A Novel Multi-objective Evolutionary Algorithm Solving Portfolio Problem

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Abstract— **With the improvement of complex and uncertain finance environment, the difficulty of portfolio problem is increasing. Whether or not the projects is successfully selected, directly affects the development of the investment companies. This paper firstly talks about the finance conditions in single term investment and then extends the investment from one term to many terms. After that, a multi-project and multi-term portfolio model through considering the remaining funds in different investment terms is proposed. The model is based on a new kind of Mean-Semi-covariance theory, which can describe the uncertainty of return and risk in investment. The portfolio investment is a multi-objective optimization problem with constraints. Multi-objective evolutionary algorithm (MOEA) with greedy repair strategy is used to deal with the infeasible individuals and makes the investment reasonable. Finally, computer simulation shows that the proposed algorithm can be considered as a viable alternative.**

Index Terms— **Multi-objective optimization, multi-project and multi-term portfolio, portfolio model, evolutionary algorithm**

I. Introduction

In the finance environment, portfolio investment actually faces a large number of risky assets. It is an open question how to distribute the limited funds reasonably. Generally speaking, the investment purpose is to get returns' maximum and risks' minimum. In order to quantify the risk, the Mean-Variance portfolio model first was proposed by Markowitz in literature [1]. This theory has become an important tool in coping the financial investment problem and decision-making.

A large number of portfolio models and algorithms have been proposed in literature [2-9]. Konno and Yamazaki proposed the mean-absolute deviation portfolio optimization model in literature [2]. Lin proposed an effective decision, he used genetic algorithm to deal with multi-objective portfolio optimization problem in literature [3]. In literature [4]. Gabriella Dellino used dynamic objectives aggregation method to solve the portfolio optimization problem. Kawakami made use of the genetic algorithm to deal with the dynamic asset portfolio optimization problem in literature [5]. Xu Bin put forward a general investment combination model and gave one solution of this model in his paper [6]. A new mean-variance model was proposed for optimal capital allocation and a fuzzy simulation was provided for solving the proposed optimization problem in literature [7]. Song Yuantao builded a model about staged investment and got the biggest benefit in literature [8]. In literature [9]. Wang Zhongye proposed a investment model based on information entropy and used genetic algorithm to make decision. Hou Linlin considered the investment sequence and introduced the combinatorial risk in his literature [10].

In the investment market, as shown in literature [11], we could find that investors confront more financial constraints than we have ever expected, if the technical resources and other factors are being considered. In fact, investors may have different investment preferences. Some of them prefer to take high risk in order to gain high return, while others incline to avoid high risk. In this case, its not proper if we only furnish one particular portfolio. Usually, investors want to obtain a series of investment portfolios and then they can choose the portfolio by their own preference. However, people must consider the funding constraints in the multi-term. The portfolio investment is a multi-objective optimization problem (MOP) with constraints in real life. In different investment terms, investors may have the remaining funds. It is not appropriate that we do not consider the remaining funds in investment. Thus this paper proposes a multi-project and multi-term portfolio model through considering the possible remaining funds in investment. Generally speaking, investment risk is closely related with the uncertainty. In general, investors would like to consider an investment as available one if the return is higher than they had expected. In other words, an investment will be regarded as so full of hazard if the investment return is lower than the expected return. It is quite common that people use variance to measure the investment risk in the proposed methods. However, the variance may exaggerate the risk in the investment. So, both of variance or absolute deviation is not the best approach to measure the investment risk. To solve this problem, this paper proposes a new method

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doi:10.4304/jsw.9.1.222-229

that can estimate the risk in investment more effectively: method of semi-covariance. It can describe the risk of investment and then makes the model more effective than some existing models.

Nowadays some methods are extremely difficult to solve multi-objective optimization problem if there are many constraints. For example, using the mutually-excluding method is complex to compute [12]. Due to the complexity of portfolio, traditional mathematical optimal methods would consume a lot of time and take too much EMS memory. Thus, we need to explore more effective algorithms. In fact, a large number of improved evolutionary algorithms have been given in literature [12-19]. Multi-objective evolutionary algorithm (MOEA) with greedy repair strategy is proposed in this paper. In investment market, the investment return always accompanies with the risk, and start-up investment funds of every project in investment need to be taken into account. The lost is denoted by the weighted sum of risk and start-up investment funds of every project in every term. Not all investment portfolios are feasible individuals. Therefore, greedy repair strategy considers ratio of the return and the loss of every project in investment which violate the constraint. It is used to repair the infeasible individuals during the evolutionary process and enlarge the return in investment as much as possible.

