A Modified Constrained Constant Modulus Approach to Blind Adaptive Multiuser Detection

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***Abstract*—An alternative blind adaptive multiuser detection is investigated based on modified constrained constant modulus (CM) criterion. It has been shown that the performance of a CM-based receiver is limited by the received power of the desired user. In this paper, we show that the limitation can be avoided using noncanonical constraint CM criterion and that in the presence of channel noise the modified CM criterion function is strictly convex by properly selecting some constant. With analyzing the extrema of the cost function, we point out how to select the constant. Moreover, a simple stochastic gradient algorithm for implementing our scheme is presented, and the convergence properties of the algorithm are analyzed. Simulation examples are given to demonstrate the performance of the proposed scheme.**

***Index Terms*—Blind multiuser detection, CDMA, constant modulus approach, multiple-access interference, stochastic gradient algorithm.**

I. INTRODUCTION

DIRECT-SEQUENCE (DS) code division multiple access (CDMA) is a promising technology for wireless environments with multiple simultaneous transmissions. Multiple-access interference (MAI) due to many simultaneous users constitutes the main limitation of DS-CDMA systems. Multiuser detection techniques can efficiently suppress MAI and substantially increase the capacity of CDMA systems. Various multiuser detection schemes have been developed over the past decade [1].

More recently, blind adaptive multiuser detection, which requires the prior knowledge of only the signature waveform and timing of the desired user, has received considerable attention [2]. The main motivation for employing a blind scheme is to avoid the requirements of training sequence, to thus offer better spectrum efficiency. The most representative methods of the blind multiuser detection include the minimum output energy (MOE) [3] and subspace approach [4].

Because of the analogy between intersymbol interference (ISI) and MAI, researchers have attempted to apply blind equalization techniques, such as the so-called Godard algorithm [5] or constant modulus algorithm (CMA) [6], to a multiuser context. It has been shown in [7] that the CM receivers can perform almost as well as the nonblind/trained receiver design if undesirable local minima can be avoided. In [8], it has also been discussed how the MMSE receivers approximate the local minima of the CMA cost function. To avoid the undesired local minima, the constrained versions of CMA should be considered.

A technique that insures global convergence of blind equalizers was developed earlier in [9], and an anchored blind equalizer was proposed there. A preliminary attempt to apply blind equalization techniques to a multiuser context appeared in [10], but this paper did not satisfactorily address the issue of discriminating between local minima. Since then, several methods have been proposed in the literature for this purpose. In [11]–[13], CM-type algorithms for multiuser detection with the use of a cross-correlation penalization term, as opposed to a hard constraint, have been presented to retrieve all user signals simultaneously, and they were shown to be, under certain conditions, free of undesired stationary points. However, these techniques are suitable only for the centralized multiuser receivers, where there is a need for the detection and separation of all the users.

In [14], a linearly constrained constant modulus (LCCM) approach to MAI suppression was investigated, where the canonical constraint as given in [3] was employed. Such a strategy is suitable for implementation both as downlink receivers and centralized station receivers. However, it has been shown in [15] that the performance of the LCCM is poor when the desired user amplitude is less than critical value $1/\sqrt{3}$. This means that the capability of LCCM to remove MAI is limited by the received power of the desired user.

In this paper, we consider a modified constrained CMA for blind multiuser detection, where a noncanonical constraint is employed. The capability of the modified CMA to suppress MAI can be strengthened by properly selecting some constant when the desired user's power becomes weaker. So the modified CMA overcomes the weakness of LCCM. In the noise situation, we show that the modified CMA cost function is strictly convex by properly selecting some constant. The extrema of the modified CMA cost function are indirectly given. According to the extreme properties of the modified cost function and numeric analysis, we point out how to select the constant. Moreover, an adaptive algorithm for implementing the blind scheme presented here is given using stochastic gradient methods, and the effect of strengths of users, as well as the level of noise on the convergence of the algorithm, is analyzed.

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This paper is organized as follows. In Section II, the signal model is introduced. The modified CMA is developed in Section III. Section IV presents an adaptive algorithm and analyzes the convergence of the algorithm. Section V provides some simulation examples. Section VI contains some conclusions.

II. Signal Model

Consider a synchronous baseband DS-CDMA system with K users. The received signal is given by

$$r(t)=\sum_{k=1}^{K}A_kb_ks_k(t)+n(t),\quad t\in[0,T] \tag{1}$$

where A_k, b_k, and $s_k(t)$ denote, respectively, the received amplitude, transmitted symbol, and normalized signaling waveform of the kth user, and $n(t)$ is the additive white Gaussian noise (AWGN) with zero mean and variance σ^2. For simplicity, it is assumed that b_k is a binary phase-shift keying (BPSK) signal, that is, $b_k\in\{\pm1\}$ is assumed to be independent equiprobable random variables, and that $s_k(t)$ is real and supported only on the interval $[0,T]$, which is of the form

$$s_k(t)=\sum_{j=0}^{N-1}c_j^k\varphi(t-jT_c),\quad t\in[0,T] \tag{2}$$

where N is the processing gain, $(c_0^k,c_1^k,\ldots,c_{N-1}^k)$ is a signature sequence of ±1's assigned to the kth user, and φ is a normalized chip waveform of duration T_c, where $NT_c=T$. The extension to the MPSK signal and complex signaling is straightforward.

At the receiver, chip-matched filtering followed by chip rate sampling yields an N-vector of chip-matched filter output samples within a symbol interval T

$$\mathbf{r}=\sum_{k=1}^{K}A_kb_k\mathbf{s}_k+\mathbf{n} \tag{3}$$

where $\mathbf{s}_k=(1/\sqrt{N})\left[c_0^k,\ldots,c_{N-1}^k\right]^{\mathrm{T}}$ is the normalized signature waveform vector of the kth user, and $\mathbf{n}$ is an AWGN vector with mean $\mathbf{0}$ and covariance matrix $\sigma^2\mathbf{I}_N$. In this paper, $\mathbf{I}_n$ denotes the $n\times n$ identity matrix. Moreover, assume that the vectors $\{\mathbf{s}_k\}_{k=1}^{K}$, also called signal vectors, are independent.

Henceforth, let user 1 be the desired user. We will use the following notations and definitions throughout this paper. For any real symmetric matrix $\mathbf{A}$, it is nonnegative definite if $\mathbf{x}^{\mathrm{T}}\mathbf{A}\mathbf{x}\geq0$ for any real vector $\mathbf{x}$, denoted by $\mathbf{A}\geq\mathbf{0}$, and it is positive definite if $\mathbf{x}^{\mathrm{T}}\mathbf{A}\mathbf{x}>0$ for any nonzero vector $\mathbf{x}$, denoted by $\mathbf{A}>\mathbf{0}$. Bold zero "$\mathbf{0}$" denotes zero matrix or vector with corresponding dimension.

