An EM Approach for Cooperative Spectrum Sensing in Multi-Antenna CR Networks

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Abstract—In this paper, a cooperative wideband spectrum sensing scheme based on the expectation maximization (EM) algorithm is proposed for the detection of a primary user (PU) system in multi-antenna cognitive radio (CR) networks. Given noisy signal observations from $N$ secondary users (SUs) over multiple subbands at a fusion center, prior works on cooperative spectrum sensing often use the set of received subband energies as decision statistics over the sensing interval. However, to achieve satisfactory performance, knowledge of the channel state information and the noise variances at all the SUs is required by these algorithms. To overcome this limitation, our proposed method, referred to as joint detection and estimation (JDE), adopts the EM algorithm to jointly detect the PU signal and estimate the unknown channel frequency responses and noise variances over multiple subbands in an iterative manner. Various aspects of this proposed EM-JDE scheme are investigated, including a reliable initialization strategy to ensure convergence under practical conditions and a distributed implementation to reduce communication overhead. Under the assumption of perfect estimation, the channel frequency responses and noise variances, we further show that the proposed EM-JDE converges to the maximum-likelihood (ML) solution, which serves as an upper bound on its performance. Monte Carlo simulations over Rayleigh fading channels show that the proposed scheme significantly improves the performance of spectrum detection by exploiting the diversity of the spatially distributed SUs with multiple antennas.

I. INTRODUCTION

In recent years, cognitive radio (CR) has emerged as a key technology paradigm to alleviate the frequency spectrum scarcity [1]. The basic idea behind CR is that unlicensed or secondary users (SUs) share the frequency spectrum opportunistically with licensed or primary users (PUs) without causing harmful interference. This can be achieved by enabling the SUs to monitor the presence of PUs over a particular band of frequencies. In the literature, several spectrum sensing techniques have been proposed, which include energy detection (ED), matched filter detection, and cyclostationary feature detection [2], [3]. The appropriate spectrum sensing technique is chosen based on a priori knowledge about the PU’s signal and the receiver complexity. For example, the matched filter is the optimal detection technique when the SNR has complete information about the PU signal, which is rarely the case in practice [4]. The cyclostationary feature detector exploits the periodicity of the modulated signal to distinguish it from the stationary noise; however, it suffers from high computational complexity [5], [6]. The ED is the optimal detection scheme if the PU’s signal is unknown; it also offers the advantage of a lower complexity [7].

In many CR applications, detailed a priori knowledge about the PU’s signal structure and modulation is not readily available and in this context, ED is the most common choice for spectrum sensing. However, ED suffers from a poor performance in wireless environments characterized by low signal-to-noise ratio (SNR), multipath fading or shadowing [8]. Multi-antenna techniques have been employed along with ED to combat the fading effects by exploiting the spatial diversity of the observations at the SU terminal in [9]; they also help to reduce the sensing time compared to single antenna ED. Another drawback of ED is its inherent susceptibility to uncertainties about the noise variance at the SU side. In [10], the authors notice that there is a minimum value of SNR, referred to as the SNR wall, below which the spectrum detection fails even with infinite sensing intervals. In [11], the authors introduce the necessary conditions for the existence of an SNR wall in ED techniques coupled with noise power estimation. In [12], the performance of multi-antenna based cooperative spectrum sensing is investigated under Rayleigh fading channels when an improved form of ED is employed, where the decision statistic is an arbitrary positive power of the amplitudes of the PU’s signal samples.

Most works presented for the spectrum sensing problem assume the perfect knowledge of the channel conditions and noise variance by the SU, and few researchers have investigated the effect of estimation errors in these parameters on the PU detection process or possible estimation techniques that can be used jointly [13]. In [14] and [15], spectrum detectors based on the generalized likelihood ratio test (GLRT) are proposed for multi-antenna CR, and their performance is examined under flat fading channel conditions, assuming unknown channel gains and noise variance. An eigenvalue-based signal detection scheme is developed in [16] under noise variance or signal correlation uncertainty. In [17], the authors study the spectrum sensing techniques for a finite-rank PU signal with unknown spatial covariance matrix. The author in [18] presents a multi-antenna spectrum sensing technique based on discrete Fourier transform (DFT) analysis of the received signals over flat fading channels, which does not require the knowledge of the noise variances at the different receive antennas.
In the aforementioned spectrum sensing schemes, the test statistics, e.g., energy measures, are derived as a function of the received signals at different SU’s antennas, making the decision process on spectral occupancy vulnerable to common channel impairments, such as time variations and multipath fading, especially for mobile networks. In this paper, to overcome this limitation, we propose a spectrum sensing scheme where a binary hypothesis test is applied on estimates of the average power transmitted by the PU over the frequency spectrum of interest during the sensing interval. Consequently, it is possible to make decision on the spectral state of occupancy of a wideband frequency spectrum. In spectrum sensing in multi-user multi-antenna CR networks, we derive the EM-JDE scheme for cooperative spectrum sensing in multi-antenna CR networks. An ML-based upper bound on the performance of the proposed scheme is developed in Section IV. In Section V, the simulation results and discussions are presented. Finally, the conclusions are drawn in Section VI.

II. SYSTEM MODEL AND PROBLEM FORMULATION

In our work, we assume a CR network comprised of N SUs, indexed by \( n \in \{0, \ldots, N - 1\} \), where each SU terminal is equipped with \( P \) receiving antennas. We consider a wideband frequency spectrum, which is divided into \( K \) subbands as illustrated in Fig. 1. Here, the concept of a subband is identical to that used in a multicarrier modulation system, where each subband represents a narrow band of frequency centered around a single sub-carrier, upon which the corresponding subband information is modulated.

The time domain measurements, after sampling using Nyquist rate, are derived; the corresponding probability of false alarm and missed detection are also derived;

4) Numerical Experiments: New simulation results for the multi-user CR network are reported. Besides the convergence behavior of the iterative algorithm with the proposed initialization, the effects of time-varying fading channels, the number of cooperating SUs, and the distributed implementation are thoroughly studied.

Our theoretical findings and simulation results demonstrate the advantages of the proposed cooperative EM-JDE scheme in improving the performance of spectrum detection by exploiting the diversity offered by spatially distributed SUs with multiple antennas.

The rest of the paper is organized as follows. The System model and problem formulation are presented in Section II. In Section III, we derive the EM-JDE scheme for cooperative spectrum sensing in multi-antenna CR networks. An ML-based upper bound on the performance of the proposed scheme is developed in Section IV. In Section V, the simulation results and discussions are presented. Finally, the conclusions are drawn in Section VI.

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{0, ⋅⋅⋅, L − 1} denotes the q-th tap of the channel impulse response between the PU and the p-th antenna of the n-th SU, assumed to be of finite length L, and v_{n,p}(l) is an additive noise process. In our work, we employ a frequency-domain detector, where a K-point discrete Fourier Transform (DFT) operation is applied on successive frames of r_{n,p}(l) to obtain the following narrow-band discrete frequency components:

\[ R_{k,n,p}(m) = \sum_{l=0}^{K-1} w(l)r_{n,p}(mK + l)e^{-j2\pi lk/K} \]  

(2)

where frequency index \( k \in \{0, \ldots, K − 1\} \), frame index \( m \in \{1, \ldots, M − 1\} \) with \( M \) being the number of frames available for detection, and \( w(l) \) is a normalized frequency analysis window [25]. Under the assumption that the frequency subbands are sufficiently narrow, in comparison to the interval of variations in the channel frequency responses (or equivalently, \( K \gg L \)), the linear convolution in (1) can be approximated as the circular convolution, such that application of DFT in the time domain is equivalent to the point-wise multiplication of the corresponding \( K \) points in the discrete frequency domain [26], that is,

\[ R_{k,n,p}(m) = H_{k,n,p}s_k(m) + V_{k,n,p}(m) \]  

(3)

where \( s_k(m) \) represents the DFT coefficient of the PU signal \( s(t) \) over the \( m \)-th time frame in the \( k \)-th subband, \( H_{k,n,p}(m) \) is the DFT coefficient of the channel \( h_{n,p}(q) \) between the PU and the \( p \)-th receive antenna of the \( n \)-th SU in the \( k \)-th subband, and \( V_{k,n,p}(m) \) is the DFT coefficient of the noise \( v_{n,p}(l) \) at the \( p \)-th receive antenna of the \( n \)-th user, as obtained over the \( m \)-th frame in the \( k \)-th subband.