The remainder of the paper is organized as follows. First of all, measurement of return and risk is described in Section II. Then, investment problem is described in Section III. After that, the model of multi-project and multi-term portfolio and the framework of proposed algorithm are shown in Section IV and Section V. Finally, simulation results and conclusions are shown in Section VI and Section VII.

II. Investment Problem

A. Background

Most of paper only discuss single large project. However, there is not only single large project and the investment may last a long term in the actual investment environment. Sometimes it is difficult for investors to raise enough funds in a term. Nevertheless, it is necessary to extend the investment from one term to many terms in the field of the financial research. Investors may have many funding limitation, so it is important to take into account the remaining funds of each term. In addition, the modern financial environment is full of complexity and mutual influence. Investors may confront another finance constrains, for example, the administrative expenses. In the investment market, the unknown factors are obstructions on the road of seeking effective method for measuring the risk because it is always uncertain.

B. Single Term Investment

1) One Project: In the investment market which has only one project, we set the net cash flow in t ($t = 1, 2, ..., n$) duration as NCF_t, the cash flow bases on probability distribution as follows:

$$P(NCF_t) = p_t \quad and \quad \sum_{t=1}^{n} p_t = 1$$

The start-up investment funds of project is denoted by K in the single term. If r is the risk-free interest rate, the net present value is denoted by

$$NPV = \sum_{t=1}^{n} NCF_t(1+r)^{\frac{-t}{365}} \quad (1)$$

and the net present value index is denoted by

$$NPVI = \frac{1}{K} \cdot NPV = \frac{1}{K} \cdot \sum_{t=1}^{n} NCF_t(1+r)^{\frac{-t}{365}} \quad (2)$$

Then the return and risk of the project in the term can be given as follows:

$$E(NPVI) = \frac{1}{K} \cdot \sum_{t=1}^{n} E(NCF_t)(1+r)^{\frac{-t}{365}} \quad (3)$$

$$D(NPVI) = \frac{1}{K^2} \cdot \sum_{t=1}^{n} D(NCF_t)(1+r)^{-2(\frac{t}{365})} \quad (4)$$

In equation(3), we know that $E(NCF_t) = NCF_t \cdot p_t$.

2) Multi-Project: In the investment market, there are k projects. If these projects can be marked as $x_1, x_2, ..., x_k$, we try to select some available projects for the investment. Taking project k for example, we want to invest it in the Tth investment term ($T = 0, 1, ..., m$). We mark the net cash flow in t duration as NCF_{k^t} ($t = 1, 2, ..., n$), the cash flow abides by probability distribution as follows:

$$P(NCF_{k^t}) = p_{k^t} \quad and \quad \sum_{t=1}^{n} p_{k^t} = 1$$

The start-up investment funds of project i in the Tth term is denoted by K_i. Afterwards, the return and risk of projects in the single term can be given as follows:

$$R = \sum_{i=1}^{k} \frac{x_i K_i}{\sum_i x_i K_i} \cdot E(NPVI_i) \quad (5)$$

$$\Sigma(NPVI_i, NPVI_j) =$$

$$E(NPVI_i - E(NPVI_i))(NPVI_j - E(NPVI_j)) \quad (6)$$

In equation (5), we know that

$$E(NPVI_i) = \frac{1}{K_i} \cdot \sum_{t=1}^{n} E(NCF_{k^t})(1+r)^{\frac{-t}{365}}$$

C. Multi-Term and Multi-project Investment

1) Assumptions and Symbols: In the investment market, if each large project i ($i = 1, 2, ..., k$) is prepared to be invested in many terms, we mark project i in the Tth term as $x_{i,T}$ ($T = 0, 1, ..., m$). Generally speaking, every project in different terms need the start-up investment funds and the funds of project i in the Tth term can be denoted by $K_{i,T}$. Furthermore, $K_{i,T}$ is different in every investment term. In every investment term, it is no needs to have the same duration t $(t = t_1, t_2, ..., t_i, ..., t_n)$. For instance, if we plan to invest the project k in its 2nd term, the term has 30 days, so it means that the duration $t = 30$.

Suppose that we mark the net cash flow of project i in its t duration as NCF_{it} in the Tth term. For example, the net cash flow of project 1 in the multi-term investment can be given as:

$NCF_{11} \to NCF_{12} \to ... \to NCF_{1t_1}, NCF_{1t_1+1} \to NCF_{1t_1+2} \to ... \to NCF_{1t_2}, ..., NCF_{1t_{n-1}+1} \to NCF_{1t_{n-1}+2} \to ... \to NCF_{1t_n}$.