III. The Modified Constrained CMA Receivers

We consider a constrained CMA receiver, which is given by the following constrained optimization problem:

$$\min_{\mathbf{w}^{\mathrm{T}}\mathbf{s}_1=d}J(\mathbf{w})=E[(\mathbf{w}^{\mathrm{T}}\mathbf{r})^2-1]^2 \tag{4}$$

where $d>0$ is some constant and $\mathbf{w}^{\mathrm{T}}\mathbf{s}_1=d$ is the noncanonical constraint. When $d=1$, the constraint is canonical form as given in [3].

Using the same analysis as given in [15], we can obtain the following proposition.

Proposition 1: 1) When $3d^2A_1^2-1\geq0$, the constrained CMA receiver possesses the ability to remove MAI. 2) When $3d^2A_1^2-1<0$, the constrained CMA receiver cannot completely remove MAI. For this case the performance is relatively poor.

According to Proposition 1, the performance of the constrained CMA receivers can be enhanced by properly selecting the constant d subject to the condition $3d^2A_1^2-1\geq0$. In what follows, we assume that the constant d satisfies the condition $3d^2A_1^2-1\geq0$. We shall analyze the solution to optimization problem (4) and show that the modified CMA cost function is convex by properly selecting constant d.

Denote $u_k=A_k(\mathbf{w}^{\mathrm{T}}\mathbf{s}_k)$ and $\mathbf{u}=[u_1,\ldots,u_K]^{\mathrm{T}}$, then $\mathbf{u}=\mathbf{A}\mathbf{S}^{\mathrm{T}}\mathbf{w}$ where $\mathbf{S}=[\mathbf{s}_1,\ldots,\mathbf{s}_K]$ and $\mathbf{A}=\mathrm{diag}(A_1,\ldots,A_K)$. Since there exists an orthonormal transformation such that the first column of matrix $\mathbf{S}$ is $[1\quad0\quad\ldots\quad0]^{\mathrm{T}}$, in what follows, for convenience and without loss of generality, assume $\mathbf{s}_1=[1\quad0\quad\ldots\quad0]^{\mathrm{T}}$. Then the constrained condition $\mathbf{w}^{\mathrm{T}}\mathbf{s}_1=d$ is equivalent to $w_1=d$ or $u_1=A_1d$. Therefore, the optimization problem (4) transfers to

$$\min_{\overline{\mathbf{w}}\in R^{N-1}}\tilde{J}(\overline{\mathbf{w}})=J(\mathbf{w}_d) \tag{5}$$

where $\overline{\mathbf{w}}=[w_2,\ldots,w_N]^{\mathrm{T}}$ and $\mathbf{w}_d=[d,\overline{\mathbf{w}}^{\mathrm{T}}]^{\mathrm{T}}$.

In analogy to the derivation given in [7] and [16], we can calculate

$$J(\mathbf{w})=E(\mathbf{w}^{\mathrm{T}}\mathbf{r})^4-2E(\mathbf{w}^{\mathrm{T}}\mathbf{r})^2+1=J_0(\mathbf{u})+\sigma^2J_1(\mathbf{w}) \tag{6}$$

where

$$J_0(\mathbf{u})=3(\mathbf{u}^{\mathrm{T}}\mathbf{u})^2-2\mathbf{u}^{\mathrm{T}}\mathbf{u}-2\sum_{k=1}^{K}u_k^4+1 \tag{7}$$

$$J_1(\mathbf{w})=(6\mathbf{u}^{\mathrm{T}}\mathbf{u}-2+3\sigma^2\mathbf{w}^{\mathrm{T}}\mathbf{w})(\mathbf{w}^{\mathrm{T}}\mathbf{w}) \tag{8}$$

Define

$$\overline{\mathbf{u}}=[u_2,\ldots,u_k]^{\mathrm{T}}=[\mathbf{b},\mathbf{P}]\mathbf{w}_d=d\mathbf{b}+\mathbf{P}\overline{\mathbf{w}} \tag{9}$$

where $[\mathbf{b},\mathbf{P}]=[A_2\mathbf{s}_2,\ldots,A_K\mathbf{s}_K]^{\mathrm{T}}$, $\mathbf{b}$ is $(K-1)$-dimensional vector, and $\mathbf{P}$ is $(K-1)\times(N-1)$ matrix. From (6), we have

$$\tilde{J}(\overline{\mathbf{w}})=J(\mathbf{w}_d)=\tilde{J}_0(\overline{\mathbf{w}})+\sigma^2\tilde{J}_1(\overline{\mathbf{w}}) \tag{10}$$

where

$$\tilde{J}_0(\overline{\mathbf{w}})=3(d^2A_1^2+\overline{\mathbf{u}}^{\mathrm{T}}\overline{\mathbf{u}})^2-2\overline{\mathbf{u}}^{\mathrm{T}}\overline{\mathbf{u}}-2\sum_{k=2}^{K}u_k^4-D \tag{11}$$

$$\tilde{J}_1(\overline{\mathbf{w}})=(6d^2A_1^2+6\overline{\mathbf{u}}^{\mathrm{T}}\overline{\mathbf{u}}-2+3\sigma^2\mathbf{w}_d^{\mathrm{T}}\mathbf{w}_d)(\mathbf{w}_d^{\mathrm{T}}\mathbf{w}_d) \tag{12}$$

in which $D=2d^2A_1^2+2d^4A_1^4-1$.

We next analyze the extreme properties of the function $\tilde{J}(\overline{\mathbf{w}})$. From (11) and (12), we can calculate the gradients

$$\begin{aligned}\nabla_{\overline{\mathbf{w}}}\tilde{J}_0 =& \mathbf{P}^{\mathrm{T}}\nabla_{\overline{\mathbf{u}}}\tilde{J}_0\\ =& \mathbf{P}^{\mathrm{T}}\left[4(3d^2A_1^2-1)\overline{\mathbf{u}}+12(\overline{\mathbf{u}}^{\mathrm{T}}\overline{\mathbf{u}})\overline{\mathbf{u}}\right.\\ &\left.-8\mathrm{diag}(u_2^2,\ldots,u_k^2)\overline{\mathbf{u}}\right] \qquad (13)\\ \nabla_{\overline{\mathbf{w}}}\tilde{J}_1 =& 2(6d^2A_1^2-2+6\overline{\mathbf{u}}^{\mathrm{T}}\overline{\mathbf{u}}+3\sigma^2\mathbf{w}_d^{\mathrm{T}}\mathbf{w}_d)\overline{\mathbf{w}}\\ &+6(\mathbf{w}_d^{\mathrm{T}}\mathbf{w}_d)(2\mathbf{P}^{\mathrm{T}}\overline{\mathbf{u}}+\sigma^2\overline{\mathbf{w}}). \qquad (14)\end{aligned}$$

