Nonparametric Estimation of
Production Risk and Risk Preference Functions

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ABSTRACT

This paper deals with estimation of risk and the risk preference function when producers face uncertainties in production (usually labeled as production risk) and output price. These uncertainties are modeled in the context of production theory where the objective of the producers is to maximize expected utility of normalized anticipated profit. Models are proposed to estimate risk preference of individual producers under (i) only production risk, (ii) only price risk, (iii) both production and price risks, (iv) production risk with technical inefficiency, (v) price risk with technical inefficiency, and (vi) both production and price risks with technical inefficiency. We discuss estimation of the production function, the output risk function, and the risk preference functions in each case. Norwegian salmon farming data is used for an empirical application of some of the proposed models. We find that salmon farmers are, in general, risk averse. Labor is found to production risk decreasing while capital and feed are found to be risk increasing.

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1. Introduction

Risk in production theory is mostly analyzed under (i) output price uncertainty and (ii) production uncertainty (commonly known as production risk). Output price can be uncertain due to a variety of reasons. Perhaps the most important factor is the presence of a time lag between use of inputs and output produced. Moreover, produced output is often sold at a later date when output price is likely to be different from the date when the production plan was made. Uncertainty in output price makes profits uncertain. Profit can also be uncertain if there output is risky, which may be affected by input quantities. That is, input quantities not only determine the volume of output produced, but some of these inputs might also be affecting variability of output (often labeled as production risk). For example, fertilizer might be risk augmenting in the production of crop while labor might decrease output risk. Here we address the implications of these risks in a framework where producers maximize expected utility of anticipated profit. In particular, we examine input allocation decisions in the presence of price uncertainty and production risk. Since input demand and output supply (as well as own and cross price elasticities, returns to scale, etc.) are affected by the presence of these uncertainties, it is desirable to accommodate uncertainty in production studies, especially in estimating the underlying production technology.

Although the theoretical work on risk in the production literature is quite extensive, there are relatively fewer empirical studies devoted to analyzing different sources of risk on production and input allocation. Most of these studies either looked at output price uncertainty (Appelbaum and Ullah (1997), Kumbhakar (2002), Sandmo (1971), Chambers (1983)) or production risk along the Just-Pope framework (Tveterås (1999, 2000), Asche and Tveterås (1999), Kumbhakar and Tveterås (2001). To examine producers' behavior under risk some parametric forms of the utility function, production function and output risk function along with specific distributional assumptions on the error term representing risk are considered in the existing literature (Love and Buccola (1991), Saha, Shumway and Talpaz (1994)). Thus, the risk studies in the production literature have some or all of these features built in, viz., (i) parametric forms of the production and risk function, (ii) parametric form of the utility function, (iii)
distributional assumption(s) on the error term(s) representing either production risk or output price uncertainty or both.

In the present paper we estimate the production function, the risk function (output risk), and risk preference functions (associated with price and production uncertainties). We derive estimates of risk preference functions that do not depend on specific functional form of the underlying utility function. In estimating these functions no distributional assumptions are made on the random terms associated with production and output uncertainties. Furthermore, we obtain estimates of producer-specific risk premium.

The rest of the paper is organized as follows. The models with price uncertainty and production risk are presented in Section 2. Extensions of these models to accommodate technical inefficiency are considered in Section 3. Section 4 describes the Norwegian salmon farming data that is used to apply the models proposed in Section 2. Empirical results are presented in Section 5. Section 6 concludes the paper with a brief summary of results.

2. Risk Models with Output Price Uncertainty and Production Risk

We assume that the production technology can be represented by a Just-Pope (1978) form, viz.,

$$ y = f(X,Z) + h(X,Z)\varepsilon, \quad \varepsilon \sim (0,1) $$

where $y$ is output, $X$ and $Z$ are vectors of variable and quasi-fixed inputs and $f(X,Z)$ is the mean output function. Since output variance is represented by $h^2(X,Z)$, the $h(X,Z)$ function is labeled as the output risk function. In this framework an input $j$ is said to be risk increasing (decreasing) if the partial derivative $h_j(X,Z) > (<) 0$.

2.1 Only production risk (Model I)

Let’s first consider the case where output and input markets are competitive and their prices are known with certainty. Production is, however, uncertain. Assume that producers maximize expected utility of anticipated normalized profit $E(U(\pi^c / p))$ to
choose optimal input quantities, which in turn determines output supply. Define anticipated profit \( \pi^e \) as

\[
\pi^e = py - wX = pf(X,Z) - wX + ph(X,Z) = \mu_x + ph(X,Z)\varepsilon
\tag{2}
\]

where \( \mu_x = pf(X,Z) - wX \), \( p \) being the output price and \( w \) the price vector of the variable inputs.

The first-order conditions (FOCs) of expected utility of anticipated normalized profit \( E(U(\pi^e / p)) \) maximization can be written as

\[
E(U'(\pi^e / p))(f_j(X,Z) - \bar{w}_j + h_j(X,Z)\varepsilon) = 0
\tag{3}
\]

where \( U'(\pi^e / p) \) is the marginal utility of anticipated normalized profit, \( f_j(X,Z) \) and \( h_j(X,Z) \) are partial derivatives of \( f(X,Z) \) and \( h(X,Z) \) functions with respect to input \( X_j \), respectively.

We can rewrite the above FOCs as

\[
f_j(X,Z) = \bar{w}_j - h_j(X,Z)\theta_1(.)
\tag{4}
\]

where

\[
\theta_1(.) = \frac{E(U'(\pi^e / p)\varepsilon)}{E(U'(\pi^e / p))}
\tag{5}
\]

The \( \theta_1(.) \) term in the first-order conditions (4) is the risk preference function associated with production risk. If producers are risk averse then \( \theta_1(.) < 0 \) (i.e., an increase in \( \varepsilon \) increases \( \pi^e / p \) which in turn reduces \( U'(\pi^e / p) \) since \( U''(\pi^e / p) < 0 \) (utility function being concave)). Similarly, \( \theta_1(.) \) is positive if producers are risk lovers and is zero for risk neutral producers.

If \( h_j(X,Z) > 0 \), then for risk averse producers the value of the (expected) marginal product of input \( X_j \) exceeds its price \( (p f_j(X,Z) > w_j) \). Consequently, a risk averse producer will use the input less relative to a risk neutral producer \( (\theta_1 = 0) \). Similarly, if producer A is more risk averse than an otherwise identical producer B, producer A will
use less of input $X_j$ than producer B. Thus, input demand functions (the solution of $X_j$ from (4)) will depend not only on observed prices but also on the risk preference functions. Consequently, anything that depends on the demand functions (e.g., own and cross price elasticities, returns to scale, technical change, etc.) is likely to be affected by the presence of risk via $\theta_j(.)$. Since input demand functions are affected, output supply will also be affected even if the producers share the same technology, face the same input and output prices.

2.2 Only output price uncertainty (Model II)

We now consider the case where output price is uncertain (Appelbaum and Ullah (1997), Sandmo (1971)) and there is no production uncertainty ($h(X,Z) = \text{constant}$). We describe output price uncertainty by postulating anticipated price ($p^e$) as, $p^e = p e^\eta$ with the assumption that $E(e^\eta) = 1$ (Zellner, Kmenta and Dreze (1966)) so that the expected value of $p^e$ is the same as the observed price, $p$. The anticipated price differs from the observed price at a point in time because the production process is not always instantaneous, and the quantity of output cannot be perfectly predicted at the time production decisions are made.

Similar to Model I we assume that producers maximize expected utility of anticipated normalized profit $E\left(\frac{U(\pi^e / p)}{p}\right)$ to determine optimal input quantities, which in turn determines output supply. The production function is the same as in (1). Define anticipated profit $\pi^e$ as

$$
\pi^e = p^e y - wX = pf(X,Z) - wX + pf(X,Z)(e^\eta - 1) + p e^\eta \varepsilon
\Rightarrow \pi^e / p = f(X,Z) - \tilde{w} X + f(X,Z)(e^\eta - 1) + e^\eta \varepsilon = \mu_x + f(X,Z)z_1 + z_2
$$

(6)

where $z_1 = (e^\eta - 1)$, $z_2 = e^\eta \varepsilon$ and $\tilde{w}_j = w_j / p$. Note that both $z_1$ and $z_2$ are zero mean random variables.