The net cash flow of project k in multi-term investment is given as:

$NCF_{k1} \to NCF_{k2} \to ... \to NCF_{kt_1}, NCF_{kt_1+1} \to NCF_{kt_1+2} \to ... \to NCF_{kt_2}, ..., NCF_{kt_{n-1}+1} \to NCF_{kt_{n-1}+2} \to ... \to NCF_{kt_n}$.

In the investment, we have the total capital limits of each investment term. The total capital limits in the Tth term is denoted by Q_T and it is various in different investment term. Moreover, because the project is integrative, it means that the project can't be separated. Hence, investors may have the remaining funds in some investment term. The remaining funds in the Tth term is denoted by U_T. It is not appropriate that investors do not consider the remaining funds in the investment or treat them as the return directly.

2) Measurement of Return and Risk: Suppose that $NCF_{i,T^{t_i}}$ is the cash flow of the project i in the Tth term and the term has t_i duration. The cash flow obeys probability distribution as follows

$$P(NCF_{i,T^{t_i}}) = p_{i,T^{t_i}} \quad (i = 1, 2, ..., k)$$

$$\sum_{T^{t_i}=1}^{t_i} p_{i,T^{t_i}} = 1$$

The mathematical expectation of project i in the Tth term is

$$E(NCF_{iT}) = \sum_{T^{t_i}=1} NCF_{i,T^{t_i}} \cdot p_{i,T^{t_i}} \tag{7}$$

If r is the risk-free interest rate as a specified value, investors can obtain the net present value index of project i in the Tth term as follows:

$$E(NPVI_{iT}) = \frac{1}{K_{i,T}} \cdot E(NPV_{iT}) \tag{8}$$

In the equation (8), the net present value is

$$NPV_{iT} = \sum_{T^{t_i}=1} NCF_{i,T^{t_i}} (1+r)^{\frac{-t_i}{365}}$$

The return of the project i in the Tth term is

$$R_{i,T} = \frac{x_{i,T} K_{i,T}}{\sum_{i=1}^{k} x_{i,T} K_{j,T}} \cdot E(NPVI_{iT})$$

Then the return and the risk of projects in the multi-term can be given as follows:

$$R = \sum_{T=0}^{m} \sum_{i=1}^{k} \frac{x_{i,T} K_{i,T}}{\sum_{i}^{k} x_{i,T} K_{j,T}} \cdot E(NPVI_{iT}) \tag{9}$$

$$\Sigma(NPVI_{iT}, NPVI_{jT}) =$$

$$E(NPVI_{iT} - E(NPVI_{iT}))(NPVI_{jT} - E(NPVI_{jT})) \tag{10}$$

As shown in Section II, it is not proper to use variance or absolute deviation for risk measuring. Thus, a new optimized method is given. The method is that let the semi-covariance be a measure which estimate the investment risk. We use $\Sigma(NPVI_{iT}, NPVI_{jT})$ to represent the covariance matrix. After the correction, the covariance matrix can be divided into two parts. The lower semi-covariance is denoted by $\Sigma(NPVI_{iT}, NPVI_{jT})^-$. The upper semi-covariance is denoted by $\Sigma(NPVI_{iT}, NPVI_{jT})^+$. They can be shown as follows:

$$\Sigma(NPVI_{iT}, NPVI_{jT})^- =$$

$$E(NPVI_{iT} - E(NPVI_{iT})^-)(NPVI_{jT} - E(NPVI_{jT})^-) \tag{11}$$

$$\Sigma(NPVI_{iT}, NPVI_{jT})^+ =$$

$$E(NPVI_{iT} - E(NPVI_{iT})^+)(NPVI_{jT} - E(NPVI_{jT})^+) \tag{12}$$

In the equation (11), we have

$$(NPVI_{iT}, NPVI_{iT})^- = \max(0, E(NPVI_{jT}) - NPVI_{jT})$$

In the equation (12), we have

$$(NPVI_{iT}, NPVI_{iT})^+ = \max(0, NPVI_{jT} - E(NPVI_{jT}))$$

III. Model Of Multi-project And Multi-term Portfolio

In the multi-project and multi-term investment, investors may consider the return and the risk of projects in the multi-term. Usually, Generally speaking, they want to get the high returns with the low risk. So the portfolio optimization model is based on maximizing the return and minimizing the risk.