Moreover, we can derive the Hessian matrices

$$\nabla^2_{\overline{\mathbf{w}}}\tilde{J}_0 = \mathbf{P}^{\mathrm{T}}\left[4(3d^2A_1^2-1)\mathbf{I}_{K-1}+12\mathbf{U}_{K-1}\right]\mathbf{P} \qquad (15)$$

$$\begin{aligned}\nabla^2_{\overline{\mathbf{w}}}\tilde{J}_1 =& 4(3d^2A_1^2-1+3\overline{\mathbf{u}}^{\mathrm{T}}\overline{\mathbf{u}}+3\sigma^2\mathbf{w}_d^{\mathrm{T}}\mathbf{w}_d)\mathbf{I}_{N-1}\\ &+24\sigma^2\overline{\mathbf{w}}\overline{\mathbf{w}}^{\mathrm{T}}+12(\mathbf{w}_d^{\mathrm{T}}\mathbf{w}_d)\mathbf{P}^{\mathrm{T}}\mathbf{P}\\ &+24\overline{\mathbf{w}}\overline{\mathbf{u}}^{\mathrm{T}}\mathbf{P}+24\mathbf{P}^{\mathrm{T}}\overline{\mathbf{u}}\overline{\mathbf{w}}^{\mathrm{T}}\\ =& 12\left[(\overline{\mathbf{u}}^{\mathrm{T}}\overline{\mathbf{u}}+\sigma^2\mathbf{w}_d^{\mathrm{T}}\mathbf{w}_d)\mathbf{I}_{N-1}\right.\\ &\left.+2\sigma^2\overline{\mathbf{w}}\overline{\mathbf{w}}^{\mathrm{T}}+\left(\mathbf{w}_d^{\mathrm{T}}\mathbf{w}_d\right)\mathbf{P}^{\mathrm{T}}\mathbf{P}\right]\\ &+4\left[(3d^2A_1^2-1)\mathbf{I}_{N-1}+6\overline{\mathbf{w}}\overline{\mathbf{u}}^{\mathrm{T}}\mathbf{P}+6\mathbf{P}^{\mathrm{T}}\overline{\mathbf{u}}\overline{\mathbf{w}}^{\mathrm{T}}\right]\\ \stackrel{\mathrm{def}}{=}& \mathbf{G}_0(\overline{\mathbf{w}})+\mathbf{G}_1(\overline{\mathbf{w}}) \qquad (16)\end{aligned}$$

where

$$\mathbf{U}_{K-1}\stackrel{\mathrm{def}}{=}\begin{bmatrix}\sum_{k=2}^K u_k^2 & 2u_2u_3 & \cdots & 2u_2u_k\\ 2u_3u_2 & \sum_{k=2}^K u_k^2 & \cdots & 2u_3u_K\\ \vdots & \vdots & \ddots & \vdots\\ 2u_Ku_2 & 2u_Ku_3 & \cdots & \sum_{k=2}^K u_k^2\end{bmatrix}. \qquad (17)$$

Note that the matrix $\mathbf{U}_{K-1}$ is nonnegative definite, i.e., $\mathbf{U}_{K-1} \geq \mathbf{0}$ (see Appendix A). The Hessian matrix of the function $\tilde{J}(\overline{\mathbf{w}})$ is as follows:

$$\nabla^2_{\overline{\mathbf{w}}}\tilde{J} = \nabla^2_{\overline{\mathbf{w}}}\tilde{J}_0+\sigma^2\nabla^2_{\overline{\mathbf{w}}}\tilde{J}_1 = \nabla^2_{\overline{\mathbf{w}}}\tilde{J}_0+\sigma^2\left[\mathbf{G}_0(\overline{\mathbf{w}})+\mathbf{G}_1(\overline{\mathbf{w}})\right]. \qquad (18)$$

When $\sigma^2=0$, $\nabla^2_{\overline{\mathbf{w}}}\tilde{J} = \nabla^2_{\overline{\mathbf{w}}}\tilde{J}_0$.

Since $3d^2A_1^2-1\geq 0$ and $\mathbf{U}_{K-1}\geq\mathbf{0}$, we have $\nabla^2_{\overline{\mathbf{w}}}\tilde{J}_0\geq\mathbf{0}$. Thus, when $d\geq 1/\sqrt{3}A_1$ and $\sigma^2=0$, the function $\tilde{J}(\overline{\mathbf{w}})$ is convex. Since $\tilde{J}(\overline{\mathbf{w}})$ is continuous in terms of σ^2, we may assume that the extrema of the function in noisy case can be deduced for small σ^2 by a slight perturbation of the noise-free extrema. Hence, for small σ^2, it also follows that $\tilde{J}(\overline{\mathbf{w}})$ is convex when $d\geq 1/\sqrt{3}A_1$. According to the equivalence of (4) and (5), we have the following proposition.

Proposition 2: 1) When $3d^2A_1^2-1\geq 0$ and $\sigma^2=0$, the function $J(\mathbf{w})$ with the constraint $\mathbf{w}^{\mathrm{T}}\mathbf{s}_1=d$ is convex. 2) When $3d^2A_1^2-1\geq 0$ and $\sigma^2\neq 0$, for small σ^2, the function $J(\mathbf{w})$ with the constraint $\mathbf{w}^{\mathrm{T}}\mathbf{s}_1=d$ is also convex.

Remark 1: It can be seen from the definition of (16) that $\mathbf{G}_0(\overline{\mathbf{w}})\geq 0$. Substituting $\overline{\mathbf{u}}=d\mathbf{b}+\mathbf{P}\overline{\mathbf{w}}$ into $\mathbf{G}_1(\overline{\mathbf{w}})$ as defined in (16), we have

$$\mathbf{G}_1(\overline{\mathbf{w}}) = 4(3d^2A_1^2-1)\mathbf{I}_{N-1}+24\mathbf{Q}(\overline{\mathbf{w}}) \qquad (19)$$

where $\mathbf{Q}(\overline{\mathbf{w}}) = d(\overline{\mathbf{w}}\mathbf{b}^{\mathrm{T}}\mathbf{P}+\mathbf{P}^{\mathrm{T}}\mathbf{b}\overline{\mathbf{w}}^{\mathrm{T}})+(\overline{\mathbf{w}}\overline{\mathbf{w}}^{\mathrm{T}}\mathbf{P}^{\mathrm{T}}\mathbf{P}+\mathbf{P}^{\mathrm{T}}\mathbf{P}\overline{\mathbf{w}}\overline{\mathbf{w}}^{\mathrm{T}})$. It is easily seen from (19) that we can select a sufficiently large value of d such that $\mathbf{G}_1(\overline{\mathbf{w}})>\mathbf{0}$ in any bounded region. Therefore, with properly selecting the constant d, we have $\nabla^2_{\overline{\mathbf{w}}}\tilde{J}>\mathbf{0}$ in any bounded region. Hence, we have the conjecture that for the noisy case we can properly select the constant d such that the function $\tilde{J}(\overline{\mathbf{w}})$ is strictly convex. This also means that we can properly select the constant d such that the function $J(\mathbf{w})$ with the constraint $\mathbf{w}^{\mathrm{T}}\mathbf{s}_1=d$ is strictly convex when $3d^2A_1^2-1\geq 0$ and $\sigma^2\neq 0$. We have not given the rigorous proof for this case, but an example to illustrate this conjecture is given in example 1 below.