The FOCs of expected utility of anticipated normalized profit $E\left(\frac{U(\pi^e / p)}{p}\right)$ maximization can be written as

$$
E\left(U'(\pi^e / p)\{f_j(X,Z) - \tilde{w}_j + f_j(X)z_1\}\right) = 0
$$

(7)
We can rewrite (7) as

\[ f_j(X, Z)(1 + \theta_2(\cdot)) = \tilde{w}_j \tag{8} \]

where

\[ \theta_2(\cdot) = \frac{E[U'(\pi^e / p) z_1]}{E[U''(\pi^e / p)]} \tag{9} \]

The \( \theta_2(\cdot) \) term in the first-order conditions (9) is the risk preference function associated with output price uncertainty. If producers are risk averse then \( \theta_2(\cdot) < 0 \) (i.e., an increase in \( \pi^e \) (for any \( \epsilon \)) increases \( \pi^e / p \) which in turn reduces \( U'(\pi^e / p) \) since \( U''(\pi^e / p) < 0 \) (utility function being concave)). Similarly, \( \theta_2(\cdot) \) is positive if producers are risk lovers and is zero for risk neutral producers.

### 2.3 Both production risk and output price uncertainty (Model III)

Now we consider the case where producers face both production risk and uncertainty in output price. Output price is assumed to be governed by the same process as in Model II, and the production function is given in (1). For simplicity we assume that \( \epsilon \) is independent of \( \eta \). Furthermore the variance of \( e^\eta \) is assumed to be constant.

With the presence of both types of uncertainties the anticipated normalized profit \( \pi^e / p \) can be written as

\[ \pi^e / p = e^\eta y - \tilde{w}X = f(X, Z) - \tilde{w}X + f(X, Z)(e^\eta - 1) + h(X, Z)(e^\eta \epsilon) = \mu_2 + f(X, Z)z_1 + h(X, Z)z_2 \tag{10} \]

The FOCs of expected utility of anticipated profit \( E(U(\pi^e / p)) \) maximization can be written as

\[ E\left(U'(\pi^e / p)\{f_j(X, Z) - \tilde{w}_j + f_j(X, Z)z_1 + h_j(X, Z)z_2\}\right) = 0 \tag{11} \]

where \( U'(\pi^e / p), f_j(\cdot) \) and \( h_j(\cdot) \) are the same as before.

We can rewrite (11) as

\[ f_j(X, Z)(1 + \tilde{\theta}_2(\cdot)) = \tilde{w}_j - h_j(X, Z)\tilde{\theta}_1(\cdot) \tag{12} \]

where
\[ \tilde{\theta}_1(.) \equiv \frac{E(U'(\pi^e/p)z_2)}{E(U'(\pi^e/p))} \]  
\[ \tilde{\theta}_2(.) \equiv \frac{E(U'(\pi^e/p)z_1)}{E(U'(\pi^e/p))} \]  

and

The \( \tilde{\theta}_1(.) \) and \( \tilde{\theta}_2(.) \) functions in (13) and (14) are called risk preference functions associated with output price uncertainty and production risk, respectively.\(^1\) If producers are risk averse then \( \tilde{\theta}_2(.) < 0 \). A similar reasoning shows that \( \tilde{\theta}_1(.) = 0 \) when producers are risk neutral (i.e., \( U''(\pi^e/p) = 0 \) which implies that the utility function is linear), and if producers are risk loving then \( \tilde{\theta}_2(.) > 0 \) (i.e., \( U''(\pi^e/p) > 0 \) which means that the utility function is convex). Finally, it can be shown, using similar arguments, that \( \tilde{\theta}_1(.) \) is negative if producers are risk averse, positive for risk loving and zero for risk neutral producers.

The model with only output price uncertainty can be obtained from the above model by assuming that there is no output risk (i.e., \( h(X,Z) \) is a constant thereby meaning that \( h_j(X,Z) = 0 \)). This means that the \( \tilde{\theta}_1(.) \) function will disappear from the FOCs. Similarly, if there is only production risk and no uncertainty in output price, then \( z_1 = 0 \), and the \( \tilde{\theta}_2(.) \) function will disappear from the FOCs. Finally, if the producers are risk neutral, then both \( \tilde{\theta}_1(.) \) and \( \tilde{\theta}_2(.) \) will disappear from the FOCs in (12).

\(^1\) Note that \( \pi^e/p \) in (10) has two sources of randomness (\( \eta \) and \( \varepsilon \)) whereas the source of randomness in \( \pi^e \) in Model I (given in equation (2)) is \( \varepsilon \). Consequently, the \( \tilde{\theta}_1(.) \) and \( \tilde{\theta}_2(.) \) functions in (13) and (14) are not exactly the same as \( \theta_1(.) \) and \( \theta_2(.) \) in (5) and (9), although we are interpreting them as risk functions associated with output price and production risk, respectively. In general, the \( \tilde{\theta}_2(.) \) and \( \tilde{\theta}_1(.) \) functions in (13) and (14) will depend on the parameters of the distributions of both \( \eta \) and \( \varepsilon \).
3. Risk Models with Production risk and technical efficiency

3.1 Only production risk (Model IV)

If the producers face production risk and are technically inefficient, the production function can be written as

\[ Y = f(X, Z) + h(X, Z)\varepsilon - g(X, Z)u, \quad h(X, Z) > 0, \quad g(X, Z) > 0, \quad u \geq 0 \]  

This model is a generalization of the Battese, Rambaldi and Wan (1997) model. If \( h(X, Z) = g(X, Z) \) then the model reduces to the Battese et al. model.

We assume that producers they maximize \( E[U(\pi^e / p)] \) conditional on \( u \).

Anticipated profit, \( \pi^e \), is

\[ \pi^e = pY - wX \Rightarrow \pi^e / p = f(X, Z) + h(X, Z)\varepsilon - g(X, Z)u - (w/p)X \]

The first-order conditions (FOC) of \( E[U(\pi^e / p)] \) maximization, given \( u \), are

\[ E[U'(\varepsilon)]f_j(X, Z) + h_j(X, Z)\varepsilon - g_j(X, Z)u - \tilde{w}_j = 0 \]

\[ \Rightarrow f_j(X, Z) - g_j(X, Z)u + h_j(X, Z)\frac{E[U'(\varepsilon)]}{E[U'(\varepsilon)]} - \tilde{w}_j = 0 \]

\[ \Rightarrow f_j(X, Z) - \tilde{w}_j - g_j(X, Z)u + h_j(X, Z)\lambda(\cdot) = 0 \]  

(16)

where \( \lambda(\cdot) = \frac{E[U'(\varepsilon)]}{E[U'(\varepsilon)]} \) is the risk preference function associated with production risk. The only difference between \( \lambda(\cdot) \) and \( \theta(\cdot) \) is that \( \lambda(\cdot) \) depends on inefficiency as well through the utility function.

3.2 Only output price uncertainty (Model V)

Now we introduce the presence of technical inefficiency into the model with only output price uncertainty. The production function is

\[ Y = f(X, Z) + h_0\varepsilon - g(X, Z)u \]

where \( h_0 \) is a constant. This is basically a stochastic frontier model in which determinants of technical inefficiency are modeled through the scaling function, \( g(X, Z) \) (see Wang and Schmidt (2002)). Since we are considering an optimizing model and output price is
uncertain, input choices will be affected by price uncertainty. Here we are interested in estimating the production function, determinants of technical inefficiency, and the risk preference function associated with output price uncertainty.

As before, we assume that producers choose $X$ by maximizing $E[U(\pi^e / p)]$ where

$$\pi^e = p^* Y - wX \Rightarrow \pi^e / p = e^\eta [f(X,Z) + h_0 e - g(X,Z)u - \tilde{w}.X].$$

We rewrite anticipated normalized profit as

$$\pi^e / p = f(X,Z) - \tilde{w}.X - g(X,Z)u e^\eta + h_0 e e^\eta + f(X,Z)(e^\eta - 1)$$

$$\pi^e / p = f(X,Z) - \tilde{w}.X - g(X,Z) \cdot (1 + z_1) + h_0 \cdot z_2 + f(X,Z) \cdot z_1$$

(17)

The FOCs of maximization $E[U(\pi^e / p)]$ with respect to the elements of $X$ (given $u$) are

$$E[U'(\cdot) f_j(X,Z) - \tilde{w}_j - g_j(X,Z) \cdot u \cdot (1 + z_1) + f_j(X,Z) \cdot z_1] = 0.$$  
$$\Rightarrow f_j(X,Z) - \tilde{w}_j - g_j(X,Z) \cdot u \cdot (1 + \lambda_2(\cdot)) + f_j(X,Z) \cdot \lambda_2 = 0$$

(18)

where $\lambda_2(\cdot) = E[U'(\cdot)z_1]/E[U'(\cdot)]$ is the risk preference function associated with price risk.

