Thermodynamics of combined-cycle electric power plants

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Published data imply an average thermal efficiency of about 0.34 for U.S. electricity generating plants. With clever use of thermodynamics and technology, modern gas and steam turbines can be coupled, to effect dramatic efficiency increases. These combined-cycle power plants now reach thermal efficiencies in excess of 0.60. It is shown how the laws of thermodynamics make this possible. © *2012 American Association of Physics Teachers.*
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I. INTRODUCTION

It is common in the study of thermodynamics to examine *reversible* heat engine models. One learns that the maximum efficiency of a heat engine operating between colder (T_c) and hotter (T_h) constant-temperature reservoirs is $\eta_{carnot} = 1 - T_c/T_h$, that of a *reversible* Carnot cycle. Of course, there are no such reversible cycles in reality. Indeed, a reversible cycle must be executed infinitely slowly and, therefore, would have a power output of zero. This point is often overlooked in textbooks.

Some modern books[1,2] take note of the fact that a relatively simple irreversible extension of the Carnot cycle, entailing irreversible energy transfers from the hot and to the cold reservoir, yields an efficiency at maximum power output of

$$\eta_{nca} = 1 - (T_c/T_h)^{1/2}. \tag{1}$$

It is easy to see that[3] $\eta_{carnot}/\eta_{nca} = 1 + (T_c/T_h)^{1/2}$, from which it follows that for $0 < T_c < T_h$,

$$\frac{1}{2}\eta_{carnot} < \eta_{nca} < \eta_{carnot}. \tag{2}$$

Such irreversible models were investigated first in the engineering literature and then independently in the physics literature, and Eq. (1) is sometimes referred to as the "Novikov–Curzon–Ahlborn" efficiency.[4–7] These models can help students understand how one type of irreversibility, namely, energy transfer through finite temperature differences, can lower the efficiencies of heat engines.

The well-known article by Curzon and Ahlborn[4] has helped spawn an entire field of study called finite-time thermodynamics (FTT). In a recent review article on FTT, Andresen wrote,[8] "The immediate inspiration for finite-time thermodynamics was the seminal paper by Curzon and Ahlborn in which they showed that a Carnot engine with heat resistance to its reservoirs has a maximal power production, and at that maximum its thermal efficiency can be described by [Eq. (1)]…This expression has three remarkable features: It is simple and generic; it is amazingly similar to the Carnot efficiency; and it is independent of the magnitude of the heat resistances…" Among the many articles written about FTT and Eq. (1), a few authors have pointed to weaknesses.[9–12] Interestingly, the efficiency η_{nca} occurs also under maximum work conditions for some *reversible* heat engine models operating between fixed lowest and highest temperatures T_c and T_h, respectively.[3]

Perhaps fortuitously, η_{nca} is a rough guide to the efficiencies of many existing fossil fuel, nuclear, and geothermal electric power plants. For example, a plant with combustion temperature 838 K and ambient temperature 298 K has an actual plant efficiency of 0.36,[4] which compares reasonably well with $\eta_{nca} = 0.40$. In contrast, $\eta_{carnot} = 0.64$, consistent with inequality (2).

Electricity is ubiquitous in developed countries. In 2010, the United States used an estimated 41.8 EJ (41.8×10^{18} J) of primary energy to generate electricity, out of a total primary energy use of 104 EJ.[13,14] That is, electricity energy generation accounts for about 40% of U.S. annual energy resource use. Electricity is so common that its availability is often taken for granted.

For many years, the average electric energy generated at fossil fuel combustion plants has been approximately one third the concomitant energy released by the fuel. In 2010, U.S. Department of Energy data shows a net generation of 3.97×10^{12} kWh of electrical energy, with an input of 11.6×10^{12} kWh of fuel energy, which implies an efficiency of 34.2%.[13] Driven by both environmental and economic considerations, considerable effort has gone into increasing the efficiencies of electric generating plants. As more efficient plants are added, the average efficiency will continue to grow.