Remark 2: It is easily verified that the solution to the optimization problem (4) lies in the space spanned by the columns of the signature waveform matrix $\mathbf{S}$. Let $\mathbf{w}=\mathbf{S}\mathbf{v}$. We have $\mathbf{u}=\mathbf{A}\mathbf{S}^{\mathrm{T}}\mathbf{w}=\mathbf{A}\mathbf{S}^{\mathrm{T}}\mathbf{S}\mathbf{v}$ or $\mathbf{v}=(\mathbf{S}^{\mathrm{T}}\mathbf{S})^{-1}\mathbf{A}^{-1}\mathbf{u}$. So the solution to the optimization problem (4) can be expressed with $\mathbf{w}=\mathbf{S}(\mathbf{S}^{\mathrm{T}}\mathbf{S})^{-1}\mathbf{A}^{-1}\mathbf{u}\stackrel{\mathrm{def}}{=}\Phi\mathbf{u}$. According to (6), we define

$$\varphi(\overline{\mathbf{u}}) = J_0(\mathbf{u}_d)+\sigma^2J_1(\mathbf{w}_d) \qquad (20)$$

where $\mathbf{u}_d=[dA_1,\overline{\mathbf{u}}^{\mathrm{T}}]^{\mathrm{T}}$ and $\mathbf{w}_d=\Phi\mathbf{u}_d$. Then the solution to optimization problem (4) corresponds to the extrema of the function $\varphi(\overline{\mathbf{u}})$, which can be expressed as follows:

$$\begin{aligned}\overline{\mathbf{u}} =& \sigma^2\left[(3d^2A_1^2-1)+3\overline{\mathbf{u}}^{\mathrm{T}}\overline{\mathbf{u}}+3\sigma^2\mathbf{w}_d^{\mathrm{T}}\mathbf{w}_d\right]\\ &\times\mathbf{C}^{-1}(\overline{\mathbf{u}})\left[\mathbf{0},\mathbf{I}_{K-1}\right]\Phi^{\mathrm{T}}\Phi\mathbf{u}_d \qquad (21)\end{aligned}$$

where

$$\begin{aligned}\mathbf{C}(\overline{\mathbf{u}}) =& \left[(3d^2A_1^2-1)+3(\overline{\mathbf{u}}^{\mathrm{T}}\overline{\mathbf{u}})+3\sigma^2\mathbf{w}_d^{\mathrm{T}}\mathbf{w}_d\right]\mathbf{I}_{K-1}\\ &-2\mathrm{diag}(u_2^2,\ldots,u_K^2). \qquad (22)\end{aligned}$$

Equation (21) can be readily derived from the gradient of the function $\varphi(\overline{\mathbf{u}})$. Also, it is easily verified that $\mathbf{C}(\overline{\mathbf{u}})>\mathbf{0}$. Note that the constrained CMA receivers can suppress the MAI if and only if $\overline{\mathbf{u}}\to\mathbf{0}$. In this sense, the CMA receiver is also called a zero-forcing receiver.

Remark 3: It is obvious from (21) that $\overline{\mathbf{u}}\to\mathbf{0}$ as $\sigma\to 0$. Furthermore, it can be shown from (21) that $\overline{\mathbf{u}}\to\infty$ as $d\to\infty$, so a sufficiently large value of d cannot be selected. For small σ, we know from Proposition 2 that d can be selected to be larger than critical value $1/\sqrt{3}A_1$. Example 2 is given below to illustrate how the norm of extrema of the constrained CM cost function changes with parameter d.

Example 1: The surfaces of function $\varphi(\overline{\mathbf{u}})$ are plotted in Fig. 1(a)–(c) for three different values $d=0.1$, $1/\sqrt{3}$ and 1, where the number of users $K=3$, processing gain $N=5$, $A_1=A_2=A_3=1$, and $\sigma^2=0.01$. It is obvious from Fig. 1(b) and (c) that the function $\varphi(\overline{\mathbf{u}})$ is strictly convex when $d\geq 1/\sqrt{3}A_1$.

Example 2: The examples are given to illustrate how the norm of extrema of function $\varphi(\overline{\mathbf{u}})$ changes with parameter d. We consider the number of users $K=5$, processing gain $N=10$, and received amplitudes $A_k=1$, $k=1,\ldots,5$. Numeric results are shown in Fig. 2(a) and (b) for three different variances $\sigma^2=1$, 0.2, and 0.1. It can be seen from Fig. 2(a) that the norm of the extrema approaches the minimum when d is near to critical value $1/\sqrt{3}$ (0.577). Fig. 2(b) shows that the norm of extrema almost linearly increases as d. So we know from Fig. 2(a) and (b) that the parameter d should be selected in the field $(1/\sqrt{3}A_1, 1/A_1]$.