### 3.3 Both production risk and price uncertainty (Model VI)

In this section we introduce both output price and production uncertainty into the analysis. The production function is the same as the one in (15), i.e.,

$$Y = f(X,Z) + h(X,Z)e - g(X,Z)u$$

Output price uncertainty is modeled as before (in Model II), i.e., $p^e = pe^\eta$ such that $E(e^\eta) = 1$ and $V(e^\eta) = \beta^2 > 0$. Here our objectives are to estimate (i) the production risk function, $h(X,Z)$; (ii) technical inefficiency, $u$ and the determinants of technical inefficiency through the scaling function, $g(X,Z)$; and (iii) the risk preference functions associated with production risk and output price uncertainty.

As before, we assume that producers choose $X$ by maximizing $E[U(\pi^e / p)]$ where

$$\pi^e = p^* Y - wX \Rightarrow \pi^e / p = e^\eta [f(X,Z) + h(X,Z)e - g(X,Z)u] - \tilde{w}.X.$$
Now we rewrite anticipated profit as

\[ \pi^e / p = f(X, Z) - \bar{w}.X - g(X, Z)u \epsilon^y + h(X, Z)\epsilon^y + f(X, Z)(\epsilon^y - 1) \]

\[ = f(X, Z) - \bar{w}.X - g(X, Z)\cdot(1 + z_i) + h(X, Z)\cdot z_2 + f(X, Z)\cdot z_1 \]  \hspace{1cm} (19)

The FOCs of maximization \( E[U(\pi^e / p)] \) with respect to the elements of \( X \) (given \( u \)) are

\[ E[U'(\cdot)]\{f_j(X, Z) - \bar{w}_j + h_j(X, Z)\cdot z_2 - g_j(X, Z)\cdot u \cdot \epsilon^y + f_j(X, Z)\cdot z_1] = 0. \]

\[ \Rightarrow f_j(X, Z) - \bar{w}_j + h_j(X, Z)\cdot \tilde{\lambda}_2 - g_j(X, Z)\cdot u \cdot (\tilde{\lambda}_1) + f_j(X, Z)\cdot \tilde{\lambda}_1 = 0 \]  \hspace{1cm} (20)

where \( \tilde{\lambda}_1 = E[U'(\cdot)z_1]/E[U'(\cdot)] \), \( \tilde{\lambda}_2 = E[U'(\cdot)z_2]/E[U'(\cdot)] \) are risk preference functions associated with price and production risks, respectively.

4. Problems with Parametric Econometric Models of Risk

Since our interest is to estimate the parameters of the mean output function, output risk function and the risk preference function, the most important task is to derive an algebraic form of the risk preference function, which is easy to implement econometrically, and imposes minimum restrictions on the structure of risk preferences on the individual producers. Certain specific forms of \( U(\cdot) \) together with some specific distributional assumptions on \( \epsilon \) give an explicit closed form solution of \( \theta(\cdot) \) (Love and Buccola, 1991; Saha, Shumway, and Talpaz, 1994). However, estimation of these models is quite complex. It is, however, possible to derive an algebraic expression for the risk preference function without assuming any distribution on \( \epsilon \) and without any specific functional form on \( U(\cdot) \) that imposes \textit{a priori} restrictions on the structure of risk aversion.\(^2\) In fact, our result would be very useful in empirical applications, especially if one is interested in estimating general forms of risk preferences without estimating a complicated system of equations (Chavas and Holt, 1996; Love and Buccola, 1991; Saha, Shumway and Talpaz, 1994).

\(^2\) This is, for example, the case in Appelbaum (1991), where constant absolute risk aversion is assumed.
4.1 Specification and estimation of Model I

If \( U(\mu_x + h(X,Z)e) \) is continuous and differentiable, and we take a linear approximation of \( U'(\mu_x + h(X,Z)e) \) at \( e = 0 \), then the risk preference function in Model I takes the following form\(^3\)

\[
\theta_i(\cdot) = -AR(\mu_x) \cdot h(X,Z), \\
\tag{21}
\]

where \( AR(\mu_x) = -U''(\mu_x)/U'(\mu_x) \) is the Arrow-Pratt measure of absolute risk aversion.

Using the above result the FOC in (4) can be expressed as

\[
f_j(X,Z) = \hat{w}_j + h_j(X,Z) \cdot AR(\mu_x) \cdot h(X,Z) \\
\tag{22}
\]

A close look at the first order condition in (22) shows that the focus of the problem is now shifted from the utility function to the AR function. In addition to the mean production and risk functions, one needs to specify a functional form on AR which will define a system of \( J \) equations in \( J \) variable inputs \((X)\) in (22). It is worth noting here that any specification of the AR function will indirectly imply some underlying utility function, viz., \( U = \int e^{-AR} d\mu_x \). That is, the AR function gives all the information possessed by the utility function (Pratt, 1964). The main advantage of working with the AR function is that one doesn't have to worry about (i) the underlying utility function (which may not be always solvable analytically), (ii) the derivation of \( \theta_i(\cdot) \) (which might not always give a closed form solution), and (iii) the solution \( \theta_i(\cdot) \) (which, although solvable for some specific utility functions, might not be easy to work with empirically). Furthermore, one can assume a functional form on AR that is flexible enough to test whether producers are risk neutral \((AR = 0)\) or not. If risk neutrality does not exit then we can also test for constant absolute risk aversion \((CARA)\), decreasing absolute risk aversion \((DARA)\), and increasing absolute risk aversion \((IARA)\) hypotheses.

\( AR \) can be parameterized to allow (test) for constant, increasing or decreasing absolute risk aversion \((CARA, IARA \text{ and } DARA, \text{ respectively})\). For example, if \( AR = \delta_1 + \delta_2\mu_x + 0.5\delta_3\mu_x^2 \) then \( CARA \Rightarrow \delta_2 = \delta_3 = 0, \) \( IARA \Rightarrow (\delta_2 + \delta_3\mu_x) > 0, \) and \( DARA \Rightarrow (\delta_2 + \delta_3\mu_x) < 0. \) Furthermore, \( \delta_1 = \delta_2 = \delta_3 = 0 \Rightarrow AR = 0 \Rightarrow \theta = 0, \) i.e., risk neutrality. These are all testable hypotheses. Some other non-linear functions can also be used to parameterize

\(^3\)See Kumbhakar and Tvetereas (2003) for a proof.
and test different forms of risk preferences. Although a parametric form on $AR$ indirectly implies some form of a utility function, it is not necessary to know the exact parametric form of the underlying utility function in specifying a functional form for $AR$.

The model outlined above (Model I) can be estimated by estimating the system consisting of the production function in (1) along with the FOCs in (22) once parametric functional forms are chosen for $f(X,Z)$, $h(X,Z)$ and the AR(.) functions; and classical error terms are added to each of the FOCs in (22). Two things are to be noted here. First, the system is highly complicated and nonlinear is parameters, and therefore a nonlinear system approach has to be used. Second, the endogenous variables are the variable inputs ($X$) and output ($Y$) which appear almost everywhere in the system. Thus a nonlinear three stage least squares or other instrumental variable approach has to be used. The exogenous variables (instruments) are the quasi-fixed inputs ($Z$) and prices ($p$ and $w$).

4.2 Specification and estimation of Model II

A similar procedure can be used to estimate Model II that incorporates only output price risk discussed in section 2.2. We use the following result to express the risk preference function in terms of the AR function.

If $U(\mu + f(X,Z)z_1 + z_2)$ is continuous and differentiable, and we take a linear approximation of $U'(\mu + f(X,Z)z_1 + z_2)$ at $z_1 = z_2 = 0$, then the risk preference function takes the following form\(^5\)

\[
\theta_2(.) = -AR(\mu) . f(X,Z), \text{ where } AR = -U''(.) / U'(.) \text{ evaluated at } \mu.
\]

Using this result we write the FOCs in (8) as

\[
f_j(X,Z)[1 - AR(\mu) . f(X,Z)] = \tilde{\nu}_j + v_j (23)
\]

where $v_j$ can be viewed as an optimization error in choosing the $j$th variable input. Thus, the estimating model consists of the production function in (1) and the FOCs in (23) that can be estimated using a non-linear system approach. This system is also heavily parametric and difficult to estimate.