In recent years, the emergence of so-called combined-cycle plants has offered the opportunity to dramatically increase the electricity-generating plant efficiencies.[15] Such plants are now providing nearly 20% of all electric energy worldwide, up from less than 5% about a decade ago.[16]

This article is directed toward teachers who might like to show students how straightforward thermodynamics explains why combined-cycle plants can achieve such sharp efficiency improvements.

II. COMBINED-CYCLE THERMODYNAMICS

A gas turbine cycle is depicted symbolically in Fig. 1. By definition, the efficiency of the gas turbine is given by

$$\eta_{gt} \equiv \frac{\dot{W}_{gt}}{\dot{Q}_h}, \tag{3}$$

where $\dot{Q}_h$ is the energy input *rate* from the high-temperature source at temperature T_h, and $\dot{W}_{gt}$ is the power output delivered to an electric generator. By energy conservation, the rate of energy transfer to the lower-temperature reservoir at the exhaust temperature T_{ex} is

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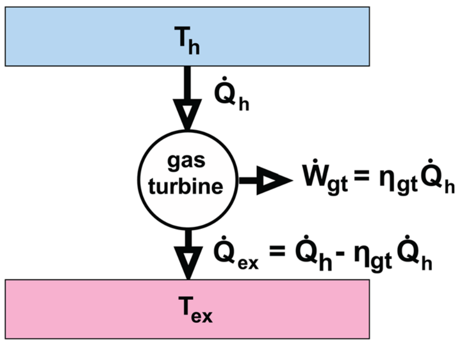

Fig. 1. Gas turbine cycle, which receives energy at rate $\dot{Q}_h$ from a source at temperature T_h, converts $\dot{W}_{gt} = \eta_{gt}\dot{Q}_h$ to useful work and sends its "waste" energy at temperature T_{ex}, at rate $\dot{Q}_{ex} = \dot{Q}_h - \eta_{gt}\dot{Q}_h$, to a steam turbine cycle.

$$\dot{Q}_{ex} \equiv \dot{Q}_h - \dot{W}_{gt}. \tag{4}$$

The gas-turbine cycle is based upon the reversible Joule–Brayton cycle, illustrated in Fig. 2. In 1851, Joule conceived an "air engine" using this cycle as a substitute for the steam engine.[17] Engineer George Brayton built a piston-driven internal combustion engine based upon the same cycle, but its efficiency was too low to be competitive.[18] The Joule–Brayton cycle models not only gas turbine power plants but also the familiar gas turbines of jet engines.[19]

Because they can burn relatively clean fuel, have relatively low capital costs, and can be started and stopped quickly, gas turbines have become popular for electricity "peaking" and emergency power generation, as well as for base load operations (providing minimum power requirements).[16] Stationary gas turbines have the flexibility to burn not only methane but also distillate oil, which though less clean than natural gas, is often preferable to coal for power plants.

The basic operation in Fig. 2 entails compression of air (1–2), providing the high pressure needed to drive the turbine. Then combustion of a fuel, typically methane (2–3), increases the temperature and energy of the gas stream. Segment 3–4 represents the combustion gases expanding as they drive the rotating turbine, and 4–1 cools and exhausts the hot gases at constant atmospheric pressure, thereby dumping "wasted" energy to the environment. Advances in metallurgy and cooling technology during recent years have enabled ever higher combustion and exhaust temperatures.