If $x_{i,T} = 1$, it means that we select project i to be invested in the Tth investment term; if $x_{i,T} = 0$, it means that we will not plan to select project i to be invested in the Tth investment term.

So if we want to get the maximization of return and minimization of risk, the MOP model can be given as follows:

$$\max(R) = \sum_{T=0}^{m} \sum_{i=1}^{k} \frac{x_{i,T} K_{i,T}}{\sum_{i}^{k} x_{i,T} K_{i,T}} E(NPVI_{iT})$$

$$\min(V) = \frac{\sum_{T}^{m} \sum_{i}^{k} \sum_{j}^{k} \frac{x_{i,T} K_{i,T}}{\sum_{i}^{k} x_{i,T} K_{i,T}} \Sigma(NPVI_{iT}, NPVI_{jT})^- \frac{x_{j,T} K_{j,T}}{\sum_{j}^{k} x_{j,T} K_{j,T}}}{\sum_{T}^{m} \sum_{i}^{k} \sum_{j}^{k} \frac{x_{i,T} K_{i,T}}{\sum_{i}^{k} x_{i,T} K_{i,T}} \Sigma(NPVI_{iT}, NPVI_{jT})^+ \frac{x_{j,T} K_{j,T}}{\sum_{j}^{k} x_{j,T} K_{j,T}}}$$

$$s.t \quad x_{1,0}K_{1,0} + x_{2,0}K_{2,0} + x_{3,0}K_{3,0} + ... + x_{k,0}K_{k,0} + x_{k+1,0}U_0 = Q_0 \tag{13}$$

$$x_{1,1}K_{1,1} + x_{2,1}K_{2,1} + x_{3,1}K_{3,1} + ... + x_{k,1}K_{k,1} + x_{k+1,1}U_1 \leq Q_1 + U_0(1+r)^{\frac{t_1}{365}} \tag{14}$$

$$x_{1,2}K_{1,2} + x_{2,2}K_{2,2} + x_{3,2}K_{3,2} + ... + x_{k,2}K_{k,2} + x_{k+1,2}U_2 \leq Q_2 + U_1(1+r)^{\frac{t_2-t_1}{365}} \tag{15}$$

... ...

$$x_{1,T}K_{1,T} + x_{2,T}K_{2,T} + x_{3,T}K_{3,T} + ... + x_{k,T}K_{k,T} + x_{k+1,T}U_T \leq Q_T + U_{T-1}(1+r)^{\frac{t_n-t_{n-1}}{365}} \tag{16}$$

$$\sum_T^m \sum_i^k x_{i,T}K_{i,T} \leq \sum_T^m Q_T \tag{17}$$

where equation (13) is capital funding constraint of the initial term in the investment, equation (14) and (15) are the funding constraints of all projects in the 1st investment term and the 2nd investment term, equation (16) and (17) are the funding constraints of all projects in the Tth investment term and the whole investment terms respectively.

Besides, we consider the minimization problem and denote it by $f(x) \in (0,1)$. If the constant value $\gamma \geq 1$, the problem of minimizing $f(x)$ can be turned to obtain the maximization of $(\gamma - f(x))$.

IV. The Framework Of Proposed Algorithm

A. Encoding

Suppose that we plan to select some projects from projects $1, 2, ..., k$, which will be invested in the T investment terms. In this algorithm, every individual is a composed of $T \times k$ matrix. The number of the row and the number of the column correspond to the project and investment term, elements of the matrix are 0 or 1.

For example, suppose $k = 25, T = 5$, it means that we plan to select some projects to invest in five investment terms. An individual is represented by a transposed matrix of 25 rows and 5 columns, we can denote it by X. In the matrix, $x_{0,4} = 0$ means project 4 is not selected in the first term ; $x_{1,10} = 1$ means project 10 is selected in the second term and $x_{2,2} = 0$ means project 2 isn't selected in the third term.

$$X = \begin{pmatrix} 1110000101100000101000 1010 \\ 0110011011010010100101110 \\ 0001111000000000110101100 \\ 1000101000000000110001010 \\ 0001111000000000110001111 \end{pmatrix}$$

X is a 5×25 matrix as follows

$$X = \begin{pmatrix} x_{0,1}x_{0,2}x_{0,3}...x_{0,25} \\ x_{1,1}x_{1,2}x_{1,3}...x_{1,25} \\ ...... \\ ...... \\ x_{4,1}x_{4,2}x_{4,3}...x_{4,25} \end{pmatrix}$$

In order to simplify the operation of the proposed algorithm, the encoding of X in this paper is as follows

$$X = \left(\; x_{0,1}, ..., x_{0,25}, x_{1,1}, ..., x_{1,25}, ...x_{4,1}, ..., x_{4,25} \; \right)$$

where $x_{1,1}$ also indicates project 1 in the second investment term.