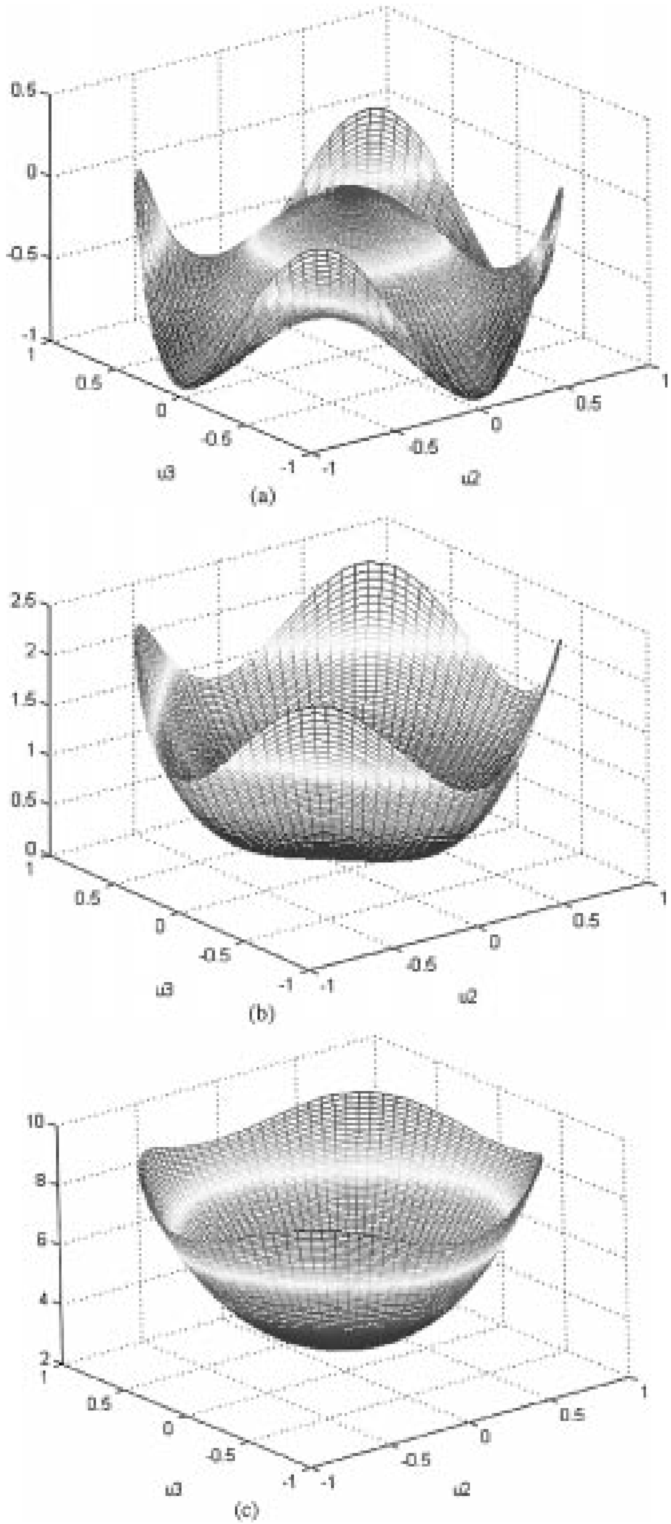

Fig. 1. Surfaces of function $\varphi(\overline{\mathbf{u}})$ for number of users $K=3$, processing gain $N=5$, $A_1=A_2=A_3=1$, and $\sigma^2=0.01$. (a) $d=0.1$. (b) $d=1/\sqrt{3}$. (c) $d=1$.

IV. Adaptive Algorithm and Convergence Analysis

In this section, we derive an adaptive algorithm to solve the optimization problem (4) using the stochastic gradient algorithm and analyze convergence properties of the algorithm. Let $\mathbf{B}$ be the $N\times(N-1)$ matrix whose columns span the orthogonal complement of vector $\mathbf{s}_1$. Then, for any $\mathbf{w}\in R^N$, $\mathbf{w}^{\mathrm{T}}\mathbf{s}_1=d$ if and only if there uniquely exists $\mathbf{x}\in R^{N-1}$ such that $\mathbf{w}=d\mathbf{s}_1+\mathbf{B}\mathbf{x}$. Therefore, the constrained optimization problem (4) can be converted into an unconstrained form

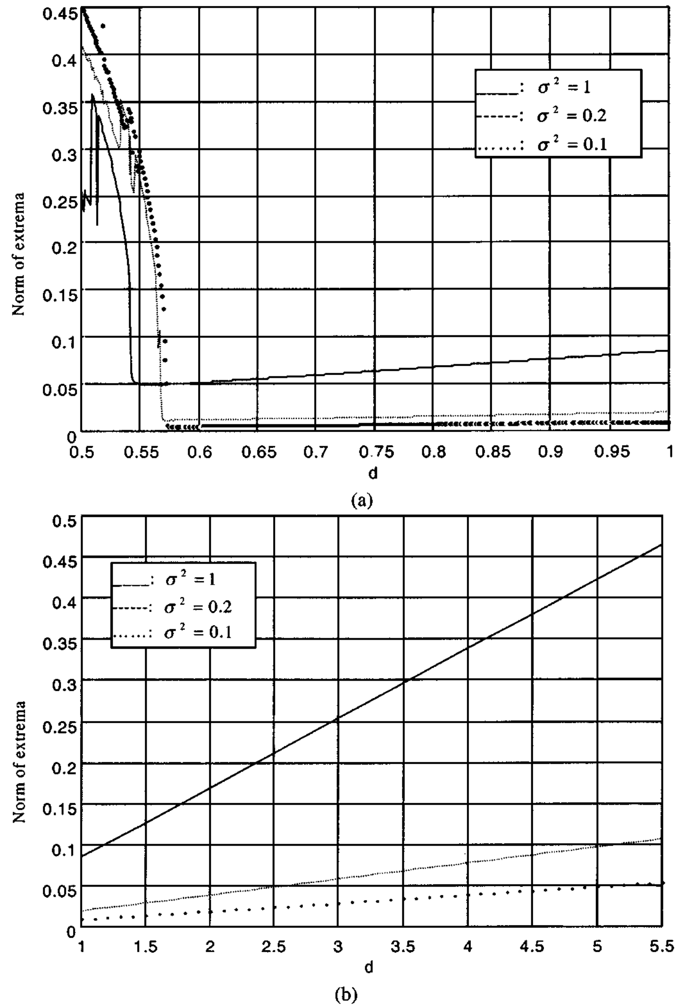

Fig. 2. Norm of the extrema of function $\varphi(\overline{\mathbf{u}})$ for $\sigma^2=1, 0.2$, and 0.1, where $K=5$, $N=10$ and $A_k=1, k=1,\ldots,5$. (a) $1/2A_1\le d\le 1/A_1$. (b) $1/A_1\le d\le 5.5/A_1$.

$$\min_{\mathbf{x}\in R^{N-1}} \eta(\mathbf{x})=J(d\mathbf{s}_1+\mathbf{B}\mathbf{x}). \tag{23}$$

Using the stochastic gradient methods, we can solve the optimization problem (23) and derive the following adaptive algorithm to solve optimization problem (4).

Algorithm:

$$\begin{aligned}\mathbf{x}_n =&\mathbf{x}_{n-1}-\mu\mathbf{B}^{\mathrm{T}}\left[(\mathbf{w}_{n-1}{}^{\mathrm{T}}\mathbf{r}_n)^2-1\right](\mathbf{w}_{n-1}{}^{\mathrm{T}}\mathbf{r}_n)\mathbf{r}_n\\ \mathbf{w}_n =&d\mathbf{s}_1+\mathbf{B}\mathbf{x}_n,\quad \mathbf{w}_0=d\mathbf{s}_1\end{aligned}$$

where μ is step-size and $d\ge 1/\sqrt{3}A_1$. In practice, it can be used that $\mathbf{B}=\mathbf{I}-\mathbf{s}_1\mathbf{s}_1^{\mathrm{T}}$, correspondingly, $\mathbf{x}\in R^N$.