The linear model (3), with multiplicative channel effect on the PU signal in each subband, is common in the wideband spectrum sensing literature, see e.g., [27]–[29]. Nevertheless, in practice the DFT operation will suffer from spectral leakage, which may cause interference between neighboring subbands. Traditionally, a properly chosen window function \( w(l) \) can be applied in (2) to allow a design trade-off between frequency resolution and leakage [25]. Specific solutions to the suppression of spectral leakage in the context of spectrum sensing have been studied in [30]–[32]. In our work, we assume that such a suppression technique has been employed to eliminate, or at least reduce the spectral leakage among successive frequency bands to a level that is comparable to that of the additive background noise. Considering the high levels of radio noise and interference often encountered in CR applications, this does not represent a very stringent requirement. Thus, the effect of the spectral leakage can be minimized or neglected, which facilitates the derivation and analysis of the EM algorithm\(^2\).

In our work, we make use of the statistical model described below for the characterization of the received signal samples \( \{R_{k,n,p}(m)\} \), which is widely adopted in the literature (see again [27]–[29] and the references therein). To begin with, we define the vectors \( S = [S_0^T, \ldots, S_{K-1}^T]^T \) and \( S_k = [S_k(0), \ldots, S_k(M-1)]^T \), where the superscript \( T \) denotes the transpose operation. Since we have no prior knowledge about the PU signal, \( S_k \) is assumed to follow a complex circular Gaussian distribution with zero mean and covariance matrix \( B_kI_M \), denoted as \( CN(0_M, B_kI_M) \), where \( 0_M \) is a \( M \times 1 \) zero vector, \( I_M \) is an identity matrix of order \( M \) and \( B_k \) is an occupancy parameter as explained below. The complex circular Gaussian assumption of the subband PU signal is widely employed in the spectrum sensing literature as it can be naturally justified in many applications. For instance, when the PU system employs a broadband form of modulation, such as multicarrier modulation [33], [34] or spread spectrum [35], the received signal in each subband in (2) is the sum of \( K \) nearly independent contributions. In this case, one can invoke the central limit theorem [36] to motivate the Gaussian assumption since in practice, the number of subbands \( K \) used for spectrum sensing can be fairly large, e.g. \( K = 2^l \) where \( l \geq 6 \). An exception to this would be when the PU system uses OFDM modulation with the same frequency plan as the one used by the wideband SU detector, and with perfect synchronization in time and frequency, but in practice this is unlikely to be the case. Notwithstanding the above, the Gaussian model corresponds to a worst-case assumption according to the principle of maximum entropy [37].

We model the subband occupancy \( B_k \) as a binary random variable, which indicates the status of the PU activity in the \( k \)-th subband: \( B_k = 0 \) when the \( k \)-th subband is vacant, while \( B_k = 1 \) when the PU signal is present\(^3\). We assume independent subband occupancy, that is, the joint probability mass function (PMF) of \( B = [B_0, \ldots, B_{K-1}]^T \) is given by \( P(B) = \prod_{k=0}^{K-1} P(B_k) \), where \( P(B_k) \) denotes the marginal PMF of \( B_k \). Also, given \( B \), signal samples from different subbands are independent, i.e. the conditional probability density function (PDF) \( f(S | B) = \prod_{k=0}^{K-1} f(S_k | B_k) \), where \( f(S_k | B_k) \) denotes the conditional PDF of \( S_k \) given \( B_k \).

The channel coefficients \( H_{k,n,p} \) in (3) are assumed to remain constant during the sensing interval and hence, are modeled as deterministic but unknown quantities. For later reference, we define the channel coefficient vectors \( H = [H_0^T, \ldots, H_{K-1}^T]^T \), \( H_k = [H_k^T, 0, \ldots, 0, H_k^T, \ldots, 0, H_k^T] \), \( H_{k,n} = [H_{k,n,0}, 0, \ldots, 0, H_{k,n,p-1}]^T \), \( V_{k,n,p}(m) \) for \( m \in \{0, \ldots, M-1\} \) in (3) are represented by the vector \( V_{k,n,p} = [V_{k,n,p}(0), \ldots, V_{k,n,p}(M-1)]^T \), which is modeled as \( CN(0_M, \varsigma_{k,n}I_M) \), where the noise variance,\(^2\)

\(^2\)At moderate SNR, and in the absence of correlation between adjacent subband occupancy by the PU, the consideration of leakage is conceptually equivalent to a slight increase in the additive background noise variance. We have been able to confirm this point for a standard DFT-based (rectangular window) by independent simulations not reported in this study.

\(^3\)Without loss of generality, to simplify the presentation, we assume that when the PU is present, the signal power in the \( k \)-th subband is normalized to unity, i.e. \( E[|S_k|^2|B_k = 1] = 1 \).
estimate the unknown parameter vector of each subband while performing the spectrum detection process concurrently. This technique, referred to as iterative JDE, has been used to solve many problems of practical interest in the wireless communications, but to the best of our knowledge, its application to spectrum sensing has not been extensively researched. Below we first apply the EM formalism to derive the proposed JDE algorithm for cooperative spectrum sensing in multi-antenna CR networks. This is followed by a discussion of related implementation aspects, including: computational complexity, distributed implementation in cooperative framework, and proposed initialization scheme.

A. Algorithm Derivation

While the EM algorithm is often developed and studied for the case of continuous parameters, here we are faced with a mixed situation in which the unknown channel and noise parameters, \( \mathbf{H} \) and \( \varsigma \), are continuous, while the spectral occupancy vector, \( \mathbf{B} \), takes on discrete (binary) values. Therefore, to simplify the application of the EM formalism and the convergence analysis of the resulting algorithm, we initially adopt a purely continuous approach in which the occupancy variables \{\( \mathcal{B}_k \)} are first treated as continuous within the interval of \([0, 1]\). In this way, the intermediate estimates of each \( \mathcal{B}_k \) obtained through the sequence of EM iterations, which are denoted by \( \hat{\mathcal{B}}_k^{(i)} \), where the iteration index \( i \in \mathbb{N} \), may be viewed as soft estimates of the occupancy in subband \( k \). This makes it possible to find closed-form expressions for the maximum of the expected conditional likelihood during the maximization step of the EM procedure. Once the sequence of soft estimates \( \hat{\mathcal{B}}_k^{(i)} \) has been judged to converge to an adequate level (as will be explained below), a hard estimate of \( \mathcal{B}_k \) is finally obtained by applying a binary test, i.e. comparing \( \hat{\mathcal{B}}_k^{(i)} \) to a properly selected threshold. The specific details of our derivation follow.

According to the EM terminology, the \emph{incomplete} data \( \mathbf{R} = [\mathbf{R}_0^T, \ldots, \mathbf{R}_{K-1}^T]^T \) consists of the observations from all the receive antennas of the \( N \) SUs over the \( K \) subbands, where we define \( \mathbf{R}_k = [\mathbf{R}_k(0)^T, \ldots, \mathbf{R}_k(M-1)^T]^T \), \( \mathbf{R}_k(m) = [\mathbf{R}_{k,0}(m)^T, \ldots, \mathbf{R}_{k,N-1}(m)^T]^T \), and \( \mathbf{R}_{k,n}(m) = [\mathbf{R}_{k,n,0}(m), \ldots, \mathbf{R}_{k,n,p-1}(m)]^T \). The so-called \emph{complete} data \( \mathbf{Y} \) is defined as a combination of \( K \) independent pairs of \( \mathbf{R}_k \) and \( \mathbf{S}_k \), i.e., \( \mathbf{Y} = [\mathbf{Y}_0^T, \ldots, \mathbf{Y}_{K-1}^T]^T \), where \( \mathbf{Y}_k = [\mathbf{R}_k^T, \mathbf{S}_k^T]^T \).

Conditioned on \( \mathbf{U} = [\mathbf{B}^T, \mathbf{H}^T, \varsigma^T]^T \), the \( K \) component vectors of \( \mathbf{Y} \) are statistically independent, and consequently

\[
f(\mathbf{Y}|\mathbf{U}) = \prod_{k=1}^{K} f(\mathbf{R}_k, \mathbf{S}_k|\mathbf{U}_k) \tag{4}
\]

where

\[
f(\mathbf{R}_k, \mathbf{S}_k|\mathbf{U}_k) = f(\mathbf{R}_k|\mathbf{S}_k, \mathbf{U}_k)f(\mathbf{S}_k|\mathbf{U}_k). \tag{5}
\]

Invoking the Gaussian assumptions on the signal and noise made in Section II, it follows that the complete data log-
likelihood function $L(Y|U)$ is given by
\[ L(Y|U) = \sum_{k=0}^{K-1} L(Y_k|U_k) \]  
where
\[ L(Y_k|U_k) = L(R_k|S_k, U_k) + L(S_k|U_k) \]
\[ = -MP \sum_{n=0}^{N-1} \ln(\pi\varsigma_k,n) - M \ln(\pi B_k) - \frac{1}{B_k} \sum_{m=0}^{M-1} |S_k(m)|^2 \]
\[ - \sum_{n=0}^{N-1} \sum_{p=0}^{P-1} \sum_{m=0}^{M-1} |R_{k,n,p}(m) - H_{k,n,p} S_k(m)|^2 \]
\[ \Delta(U|\hat{U}^{(i)}) = \sum_{k=0}^{K-1} \Delta(U_k|\hat{U}^{(i)}) \]  
where $\Delta(U_k|\hat{U}^{(i)})$ is expressed as (9), shown on top of the next page, and $E[\cdot]$ denotes the expectation operator.