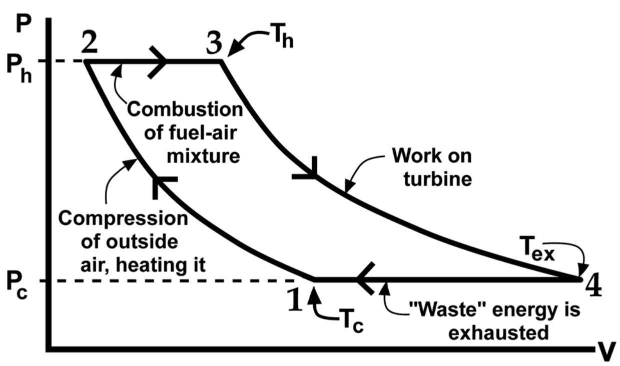

Fig. 2. Joule–Brayton reversible cycle, shown on a pressure (P) vs volume (V) graph. Beginning in state 1 of Fig. 2, air is compressed in a rotating compressor, bringing it to state 2. The air then enters a combustion chamber where it is mixed with fuel and burned. This is approximated as the isobaric segment 2–3. The combustion gases are then sent over a rotating turbine where they expand, doing work on the turbine—approximated as the adiabatic expansion 3–4. State 4 has temperature T_{ex}, typically well above that of the ambient outdoor temperature. In a simple gas turbine, combustion gases at this temperature are exhausted to the environment. This process, approximated as isobaric compression, is path 4–1. In the combined-cycle system the "waste" energy in step 4–1 is recovered to feed a steam turbine system, increasing the overall thermal efficiency. Keeping T_h and T_c fixed, and varying T_2 and $T_4 \equiv T_{ex}$ to achieve maximum work, the resulting efficiency[3] is $\eta_{\rm nca}$ in Eq. (1).

Modern gas turbines with ultra-high combustion temperatures have efficiencies of about 0.4, roughly the same as the most advanced coal-burning plants. However, they not only have the advantage of much higher inlet temperatures than steam turbines but also characteristically have the disadvantage of much higher exhaust temperatures, $T_4 \equiv T_{ex}$ in Fig. 2. Thus, a gas turbine cycle can dump substantial amounts of wasted energy to the environment, which limits its efficiency. For high-efficiency operation, one generally wants a high inlet (maximum) temperature, which gas turbines have, but also a low exhaust temperature, near that of the environment, which gas turbines lack.

Bejan cites various sources of irreversibility that plague gas turbines and explains how clever engineering designs, entailing regenerative heat exchangers, reheaters, and intercoolers, can bring higher efficiencies.[20] However, even greater efficiency gains are possible for electricity generation by using the high-temperature exhaust of the gas turbine to power a steam cycle, which inherently has a much lower exhaust temperature. For example, a gas turbine with inlet and exhaust temperatures 1673 K and 873 K, respectively, might use the exhaust gases to heat a steam turbine that has exit temperature 350 K, achieving the overall temperature range, 1673 K → 350 K. A single gas turbine cycle cannot match this.

In this regard, Bejan writes,"…gas-turbine cycles are better suited for efficient operation at high temperatures than steam-turbine cycles. On the other hand, the steam-turbine cycle is more attractive from the point of view of minimizing the temperature gap between the cold end of the cycle and the low-temperature reservoir… The engineering challenge that remains is to mesh optimally the two cycles along that seam of intermediate temperatures where the upper (warmer) cycle must act as a heat source for the lower one." The bottom line is that existing combined cycles are more efficient than any currently achievable single cycle.

In the simplest *combined-cycle* design, the gas turbine drives one electric generator and the steam turbine runs another, as illustrated in Figs. 3 and 4. Ignoring losses in the heat exchanger, the inlet temperature to the steam turbine is T_{ex}. Combining Eqs. (3) and (4), the "waste energy rate,"

$$\dot{Q}_{ex} = \dot{Q}_h - \eta_{gt}\dot{Q}_h, \tag{5}$$

in the exhaust gases becomes the input power for the steam turbine cycle, whose efficiency we call $\eta_{st} \equiv \dot{W}_{st}/\dot{Q}_{ex}$. Thus, the power output delivered by the steam turbine is

$$\dot{W}_{st} = \eta_{st}\dot{Q}_{ex} = \eta_{st}(\dot{Q}_h - \eta_{gt}\dot{Q}_h). \tag{6}$$

Combining Eqs. (3) and (6), the total power delivered by the gas and steam turbine combination is

$$\dot{W}_{tot} = \dot{W}_{gt} + \dot{W}_{st} = [\eta_{gt} + \eta_{st}(1 - \eta_{gt})]\dot{Q}_h, \tag{7}$$

and the combined-cycle efficiency is,[21]