Let the vector $\mathbf{x}_{\mathrm{opt}}$ be the solution to the optimization problem (23). Then the solution to the optimization problem (4) is $\mathbf{w}_{\mathrm{opt}}=d\mathbf{s}_1+\mathbf{B}\mathbf{x}_{\mathrm{opt}}$. Denote the tap vector error $\mathbf{e}_n=\mathbf{x}_n-\mathbf{x}_{\mathrm{opt}}$. We have

$$\mathbf{e}_n=\mathbf{e}_{n-1}-\mu\mathbf{B}^{\mathrm{T}}\left[(\mathbf{w}_{n-1}{}^{\mathrm{T}}\mathbf{r}_n)^2-1\right]\left(\mathbf{w}_{n-1}{}^{\mathrm{T}}\mathbf{r}_n\right)\mathbf{r}_n \tag{24}$$

and taking expectation of both sides gives

$$E\{\mathbf{e}_n\} = E\{\mathbf{e}_{n-1}\} - \mu \mathbf{B}^{\mathrm{T}} \nabla J(\mathbf{w}_{n-1}) \tag{25}$$

where $\nabla J(\mathbf{w}) = E\{[(\mathbf{w}^{\mathrm{T}}\mathbf{r})^2 - 1](\mathbf{w}^{\mathrm{T}}\mathbf{r})\mathbf{r}\}$ is the gradient of the cost function $J(\mathbf{w})$. Since $\mathbf{B}^{\mathrm{T}}\nabla J(\mathbf{w}_{\mathrm{opt}}) = \mathbf{0}$ and $\mathbf{w}_n - \mathbf{w}_{\mathrm{opt}} = \mathbf{B}\mathbf{e}_n$, we have

$$\begin{aligned} E\{\mathbf{e}_n\} =& E\{\mathbf{e}_{n-1}\} - \mu \mathbf{B}^{\mathrm{T}} \left[\nabla J(\mathbf{w}_{n-1}) - \nabla J(\mathbf{w}_{\mathrm{opt}})\right] \\ =& E\{\mathbf{e}_{n-1}\} - \mu \mathbf{B}^{\mathrm{T}} \nabla^2 J(\mathbf{w}_{\mathrm{opt}} + \theta \mathbf{B}\mathbf{e}_{n-1}) \mathbf{B} E\{\mathbf{e}_{n-1}\} \\ =& \left[\mathbf{I}_{N-1} - \mu \mathbf{B}^{\mathrm{T}} \nabla^2 J(\mathbf{w}_{\mathrm{opt}} + \theta \mathbf{B}\mathbf{e}_{n-1})\mathbf{B}\right] E\{\mathbf{e}_{n-1}\} \end{aligned} \tag{26}$$

where $0 \le \theta < 1$, and $\nabla^2 J(\mathbf{w}) = E\{[3(\mathbf{w}^{\mathrm{T}}\mathbf{r})^2 \mathbf{r}\mathbf{r}^{\mathrm{T}} - \mathbf{r}\mathbf{r}^{\mathrm{T}}]\}$ is the Hessian matrix of cost function $J(\mathbf{w})$. According to Proposition 2, we can properly choose constant d such that

$$\mathbf{D} \overset{\mathrm{def}}{=} \mathbf{B}^{\mathrm{T}} \nabla^2 J(\mathbf{w}_{\mathrm{opt}} + \theta \mathbf{B}\mathbf{e}_{n-1})\mathbf{B} > \mathbf{0}. \tag{27}$$

Therefore, the above algorithm converges to the optimal solution if and only if

$$0 < \mu < \frac{2}{\lambda_{\max}^{\mathbf{D}}} \tag{28}$$

where $\lambda_{\max}^{\mathbf{D}}$ is the largest eigenvalue of the matrix $\mathbf{D}$. Next, we further analyze the effect of strengths of users, as well as the level of noise on the convergence. The analysis is analogous to that given in [3]. Assume that the condition (28) is satisfied. Then, for large n, it can be derived (see Appendix B) that

$$\mathbf{D} \approx \left[(3d^2 A_1^2 - 1) + 3\sigma^2 \mathbf{w}_n^{\mathrm{T}} \mathbf{w}_n\right] \mathbf{B}^{\mathrm{T}} \mathbf{R} \mathbf{B} \overset{\mathrm{def}}{=} \alpha \mathbf{D}_0 \tag{29}$$

where $\mathbf{R} = \sum_{k=1}^{K} A_k^2 \mathbf{s}_k \mathbf{s}_k^{\mathrm{T}} + \sigma^2 \mathbf{I}_N$, $\alpha = (3d^2 A_1^2 - 1) + 3\sigma^2 \mathbf{w}_n^{\mathrm{T}} \mathbf{w}_n$, and $\mathbf{D}_0 = \mathbf{B}^{\mathrm{T}}\mathbf{R}\mathbf{B}$. From Remark 2, we can calculate $\mathbf{w}_n^{\mathrm{T}} \mathbf{w}_n = \mathbf{u}_n^{\mathrm{T}} \mathbf{A}^{-1} (\mathbf{S}^{\mathrm{T}}\mathbf{S})^{-1} \mathbf{A}^{-1} \mathbf{u}_n \approx d^2 [(\mathbf{S}^{\mathrm{T}}\mathbf{S})^{-1}]_{11}$.

Let $\mathbf{B} = \mathbf{I} - \mathbf{s}_1 \mathbf{s}_1^{\mathrm{T}}$. Then we can observe that K eigenvectors of $\mathbf{D}_0$ lie in the space spanned by the signal vectors $\mathbf{s}_1, \ldots, \mathbf{s}_K$, and the remaining $N - K$ eigenvectors of $\mathbf{D}_0$ are orthogonal to the signal space. The eigenvalue associated with these latter eigenvectors is σ^2, and $\mathbf{s}_1$ is an eigenvector of $\mathbf{D}_0$ with eigenvalue $\lambda_1 = 0$. Denote $\mathbf{u}_k = \mathbf{s}_k - \rho_{1k}\mathbf{s}_1$ as the orthogonal projection of $\mathbf{s}_k$ onto $\mathbf{s}_1$, where $\rho_{1k} = \mathbf{s}_1^{\mathrm{T}} \mathbf{s}_k$. It is obvious that $\mathbf{B}\mathbf{u}_k = \mathbf{B}\mathbf{s}_k = \mathbf{u}_k$. If the signal vectors are approximately orthogonal, then $\mathbf{u}_k^{\mathrm{T}} \mathbf{s}_j \approx 0$ for $k \ne j$, furthermore

$$\begin{aligned} \mathbf{D}_0 \mathbf{u}_k =& \mathbf{B}^{\mathrm{T}}\mathbf{R}\mathbf{B}\mathbf{u}_k = \mathbf{B}^{\mathrm{T}}\mathbf{R}\mathbf{u}_k = \left[\sum_{k=1}^{K} A_k^2 \mathbf{B}\mathbf{s}_k \mathbf{s}_k^{\mathrm{T}} + \sigma^2 \mathbf{B}\right] \mathbf{u}_k \\ \approx& \left(A_k^2 \mathbf{s}_k^{\mathrm{T}} \mathbf{u}_k + \sigma^2\right) \mathbf{u}_k = \left[A_k^2 \left(1 - \rho_{1k}^2\right) + \sigma^2\right] \mathbf{u}_k \end{aligned} \tag{30}$$

so that the eigenvalues of $\mathbf{D}$ can be approximated as

$$\lambda_k^{\mathbf{D}} \approx \begin{cases} \alpha \left[A_k^2 \left(1 - \rho_{1k}^2\right) + \sigma^2\right], & k = 2, \ldots, K \\ \alpha \sigma^2, & k = K+1, \ldots, N \end{cases}. \tag{31}$$