In the expectation-step (E-step) of the EM algorithm, we compute the conditional expectation of (6) given $R$ and the estimation of $U$ at the $i$-th iteration, i.e., $U = \hat{U}^{(i)}$, where $\hat{U}^{(i)} = \left(\hat{B}^{(i)}, \hat{H}^{(i)}, \hat{\varsigma}^{(i)}\right)^T$, and $\hat{B}^{(i)}$, $\hat{H}^{(i)}$ and $\hat{\varsigma}^{(i)}$ are the EM estimates of $B$, $H$ and $\varsigma$ at the $i$-th iteration, respectively. The result of this conditional expectation is a function of the unknown parameter vector $U_k$, which we denote by $\Delta(U|\hat{U}^{(i)})$. Hence, we have
\[ \Delta(U|\hat{U}^{(i)}) = \sum_{k=0}^{K-1} \Delta(U_k|\hat{U}^{(i)}) \]  
\[ \Delta(U_k|\hat{U}^{(i)}) = \left[ B_k, H_k, \varsigma_k \right]^T \]  
where $\Delta(U_k|\hat{U}^{(i)})$ is expressed as (9), shown on top of the next page, and $E[\cdot]$ denotes the expectation operator.

In the maximization-step (M-step) of the algorithm, the updated parameter estimate, $\hat{U}^{(i+1)}$, is obtained by maximizing $\Delta(U|\hat{U}^{(i)})$ in (8) with respect to $U$. Here, since the samples of each process are statistically independent across the frequency index, $U_k$ can be estimated individually by maximizing its corresponding term, i.e., $\Delta(U_k|\hat{U}^{(i)})$ in (9). Notice that the occupancy parameter $B_k$ in the conditional expectation in (9) is actually decoupled from the channel and variance parameters $H_{k,n,p}$ and $\varsigma_{k,n}$ and therefore, we can first estimate $\hat{B}_k$, followed by the estimation of $H_k$ and $\varsigma_k$.

To begin with, $\hat{B}_k^{(i)}$ can be updated by maximizing (9) with respect to $B_k$. By neglecting the terms in (9) that are independent of $B_k$, we obtain
\[ \hat{B}_k^{(i+1)} = \arg \max_{B_k \in [0,1]} g(B_k, \sum_{m=0}^{M-1} E[|S_k(m)|^2|R, \hat{U}^{(i)}]) \]  
where
\[ g(B_k, \phi) \triangleq -M \ln(\pi B_k) - \frac{1}{B_k} \phi. \]  
As explained earlier, the parameter space for $B_k$ is discrete, and the convergence of the EM algorithm in this case is not always guaranteed. To overcome this problem, we artificially extend the search range of the M-step maximization from the discrete space $\{0,1\}$ to the continuous space $[0,1]$, and therefore the solution to the M-step update (10) can be given by
\[ \hat{B}_k^{(i+1)} = \min \left\{ \frac{1}{M} \sum_{m=0}^{M-1} E[|S_k(m)|^2|R, \hat{U}^{(i)}], 1 \right\}. \]  
Under this continuous parameter space assumption, the convergence of the resulting EM estimator to a stationary point follows from well-known results on the analysis of the conventional EM algorithm [39], [40]. To compute $\hat{B}_k^{(i+1)}$ in (12), we note that $E[|S_k(m)|^2|R, \hat{U}^{(i)}] = E[S_k(m)|R, \hat{U}^{(i)}]^2 + \text{Var}[S_k(m)|R, \hat{U}^{(i)}]$, and $\text{Var}[\cdot]$ is the variance operator. Since $R$ and $S$ are jointly Gaussian, the conditional mean and variance of $S_k(m)$ given $R$ and $\hat{U}^{(i)}$ can be expressed in the following forms [41]
\[ E[S_k(m)|R, \hat{U}^{(i)}] = \hat{B}_k^{(i)} H_k^{(i)} \Gamma_k^{(i)-1} R_k(m) \]  
\[ \text{Var}[S_k(m)|R, \hat{U}^{(i)}] = \hat{B}_k^{(i)} - \hat{B}_k^{(i)} H_k^{(i)} \Gamma_k^{(i)-1} H_k^{(i)} \hat{B}_k^{(i)} \]  
where the superscript $^H$ denotes the conjugate transpose and $\Gamma_k^{(i)} = \hat{B}_k^{(i)} H_k^{(i)} \hat{B}_k^{(i)} + \Sigma_k^{(i)}$. The matrix $\Sigma_k^{(i)}$, which represents the $i$-th iterative estimate of the noise covariance matrix in the $k$-th subband for the $N$ SUs, is given by
\[ \Sigma_k^{(i)} = \begin{pmatrix} \hat{\varsigma}_{k,0} & \cdots & 0_P \\ 0_P & \Sigma_{k,1} & \cdots & 0_P \\ \vdots & \vdots & \ddots & \vdots \\ 0_P & \cdots & \Sigma_{k,N-1} \end{pmatrix} \]  
where $\hat{\varsigma}_{k,n} = \varsigma_{k,n} I_P$. From (12)–(14), it follows that the EM algorithm provides an iterative estimate of the average transmission power of the PU system in each subband, i.e., an iterative energy detector. In turn, this can be used to provide binary-valued subband occupancy decisions through the application of a final hard limiter, as will be explained shortly.

Next, we obtain the estimate of $H_k$ at the $(i+1)$-th iteration as follows. Each element of $H_{k,n,p}^{(i+1)}$ is obtained by maximizing its corresponding summand in the right-hand side of (9), which yields
\[ \hat{H}_{k,n,p}^{(i+1)} = \arg \min_{H_{k,n,p}} \sum_{m=0}^{M-1} E[|R_{k,n,p}(m) - H_{k,n,p} S_k(m)|^2|R, \hat{U}^{(i)}] \]  
and subsequently
\[ \hat{H}_{k,n,p}^{(i+1)} = \frac{\sum_{m=0}^{M-1} R_{k,n,p}(m) E[S_k(m)|R, \hat{U}^{(i)}]}{\sum_{m=0}^{M-1} E[|S_k(m)|^2|R, \hat{U}^{(i)}]} \]  
where the superscript $^*$ denotes the complex conjugate. Introducing $R_{k,n,p} = [R_{k,n,p}(0), \ldots, R_{k,n,p}(M-1)]^T$, $\hat{H}_{k,n,p}^{(i+1)}$ in (17) can be represented in a more compact form as
\[ \hat{H}_{k,n,p}^{(i+1)} = E[S_k^H S_k|R, \hat{U}^{(i)}]^{-1} E[S_k^H R_{k,n,p}|R, \hat{U}^{(i)}] H_{k,n,p} \]  
which can be seen as an iterative least-squares (LS) channel estimation of $H_{k,n,p}$ at the $(i+1)$-th iteration. During the
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\[
\Delta(U_k|\hat{U}^{(i)}) = -MP \sum_{n=0}^{N-1} \ln(\pi_{k,n}) - M \ln(\pi_{B_k}) - \frac{1}{B_k} \sum_{m=0}^{M-1} E[|S_k(m)|^2|\mathbf{R}, \hat{U}^{(i)}] \\
\quad - \frac{1}{\varsigma_{k,n}} \sum_{n=0}^{N-1} \sum_{p=0}^{P-1} \sum_{m=0}^{M-1} E[|R_{k,n,p}(m) - H_{k,n,p}S_k(m)|^2|\mathbf{R}, \hat{U}^{(i)}],
\]  