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\(^4\) See Kumbhakar and Tveteras (2003) for details.

\(^5\) The proof is similar to Kumbhakar and Tveteras (2003).
4.3 Specification and estimation of Model III

To estimate Model III that incorporates both production and output price risk discussed in section 2.3, we express the risk preference functions (specified in (13) and (14)) in terms of the AR function.

If \( U(\mu_{\pi} + f(X,Z)z_1 + h(X,Z)z_2) \) is continuous and differentiable, and we take a linear approximation of \( U'(\mu_{\pi} + f(X,Z)z_1 + h(X,Z)z_2) \) at \( z = z_1 = 0 \), then the risk preference functions are

\[
\tilde{\theta}_z(\cdot) = -AR(\mu_{\pi})f(X,Z),
\]

\[
\tilde{\theta}_h(\cdot) = -AR(\mu_{\pi})h(X,Z)
\]

Using this result we write the FOCs in (12) as

\[
f_j(X,Z)[1 - AR(\mu_{\pi})(X,Z)] = \tilde{w}_j + h_j(X,Z)h(X,Z).AR(\mu_{\pi}) + \eta_j
\]

where \( \eta_j \) can be viewed as an optimization error in choosing the \( j \)th variable input. Thus, the estimating model consists of the production function in (1) and the FOCs in (24) that can be estimated using a non-linear system approach.

4.4 Specification and estimation of Model IV

To derive an estimable expression of \( \lambda_t(\cdot) \), we express it, as before, in terms of the AR(.) function. For this, first, we expand \( U'(\pi^e/p) \) around \( \varepsilon = 0 \), i.e.,

\[
U'(\pi^e/p) = U'(q(X,Z,u)) + U''(q(X,Z,u)).h(X,Z).\varepsilon + \ldots
\]

where \( q(X,Z,u) = f(X,Z) - g(X,Z)\cdot u - \tilde{w}.X \)

Thus,

\[
\begin{align*}
E[U'(\cdot)] &= U'(q(X,Z,u)) \\
E[U'(\cdot)\cdot \varepsilon] &= U''(q(X,Z,u)).h(X,Z)
\end{align*}
\]

ignoring higher order terms

\[
\Rightarrow \lambda_t(\cdot) = \frac{U''(q(X,Z,u)).h(X,Z)}{U'(q(X,Z,u))} = -AR(X,Z,u)\cdot h(X,Z)
\]

(25)

where \( AR(X,Z,u) = -U''(\cdot)/U'(\cdot) \) is the Arrow-Pratt absolute risk aversion function evaluated at \( q(X,Z,u) \). For risk averse producers \( \lambda_t(\cdot) < 0 \Rightarrow AR(\cdot) > 0 \).

Using the above expression for \( \lambda_t(\cdot) \), we write (16) as:
\[ f_j(X,Z) - \tilde{w}_j - g_j(X,Z)u + h_j(X,Z)[AR(X,Z,u)\cdot h(X,Z)] = v_j \]

\[ \Rightarrow f_j(X,Z) - \tilde{w}_j - AR(X,Z,u)\cdot h_j(X,Z)\cdot h(X,Z) = v_j + g_j(X,Z)u \] (26)
where the error term \( v_j \) in (26) can be viewed as optimizing error associated with the \( j \)th variable input.

Estimation of the above model can be done in either two steps or a single-step. We first discuss the two-step procedure.

**Two-step procedure:**

**Step 1:** Use the maximum likelihood method to estimate the production function in (15) with the following distributional assumptions on \( u \) and \( \varepsilon \).\(^6\)

- (i) \( u \sim \text{i.i.d. } N^+\left(\mu, \sigma_u^2\right) \)
- (ii) \( \varepsilon \sim \text{i.i.d. } N(0, 1) \)
- (iii) \( u \) and \( \varepsilon \) are independent.

In specifying the variance of \( \varepsilon \) to unity we assume that the \( h(X,Z) \) function has a constant. Based on the above distributional assumptions, the likelihood function can be derived by making a few changes to the one derived in Battese et al. (1997).\(^7\) By specifying parametric functional forms for \( f(X,Z), h(X,Z) \) and \( g(X,Z) \) one can obtain estimates of the parameters in \( f(X,Z), h(X,Z) \) and \( g(X,Z) \), as well as \( \mu \) and \( \sigma^2 \).

These parameters can then be used to estimate \( u \) (for each observation) from either the mean or mode of \( u \mid \varepsilon^* \) where \( \varepsilon^* = h(X,Z)\varepsilon - g(X,Z)u \) (see the appendix). It is straightforward to show that the conditional distribution of \( u \) is truncated normal. Once \( u \) is estimated technical efficiency (TE) can be estimated from

\[ TE = \frac{E(Y \mid X,Z,u)}{E(Y \mid X,Z,u = 0)} = 1 - \frac{g(X,Z)u}{f(X,Z)} \] (27)

**Step 2:** Step 1 gives estimates of \( f(X,Z), g(X,Z), \) and \( h(X,Z) \) as well as the estimates of \( u \). These estimates can be used in (26) to compute \( \lambda_i(\cdot) \) and \( AR \) as follows:

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\(^6\) Note that the production function (15) is more general than the one used by Battese et al. (1997).

\(^7\) The Battese et al. model can be obtained by imposing the restriction \( h(X,Z) = g(X,Z) \) which is a testable hypothesis.
\[
\sum_j (f_j(X,Z) - \bar{w}_j - g_j(X,Z)u) = \sum_j v_j - \lambda_1(X,Z,u) \sum_j h_j(X,Z)
\]
\[
\Rightarrow \hat{\lambda}_1(X,Z,u) = - \sum_j (f_j(X,Z) - \bar{w}_j - g_j(X,Z)u) / \sum_j h_j(X,Z)
\]
\[
\Rightarrow \hat{AR}(X,Z,u) = - \hat{\lambda}_1(X,Z,u) / h(X,Z),
\]
assuming that \( \sum_j v_j = 0 \). These estimates are observation specific. Thus, one can obtain estimates of risk preference (and absolute risk aversion) for each observation.

An alternative strategy is to assume a functional for AR and estimate the parameters of it from the FOCs in (26), which is rewritten as
\[
[\hat{f}_j(X,Z) - \hat{w}_j - \hat{g}_j(X,Z)u] / [\hat{h}_j(X,Z) \cdot \hat{h}(X,Z)] = \hat{AR}(X,Z,u) + v_j \quad j = 1, \ldots, J
\]
where \( v_j \) is an error term.

For example, if the AR function is assumed to be linear, i.e.,
\[
AR = b_0 + b_1 q(X,Z,u) = b_0 + b_1 \left( f(X,Z) - \bar{w}.X - g(X,Z)u \right),
\]
one can substitute AR from (30) into (29) and estimate \( b_0 \) and \( b_1 \) parameters from the system of \( J \) equations in (29), using the estimated values of \( f(X,Z) \), \( g(X,Z) \), and \( u \). It is to be noted that the \( X \) variables are endogenous variables. This means that one should use instruments for the \( X \) variables.

**Single-step ML procedure:**

We write the FOCs in (29) as
\[
\psi_j(X,Z) = m_j(X,Z)u + v_j, \quad j = 1, \ldots, J,
\]
where
\[
\psi_j(X,Z) = f_j(X,Z) - \bar{w}_j - h(X,Z).h_j(X,Z) \left[ b_0 + b_1 \left( f(X,Z) - \bar{w}.X \right) \right]
\]
and
\[
m_j(X,Z) = g_j(X,Z) - b_1 h_j(X,Z) h(X,Z).g(X,Z).
\]
The above FOCs together with the production function in (15) constitute the full system of \( (J+1) \) equations with \( (J+1) \) endogenous variables, which is written compactly as
\[
\begin{bmatrix}
Y - f(X, Z) \\
\Psi_1(X, Z) \\
\Psi_2(X, Z) \\
\vdots \\
\Psi_j(X, Z)
\end{bmatrix} = \begin{bmatrix}
h(X, Z) \varepsilon \\
v_1 \\
v_2 \\
\vdots \\
v_j
\end{bmatrix} - u \begin{bmatrix}
g(X, Z) \\
-m_1(X, Z) \\
-m_2(X, Z) \\
\vdots \\
-m_j(X, Z)
\end{bmatrix}
\]

The problem of dealing with this system is that the likelihood function (based on the distributions on \( \varepsilon, \nu \) and \( u \)) cannot be expressed in a closed form. This is because the Jacobian of the transformation will depend on \( u \). Because of this problem we don’t discuss the full ML method here.