Note that

$$(K-1)\lambda_{\max}^{\mathbf{D}} + \alpha (N - K)\sigma^2 \ge \sum_{i=1}^{N} \lambda_i^{\mathbf{D}} = \mathrm{tr}(\mathbf{D}) \tag{32}$$

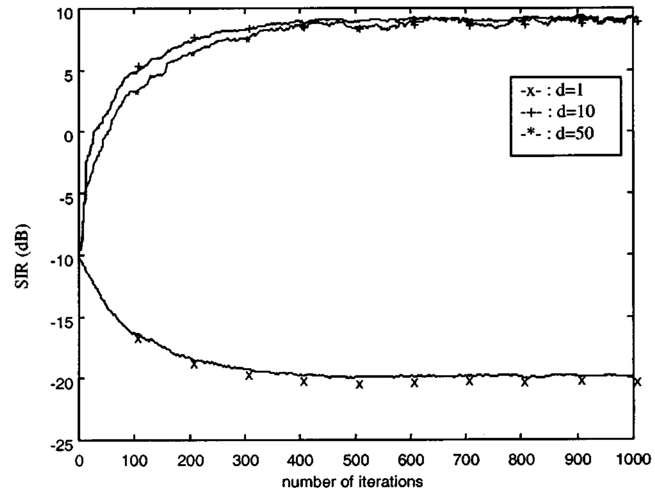

Fig. 3. Performance of the constrained CMA receiver for $d = 1$, 10, and 50, respectively. The number of users is $K = 5$ and the processing gain $N = 10$. The desired user is user 1, and $A_1 = 0.1$. There are four 20-dB MAIs. The SNR is 10 dB. The data plotted are the average over 50 simulations.

and

$$\begin{aligned} \mathrm{tr}(\mathbf{D}) \approx \alpha \mathrm{tr}(\mathbf{D}_0) =& \alpha \mathrm{tr} \left[\sum_{k=2}^{K} A_k^2 \mathbf{B}^{\mathrm{T}} \mathbf{s}_k \mathbf{s}_k^{\mathrm{T}} \mathbf{B} + \sigma^2 \mathbf{B}^{\mathrm{T}} \mathbf{B}\right] \\ =& \alpha \left[\sum_{k=2}^{K} A_k^2 \left(1 - \rho_{1k}^2\right) + \sigma^2 (N-1)\right]. \end{aligned} \tag{33}$$

We have the following approximate inequality:

$$\frac{1}{\lambda_{\max}^{\mathbf{D}}} \le \frac{1}{\alpha \left[\frac{1}{K-1} \sum_{k=2}^{K} A_k^2 \left(1 - \rho_{1k}^2\right) + \sigma^2\right]}. \tag{34}$$

Moreover, it can be approximately derived from (31) that

$$\frac{1}{\lambda_{\max}^{\mathbf{D}}} \ge \frac{1}{\alpha \left(A_{\max}^2 + \sigma^2\right)} \tag{35}$$

where $A_{\max} = \max_k A_k$.

The inequalities (34) and (35) show that the convergence of the algorithm given here is affected by constant d, amplitude A_k, as well as noise variance σ^2 and that the stability condition (28) is satisfied with taking $\mu < 2/\alpha(A_{\max}^2 + \sigma^2)$.

V. Simulation Examples

In this section, we provide simulation examples to illustrate the performance of the modified CMA receiver. The performance measure is the output signal-to-interference ratio (SIR)

$$\mathrm{SIR}(n) = \frac{E^2\left\{\mathbf{w}_n^{\mathrm{T}} \mathbf{r}_n\right\}}{\mathrm{Var}\left\{\mathbf{w}_n^{\mathrm{T}} \mathbf{r}_n\right\}} = \frac{A_1^2 \left[\mathbf{w}_n^{\mathrm{T}} \mathbf{s}_1\right]^2}{\sum_{k=2}^{K} A_k^2 \left[\mathbf{w}_n^{\mathrm{T}} \mathbf{s}_k\right]^2 + \sigma^2 \mathbf{w}_n^{\mathrm{T}} \mathbf{w}_n}. \tag{36}$$

We consider a synchronous CDMA system with processing gain $N = 10$ and number of users $K = 5$. The spreading sequences $\{c_j^k, j = 0, \ldots, N-1\}$ are randomly generated. The desired user is user 1. The received amplitude of user 1 is $A_1 = 0.1$. There are four 20-dB multiple-access interferers, i.e., $A_k^2/A_1^2 = 100$ ($k = 2 \sim 5$). The signal-to-noise ratio (SNR) is 10 dB. Numeric results are shown in Figs. 3 and 4, respectively.

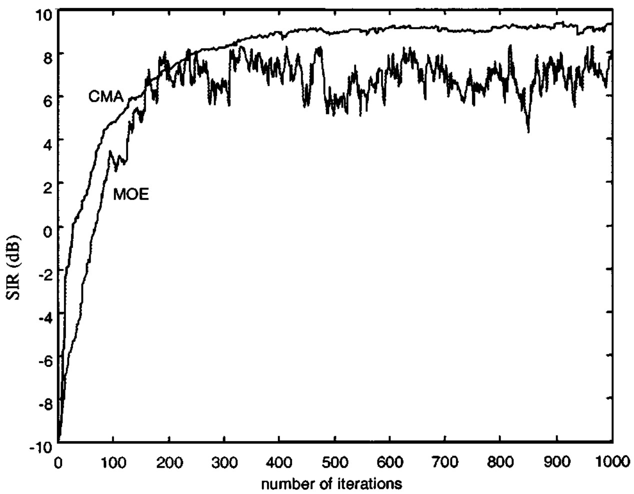

Fig. 4. Performance comparison between the constrained CMA receiver ($d = 10$) and the MOE receiver. Number of users $K = 5$ and the processing gain $N = 10$. The desired user is user 1, and $A_1 = 0.1$. There are four 20-dB MAIs. The SNR is 10 dB. The data plotted are the average over 50 simulations.