(9)

sensing periods where the k-th subband is vacant, the SUs only receive noise samples. Therefore, it is not necessary to estimate the channels, and the channel estimation step has to be dropped from the EM iterative loop. We use the following approach to make a decision on the EM channel estimation step. We compare the value of \( \hat{B}_k^{(i)} \) after running the EM algorithm for a few iterations with a preset threshold, \( \delta_B > 0 \). The value of \( \delta_B \) is chosen to be less than the mid-value between the minimum and maximum values of \( B_k \), i.e., in our case, \( \delta_B < 0.5 \). The selection tends to be conservative in the sense that it prevents stopping the channel estimation unnecessarily due to an erroneous missed detection. Consequently, the values of \( \hat{B}_k^{(i)} \) after these few iterations can be classified within two regions: \( \mathcal{R}_0 \) with \( 0 \leq \hat{B}_k^{(i)} < \delta_B \) and \( \mathcal{R}_1 \) with \( \delta_B < \hat{B}_k^{(i)} \leq 1 \). The channel estimation step is excluded from the EM iterative loop if \( \hat{B}_k^{(i)} \) is located within \( \mathcal{R}_0 \). To realize this condition without hindering the update of \( \hat{B}_k^{(i)} \) in the EM algorithm, \( \hat{H}_{k,n,p}^{(i+1)} \) in (17) can be redefined as follows

\[
\hat{H}_{k,n,p}^{(i+1)} = \frac{\sum_{m=0}^{M-1} R_{k,n,p}(m)E[S_k(m)|\hat{U}^{(i)}]|\hat{U}^{(i)}] + \sum_{m=0}^{M-1} E[|S_k(m)|^2|\mathbf{R}, \hat{U}^{(i)}]}
\]

(19)

where

\[
\varrho = \begin{cases} 
\varrho, & \hat{B}_k^{(i)} \in \mathcal{R}_0 \\
0, & \hat{B}_k^{(i)} \in \mathcal{R}_1 
\end{cases}
\]

(20)

and \( \varrho \) is a large positive number. Note that the classification in (20) is based on the soft estimate \( \hat{B}_k^{(i)} \) at the \( i \)-th iteration, and therefore may involve error. To minimize the effects of this error, variations in the values of \( \hat{B}_k^{(i)} \) over successive stages, i.e., groups of \( T \) iterations as explained in Section III.B.4, can be exploited to improve the exactness of the decision in comparing \( \hat{B}_k^{(i)} \) with threshold \( \delta_B \). This stems from the fact that when the k-th subband is vacant, the soft estimate of \( B_k \) tends to decline towards zero, but not monotonically while in the case when the k-th subbed is occupied by the PU system, \( \hat{B}_k^{(i)} \) will tend to increase towards the true value of 1. Therefore, in our implementation, the decision in (20) is performed only at the end of a stage, such that the descent in the values of \( \hat{B}_k^{(i)} \) can be more accurately detected. We remark that this approach is empirical in nature but it works well in simulation experiments. However, a formal proof of its advantages is not currently available, which remains an interesting topic of future research.

Finally, we update \( \varsigma_{k,n}^{(i)} \) as follows. In (9), we substitute \( \hat{H}_{k,n}^{(i+1)} \) for \( H_{k,n} \) and maximize the objective in (9) with respect to \( \varsigma_{k,n} \). That is,

\[
\varsigma_{k,n}^{(i+1)} = \arg\min_{\varsigma_{k,n}} \left( MP \ln(\varsigma_{k,n}) + \frac{1}{\varsigma_{k,n}} \sum_{p=0}^{P-1} \sum_{m=0}^{M-1} \nu_{k,n,p}(m) \right)
\]

(21)

where

\[
\nu_{k,n,p}(m) = \begin{cases} 
E[|R_{k,n,p}(m) - \hat{H}_{k,n,p}S_k(m)|^2|\mathbf{R}, \hat{U}^{(i)}] \\
- R_{k,n,p}(m) + \hat{H}_{k,n,p}S_k(m) \]

(22)

This yields

\[
\varsigma_{k,n}^{(i+1)} = \frac{1}{MP} \sum_{p=0}^{P-1} \sum_{m=0}^{M-1} \nu_{k,n,p}(m).
\]

(23)

Up to this point, as explained earlier, the proposed EM-JDE produces a soft estimate of \( B_k \) at each iteration. Finally, a hard (i.e., binary) estimation of \( B_k \) is performed once the sequence of EM iterative steps for the soft estimates is judged to have reached an adequate level of convergence. In our case, the convergence condition is defined by comparing the difference between the estimates of \( \hat{B}_k^{(i)} \) in two successive iterations with a small positive threshold \( \epsilon \). That is the EM iterations stop if \( |B_k^{(i+1)} - B_k^{(i)}| \leq \epsilon \), where \( \epsilon \to 0^+ \). Then, a hard limiting with threshold \( \gamma_k \) is applied on the EM estimate of \( B_k \) after convergence, which we simply denote as \( B_k^{(\infty)} = \lim_{i \to \infty} B_k^{(i)} \). The corresponding test can be expressed as

\[
B_k^{(\infty)} = \begin{cases} 
1, & \hat{B}_k^{EM} = 1 \\
0, & \hat{B}_k^{EM} = 0 \\
\gamma_k, & \hat{B}_k^{EM} \in \{0,1\}
\end{cases}
\]

(24)

where \( \hat{B}_k^{EM} \in \{0,1\} \) is the final binary estimate of the k-th subband occupancy. The choice of \( \gamma_k \), as in other binary detection problems, gives rise to a practical trade-off between the probability of false alarm, \( P_{f,k}(\gamma_k) \) and the probability of missed detection, \( P_{m,k}(\gamma_k) \). Specifically, as \( \gamma_k \) increases from 0 to 1, \( P_{f,k}(\gamma_k) \) decreases from 1 to 0, while \( P_{m,k}(\gamma_k) \) increases from 0 to 1. The various ROC curves shown in Section V are indeed obtained by varying \( \gamma_k \) in this way. In particular, if a desired level of \( P_{f,k}(\gamma_k) \) is needed in a given application, a rough estimate of \( \gamma_k \) can be obtained from the asymptotic performance analysis given in Section IV-B, as per (47); this value can then be refined through simulations or experimental measurements.

Remark 1 (Assumption on Subband Occupancy): In many
applications, there will be correlation in the occupancy of adjacent subbands, as evidenced in recent experimental studies [42]. In fact, the modeling of the subband occupancy highly depends on the operation of the primary system. If the PU represents a multi-user multicarrier modulation-based network, e.g., OFDMA, where each user operates independently on a subset of dedicated subcarriers, the subband occupancy could be assumed to be independent, especially if adaptive loading is employed. However, if the PU is a broadcast television or WLAN system, the PU signal usually occupies a range of contiguous subbands, and therefore, the subband occupancy becomes correlated. For the latter case, investigating cooperative wideband spectrum sensing with correlated subband occupancy represents an interesting, yet challenging extension of the present work. To be specific, since the random variables \{\mathcal{B}_k\} are no longer independent, to solve the M-step in (10)–(12), a joint optimization problem over multiple discrete variables needs to be considered (see, e.g., [27], [43]), which is quite computationally intensive. Therefore, in this work, we focus on the former case, which facilitates the derivation of a low-complexity algorithm. However, investigating cooperative wideband spectrum sensing with correlated subband occupancy, possibly by exploiting a simpler but suboptimal mathematical framework, remains an interesting research direction for our future work.

B. Implementation Aspects

1) Computational Complexity: Using the EM algorithm, the \(N_d\)-dimensional optimization problem for each subband \(k\) is decomposed into \(N_d\) independent one-dimensional optimization problems, where \(N_d = 1 + N(P + 1)\) is the number of unknown parameters of subband \(k\), leading to a computationally feasible scheme. Furthermore, at each iteration of the EM algorithm, the solution of these one-dimensional optimization problems is obtained in closed-form where the computation of the conditional moments in (13) and (14) dominates the computational complexity of the proposed EM-JDE scheme. By exploiting the specific structure of the matrix \(\Gamma^{(k)}\) as a sum of a diagonal matrix plus a rank-1 modification term and invoking the Sherman-Morrison formula [44], it can be shown that this step requires \(O(NP)\) mathematical operations.

2) Distributed EM-JDE Implementation: The proposed EM-JDE scheme adopts centralized spectrum sensing, where the observations from multiple SUs need to be reported to the FC. The advantage of centralized processing is that the CR units benefit from a much reduced hardware complexity since the decision-making process is performed at the FC. However, the communication overhead between the SUs and the FC is increased, since each SU must transmit their \(KP\) complex-valued observed frequency samples per sensing frame for \(M\) consecutive frames, before the EM algorithm can be run by the FC. This overhead can be reduced by adopting a distributed, or localized implementation for the EM-JDE, as explained below.

