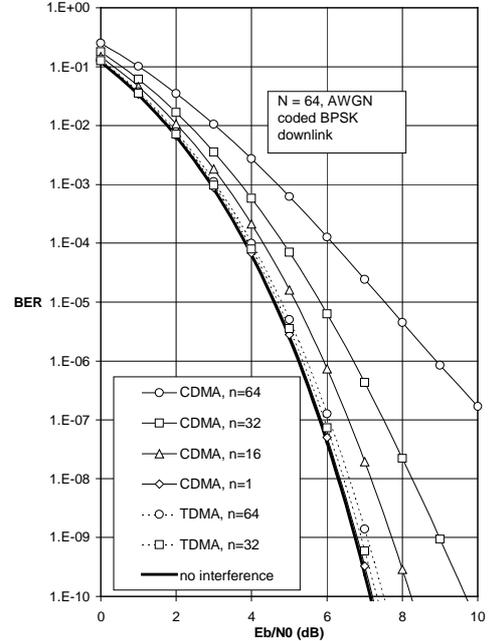


Fig. 3 : Downlink BER performance for coded transmission over AWGN channel

Fig. 2 gives the BER performance on additive white Gaussian noise (AWGN) channels in the absence of channel coding. We observe that even with $n = 16$ users (i.e. one quarter of maximum capacity), CDMA exhibits a BER floor. As for TDMA, its SNR degradation is modest and remains below 2 dB at the BER of 10^{-8} for $n = 16$. Next, Fig. 3 shows the results obtained using the industry-standard convolutional code with rate $\frac{1}{2}$ and constraint length $K = 7$. The degradations are smaller in this case, but here too TDMA outperforms CDMA. Finally, Fig. 4 shows the BER results over a flat Rayleigh fading channel with the same convolutional code as in Fig. 4. Ideal interleaving is assumed so that channel attenuation is uncorrelated from one symbol to the next. Again, CDMA exhibits a BER floor, while the degradation of TDMA remains very modest.

Before closing this section, let us point out that it is not surprising to see TDMA outperform CDMA, particularly when the propagation loss is proportional to the fourth-power of the distance. The reason is that interference in TDMA with a frequency reuse factor of 4 originates from cells

whose centers are at a distance of $4R$ or higher from the serving base station, whereas users in CDMA are faced with interference from all neighboring cells whose centers are only at a distance of $2R$, and second-nearest neighbor cells whose centers are at a distance of $\sqrt{12}R$. Interference from these cells is much less attenuated than that from the third-nearest neighbor cells which interfere in TDMA.

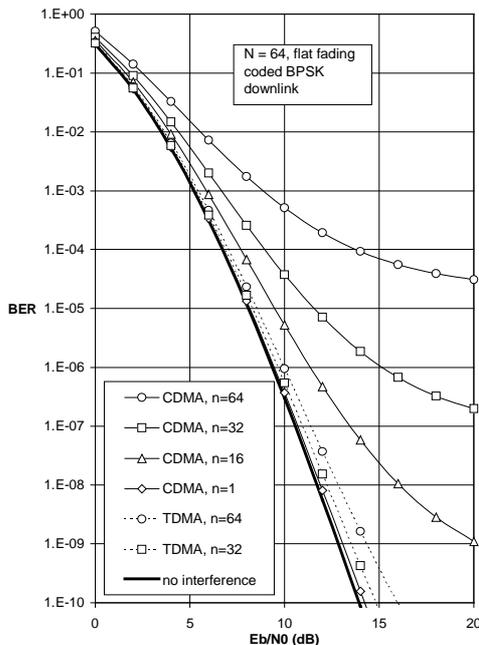


Fig. 4 : Downlink BER performance for coded transmission over flat Rayleigh fading channel

5. CONCLUSIONS

We have compared two popular cellular system concepts with a maximum capacity of N users per cell. One of them is based on TDMA with a frequency reuse factor of 4 between cells. The other is based on a two-layer CDMA with $4N$ chips per symbol and N orthogonal sequences per cell. Using a propagation model in which the signal attenuation is proportional to the fourth-power of the distance, we have shown that on the downstream channel, TDMA has an SIR advantage of approximately 10 dB over CDMA. Our BER performance results on AWGN channels and flat Rayleigh fading channels confirmed the superiority of TDMA, particularly at high user densities. At small user densities, the interference power is small compared to the noise power, and therefore TDMA and CDMA have similar BER performance.

Note that CDMA and TDMA have a similar BER

performance when they operate at the same SIR. Since there is a gap of 10 dB between the two systems considered in this paper, two-layer CDMA needs to use $40N$ chips per symbol to achieve the same SIR as the TDMA with a frequency reuse factor of 4. In turn, such a CDMA system requires 10 times the bandwidth of TDMA which accommodates the same number of users per cell and achieves the same SIR. This result may seem surprising, because it contradicts the usual claims that CDMA achieves a higher capacity than TDMA. But the fact that CDMA uses the same frequency in all cells does not imply that it can support a higher number of users in a given geographic area and a given total frequency bandwidth, and the results reported in the present paper show that the opposite is true when the same SIR is set for both schemes. We conclude that capacity is precisely the price paid for the principal attractive feature of CDMA that it frees operators from frequency planning.

Finally, we also mention that the 10 dB advantage of TDMA over CDMA in terms of SIR also holds for the uplink channel [6]. In both studies, we have purposely used a very simple model (nondispersive channel, perfect synchronization and power control) in order to separate the multiple access capacity problem from other system issues that one has to solve in practice.

REFERENCES

- [1] R. C. Dixon, "Spread Spectrum Systems," 2nd Edition, John Wiley and Sons, New York, 1984.
- [2] J. G. Proakis, "Digital Communications," 2nd Edition, McGraw Hill, New York, 1989.
- [3] N. Ahmed and K. R. Rao, "Orthogonal Transforms for Digital Signal Processing," Springer-Verlag, Berlin-Heidelberg, 1975.
- [4] T. S. Rappaport, "Wireless Communications: Principles and Practice," IEEE Press, New York, and Prentice Hall, New Jersey, 1996.
- [5] F. Adachi, M. Sawahashi, and H. Suda, "Wideband DS-SS-CDMA for Next-Generation Mobile Communications Systems," IEEE Communications Magazine, vol. 36, no. 9, pp. 56-69, September 1998.
- [6] H. Sari, H. Steendam and M. Moeneclaey, "On the Downlink capacity of cellular CDMA and TDMA over Nondispersive Channels," Proc. 1999 IEEE 49th Vehicular Technology Conf., vol. 2, pp. 1638-1642, Houston, TX, May 1999.