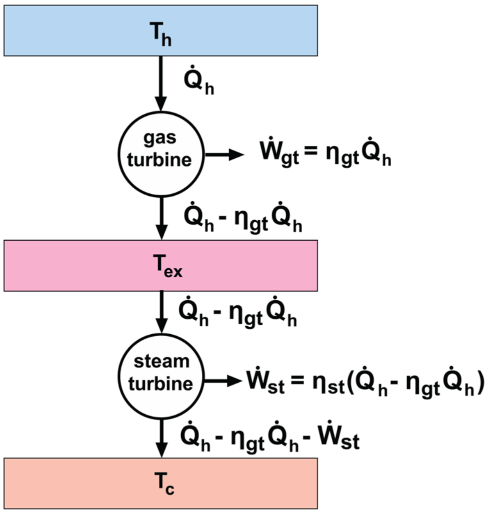

Fig. 3. The combined cycle, in which a gas turbine receives energy at rate $\dot{Q}_h$ from a source at temperature T_h, converts $\dot{W}_{gt} = \eta_{gt}\dot{Q}_h$ to useful power, and sends its "waste energy," at rate $\dot{Q}_h - \eta_{gt}\dot{Q}_h$, to a steam turbine that produces power $\dot{W}_{st} = \eta_{st}(\dot{Q}_h - \eta_{gt}\dot{Q}_h)$. In an actual combined cycle, the energy delivered to the steam turbine entails use of a heat exchanger and is less than $\dot{Q}_h - \eta_{gt}\dot{Q}_h$. Also, the temperature of the steam reaching the turbine is $T'_{ex} < T_{ex}$.

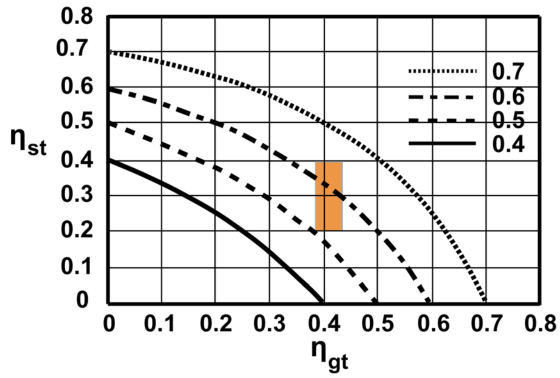

Fig. 5. Steam turbine efficiency η_{st} vs gas turbine efficiency η_{gt} implied by Eq. (8), for combined-cycle efficiencies 0.4–0.7. The shaded rectangle delineates the approximate region of combined-cycle efficiencies expected with current gas and steam turbine efficiencies.

$$\eta_{cc} = \frac{\dot{W}_{\text{tot}}}{\dot{Q}_h} = \eta_{gt} + \eta_{st} - \eta_{gt}\eta_{st}. \tag{8}$$

Equation (8) is not new,[16,22] but neither it nor any discussion of combined cycles seems to appear in physics textbooks or teaching journals. The latter equation can be used to graph η_{st} vs η_{gt} for various values of the combined-cycle efficiency, η_{cc}, as shown in Fig. 5. The shaded rectangle in this graph illustrates that overall efficiencies above 0.50 are possible with η_{st} as low as 0.20 if the corresponding η_{gt} is at least 0.38. With a steam turbine efficiency of 0.32 and a gas turbine efficiency of 0.42, the combined-cycle efficiency is $\eta_{cc} = 0.60$. If the steam turbine efficiency is 0.40, then $\eta_{cc} = 0.64$.

Figure 6, adapted in part from Ref. 23, shows η_{gt} and η_{cc} vs T_{ex}. This graph also makes it clear that for an inlet temperature 1673 K, the combined cycle achieves a maximum efficiency $\eta_{cc} \approx 0.60$, when $T_{ex} \approx 900$ K.[24] Using the η_{gt} and η_{cc} curves in Fig. 6 together with Eq. (8), one can calculate the implied steam turbine efficiency η_{st}. The average of the values implied for temperatures 1473 K and 1673 K is shown as a long-dashed curve in the figure, along with the prediction (dotted curve) of η_{nca}, the Novikov–Curzon–Ahlborn efficiency for the steam turbine. The latter curve is calculated using $T_c = 395$ K in Eq. (1), chosen to give good agreement with the curve calculated for η_{st}. This shows that for $T_{ex} \approx 900$ K, the maximum combined-cycle efficiency $\eta_{cc} = 0.60$ comes about from a gas turbine efficiency $\eta_{gt} = 0.41$ and steam turbine efficiency $\eta_{st} = 0.32$.