The block diagram of the distributed implementation of the proposed EM-JDE is presented in Fig. 3. In the proposed distributed implementation, each SU generates an iterative soft estimate of \(\mathcal{B}_k\) locally by running a simplified version of the EM algorithm (the derivation is similar to the one in Section III-A, and therefore omitted for brevity), thereby producing the sequence \(\hat{\mathcal{B}}^{(k)}_{k,n}\). Then, each SU reports its estimate of \(\mathcal{B}_k\) after convergence, i.e., \(\hat{\mathcal{B}}^{(\infty)}_{k,n}\), to the FC to make a decision on the spectral occupancy. Using these reported estimates from the \(N\) SUs, where error free transmission is assumed, the decision statistic on the \(k\)-th subband is defined as

\[
\hat{\mathcal{B}}_k = \frac{1}{N} \sum_{n=0}^{N-1} \hat{\mathcal{B}}^{(\infty)}_{k,n}.
\]  (25)

Compared to the centralized implementation described in the previous section, the distributed implementation significantly reduces the communication overhead between the FC and the SUs to \(K\) complex values per SU per sensing period of \(M\) frames. However, this is achieved at the expense of increasing the hardware complexity of the CR units used by the SUs. The performance of the above low-overhead distributed approach is compared with that of the centralized spectrum sensing in Section V.

3) Initialization: In theory, the EM algorithm monotonically increases the log-likelihood function of the observed data at each iteration. Therefore, it is guaranteed to converge to a stationary point of the likelihood function, which can be a local maxima or a saddle point, see, e.g., [39], [40]. In practice, the convergence of the EM algorithm to a global maxima (i.e., the ML solution) can be achieved by using reliable initialization of the unknown parameters. This can also guarantee fast convergence to the ML solution within a reasonable number of iterations. Below, we present an initialization strategy that ensures good convergence under practical conditions of operation in CR network with multiple SUs operating over time-varying channels.

Starting with \(\hat{\xi}_{k,n}^{(0)}\), it is assumed that the FC has a priori

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Figure 3: Block diagram of the proposed EM-JDE algorithm with distributed implementation at each SU.
knowledge about the recent history of the PU’s activity, i.e., the FC can record the intervals of presence and absence of the PU signal in the time-frequency plane (based, e.g., on some basic form of energy detection or other available data) to determine the most likely intervals over which the k-th subband is not occupied, and then report this information to the SUs. By targeting such an idle period of the PU activity, the initial estimate of \( \hat{s}_{k,n} \) at the n-th SU is determined by computing the sample variance of the observations from the \( P \) receive antennas, as follows\(^5\)

\[
\hat{s}_{k,n}(0) = \frac{1}{J_n P} \sum_{j=0}^{J_n-1} \sum_{p=0}^{P-1} |V_{k,n,p}(j)|^2 \tag{26}
\]

where \( J_n \) is the number of available frames during an idle period. In the centralized implementation, the values of \( \hat{s}_{k,n}(0) \) from the \( N \) SUs are reported along with the frequency samples \( R_{k,n,p}(m) \) to the FC, where the complete EM-JDE algorithm can be run; while in the distributed implementation, the value of \( \hat{s}_{k,n}(0) \) computed at the n-th SU is used locally to run the distributed version of the EM-JDE, as illustrated in Fig. 3. Following initialization for the first block of \( M \) frames based on a priori knowledge of the PU’s activity as above, initialization for subsequent blocks can be based on the noise variance estimates obtained via application of the EM-JDE algorithm in the previous frame.

The estimate of \( \hat{B}_k \) is initialized by the FC as follows. Let \( \hat{k}_{k,n}(0) \) represent the sample variance of the received signals at the \( P \) antennas of the n-th SU normalized by \( \hat{s}_{k,n}(0) \), as given by

\[
\hat{k}_{k,n}(0) = \frac{1}{MP_{k,n}(0)} \sum_{m=0}^{M-1} \sum_{p=0}^{P-1} |R_{k,n,p}(m)|^2 \tag{27}
\]

On the one hand, when the k-th subband is occupied by the PU (i.e., \( B_k = 1 \)), \( \hat{k}_{k,n}(0) \) represents an initial estimate of the received SNR at the n-th SU in that subband. On the other hand, when the k-th subband is idle (i.e., \( B_k = 0 \)), we have \( R_{k,n,p}(m) = V_{k,n,p}(m) \) and then \( \hat{k}_{k,n}(0) \approx 1 \). Each SU transmits its variance estimate \( \hat{k}_{k,n}(0) \) to the FC, which subsequently computes the initial estimate

\[
\hat{B}_k(0) = \min \left\{ \frac{1}{N} \sum_{n=0}^{N-1} |\hat{k}_{k,n}(0) - 1|, 1 \right\}. \tag{28}
\]

As a result, \( \hat{B}_k(0) \) is more likely to take values close to zero when the k-th subband is vacant, and values larger than zero when the PU signal is present.

Finally, we discuss the initialization of the channel estimates. In a traditional (i.e. non-opportunistic) communication network, the source can transmit a short sequence of known training symbols to help the receiver initiate the channel estimation [46]. In CR networks, this approach is not feasible since the SUs have no a priori information about the PU signal. One possible alternative is simply to initialize the unknown channel coefficients to zero, that is \( \hat{H}_{k,n,p}(0) = 0 \), where \( 0_P \) denotes a \( P \times 1 \) zero vector. However, the zero initialization might increase the probability of missed detection, which in turn leads to higher interfering rate with the PU. In this work, we consider a more practical approach where each SU has only very limited knowledge of the PU-to-SU channels. The true but unknown channel between the PU and the p-th receive antenna of the n-th SU in the k-th subband can be represented by the following additive model [47]:

\[
H_{k,n,p} = \hat{H}_{k,n,p} + \Delta H_{k,n,p} \tag{29}
\]

where \( \hat{H}_{k,n,p} \) is the available channel estimate at the SU, which can be inaccurate, and \( \Delta H_{k,n,p} \) captures the underlying channel uncertainty. Specifically, the uncertainty \( \Delta H_{k,n,p} \) is assumed to take values from the following bounded set

\[
\mathcal{H}_{k,n,p} \triangleq \{ \Delta H : |\Delta H| \leq \epsilon_{k,n,p} \} \tag{30}
\]

where \( \epsilon_{k,n,p} > 0 \) specifies the radius of \( \mathcal{H}_{k,n,p} \), and therefore reflects the degree of uncertainty associated with the available channel estimate \( \hat{H}_{k,n,p} \). Such a model has been extensively used in transceiver design for CR networks [48, 49], where \( \hat{H}_{k,n,p} \) can be obtained by calculating the deterministic path loss coefficients between PU and the different SU antennas while the size of the uncertainty region can be derived from the fading channel statistics. Therefore, for each triplet \( (k,n,p) \), an initial guess of the channel frequency response can be obtained as

\[
\hat{H}_{k,n,p}(0) = \hat{H}_{k,n,p} + \Delta H_{k,n,p}, \tag{31}
\]

where \( \Delta H_{k,n,p} \) is randomly generated from the set \( \mathcal{H}_{k,n,p} \). To further reduce the probability of missed detection of the proposed EM-JDE algorithm, multiple initializations are generated according to (31) in parallel, and the one with the largest value of the corresponding complete data log-likelihood function, i.e., \( L(Y|U) \) in (6), is selected at the expense of using additional computing resources.

4) Operation of EM-JDE Algorithm: In our implementation of the proposed EM-JDE scheme, we propose the following strategy to operate the iterative algorithm with the goal of enhancing its convergence and stability. In this strategy, the sequence of EM iterations is divided into \( S \) consecutive stages, indexed by \( s \in \{1, \ldots, S\} \), where each stage comprises \( T \) iterations, indexed by \( i = (s-1)T + j \), where \( j \in \{1, \ldots, T\} \). In each stage, we fix the value of \( \hat{B}_k^{(i)} = \hat{B}_k(0) \) for the first \( T-1 \) iterations (i.e., for \( j = 1, \ldots , T-1 \)), while the values of \( \hat{z}_{k,n}^{(i)} \) and \( \hat{s}_{k,n}^{(i)} \) are updated using the EM formulas derived in Section III-A with every iteration \( i \). At the last iteration of each stage (i.e., when we reach \( j = T \)), \( \hat{B}_k^{(i+1)} = \hat{B}_k^{(i)} \) is updated along with \( \hat{H}_{k,n,p}^{(i)} \) and \( \hat{z}_{k,n}^{(i+1)} \) using the most recent estimates \( \hat{H}_{k,n,p}^{(i)} \) and \( \hat{s}_{k,n}^{(i)} \). This means that the values of \( \hat{B}_k^{(i)} \) and \( \hat{s}_{k,n}^{(i)} \) always change with the iteration index \( i \), while the value of \( \hat{B}_k^{(i)} \) only changes with the stage order \( s \), i.e. when \( i = sT \). Indeed,

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with imperfect knowledge of these quantities, the iterative EM estimate \( \hat{B}_k^{(i)} \) might not converge sufficiently rapidly to the true value \( B_k \) due to the uncertainty in the channel and noise parameters. We have found experimentally that by devoting more iterations for improving the channel and noise estimation before updating \( \hat{B}_k^{(i+1)} \), we can ascertain a unique slope of the EM estimate \( \hat{B}_k^{(i)} \) towards one of the two limiting values, i.e., 0 or 1. Specifically, this increases the likelihood that at the end of a given iteration, \( \hat{B}_k^{(i)} > \hat{B}_k^{(i-1)} \) when \( B_k = 1 \) and \( \hat{B}_k^{(i)} < \hat{B}_k^{(i-1)} \) when \( B_k = 0 \). Hence, the stability of the EM algorithm is improved, even with imperfect channel and noise variance estimation.