B. Proposed Algorithm

1) Subregion Strategy: As is well known, evolutionary algorithm has the problem of premature. It is important to maintain the diversity of population. If we do not consider to use the subregion strategy, some points will be easy to be eliminated in the evolution process. The increasing investment return has accompanied with risk in finical investment. It is difficult to furnish portfolios which may have the maximization of return with a reasonable value of relative risk. If we utilize the subregion strategy, points will be divided into different subregion so that they can be preserved. Thus it is necessary to use the subregion strategy to preserve these points, and the points in the same subregion may have the maximization of return with reasonable value of risk. Then they may offer help in furnishing the Pareto optimal solution. In addition, the number of individuals in every subregion is less than the population size. Therefore, the subregion strategy can be used to decrease the complexity of algorithm.

The objective space is divided into M subregions by using the subregion strategy, refer to literature [16]and M center vectors are distribute uniformly in the space. Then every subregion is independently optimized and corresponds to an external set, the set is denoted by H_h and is employed to preserve some individuals ever found in this subregion ($h = 1, 2, ..., M$). In this paper, two main objectives are denoted by $f_s(X)$, where $s = 1, 2$. The weight of individual X was denoted by set $W^s = ({w_1}^s, {w_2}^s, ..., {w_{popsize}}^s)$. We classify the weight vectors by Tchebycheff method. We can express the fitness function $G(X) = max\{W^s g_s(X)\}$, where $g_s(X) = {f_s}^* - f_s(X)$ and ${f_s}^* = max\{f_s(X)\}$. The subregion strategy is used to maintain the diversity of population with a purpose of preventing premature in the evolutionary process.

2) Greedy Repair Strategy: It is important to deal with the infeasible individuals in multi-objective optimization problem with constraints. So we choose greedy repair strategy to repair the infeasible individuals in order to effectively deal with the constraint.

Suppose that X is a set of infeasible individual such that $\sum_T^m \sum_i^k x_{i,T}K_{i,T} > \sum_T^m Q_T$, $T = 0, 1, ..., m$. The start-up investment funds of project i in the Tth investment term is denoted by $K_{i,T}$. If $\sum_T^m \sum_i^k x_{i,T}K_{i,T} > \sum_T^m Q_T$, a way to make X as a feasible individual is to remove some projects from the investment term regularly. But not all investment portfolios are the infeasible individuals, the individuals which don't violate the constraint do not need to be considered in the investment. The greedy repair strategy is used frequently and described as follows

Step 1) If X is infeasible individual, then go to Step 2.

Step 2) Set $X = \{(i, T) \mid x_{i,T} = 1\}$.

Step 3) Select $x \in X$ such that:

$$x = max \frac{x_{i,T} R_{i,T}}{K_{i,T}} \tag{18}$$

Step 4) Set $x_{i,T} = 1$ and Stop .

If one project is removed from the investment term, in investment market, the return and risk of the investment must be changed. Because the investment return has always accompanied with risk. Some investors may want to get high returns but they do not like the risk. And at the same time, investors may pay some attention on the start-up investment funds of every project in each investment term. So only project is selected to be invested in one investment term, the investment portfolio will be affected by the project's risk, the return and start-up investment funds. Hence the greedy repair strategy is based on the ratio between profit and loss of every individuals. The lost funding of project i in Tth term is denoted by the sum of start-up investment funds and risk. According to this idea, an improved greedy repair strategy can be given and transformed formula as

$$x = max \frac{x_{i,T} R_{i,T}}{aK_{i,T} + bV_{i,T}} \tag{19}$$

In equation (19), $(aK_{i,T} + bV_{i,T})$ denotes the lost funding in the investment, where $a + b = 1$ and $a, b \in (0, 1)$. The lost funding of project i in Tth term is denoted by the sum of start-up investment funds and risk. As is known, $R_{i,T}$ is the return of project j in Tth investment term, $V_{i,T}$ is the risk of project j in Tth investment term.

Additionally, investors can evaluate a and b by their own preferences. In this paper, we provide $a = 0.8$, $b = 0.2$. It means investors like considering about the start-up investment funds of every project in every investment term. Considering the start-up investment funds an risk could allow the portfolios to become more feasible. As a consequence, the major purpose of using greedy repair strategy is to repair the infeasible individuals, and keep the higher return in every investment term as much as possible. Feasible individuals are penetrated into the next generation, and the proposed greedy repair strategy can improve the quality of population individuals.