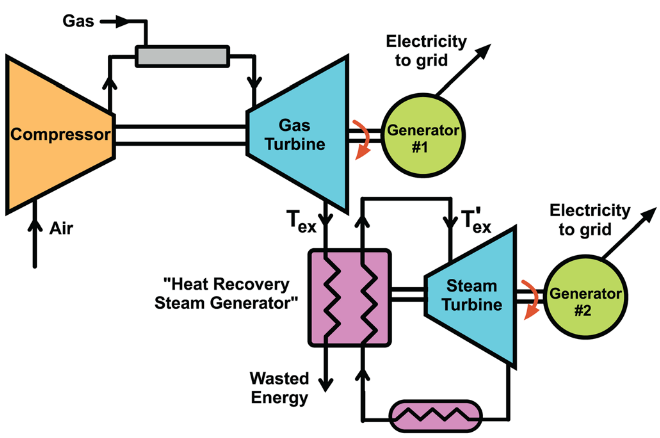

Fig. 4. Schematic diagram of a combined-cycle power plant. Combustion gas at temperature T_{ex} from a gas turbine engine is fed through a "heat recovery steam generator" (HRSG) to boil water at temperature T'_{ex}. The resulting steam drives a turbine. The gas and steam turbine engines are each coupled mechanically to electric generators. In some systems, a single shaft is used for both turbines, with only one electric generator.

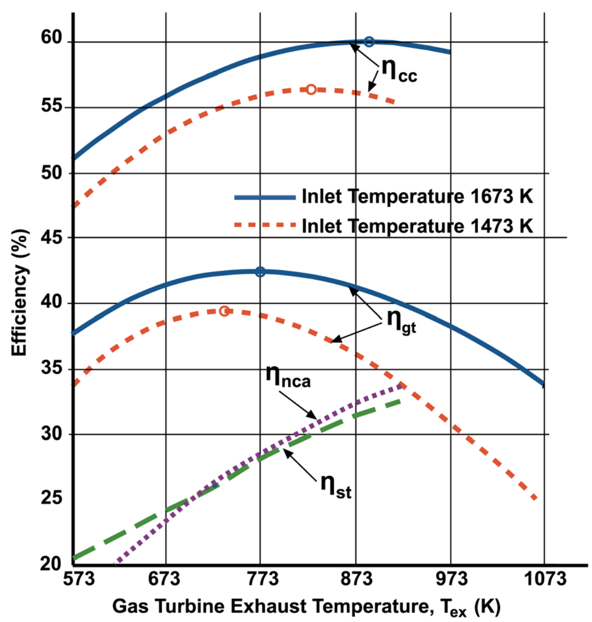

Fig. 6. Thermal efficiencies η_{gt} and η_{cc} vs the gas turbine's exhaust temperature T_{ex} (adapted from Kehlhofer *et al.*, Combined-Cycle Gas and Steam Turbine Power Plants, 2nd Edition. Copyright © 1999 by PennWell) for inlet temperatures 1473 K and 1673 K. Also graphed as functions of T_{ex} are: (i) the steam turbine efficiency η_{st}, the average for the two inlet temperatures implied by η_{cc}, and η_{gt} using Eq. (8), and (ii) the Novikov–Curzon–Ahlborn efficiency η_{nca} for low temperature $T_c = 395$K. The circles show relative maxima of the combined-cycle and gas turbine curves. For each inlet temperature, the maximum of η_{cc} occurs for a higher exhaust temperature T_{ex} than does the corresponding maximum of η_{gt}.

To gain perspective on the latter combined-cycle efficiency, note that a Carnot cycle operating between 395 K and 1673 K would have efficiency $\eta_{carnot} = 0.76$; the corresponding Novikov–Curzon–Ahlborn engine would have $\eta_{nca} = 0.51$. If the Carnot cycle operates between an *ambient* temperature of 298 K and 1673 K, then $\eta_{carnot} = 0.82$ and $\eta_{nca} = 0.58$.