IV. UPPER BOUND ON THE ASYMMETRIC PERFORMANCE OF EM-JDE

The performance of spectrum sensing schemes is evaluated using the Neyman-Pearson criterion, for a given probability of false alarm in the \( k \)-th subband, say \( P_f,k(\gamma_k) = \alpha_k \) with \( \alpha_k \in [0,1] \), the optimum threshold in the decision-making process, \( \gamma_k^{\text{opt}} \), and subsequently the optimum probability of detection, \( P_d,k(\gamma_k^{\text{opt}}) \), are provided [41]. This analytical evaluation cannot be applied in iterative EM schemes since the derivation of a closed-form expression for the final decision statistics, i.e., \( \hat{B}_k^{\infty} \), is not apparently feasible. Therefore, we present an analytical evaluation of the EM-JDE assuming a perfect knowledge of the channel state information and noise variances by the SUs. In this case, the receiver operating characteristic (ROC) curve represents an upper bound on the performance of the EM-JDE scheme. The derivations can be summarized as follows. First, we present the EM estimation of \( B_k \) assuming that \( \varsigma \), and \( \mathbf{H}_k \) are perfectly known by the SU, which enables us to express \( B_k^{(i+1)} \) in terms of \( B_k^{(i)} \). Then, we obtain an explicit expression of \( B_k^{\infty} \) by deriving the ML solution of the same problem, which is denoted by \( \hat{B}_k^{\text{ML}} \), and proving that \( \lim_{i \to \infty} \hat{B}_k^{(i)} = \hat{B}_k^{\text{ML}} \). Finally, using \( \hat{B}_k^{\text{ML}} \), closed-form expressions of \( \gamma_k^{\text{opt}} \) and \( P_d,k(\gamma_k^{\text{opt}}) \) are derived.

The analysis presented below is important for several reasons. First, it sheds light on the convergence properties of the proposed EM-JDE scheme under idealized conditions. Second, the closed form expressions of \( P_f,k(\gamma_k) \) and \( P_d,k(\gamma_k) \) obtained for the ideal ML solution can be used as an upper bound on the performance of the proposed scheme, allowing to set preliminary values of the detection thresholds \( \gamma_k \) to achieve a specified false alarm rate.

A. EM-Based Spectrum Sensing

In this part, we determine the spectrum occupancy assuming the perfect knowledge of channel frequency responses \( \mathbf{H} \) and noise variances \( \varsigma \). Therefore, the only unknown parameter in the \( k \)-th subband is the occupancy parameter \( B_k \), and we denote the EM solution in this case as the ideal EM-based spectrum sensing (IEM-SS). Following the same procedure as in Section III-A, \( \hat{B}_k^{(i+1)} \) is obtained as (10)

\[
\hat{B}_k^{(i+1)} = \frac{1}{M} \sum_{m=0}^{M-1} E[|S_k(m)|^2 | \mathbf{R}, \hat{B}_k^{(i)}] 
\]

where \( E[|S_k(m)|^2 | \mathbf{R}, \hat{B}_k^{(i)}] \) can be derived using (13) and (14) with \( \hat{H}_k^{(i)} \) and \( \varsigma_k^{(i)} \) replaced by their true values. To justify the convergence of \( \hat{B}_k^{(i+1)} \) to \( \hat{B}_k^{\text{ML}} \) as \( i \to \infty \), we first derive a closed-form expression for the ML estimator of \( \mathbf{B} \), referred to as the ideal ML-based spectrum sensing (IdML-SS), as follows.

The log-likelihood function of \( \mathbf{R} \) given \( \mathbf{B} \) (assuming that \( \mathbf{H} \) and \( \varsigma \) are known) is

\[
L(\mathbf{R} | \mathbf{B}) = \sum_{k=0}^{K-1} L(\mathbf{R}_k | B_k) 
\]

where

\[
L(\mathbf{R}_k | B_k) = -MN P \ln(\pi) - M \ln(\det(\mathbf{T}_k)) 
- \sum_{m=0}^{M-1} \mathbf{R}_k(m)^H \mathbf{R}_k(m). 
\]

In (34), \( \mathbf{T}_k = B_k \mathbf{H}_k \mathbf{H}_k^H + \varsigma_k \) with \( \varsigma_k \) defined as in (15) but using the true values of \( \varsigma_k,n,i.e., \varsigma_k,n(i) = \varsigma_k,n \), and \( \det(\cdot) \) denotes the matrix determinant. Using the matrix determinant lemma [50], \( \det(\mathbf{T}_k) \) is reduced to

\[
\det(\mathbf{T}_k) = (1 + B_k \mathbf{H}_k^H \varsigma_k^{-1} \mathbf{H}_k) \det(\varsigma_k) 
= (1 + B_k \mathbf{H}_k^H \varsigma_k^{-1} \mathbf{H}_k) \prod_{n=0}^{N-1} \varsigma_k,n. 
\]

Also, by using the Sherman-Morrison formula [44], \( \Gamma_k^{-1} \) can be expressed as

\[
\Gamma_k^{-1} = \varsigma_k^{-1} - \frac{B_k \varsigma_k^{-1} \mathbf{H}_k \mathbf{H}_k^H \varsigma_k^{-1}}{1 + B_k \mathbf{H}_k^H \varsigma_k^{-1} \mathbf{H}_k}. 
\]

Substituting (35) and (36) in (34), and neglecting the terms independent of \( B_k \), we obtain

\[
L(\mathbf{R}_k | B_k) 
= -M \ln(1 + B_k \mathbf{H}_k^H \varsigma_k^{-1} \mathbf{H}_k) + \frac{B_k}{1 + B_k \mathbf{H}_k^H \varsigma_k^{-1} \mathbf{H}_k} \times \sum_{m=0}^{M-1} \mathbf{R}_k(m)^H \varsigma_k^{-1} \mathbf{H}_k \mathbf{H}_k^H \varsigma_k^{-1} \mathbf{R}_k(m). 
\]

Since the subband occupancies, \( \{B_k\} \), are assumed to be statistically independent, the maximization process of (33) with respect to \( \mathbf{B} \) is done separately for each subband. The ML estimate of the \( k \)-th subband occupancy is obtained by maximizing the log-likelihood function (37) with respect to \( B_k \), that is, 

\[
B_k^{\text{ML}} = \arg \max_{B_k} L(\mathbf{R}_k | B_k). 
\]

To simplify its derivation, let us introduce random variable \( \Upsilon_k(m) = \mathbf{R}_k(m)^H \varsigma_k^{-1} \mathbf{H}_k \), and also define \( \Psi_k = \mathbf{H}_k^H \varsigma_k^{-1} \mathbf{H}_k \). Then, \( B_k^{\text{ML}} \) is obtained by taking the derivative of (37) with respect to \( B_k \) and equating the resultant equation to 0 as follows:

\[
-\frac{M \Psi_k}{1 + B_k \Psi_k} + \left( \frac{1}{1 + B_k \Psi_k} - \frac{B_k \Psi_k}{(1 + B_k \Psi_k)^2} \right) \times \sum_{m=0}^{M-1} |\Upsilon_k(m)|^2 = 0
\]
which yield
\[
\hat{B}_{k,\text{ML}}^i = \frac{1}{M \Psi_k} \sum_{m=0}^{M-1} (| \mathbf{Y}_k(m) |^2 - \Psi_k). \tag{39}
\]

Using (36) with \( B_k = \hat{B}_{k,\text{ML}}^i \), (13) is reduced to
\[
E[S_k(m)|\mathbf{R}, \hat{B}^{(i)}] = \hat{B}_{k}^{(i)} \mathbf{H}_k^H \left( \sum_{\kappa=1}^{K} \frac{\hat{B}_{k}^{(i)} \Sigma_k^{-1} \mathbf{H}_k \mathbf{H}_k^H \Sigma_k^{-1}}{1 + \hat{B}_{k}^{(i)} \Psi_k} \right) \mathbf{R}_k(m).
\]