3) Crossover Operator and Mutation Operator: We perform the crossover between an individual and an individual which is randomly selected from the corresponding external set. It is helpful in exploring great individuals and wide area. The crossover operator uses one-point crossover strategy. Crossover probability is denoted by p_c. Individuals are selected to perform the crossover according to a random number in $[0, 1]$. $popsize$ denote the population size. A random number pos denote the crossover point, where $pos \in [1, j]$ and j is the length of chromosome. There are $j - 1$ crossover positions. The coupled individuals exchange partial chromosomes with each other at the crossover point. So we can get the new offsprings.

The individuals which from the same subregion and the corresponding external set are selected for mutation operator. It exchanges the information among different subregions to discover the new individuals. The mutation operator uses uniform mutation strategy. It selects a single parent $X = (x_1, ..., x_w, ..., x_p)$ and generates a single offspring $X' = (x_1, ..., x_w', ..., x_p)$, where $w \in (1, p)$. Mutation probability is smaller than crossover probability, and it is denoted by p_m. Individuals are selected to perform the mutation. In this paper, if $x_w = 1$, after the mutation, $x_w' = 0$; if $x_w = 0$, after the mutation, $x_w' = 1$. Then we can obtain the new offsprings. Mutation operator plays an important part in the evolution process as the solutions are allowed to shift freely in the search space. It can enhance the search ability and exploit the optimum offspring avoiding a local optimum.

4) Selection Strategy: This paper adopts selection strategy, refer to [17]. An external set is introduced for each subregion and is used to store individuals ever found in this subregion. The dominated individuals are eliminated at once in some algorithms. But it is not helpful to utilize the dominated individuals to construct the simulative descent direction. We generate new individuals and update the external sets and subregions in the evolutionary process.

C. Steps For The Proposed Algorithm

(1) Setting parameter: set size of the population $popsize$, the number of subregions M and iterations $maxgen$, crosser probability p_c and mutation probability p_m.

(2) Initialization: generate weight vectors W^s and initial population randomly, repair infeasible individuals by formula (19), divide population into M subregions, calculate fitness value of individuals and select individuals of having the best value into sub population.

(3) Performing crosser and mutation: modify the population and generate new offspring, classify them into different subregions.

(4) Updating: update the current population and external sets.

(5) Stopping criteria: repeat (3) until satisfy the stopping criteria.

V. Computer Simulation

This paper uses similar examples as literature [6] and literature [20]. Example I: there is an investment company. The company plan to select some projects to be invested in the investment market. According to the method: coefficient of variation, where $C.V \le 1.0$, we have 10 large projects successfully passed the assessment. If these projects can be invested in five investment terms, we can suppose $T = 0, 1, 2, 3, 4$. Every term contains $\triangle t_1/365 = 2$, $\triangle t_i/365 = 3, i = 2, 3, 4, 5$. Inputs are given at the beginning and outputs are furnished at the end. The cash flows $NCF_{11}, NCF_{12}, ...$, in different terms have been completely given. The risk-free rate is denoted by r, where $r = 0.05$. Assume that the rate will not be changed in the multi-term investment. The total investment funds Q_T in the multi-term investment and start-up investment funds $K_{i,T}$ of every project are given by Table I and Table II.

TABLE I.
TOTAL INVESTMENT FUNDS

$Term$	$Initial$	$1st$	$2nd$	$3rd$	$4th$
$Funds$	84	120	253	98	161

TABLE II.
START-UP INVESTMENT FUNDS

$Project$	$Initial\ term$	$1st$	$2nd$	$3rd$	$4th$
1	10	15	20	18	25
2	20	23	24	25	30
3	30	35	40	45	40
4	08	10	12	06	15
5	15	20	25	30	15
6	18	25	27	20	25
7	25	30	35	35	40
8	40	38	40	45	55
9	35	40	45	38	37
10	30	33	38	35	48

In this paper, according to the equation in Section III , Table I and Table II, we can get Table III, IV and V by using the tool of EXCEL. Upper semi-covariance is given by Table III, lower semi-covariance is given by Table IV and the net present value index is given by Table V.