### 4.5 Specification and estimation of Model V

To derive an estimable expression for \( \lambda_2(\cdot) \) we take a Taylor series expansion of \( U(\cdot) \) at \( z_1 = z_2 = 0 \), given \( u \). This gives

\[U'(\pi^e/p) = U'(q(X, Z, u)) + U''(q(X, Z, u)) \cdot h_0 \cdot z_2 + U''(q(X, Z, u)) \cdot [f(X, Z) - g(X, Z)u] \cdot z_1\]

where

\[q(X, Z, u) = f(X, Z) - g(X, Z) \cdot u - \bar{w} \cdot X.\]

As before we assume that \( \eta \) and \( \varepsilon \) are independent. Thus,

\[E[U'(\cdot)] = U'(q(X, Z, u))\]

and

\[E[U'(\cdot) \cdot z_1] = U''(q(X, Z, u)) \cdot [f(X, Z) - g(X, Z)u]\]

\[\Rightarrow \lambda_2(\cdot) = \frac{U''(q(X, Z, u)) \cdot [f(X, Z) - g(X, Z)u]}{U'(q(X, Z, u))} = -AR(X, Z, u) \cdot [f(X, Z) - g(X, Z)u]\]

using the result \( AR(\cdot) = -\frac{U''(\cdot)}{U'(\cdot)} \) evaluated at \( q(X, Z, u) \).

Using the above results, we rewrite the FOCs in (18) as
\[ f_j(X,Z) - \tilde{w}_j - g_j(X,Z)u = AR(.)[f(X,Z) - g(X,Z)u]\]  \hspace{1cm} (32)

Write (32) more compactly as

\[ \Psi_{ij}(X,Z,u) = AR(.) \hspace{1cm} j = 1,\ldots,J \]  \hspace{1cm} (33)

when

\[ \Psi_{ij}(X,Z,u) = f_j(X,Z) - \tilde{w}_j - g_j(X,Z)u, \] \hspace{1cm} and

\[ m_{ij}(X,Z) = [f(X,Z) - g(X,Z)u][f_j(X,Z) - g_j(X,Z)u]. \]

Given the complexity of the model we suggest a two-step procedure. In step one we estimate the production function in (15) following the procedure discussed in the previous section. By specifying parametric functional forms for \( f(X,Z) \) and \( g(X,Z) \) together with the distributions on \( u \) and \( \varepsilon \), one can obtain ML estimates of the parameters in \( f(X,Z) \) and \( g(X,Z) \), as well as \( \mu, \sigma^2 \) and \( h_0 \). These estimators are consistent.

**Step 2:** Use the estimated/predicted values from step 1 to compute \( \Psi_j \) and \( m_j \).

Assume a functional form for AR, e.g., \( AR = b_0 + b_1(f(X,Z) - \tilde{w}_X - g(X,Z)u) \). Using this specification, we rewrite (33) as

\[ \Psi_{ij}(X,Z,u) = b_0 + b_1(f(X,Z) - \tilde{w}_X - \tilde{g}(X,Z)\tilde{u}) = m_{ij}(X,Z)u + \eta_j \hspace{1cm} j = 1,\ldots,J \]  \hspace{1cm} (34)

where \( \eta_j \) is an error term appended to the \( j \)th FOC. The above nonlinear system of \( J \) equations can be used to estimate \( b_0 \) and \( b_1 \). The \( Z, \tilde{w}, \) and \( p \) variables can be used as instruments in estimating the above system. Once \( b_0 \) and \( b_1 \) are estimated \( AR(.) \) can be computed for each observation.

**4.6 Specification and estimation of Model VI**

As before first we derive estimable expressions for \( \tilde{\lambda}_1(.) \) and \( \tilde{\lambda}_2(.) \) by taking a linear Taylor series expansion of \( U'(.) \) at \( z_1 = z_2 = 0 \), given \( u \). This gives
\[ U'(\pi'/p) = U'(q(X,Z,u)) + U'(q(X,Z,u)) \cdot h(X,Z) \cdot z_2 + U'(q(X,Z,u)) [f(X,Z) - g(X,Z)u] \cdot z_1 \]

where \( q(X,Z,u) = f(X,Z) - g(X,Z) \cdot u - \tilde{w}X \).

Thus,
\[
E[U'(\cdot) \cdot z_1] = U'(q(X,Z,u)) \cdot [f(X,Z) - g(X,Z)u] 
\]

and
\[
E[U'(\cdot) \cdot z_2] = U'(q(X,Z,u)) \cdot h(X,Z) 
\]

\[
\Rightarrow \tilde{z}_1(\cdot) = \frac{U'(q(X,Z,u)) \cdot [f(X,Z) - g(X,Z)u]}{U'(q(X,Z,u))} = -AR(X,Z,u) \cdot [f(X,Z) - g(X,Z)u] 
\]

\[
\Rightarrow \tilde{z}_2(\cdot) = \frac{U'(q(X,Z,u)) \cdot h(X,Z)}{U'(q(X,Z,u))} = -AR(X,Z,u) \cdot h(X,Z), 
\]

when \( AR(\cdot) = -\frac{U'(\cdot)}{U'(\cdot)} \) is evaluated at \( q(X,Z,u) = f(X,Z) - g(X,Z) \cdot u - \tilde{w}X \).

Using the above results, we rewrite the FOCs in (20) as

\[
f_j(X,Z) - \tilde{w}_j - g_j(X,Z)u = AR(\cdot) \left[ f_j(X,Z) (f(X,Z) - g(X,Z)u) \right] 
+ h_j(X,Z) \cdot h(X,Z) - g_j(X,Z)u \left[ f_j(X,Z) - g(X,Z)u \right] 
\]

\[= AR(\cdot) \left[ (f(X,Z) - g(X,Z)u)(f_j(X,Z) - g_j(X,Z)u) \right] + h_j(X,Z) \cdot h(X,Z) \] (35)

Write (35) more compactly as

\[
\Psi_j(X,Z,u) \left[ m_j(X,Z) + r_j \right] = AR(\cdot), \quad j = 1, \ldots, J 
\] (36)

where \( \Psi_j(X,Z,u) \) and \( m_j(X,Z) \) are defined beneath equation (33). Finally,

\[ r_j = h_j(X,Z) \cdot h(X,Z) \]

Given the complexity of the model we suggest a two-step procedure. In step one we estimate the production function in (15) following the procedure discussed in the
previous section. By specifying parametric functional forms for \( f(X,Z) \), \( h(X,Z) \) and \( g(X,Z) \) together with the distributions on \( u \) and \( \varepsilon \), one can obtain ML estimates of the parameters in \( f(X,Z) \), \( h(X,Z) \) and \( g(X,Z) \), as well as \( \mu \) and \( \sigma^2 \). These estimators are consistent.

**Step 2:** Use the estimated/predicted values from step 1 to compute \( \Psi_j \) and \( m_j \).

Assume a functional form for AR, e.g., \( AR = b_0 + b_1(f(X,Z) - \tilde{w}X - g(X,Z)u) \). Using this specification, we rewrite (31) as

\[
\Psi_j(X,Z,u)[\hat{m}_j(X,Z)u] = \left[ b_0 + b_1\left(\hat{f}(X,Z) - \tilde{w}.X - \hat{g}(X,Z)\hat{u}\right)\right] + \eta_j \quad j = 1,\ldots,J
\]

where \( \eta_j \) is an error term appended to the \( j \)th FOC. The above nonlinear system of \( J \) equations can be used to estimate \( b_0 \) and \( b_1 \). The \( Z \), \( w \), and \( p \) variables can be used as instruments in estimating the above system. Once \( b_0 \) and \( b_1 \) are estimated \( AR(,) \), \( \lambda_1(,) \), and \( \lambda_2(,) \) can be computed for each observation.