Substituting (40) and (41) in (32), we have
\[
\hat{B}_{k}^{(i)} = \frac{1}{M} \sum_{m=0}^{M-1} \frac{\hat{B}_{k}^{(i)} | \mathbf{Y}_k(m) |^2}{1 + \hat{B}_{k}^{(i)} \Psi_k}. \tag{42}
\]

Substituting \( \hat{B}_{k}^{(i)} = \hat{B}_{k}^{(i+1)} = \hat{B}_{k}^{(\infty)} \) in (42) and solving the resultant equation, we obtain \( \hat{B}_{k}^{(\infty)} = \hat{B}_{k,\text{EM}}^i \) as given by (39).

\[\text{B. Performance Evaluation}\]

In this part, we derive closed-form expressions for the probability of false alarm and missed detection, i.e., \( P_{f,k}(\gamma_k) \) and \( P_{d,k}(\gamma_k) \) respectively, for the IdML-SS. Based on the analysis in Part A, it follows that these expressions will also be applicable to the IdEM-SS approach in the limit of large \( \iota \), assuming that \( \mathbf{H}_k \) and \( \varsigma_k \) are known. The desired performance metrics are derived by considering the binary test in (24), with \( \hat{B}_{k,\text{EM}}^i \) now replaced by \( \hat{B}_{k,\text{ML}}^i \), and making use of the expression of \( \hat{B}_{k,\text{ML}}^i \) in (39).

Conditioned on \( \mathbf{H}_k \) and \( \varsigma_k \), the random variable \( \mathbf{Y}_k(m) = \mathbf{R}_k^H(m) \Sigma_k^{-1} \mathbf{H}_k \) has a complex Gaussian distribution with zero mean and variance \( E[| \mathbf{Y}_k(m) |^2] = E[| \mathbf{R}_k(m) |^2 \Sigma_k^{-1} \mathbf{H}_k \mathbf{H}_k^H \Sigma_k^{-1} \mathbf{R}_k(m)] \), whose expression can be derived as follows. We first expand \( | \mathbf{Y}_k(m) |^2 \) as the sum
\[
| \mathbf{Y}_k(m) |^2 = \sum_{n=0}^{N-1} \sum_{n'=0}^{N-1} \varsigma_{k,n}(m)^* \varsigma_{k,n'}(m) \tag{43}
\]

where \( \varsigma_{k,n}(m) = \varsigma_{k,n}(m)^* \mathbf{H}_k(m) \mathbf{H}_k^H \mathbf{H}_k \mathbf{H}_k^H \mathbf{H}_k(m)^* \). Then, we derive the expected value of the cross-term \( \varsigma_{k,n}(m)^* \varsigma_{k,n'}(m) \), which is given by (see the Appendix)
\[
E[\varsigma_{k,n}(m)^* \varsigma_{k,n'}(m)^*] = \varsigma_{k,n}^{*2} \mathbf{H}_k(m)^2 \mathbf{H}_k(m)^2 + \varsigma_{k,n} \varsigma_{k,n'} \delta_{n,n'} \tag{44}
\]

where \( \| \cdot \| \) returns the 2-norm of a vector. Using (44),

\[
E[| \mathbf{Y}_k(m) |^2] = \sum_{n=0}^{N-1} \varsigma_{k,n}^{*2} \mathbf{H}_k(m)^2 = \sum_{n=0}^{N-1} \varsigma_{k,n}^2 \mathbf{H}_k(m)^2 + \varsigma_{k,n} \varsigma_{k,n'} \delta_{n,n'} \tag{45}
\]

Let \( | \mathbf{Y}_k(m) |^2 = \sum_{n=0}^{M-1} | \mathbf{Y}_k(m) |^2 \), which is a sum of \( M \) statistically independent terms, where each term \( | \mathbf{Y}_k(m) |^2 \), once normalized by half its variance, follows a chi-square distribution with 2 degrees of freedom, denoted as \( \chi^2_2 \). Therefore, \( \frac{E[| \mathbf{Y}_k(m) |^2]}{2} \) has a chi-square distribution with 2M degrees of freedom, i.e., \( \chi^2_{2M} \). Under the hypothesis \( B_k = 0 \),
\[
E[| \mathbf{Y}_k(m) |^2] = \frac{E[| \mathbf{Y}_k(m) |^2]}{2} = \sum_{n=0}^{N-1} \varsigma_{k,n}^2 \mathbf{H}_k(m)^2 \mathbf{H}_k(m)^2 + \varsigma_{k,n} \varsigma_{k,n'} \delta_{n,n'} \tag{46}
\]

where \( \Gamma(c, y) = \int_0^\infty x^{c-1} e^{-x_y} dx = w \) is the normalized upper incomplete gamma function, and \( \Gamma(c) = \int_0^\infty x^{c-1} e^{-x} dx \) represents the complete gamma function. Under the constraint, \( P_{f,k}(\gamma_k) = \alpha_k \), the optimum threshold is given by
\[
\gamma_k^{\text{opt}} = \frac{1}{M \Psi_k^2} \left( \Gamma(c, w) \sum_{n=0}^{N-1} \varsigma_{k,n} \mathbf{H}_k(m)^2 \mathbf{H}_k(m)^2 \mathbf{H}_k(m)^2 \mathbf{H}_k(m)^2 + \varsigma_{k,n} \varsigma_{k,n'} \delta_{n,n'} \right) \tag{47}
\]

where \( \Gamma(c, w) \) is the inverse incomplete gamma function of \( \Gamma(c, y) \) [51]. Similarly, under the hypothesis \( B_k = 1 \), the optimum probability of detection under the constraint that \( P_{d,k}(\gamma_k) = \alpha_k \), i.e., according to the Neyman-Pearson formulation of the binary hypothesis testing problem is obtained by [52]
\[
P_{d,k}(\gamma_k^{\text{opt}}) = \left( \frac{E[| \mathbf{Y}_k(m) |^2]_{B_k=1}}{\gamma_k^{\text{opt}} M \Psi_k^2 + \Psi_k} \right)^{1/2} \tag{48}
\]

where
\[
E[| \mathbf{Y}_k(m) |^2]_{B_k=1} = \sum_{n=0}^{N-1} \varsigma_{k,n} \mathbf{H}_k(m)^2 = \sum_{n=0}^{N-1} \varsigma_{k,n} \mathbf{H}_k(m)^2 + \varsigma_{k,n} \varsigma_{k,n'} \delta_{n,n'} \tag{49}
\]
We remark that by using the threshold derived in (47), the theoretical receiver operating characteristic (ROC) obtained from (46) and (48) acts as an upper bound for the proposed EM-based JDE with unknown CSI and noise variance. Therefore, the “ideal” threshold (47) in itself can actually provide some insights on how to determine a practical threshold, although not necessarily optimal for the EM-JDE. From (47), we observe that the threshold can be sensitive to the sensing interval $M$, the probability of false alarm via $\alpha_k$, the entries of the instantaneous fading channel vector $H_k$ and the noise variance $\varsigma_{k,n}$.

V. SIMULATION EXPERIMENTS

The performance of the proposed JDE scheme based on the EM algorithm is evaluated through its ROC curves. Throughout our simulations, we assume a CR network of $N$ SUs, where each SU is equipped with $P$ receive antennas and operates in a wideband frequency spectrum with $K$ subbands. Since the estimation of the unknown parameters is performed independently for each subband, our results are presented for a selected subband, e.g., $k = 0$. Also, $B_k$ is estimated on a block-by-block basis in which the channel vector $H_k$ has a constant value within a block of $M$ samples, and changes independently from a block to the next. Our results are produced for $M = 50$ samples, and the channel vector $H_k$ is modeled as complex Gaussian vector with zero mean and covariance $\sigma_H^2 \mathbf{I}$. We assume that the noise variance $\varsigma_{k,n}$ is constant over the $P$ receive antennas of each SU. In this case, the average SNR per receive antenna of each SU in the $k$-th subband is given by $\text{SNR} = \frac{B_k \sigma_H^2}{\varsigma_{k,n}}$. For each choice of the detection threshold $\gamma_k$ in (24), $10^5$ trials are used to estimate the performance metrics of the proposed EM-JDE scheme. Except for the results in Fig. 4 where $T = 1$, we run the EM algorithm for $S = 4$ stages, where each stage comprises $T = 5$ iterations; initialization is performed as explained in Section III.B.3.