Fig. 3 shows the output SIR of the constrained CMA receiver versus number of iterations for three different constants d, respectively. It can be seen from Fig. 3 that the performance of the CMA receiver is very poor when $d = 1$, which corresponds to the canonical constraint. On the other hand, we know from Fig. 3 that the constrained CMA receiver exhibits a good performance when $d = 10$ and 50, respectively. At this moment, it follows that $3d^2A_1^2 - 1 > 0$.

Fig. 4 shows the output SIR of the constrained CMA receiver ($d = 10$) and the MOE receiver, respectively. It can be seen from Fig. 4 that the modified CMA receiver outperforms the MOE receiver when adaptive algorithms of two receivers reach convergence.

VI. Conclusions

In this paper, we have presented an alternative blind adaptive multiuser detection based on modified constrained CMA. The performance of the modified CMA can be strengthened by properly selecting constant d. In the noise case, we have shown that the modified CMA cost function is strictly convex under some conditions. A simple stochastic gradient algorithm for implementing our scheme has been given, and the effect of users' powers, as well as the level of noise on the convergence of the algorithm, was analyzed. Simulation examples demonstrated the efficiency of the modified CMA and also showed that the modified CMA receiver performs better than MOE receiver when adaptive algorithms of the two receivers reach convergence.

Appendix A
Proof of the Nonnegative Definiteness of Matrix $\mathbf{U}_{K-1}$

We define the matrix

$$\mathbf{A}_K = \begin{bmatrix} \sum_{k=1}^K a_k^2 & 2a_1a_2 & \cdots & 2a_1a_K \\ 2a_2a_1 & \sum_{k=1}^K a_k^2 & \cdots & 2a_2a_K \\ \vdots & \vdots & \ddots & \vdots \\ 2a_Ka_1 & 2a_Ka_2 & \cdots & \sum_{k=1}^K a_k^2 \end{bmatrix}.$$

It is easily verified from induction that for any vector $\mathbf{x}$, we have the equation

$$\mathbf{x}^{\mathrm{T}}\mathbf{A}_K\mathbf{x} = \left(\sum_{i=1}^K a_ix_i\right)^2 + \sum_{1\le i<j\le K}(a_ix_j + a_jx_i)^2.$$

Thus, the matrix $\mathbf{A}_K$ is nonnegative definite. So $\mathbf{U}_{K-1}$ is nonnegative definite.

Appendix B
Approximate Calculation of Matrix $\mathbf{D}$

Ideally, in the asymptotic case, we have that $u_k^{(n)} = A_k\mathbf{s}_k^{\mathrm{T}}\mathbf{w}_n \approx 0,\ k = 2,\ldots,K$. Then $\mathbf{w}_n^{\mathrm{T}}\mathbf{r} \approx A_1b_1\mathbf{w}_n^{\mathrm{T}}\mathbf{s}_1 + \mathbf{w}_n^{\mathrm{T}}\mathbf{n} = dA_1b_1 + \mathbf{w}_n^{\mathrm{T}}\mathbf{n}$ and $\left(\mathbf{w}_n^{\mathrm{T}}\mathbf{r}\right)^2 \approx d^2A_1^2 + \left(\mathbf{w}_n^{\mathrm{T}}\mathbf{n}\right)^2 + 2dA_1b_1\mathbf{w}_n^{\mathrm{T}}\mathbf{n}$. Furthermore,

$$\begin{aligned} E\left[\left(\mathbf{w}_n^{\mathrm{T}}\mathbf{r}\right)^2\mathbf{r}\mathbf{r}^{\mathrm{T}}\right] \approx& d^2A_1^2\mathbf{R} + E\left[\left(\mathbf{w}_n^{\mathrm{T}}\mathbf{n}\right)^2\mathbf{r}\mathbf{r}^{\mathrm{T}}\right] \\ =& d^2A_1^2\mathbf{R} + \sigma^2\left(\mathbf{w}_n^{\mathrm{T}}\mathbf{w}_n\right)\sum_{k=1}^K A_k^2\mathbf{s}_k\mathbf{s}_k^{\mathrm{T}} \\ &+ E\left[\left(\mathbf{w}_n^{\mathrm{T}}\mathbf{n}\right)^2\mathbf{n}\mathbf{n}^{\mathrm{T}}\right] \\ =& d^2A_1^2\mathbf{R} + \sigma^2\left(\mathbf{w}_n^{\mathrm{T}}\mathbf{w}_n\right)\mathbf{R} + 2\sigma^4\mathbf{w}_n\mathbf{w}_n^{\mathrm{T}} \end{aligned}$$

where $\mathbf{R} = E[\mathbf{r}\mathbf{r}^{\mathrm{T}}] = \sum_{k=1}^K A_k^2\mathbf{s}_k\mathbf{s}_k^{\mathrm{T}} + \sigma^2\mathbf{I}_N$. So

$$\begin{aligned} \nabla^2 J(\mathbf{w}_n) =& E\left\{\left[3\left(\mathbf{w}_n^{\mathrm{T}}\mathbf{r}\right)^2\mathbf{r}\mathbf{r}^{\mathrm{T}} - \mathbf{r}\mathbf{r}^{\mathrm{T}}\right]\right\} \\ =& E\left\{\left[3\left(\mathbf{w}_n^{\mathrm{T}}\mathbf{r}\right)^2\mathbf{r}\mathbf{r}^{\mathrm{T}}\right]\right\} - E[\mathbf{r}\mathbf{r}^{\mathrm{T}}] \\ \approx& (3d^2A_1^2 - 1)\mathbf{R} + 3\sigma^2\left(\mathbf{w}_n^{\mathrm{T}}\mathbf{w}_n\right)\mathbf{R} + 6\sigma^4\mathbf{w}_n\mathbf{w}_n^{\mathrm{T}}. \end{aligned}$$

Therefore,

$$\begin{aligned} \mathbf{D} =& \mathbf{B}^{\mathrm{T}}\nabla^2 J(\mathbf{w}_{\mathrm{opt}} + \theta\mathbf{B}\mathbf{e}_{n-1})\mathbf{B} \\ \approx& \alpha\mathbf{B}^{\mathrm{T}}\mathbf{R}\mathbf{B} + 6\sigma^4\mathbf{B}^{\mathrm{T}}\mathbf{w}_n\mathbf{w}_n^{\mathrm{T}}\mathbf{B} \end{aligned}$$

where $\alpha = (3d^2A_1^2 - 1) + 3\sigma^2\mathbf{w}_n^{\mathrm{T}}\mathbf{w}_n$. Omitting the high-order term of noise, then $\mathbf{D} \approx \alpha\mathbf{B}^{\mathrm{T}}\mathbf{R}\mathbf{B}$.

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