Overall, it appears that estimation of the previously described systems in a parametric framework is highly complicated. Our computational experiences with some of these models (in unreported working papers) has been somewhat disappointing. Even estimating a production function of the form \( y = f(x) + g(x)\varepsilon \) is, in some instances, a delicate matter that involves issues of convergence, stability of estimates etc. The systems of first order conditions are also ill-behaved in many instances and, as a result, the parametric approach is not only implausible in terms of assumptions but also highly unstable from the numerical point of view.

5. Nonparametric estimation of Models I-III.

5.1. Estimation of \( f(X,Z) \) and \( h(X,Z) \) functions and their partial derivatives

Suppose \( \tilde{X} \in R^d \) is a vector of explanatory variables (that include both variable \( (X) \) and quasi-fixed inputs \( (Z) \)), and \( Y \) denotes output (the dependent variable). We assume that the production function is of the form
\[ Y = f(\vec{X}) + h(\vec{X})e \equiv f(\vec{X}) + v \]  
\[ (38) \]

where \( f : R^d \rightarrow R \) is an unspecified functional form, and \( v \) is an error term. Our objective is to obtain estimates of \( f(\vec{X}) \) and \( h(\vec{X}) \) as general as possible. So we do not consider separable specifications that are popular when dimensionality reductions are desired. We use the multivariate kernel method to obtain an estimate of \( f(\vec{X}) \) at a particular point \( f(\vec{X}) \) as follows. First, we estimate the density of \( \vec{X} \) (\( \tilde{p}(X) \)) as:

\[
\tilde{p}(\vec{X}) = (Nh)^{-1} \sum_{i=1}^{N} K_h(\vec{X} - \vec{X}_i) = (Nh)^{-1} \sum_{i=1}^{N} \prod_{j=1}^{d} K(Z_j - Z_i) 
\]
\[ (39) \]

where \( K_h(w) = \exp(-\frac{1}{2h^2}(w-w)\tilde{\Sigma}_X^{-1}(w-w)) \) is the \( d \)-dimensional normal kernel, \( h > 0 \) is the bandwidth parameter, \( K(w) = \exp(-\frac{1}{2}w^2) \) is the standard univariate normal kernel, \( \tilde{\Sigma}_X \) is the sample covariance matrix of \( \vec{X}_i \) (\( i = 1, ..., d \)),

\[
Z_i = A(\vec{X}_i - \bar{X}) / \lambda, \\
A\tilde{\Sigma}_X A = I_d, \\
\bar{X} = N^{-1} \sum_{i=1}^{N} \vec{X}_i,
\]

and \( \lambda \) is a smoothing parameter. The optimal choices for \( h \) and \( \lambda \) are

\[
h = \lambda^d |\tilde{\Sigma}_X|^{1/2}, \\
\lambda = \left( \frac{4}{(2d + 1)N} \right)^{d+4}.
\]

The unknown function is then estimated as

\[
\tilde{f}(\vec{X}) = (Nh)^{-1} \sum_{i=1}^{N} W_{hi}(\vec{X}) Y_i 
\]
\[ (40) \]

where
\[ W_{hi}(\tilde{X}) \equiv K_h(\tilde{X} - \tilde{X}_i) / \tilde{p}(\tilde{X}) \]

(see Hardle (1990, pp. 33-34). The estimates are adjusted near the boundary using the procedures discussed in Rice (1984), Hardle (1990, pp. 130-132), and Pagan and Ullah (1999, Chapter 3).

First derivatives of \( f(\tilde{X}) \) with respect to \( X \) are obtained from

\[
\frac{\partial \tilde{f}(\tilde{X})}{\partial X} = (Nh)^{-1} \sum_{i=1}^{N} \partial W_{hi}(\tilde{X})Y_i / \partial X.
\]

More specifically,

\[
\frac{\partial \tilde{f}(\tilde{X})}{\partial X_j} = -(Nh)^{-1} \left[ \sum_{i=1}^{N} G_{ji} K_h(\tilde{X} - \tilde{X}_i)Y_i - \tilde{f}(\tilde{X}) \sum_{i=1}^{N} G_{ji} K_h(\tilde{X} - \tilde{X}_i) \right] / \tilde{p}(\tilde{X})
\]

where

\[
G_{ji} = \lambda^{-2} \sum_{k=1}^{d} \bar{\sigma}_{jk} (\tilde{X}_j - \tilde{X}_j)
\]

and

\[
\bar{\Sigma}_X = \left[ \bar{\sigma}_{jk} , j, k = 1, \ldots, d \right].
\]

Given the estimate of \( \tilde{f}(\tilde{X}_i) \) one can obtain the residuals \( e_i \) from \( e_i = y_i - \tilde{f}(\tilde{X}_i) \). An estimate of the variance can then be obtained from

\[
\tilde{\sigma}^2(\tilde{X}) = (Nh)^{-1} \sum_{i=1}^{N} W_{hi}(\tilde{X})e_i^2
\]

(see Hardle (1990, p. 100), Pagan and Ullah (1999, pp. 214-215)). Since \( g(\tilde{X}) = \tilde{\sigma}(\tilde{X}) \), estimates of the \( g(\tilde{X}) \) function and its gradient \( \partial g(\tilde{X}) / \partial X \) can be obtained. Alternatively, the \( g(\tilde{X}) \) can be obtained from a nonparametric regression of \(|e_i|\) on \( X_i \) in a second step. The gradient of \( g(\tilde{X}) \) could then be obtained by a procedure similar to the one used to obtain the gradient of \( f(\tilde{X}) \) in (41).
The asymptotic properties of this procedure are well established. However, the non-parametric procedure has not been used so far in applied studies, especially in agricultural economics where strong parametric and distributional assumptions are still in use. We wish to reiterate that the great advantage of our approach is that the technology and risk properties can be recovered without strong and questionable assumptions. Moreover, as we detail below, aspects of risk preference can be easily recovered in the following manner.

5.2. Estimation of risk preference functions and risk premium

To estimate the risk preference function $\theta \equiv \theta(\bar{X}, \bar{w})$ in Model I we rewrite the relationship in (4) as

$$D_1 \equiv \frac{1}{J} \sum_j \left[ \frac{\hat{f}_j(\bar{X}) - \hat{w}_j}{-\hat{g}_j(\bar{X})} \right] = \theta(\bar{X}, \bar{w}) \quad (43)$$

Equation (43) can be computed easily since all its components have been estimated. Therefore, fully non-parametric estimates of $\theta$ can be obtained at no cost.

In Model II the risk preference function can be expressed (using (9)) as

$$D_2 \equiv \frac{1}{J} \sum_j \left[ \frac{\bar{w}_j}{\hat{f}_j(\bar{X})} - 1 \right] = \theta_2(\bar{X}, \bar{w}) \quad (44)$$

The above equation can be, again, easily computed under fully non-parametric conditions.

To estimate risk preference functions in Model III, we write the FOCs in (12) as
\[ \delta_j = \tilde{f}_j(\tilde{X}) / \tilde{f}_i(\tilde{X}) = \left[ \tilde{w}_j - \tilde{g}_j(\tilde{X})\tilde{\theta}_i(\tilde{w}, \tilde{X}) \right] / \left[ \tilde{w}_i - \tilde{g}_i(\tilde{X})\tilde{\theta}_i(\tilde{w}, \tilde{X}) \right] \]

\[ \Rightarrow D_3 = 1 + \sum_{j=2}^{\infty} \delta_j = \frac{1}{f} \sum_{j=1}^{f} \left[ \tilde{f}_j(\tilde{X}) / \tilde{f}_i(\tilde{X}) \right] = \frac{1}{f} \sum_{j} \left[ \tilde{w}_j - \tilde{g}_j(\tilde{X})\tilde{\theta}_i(\tilde{w}, \tilde{X}) \right] / \left[ \tilde{w}_i - \tilde{g}_i(\tilde{X})\tilde{\theta}_i(\tilde{w}, \tilde{X}) \right] \]

\[ \equiv \varphi(\tilde{w}, \tilde{X}) + \zeta \] (45)

where \( \zeta \) is an error term. Once the \( \varphi(\cdot) \) function is estimated non-parametrically, we can recover \( \tilde{\theta}_i(\tilde{w}, \tilde{X}) \) from

\[ \tilde{\theta}_i(\tilde{w}, \tilde{X}) = \frac{\sum_j \left[ \tilde{w}_j - \varphi(\tilde{w}, \tilde{X})\tilde{w}_i \right]}{\sum_j \left[ \tilde{g}_j(\tilde{X}) - \varphi(\tilde{w}, \tilde{X})\tilde{g}_i(\tilde{X}) \right]} \]