III. PRESENT STATUS AND FUTURE POSSIBILITIES

Progress obtaining ever higher efficiencies has been substantial since the first gas turbine electric power plant in 1939. That plant, in Switzerland, had a thermal efficiency of 0.18, with an inlet temperature of 810 K and an exhaust temperature of 550 K. In contrast, modern day gas turbines have efficiencies of about 0.40, inlet temperatures up to about 1800 K, and exhaust temperatures of 900 K. As we have seen, such high exhaust temperatures make it possible to couple gas turbines to steam turbines in combined-cycle power plants that have efficiencies of about 0.60.

Gas turbines are extremely complex, having over 7000 parts, and the turbine blade tips have tangential speeds exceeding 1700 km/h (472 m/s), well above the speed of sound in air. Clearly, enduringly strong materials are necessary to achieve this.

Modern combined-cycle power plants have relatively low emissions of carbon dioxide and nitrogen oxides. Most important, they have the advantage of being brought on-line in 5–30 min, and being highly adjustable to electricity load demands while maintaining high efficiencies. This makes them suitable for integration with solar and wind energy systems, which are intermittent.

Integrated gasification combined-cycle plants have been built and more are planned. These are intended to take advantage of the large existing supply of coal, which can be gasified, with the resulting "synthetic natural gas" (methane) used in the gas turbine. It remains to be seen if carbon dioxide can be captured and other pollutants from the coal can be controlled.

IV. CONCLUSIONS

The example of combined cycle power plants shows how relatively straightforward thermodynamics, along with advances in the thermal properties of materials and clever design, can be used to dramatically increase the efficiency of electricity generation. Modern combined-cycle electric generating plants have changed the landscape of electric power generation and will likely continue to do so for many years. The combined-cycle power plant is a practical, easily understood, socially relevant example of thermodynamics, suitable for inclusion in general physics and upper division thermodynamics, and courses on energy and environment.

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[13]Monthly Energy Review—August 2011, DOE/EIA/0035(2011/08), U.S. Energy Information Administration (EIA), <http://www.eia.gov/mer>.

[14]The U.S. EIA calculates the primary energy equivalent for solar thermal, solar photovoltaic, hydroelectric and wind-generated electricity using the typical fossil-fueled plants heat rate of 10300 MJ/kWh (electricity) to approximate the quantity of fossil fuels replaced by these sources. Note that EIA reports primary energy in BTU; I've converted to joules, with 1 BTU = 1055 J and 1 Quadrillion BTU = 1.055 EJ.

[15]Although the focus here is on power plants burning fossil fuels, these are by no means the only, or necessarily preferred, methods for large scale electricity generation. The long-term future is likely to bring a diverse mix of technologies, including renewable solar-thermal, photovoltaic, wind energy, geothermal electricity, fuel cells running on hydrogen, and nuclear power. In the near-to-medium term, however, it is likely that combined-cycle systems will have the largest impact.

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[20]See Ref. 10, Secs. 8.6 and 8.7.

[21]Here, η_{cc} does not account for the imperfect efficiency of the heat exchanger that links the low-temperature end of the gas turbine with the high-temperature end of the steam turbine, or of the efficiency of the electrical generators.

[22]See p. 418 of Bejan's book, Ref. 10, where the derivation of this equation is the objective of a homework problem.

[23]R. Kehlhofer, R. Bachmann, H. Nielsen, and J. Warner, *Combined-Cycle Gas and Steam Turbine Power Plants*, 2nd ed. (PennWell, Tulsa, 1999). Figure 6 was prepared using this edition. I learned of the newer edition, Ref. 16, only after writing this article.

[24]If the "heat recovery steam generator" (HRSG) in Fig. 4 has an efficiency of 0.85, and the gas turbine efficiency is 0.40, this implies a HRSG loss of $(1 - 0.85) \times 0.60 = 0.09$ of the input energy. Additionally, if the electrical generators bring a loss of, say, 0.02 of the input energy, then using these numbers as a rough guide, a gross efficiency of about 0.67 for the combined gas and steam turbines is needed to achieve a net efficiency that approaches 0.60.