In Fig. 4, 5 and 6, we first study the performance of the proposed spectrum sensing techniques for the basic CR configuration with $N = 1$ SU equipped with $P = 2$ antennas. Subsequently, some of these parameters will be varied. In Fig. 4, we show the convergence of the proposed EM-JDE scheme for the basic CR configuration; the value of the log-likelihood function of the complete data (9) is plotted in Fig. 4 (a), while mean-square error (MSE) of channel estimation and noise power estimation, which are defined as

$$\frac{1}{P} \sum_{p=1}^{P} E \left[ |H_{k_0,0,p} - \hat{H}_{k_0,0,p}|^2 \right]$$

(50)

respectively, are plotted in Fig. 4 (b) and Fig. 4 (c). The results show that the value of log-likelihood function increases monotonically and reaches a local maximum within a few iterations, while simultaneously, the channel and noise parameter estimates converge to stationary points.

For the same configuration as above, Fig. 5 examines the performance of spectrum sensing techniques over time-variant Rayleigh fading channels ($P_{f,k}(\gamma_k) = 0.05$).

![Figure 4: Convergence behavior of the proposed EM-JDE scheme for a CR scenario with $N = 1$ SU equipped with $P = 2$ antennas. (a) Log-likelihood function of the complete data in (6); (b) MSE of channel estimation in (50); (c) MSE of noise variance estimation in (51) (SNR = −3dB).](image)

![Figure 5: Probability of missed detection versus SNR of the proposed EM-based spectrum sensing schemes for $N = 1$ SU with $P = 2$ antennas over time-varying Rayleigh fading channels ($P_{f,k}(\gamma_k) = 0.05$).](image)
a performance benchmark. We first note that the IdML-SS outperforms the optimum detector [14] with significant gains in PU’s signal detection. The better performance of the IdML-SS is due to the fact that it uses estimates of the average transmit power as decision statistics (see equation (39)), which in turn is independent of the time-varying channel gains. The proposed IdEM-SS also outperforms the optimum detector [14] by a wide margin in the time-variant case, with the probability of missed detection converging to that of the IdML-SS as the number of iterations $i$ increases. On this figure, as a lower bound on the missed detection performance of the spectrum sensing detectors, we also present results for the case of time-invariant channels, where $H_k$ is constant over all sensing intervals. Both the IdML-SS and optimum detector achieve the same performance in this ideal situation. Comparing the results for time-varying channels to those for time-invariant channels, we conclude that user mobility can have a major impact on, that is, significantly degrade the achievable detection performance of spectrum sensing schemes.

Still for the case $N = 1$ SU with $P = 2$ antennas, Fig. 6 evaluates the performance of the proposed EM-JDE scheme over time-variant Rayleigh fading channels, in the practical case where $H_k$ and $\varsigma_k$ are unknown by the SU. For the purpose of comparison, we also include the ROC curve of the blind GLRD proposed in [14]. The results show that the EM-JDE scheme enhances the spectrum detection process compared with the blind GLRD. As a benchmark on the performance of the proposed scheme, we add the ROC curve of the IdEM-SS with the same simulation parameters assuming perfect channel

The optimum detector in [14] refers to the Neyman-Pearson detector under the assumption of known and time-invariant channel gain and noise variance parameters. This detector implements a likelihood ratio test (LRT) based on a Gaussian signal model, where the detection threshold $\gamma_k$ is adjusted to minimize the probability of missed detection $P_{\text{m,k}}(\gamma_k)$, subject to a constraint on the probability of false alarm, i.e., $P_{\text{f,k}}(\gamma_k) < \alpha$.

Figure 6: ROCs of different spectrum sensing schemes for CR scenario with $N = 1$ SU equipped with $P = 2$ antennas (SNR = −3dB).

Figure 7: ROC of the proposed EM-JDE scheme for a multi-user CR network with $N \in \{1, 2, 3, 4\}$ SUs, each equipped with $P = 2$ antennas (SNR = −3dB).

Figure 8: ROC of the proposed EM-JDE scheme with centralized and distributed implementations for a CR network with $N = 3$ SUs, each with $P = 2$ antennas (SNR = −3dB).
Figure 9: The effect of $M$ on the performance of the proposed EM-JDE scheme for a CR network with $N = 2$ SUs, each with $P = 2$ antennas ($P_{f,k}(\gamma_k) = 0.05$, SNR = $-3$dB).

Figure 10: The effect of $P$ on the performance of the proposed EM-JDE scheme for a CR network with $N = 2$ SUs, each with $P = 2$ antennas ($P_{f,k}(\gamma_k) = 0.05$, SNR = $-3$dB).

performance of the two schemes almost match with each other. Considering the challenges of physical implementations, one needs to consider the tradeoff between the communications overhead and the hardware complexity in choosing between these two different options.

Fig. 10 studies the effect of increasing the number of samples available for sensing, as represented here by the parameter $M$, on the performance of the proposed EM-JDE scheme for a CR network with $N = 2$ SUs, each equipped with $P = 2$ antennas. The plotted curves represent the relationship between the probability of missed detection in subband $k$, $P_{m,k}(\gamma_k) = 1 - P_{d,k}(\gamma_k)$, against $M$ for a fixed value of the probability of false alarm, $P_{f,k}(\gamma_k) = 0.05$. The results show that the values of $P_{m,k}(\gamma_k)$ are reduced significantly by increasing $M$.

Finally, Fig. ?? shows the effect of increasing the number of antennas, i.e., $P$, on the performance of the proposed EM-JDE scheme for a CR network with $N = 2$ SUs. Given $P_{f,k}(\gamma_k) = 0.05$, $P_{m,k}(\gamma_k)$ is plotted versus different values of $P$ in the case of time-variant Rayleigh fading channels. From the results, we notice that increasing the number of antennas at the SUs can significantly enhance the ability of the proposed scheme to efficiently sense the available spectrum in cooperative CR networks.

VI. CONCLUSIONS

In this paper, a cooperative spectrum sensing scheme based on the EM algorithm for multi-antenna CR networks was proposed. In this scheme, a binary hypothesis test is applied on estimates of the average power transmitted by the PU over a wideband frequency spectrum during the sensing interval, making the decision on the spectral occupancies independent of the channels states. However, knowledge of CSI and noise variance at each SU is crucial to obtain reliable estimates of the PU’s transmitted power over different frequency subbands. Therefore, the FC employs the EM algorithm in a non-trivial way to jointly estimate the unknown continuous parameters in each subband along with the PU detection, thereby forming an EM-JDE scheme for multi-user multi-antenna CR networks. Various aspects of this proposed EM-JDE scheme were investigated, including a reliable initialization strategy to ensure convergence under practical conditions and a distributed implementation to reduce communication overhead. We also introduced an analytical evaluation of the IdML-SS based on the Neyman-Pearson criterion as a lower bound on the missed detection performance of cooperative spectrum sensing. The results show that the proposed EM-JDE scheme can provide a significant improvement in the PU spectrum detection, especially for time-variant channels. This research work extends the application of advanced joint estimation and detection schemes, which are widely employed in the design of modern wireless communication systems, to the topic of spectrum sensing in wideband CR networks.

APPENDIX

In this part, we derive a closed-form expression for $E[\zeta_{k,n}(m)\zeta_{k,n'}(m)]$ in (44). First, we expand the product $\zeta_{k,n}(m)\zeta_{k,n'}(m)$ as

$$\zeta_{k,n}(m)\zeta_{k,n'}(m) = \zeta_{k,n}^{-1} R_{k,n}(m)^{H H_{k,n}^{H} R_{k,n'}(m)}$$

$$= \zeta_{k,n}^{-1} \sum_{p=0}^{P-1} \sum_{p'=0}^{P-1} H_{k,n,p}^{H} R_{k,n',p'} R_{k,n,p}(m) R_{k,n',p'}^{*}(m)$$

(52)

The expectation of the cross term $R_{k,n,p}(m) R_{k,n',p'}^{*}(m)$ is given by

$$E[R_{k,n,p}(m) R_{k,n',p'}^{*}(m)] = H_{k,n,p}^{H} B_{k} + \zeta_{k,n} \delta_{n,n'} \delta_{p,p'}$$

(53)

Using (53), we can obtain $E[\zeta_{k,n}(m)\zeta_{k,n'}(m)]$ in (54).
\[
E[\zeta_{k,n}(m)|\zeta_{k,n'}(m')] = \zeta_{k,n}^{-1} \sum_{p=0}^{P-1} \sum_{p'=0}^{P-1} |H_{k,n,p}|^2 |H_{k,n',p'}|^2 + \zeta_{k,n} \sum_{p=0}^{P-1} |H_{k,n,p}|^2 \delta_{n,n'}.
\]

REFERENCES