The \( \tilde{\theta}_2(\tilde{w}, \tilde{X}) \) function can then be estimated from

\[ \tilde{\theta}_2(\tilde{w}, \tilde{X}) = \frac{\sum_j \left[ \tilde{w}_j - \tilde{g}_j(\tilde{X})\tilde{\theta}_1(\tilde{w}, \tilde{X}) \right]}{\sum_j \tilde{f}_j(\tilde{X}) - 1}. \]

6. An Application to Norwegian Salmon Farming

Some models presented in the preceding sections are applied to Norwegian salmon farms. Norway, the UK and Chile are the largest producers of farmed Atlantic salmon (Bjørndal, 1990). Salmon farming is more risky than most other types of meat production due to the salmon's high susceptibility to the marine environment it is reared in. Biophysical factors such as fish diseases, sea temperatures, toxic algae, wave and wind conditions, and salmon fingerling quality are major sources of output risk.

It is believed that the effect of biophysical shocks on output risk can be influenced through the choice of input levels, although fish farmers cannot prevent occurrences of such exogenous shocks. The most important input in salmon farming is fish feed. Feed is expected to increase the level of output risk, ceteris paribus. The salmon is not able to digest all the feed, and the residue is released into the environment as feed waste or faeces. This organic waste consumes oxygen, and thus competes with the salmon for the limited amount of oxygen available in the cages. In addition, feed waste also leads to
production of toxic by-products, such as ammonia. Furthermore, production risk is expected to increase with the quantity of fish released into the cages, due to the increased consumption of oxygen and production of ammonia. We do not have any strong a priori presumptions on the risk effects of capital.

Since 1982 the Norwegian Directorate of Fisheries has compiled salmon farm production data. In the present study we use 2,447 observations on such farms observed during 1988-1992. The output \((y)\) is sales (in thousand kilograms) of salmon and the stock (in thousand kilograms) left at the pen at the end of the year. The input variables are: feed \((F)\), labor \((L)\), and capital \((K)\). Feed is a composite measure of salmon feed measured in thousand kilograms. Labor is total hours of work (in thousand hours). Capital is the replacement value (in real terms) of pens, buildings, feeding equipment, etc. Price of salmon is the market price of salmon per kilogram in real Norwegian Kronors (NOK). The wage rate (in real NOK) is obtained by dividing labor cost by hours of labor. Price of feed is obtained by dividing the cost of feed by the quantity of feed.

In the present study we are treating labor and feed as variable inputs. Capital is treated as quasi-fixed input primarily because price data on it is not available. Moreover, since capital stock adjustment is not instantaneous it is perhaps better to treat capital variable as a quasi-fixed input, especially in the static model like the one used in the present study.

First we report the estimated elasticities of the mean output function \((f(X))\) with respect to labor, capital, and feed. We plot the empirical distribution of these elasticities for labor, capital and feed in Figure 1. The mean values of these elasticities are: 0.029, 0.017, and 0.253, respectively. It can be seen that none of the distributions is symmetric. In fact they are all skewed to the right. Thus the median values of these elasticities are less than their mean values (median elasticities of the mean output with respect to labor, capital and feed are 0.017, 0.007 and 0.158, respectively). The standard deviations of these elasticities are: 0.078, 0.046, and 0.282, respectively. Although some of these elasticities are negative, this happens for a small proportion of salmon farmers. Alternatively, it is quite justifiable to do restricted estimation, and replace any negative

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8 We thank R. Tveterås for providing the data. Details on the sample and construction of the variables used here can be found in the Ph.D.dissertation (Tveterås, 1996).
elasticity for some farmer with its lowest allowable bound (zero), see Pagan and Ullah (1999, pp. 175-176).

Farm age is found to have a negative effect on mean output. The elasticity with respect to age is expected to be positive, especially when one associates age of the farmer with experience, knowledge and learning. With an increase in experience and knowledge one would expect output to increase, ceteris paribus. However, salmon farm studies show that the marine environment around the farm tends to become more disease prone over time due to accumulation of organic sediments below the cages, leading to oxygen loss and increased risk of fish diseases. Hence, the farm age variable may capture both the positive learning effect and the negative disease proneness effect. According to our results, the negative disease proneness effect seems to dominate. The median (mean) value of age elasticity is \(-0.003\) (\(-0.002\)) with a standard deviation of 0.004. Similar result is found in parametric studies.

In production models the time variable is included to capture exogenous technical change (a shift in the production function, ceteris paribus). In the present model one can define technical progress in terms of the mean output function \(f(X)\), i.e., \(TC = \partial \ln f(X) / \partial t\). Based on this formula we find mean technical progress at the rate of 4.6% per year. The frequency distribution of TC is given in Figure 1. The distribution is skewed to the left. It seems that the average rate of TC for most of the farms is around 6%. The median value of TC is 5.3% with a standard deviation of 0.026. A notable feature of this distribution is that it is bimodal. The two modal values of TC are 2.5% and 7.5% per annum, respectively. Although the mean TC is around 6% per year, some farms experienced technical progress at the rate of 2.5% while other “leading” farms experienced a much higher rate.

\[ \text{These elasticities are positive for most of the data points. There are, however, some farms for which the elasticities are negative, especially for capital. This type of violation of the properties of the underlying production technology (viz., positive marginal product) happens when one uses a flexible parametric production function such as the translog.} \]
For a risk neutral producer the input elasticities (labor, feed and capital) can be interpreted as the cost share of the input to the value of output (revenue). This is, however, not the case for a non-risk neutral producer. It can be easily verified from the FOCs that the value of the marginal product of an input deviates from its price thereby meaning that cost share (in total revenue) of an input differs from its elasticity. For example, it can be seen from (6) that if a producer is risk averse input elasticity exceeds the cost share for a risk augmenting input.

In farmed salmon production risk plays an important part. Consequently, it is important to know which input(s) is (are) risk increasing (decreasing). For this we estimate the partial derivatives of the production risk, \( g(\lambda) \) function. Based on the estimates of the risk functions we find that labor is, in general, risk reducing. Labor plays a particularly important role in production risk management. Farm workers’ main tasks are monitoring of the live fish in the pens, biophysical variables (sea temperature, salinity, oxygen concentration, algae concentrations, etc.), and the condition of the physical production equipment (pens, nets, feeding equipment, anchoring equipment, etc.). Thus workers’ ability to detect and diagnose abnormal fish behavior, detect changes in biophysical variables and make prognoses on future development, are crucial to mitigate adverse production condition and reduce production risk. We found (as expected) feed to increase the level of output risk, \( ceteris paribus \). The feed is not all digested and the residue is released into the environment as feed waste or faeces. This organic waste consumes oxygen, and thus competes with the salmon for the limited amount of oxygen available in the cages. In addition, feed waste also leads to production of toxic by-products, such as ammonia.

In Figure 2 we report the frequency distribution of elasticities of the risk function with respect to labor, capital, feed, age and time. The mean (median) values of these elasticities for labor, capital, feed, age and time are: -0.049 (-0.043), 0.016 (0.011), 0.085 (0.016), -0.001 (-0.001), and 0.002 (0.002), respectively. The risk part of the production technology seems to be quite insensitive to changes in the age (experience) of farmers. Similarly, no significant change in production risk has taken place over time.
Elasticities of the mean output and risk functions for each input are derived from the estimates of the \( f(X) \) and \( g(X) \) functions and their partial derivatives. Since we used a multi-step procedure in which the \( f(X) \) and \( g(X) \) functions and their partial derivatives are estimated in the first step, the estimated elasticities in Models I-III are the same. We use the estimated values of \( f(X) \) and \( g(X) \) and their partial derivatives to obtain estimates of the risk preference functions \( \theta_2(.) \) and \( \theta_1(.) \), and estimates of risk premium (RP) in the second step. The estimated values of \( \theta_2(.) \), \( \theta_1(.) \) and RP depend on type of risk an individual farm faces. Two farms with different values of \( \theta_2(.) \) and \( \theta_1(.) \) are not directly comparable, unless both \( \theta_2(.) \) and \( \theta_1(.) \) for one farm is higher (lower) than the other. On the other hand, the RP measures among models with different sources of uncertainty and different values of \( \theta_2(.) \) and \( \theta_1(.) \) are directly comparable. Since RP gives a direct and more readily interpretable result, reporting of RP is often preferred. Given that the RP measure is dependent on units of measurement, a relative measure of RP (defined as RRP = \( \text{RP}/\mu_\pi \)) is often reported. Relative risk premium (RRP) is independent of the units of measurement. RRP also takes farm heterogeneity into account by expressing RP in percentage terms.