TABLE III.
UPPER SEMI-COVARIANCE

$Term$	$Initial$	$1st$	$2nd$	$3rd$	$4th$
$Initial$	1.56001	0.00000	0.0	0.05281	0.00000
$1st$	0.00000	0.06056	0.00000	0.04503	0.06621
$2nd$	0.00000	0.00000	1.20978	0.00000	0.00000
$3rd$	0.05281	0.04503	0.00000	0.16843	0.22450
$4th$	0.00000	0.06621	0.00000	0.22450	1.65436

TABLE IV.
LOWER SEMI-COVARIANCE

$Term$	$Initial$	$1st$	$2nd$	$3rd$	$4th$
$Initial$	0.00000	0.05061	0.30898	0.00000	0.26439
$1st$	0.05061	0.00000	0.01126	0.00000	0.00000
$2nd$	0.30898	0.01126	0.00000	0.31654	0.51689
$3rd$	0.00000	0.00000	0.31654	0.00000	0.00000
$4th$	0.26439	0.00000	0.51689	0.00000	0.00000

With the help of Matlab7.0, we can use the parameters as follows: the number of iterations is given as $maxgen = 1500$, the population size is given as $popsize = 300$, crossover probability is given as $p_c = 0.8$, and mutation probability is given as $p_m = 0.05$.

This paper considers the structure of model in Section IV and Section V, we can get Figure 1. We consider

TABLE V.
NET PRESENT VALUE INDEX

$Project$	$Initial$	$1st$	$2nd$	$3rd$	$4th$
1	1.36594	1.07845	0.90055	1.39878	1.23368
2	2.45204	1.11645	1.55111	1.24839	1.14300
3	1.06478	1.12471	5.03689	1.12561	0.98967
4	1.13109	1.37653	1.50422	1.51784	1.53480
5	0.84234	1.12734	1.08757	1.21720	2.21834
6	1.15826	1.08426	1.33762	1.36065	1.13004
7	1.11121	1.14391	1.16814	1.15824	0.81519
8	1.09338	1.19024	1.16193	1.13716	0.74639
9	1.19343	1.13267	1.14291	1.05786	0.96679
10	1.17438	1.10146	1.14136	1.16192	0.69489

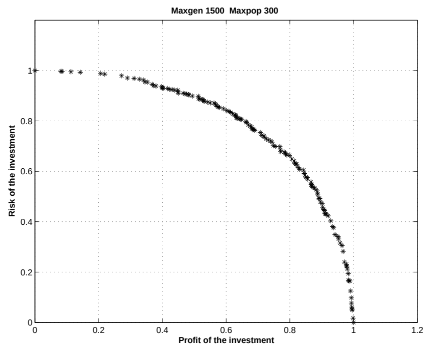

Figure 1. Return and risk in the investment

horizontal and vertical coordinates as the return of investment and risk of investment. In this situation, we can observe the investment return is increasing and the investment risk is increasing. It is able to get a series of investment portfolios, and the portfolios constitute Pareto effective surface. In the investment, investors may have different attitude to the risk and return of projects. Some of them may want to get high return even with high risk, while others may dislike high risk and they can accept the low return. So from Figure 1, they can choose portfolio which they prefer. Before investors determine the investment decision-making, it is better to talk with them and find out their preference. If investor wants to get high return and does not mind to take the high risk, he can choose the point which is the longest distance with vertical coordinate. If the investor doesn't like the high risk, he can choose the point which is the longest distance with horizontal coordinate.

In addition, from Figure 1, for example, we take two portfolios in the investment.

TABLE VI.
INVESTMENT PORTFOLIO

$Term$	$Portfolio1$	$Portfolio2$
$Inital$	1100110000	1001100010
$1st$	0101110100	1010101000
$2nd$	1001101101	0100110110
$3rd$	1100110000	0000100011
$4th$	1101110010	1011110010
$Return$	0.532061	0.530556
$Risk$	0.429770	0.423282

As shown in Table VI, the number of invested projects in portfolio1 is more than portfolio2, but their return and risk are the same. In other words, the return and risk of portfolio do not change greatly with numbers of projects in the investment. However, portfolio1 may produces more administrative expenses than portfolio2. If an identity matrix is denoted by $I_{i,T}$, we give an account of another objective $\min(N) = \sum_{T}^{m}\sum_{i}^{k} x_{i,T} \cdot I_{i,T}$ into account.

This situation will obviously be seen when the number of invested projects increase more. We will try to research this situation in more detail on another paper.

Example II: One company plan to select some projects to be invested in the investment market. We have 15 large projects successfully passed the assessment. If these projects can be invested in four investment terms, we can suppose $T = 0, 1, 2, 3$. Inputs are given at the beginning and outputs are furnished at the end. The risk-free rate is denoted by $r = 0.05$ and the rate will not be changed in the investment. We give the parameters as shown in Table VII, Table VIII, TableIX, TableX. In addition, duration t in each investment term is different.

TABLE VII.
RELATED COEFFICIENT

$Term$	$Initial$	$1st$	$2nd$	$3rd$
$Initial$	0.00000	0.25000	0.30000	−0.50000
$1st$	0.25000	0.00000	0.70000	0.10000
$2nd$	0.30000	0.70000	0.00000	−0.50000
$3rd$	−0.50000	0.10000	−0.5000	0.00000

TABLE VIII.
START-UP INVESTMENT FUNDS

$Project$	$Initial\ term$	$1st$	$2nd$	$3rd$
1	20	30	35	00
2	20	35	20	00
3	30	35	40	00
4	25	18	00	00
5	15	30	00	00
6	28	25	27	20
7	25	30	35	35
8	40	38	00	45
9	35	40	45	38
10	30	33	38	35
11	10	15	20	18
12	20	23	24	25
13	20	35	40	45
14	08	00	12	06
15	15	20	00	30

As shown in Example I, with the help of Matlab7.0, we use the parameters as follows: the number of iterations is given as $maxgen = 1500$, the population size is given as $popsize = 300$, crossover probability is given as $p_c = 0.7$, and mutation probability is given as $p_m = 0.06$.

Besides, we are able to get a series of investment portfolios and we also can find the investment return is increasing and the investment risk is increasing too. We can get a series of investment portfolios, and the portfolios constitute Pareto effective surface as shown in Figure II. In the investment, investors can choose portfolio which they prefer. For example, if investor wants to get high return and does not mind high risk, he can choose the portfolio1 as shown in Table XI; if the investor doesnt accept the high risk, he can choose the portfolio2 as shown in Table XI.

TABLE IX.
DURATION IN INVESTMENT TERMS

$Project$	$\triangle t_1/365$	$\triangle t_2/365$	$\triangle t_3/365$	$\triangle t_4/365$
1	2	3	2	0
2	3	2	2	0
3	2	2	2	0
4	3	3	0	0
5	3	2	0	0
6	2	2	2	2
7	2	3	3	2
8	3	2	0	2
9	3	2	2	3
10	3	3	2	2
11	2	2	2	2
12	2	2	2	2
13	3	3	2	2
14	2	0	2	3
15	2	2	0	3

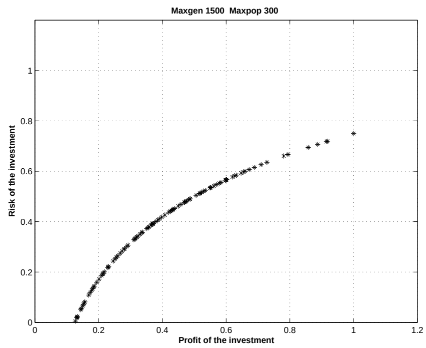

Figure 2. Return and risk in the investment

VI. CONCLUSIONS

This paper firstly talks about conditions in the single term investment and then extends the investment from one term to many terms in the field of the financial research. After that, it considers the funding constraints in finance environment and proposes a new multi-project and multi-term portfolio model. The model is about multi-objective optimization problem through considering the possible remaining funds in every investment term. Besides, it is not appropriate that we do not consider them in investment. Generally speaking, the investment risk is closely related with uncertainty. It is not appropriate to use variance to measure the investment risk. So in this paper, the model is based on a new kind of Mean-Semi-covariance theory. Then multi-objective evolutionary algorithm with greedy repair strategy is used to deal with the infeasible individuals. Finally, computer simulation proofs that the algorithm can be consider as a viable alternative. However, the influencing factors of investment may do changes all the time. We must think about variety of finance conditions in investment and we should develop another effective way to measure the investment risk. On

TABLE X.
INVESTMENT PORTFOLIO

$Term$	$Portfolio1$	$Portfolio2$
$Inital$	110011001011011	100110001011101
$1st$	010101010011001	101010100100000
$2nd$	110000100111010	010001001011010
$3rd$	000001101111110	000001010111101
$Return$	0.72158	0.34583
$Risk$	0.69353	0.30452

the whole, further research is required to the investment model and algorithm.

ACKNOWLEDGMENT

This work was supported in part by the Natural Science Foundation of China (60974077), and in part by the Natural Science Foundation of Guangdong Province (S2011030002886, S2012010008813), and in part by projects of science and technology of the department of education of Guangdong province (2012KJCX0042), and in part by Zhongshan projects of science and technology (20114A223).

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