The frequency distributions of RRP for Models I-III are reported in Figure 3. These are all skewed to the right. Predicted values of RRP from Model III are much smaller for most of the farms. The mean (median) values of RRP associated with Models I-III are: 0.252 (0.224), 0.171 (0.145) and 0.087 (0.0522), respectively. RP shows how much a risk averse farm is willing to pay to insure against uncertain profit due to production risk and/or output price uncertainty. The RRP, on the other hand, shows what percent of mean profit a risk averse farm is willing to pay as insurance. The above results show that on average a farm is willing to pay 5.22% of the mean profit as an insurance against possible profit loss due to both production risk and output price uncertainty (Model III).

Numerical values for the means and standard deviations of elasticities, \( \theta \)'s and RRP's are reported in Tables 1 and 2. In Table 2, also reported are 95% confidence intervals for \( \theta \)'s
and RRPs. These confidence intervals are somewhat wide indicating the presence of considerable heterogeneity among salmon farmers regarding their attitudes towards risk.
Appendix: Estimation of technical inefficiency (Model IV)

In this appendix we derive estimators of technical inefficiency and technical efficiency (TE).

\[ TE = \frac{E(Y \mid u)}{E(Y \mid u = 0)} = \frac{f(X, Z) - g(X, Z)u}{f(X, Z)} = 1 - \frac{g(X, Z)}{f(X, Z)} \cdot u = 1 - TI . \]

**Production Function:**
\[ y = f(X, Z) + h(X, Z)\varepsilon - g(X, Z)u \equiv f(X, Z) + \nu - u^4 \]
where \( \nu = h(X, Z)\varepsilon \) and \( g(X, Z)u = u^4 \).

I. \( \nu \sim N(0, h^2(X, Z)) = N(0, \sigma^2_\nu) \)

II. \( u^4 \sim N^+(\mu \cdot g(X, Z), \sigma^2_u \cdot g^2(X, Z)) = N^+(\mu_0, \sigma^2_\nu) \)

With these distributional assumptions the model is similar to the normal, truncated normal model proposed by Stevenson (1980). Following Kumbhakar and Lovell (pp. 85-86) we get

\[ u^4 | \varepsilon^4 \sim N^+(\tilde{\mu}, \sigma^2_\nu), \quad \varepsilon^4 = \nu - u^4 \]
\[ \tilde{\mu} = -\frac{\sigma^2_\nu \varepsilon^4 + \mu_0 \sigma^2_\nu}{\sigma^2_\nu + \sigma^2_\varepsilon}, \quad \sigma^2_* = \frac{\sigma^2_\nu \cdot \sigma^2_\varepsilon}{\sigma^2_\nu + \sigma^2_\varepsilon} \]
\[ E(u^4 \mid \varepsilon^4) = \sigma_* \left[ \frac{\tilde{\mu}}{\sigma_*} + \frac{\phi(\tilde{\mu} / \sigma_*)}{\Phi(\tilde{\mu} / \sigma_*)} \right] \]
\[ M(u^4 \mid \varepsilon^4) = \left\{ \begin{array}{ll} \tilde{\mu} & \text{if } \tilde{\mu} \geq 0 \\ 0 & \text{otherwise} \end{array} \right. \]

where \( \frac{\tilde{\mu}}{\sigma_*} = -\frac{\sigma^2_\nu \varepsilon^4 + \mu_0 \sigma^2_\nu}{\sigma^2_\nu + \sigma^2_\varepsilon} \) \( \sqrt{\frac{\sigma^2_\nu + \sigma^2_\varepsilon}{\sigma_0 \cdot \sigma_\nu \cdot \sqrt{\sigma^2_0 + \sigma^2_\varepsilon}} = -\frac{\sigma^2_\nu \varepsilon^4 + \mu_0 \sigma^2_\nu}{\sigma_0 \cdot \sigma_\nu \cdot \sqrt{\sigma^2_0 + \sigma^2_\varepsilon}} \)

Note that \( \varepsilon^4 = Y - f(X, Z), \quad \mu_0 = \mu \cdot g(X, Z), \quad \sigma^2_\nu = \sigma^2 \cdot g^2(X, Z) \) and \( \sigma^2_\varepsilon = h^2(X, Z) \).

Estimates of all these functions can be obtained using the estimated parameters. Using the estimated values of \( u^4 \), one can obtain estimates of \( u \) for each observation from
\[ u^4 = g(x, z) \cdot u \Rightarrow E(u^4 \mid \varepsilon^4) = g(x, z)E(u \mid \varepsilon^4) \Rightarrow E(u \mid \varepsilon^4) = E(u^4 \mid \varepsilon^4) / g(x, z) \]
and, \( \hat{TE} = 1 - \frac{E(u^4 \mid \varepsilon^4)}{f(x, z)} \).
Table 1. Elasticities of the mean production and production risk functions

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Median</th>
<th>std. dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Labor</td>
<td>0.029</td>
<td>0.017</td>
<td>0.078</td>
</tr>
<tr>
<td>Capital</td>
<td>0.017</td>
<td>0.007</td>
<td>0.046</td>
</tr>
<tr>
<td>Feed</td>
<td>0.253</td>
<td>0.158</td>
<td>0.282</td>
</tr>
<tr>
<td>Time</td>
<td>0.046</td>
<td>0.053</td>
<td>0.026</td>
</tr>
<tr>
<td>Age</td>
<td>-0.002</td>
<td>-0.003</td>
<td>0.0036</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Median</th>
<th>std. dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Labor</td>
<td>0.0493</td>
<td>-0.0427</td>
<td>0.044</td>
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<tr>
<td>Capital</td>
<td>0.0163</td>
<td>0.0109</td>
<td>0.028</td>
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<tr>
<td>Feed</td>
<td>0.0851</td>
<td>0.0159</td>
<td>0.216</td>
</tr>
<tr>
<td>Time</td>
<td>0.0024</td>
<td>0.0021</td>
<td>0.0038</td>
</tr>
<tr>
<td>Age</td>
<td>-0.0009</td>
<td>-0.0011</td>
<td>0.0014</td>
</tr>
</tbody>
</table>

Table 2. Risk functions and relative risk premium

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Median</th>
<th>std. dev.</th>
<th>95% confidence interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model I</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\theta_1$</td>
<td>-2.869</td>
<td>-2.888</td>
<td>0.435</td>
<td>-3.970 -2.810</td>
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<tr>
<td>RRP</td>
<td>0.252</td>
<td>0.224</td>
<td>0.124</td>
<td>0.122 0.592</td>
</tr>
<tr>
<td>Model II</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\theta_2$</td>
<td>-0.219</td>
<td>-0.205</td>
<td>0.097</td>
<td>-0.420 0.080</td>
</tr>
<tr>
<td>RRP</td>
<td>0.171</td>
<td>0.145</td>
<td>0.094</td>
<td>0.098 0.410</td>
</tr>
<tr>
<td>Model III</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\theta_1$</td>
<td>-0.577</td>
<td>-0.402</td>
<td>2.389</td>
<td>-5.240 4.150</td>
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<tr>
<td>$\theta_2$</td>
<td>-0.053</td>
<td>-0.050</td>
<td>0.080</td>
<td>-0.231 0.212</td>
</tr>
<tr>
<td>RRP</td>
<td>0.087</td>
<td>0.052</td>
<td>0.096</td>
<td>0.0220 0.342</td>
</tr>
</tbody>
</table>
Figure 1. Distributions of $f(x)$ elasticities
Figure 2. Distributions of $g(x)$ elasticities

- **Labor**

- **Capital**

- **Feed**

- **Time**

- **Age**
Figure 3. Distributions of risk functions
Figure 4. Distributions of relative risk premium (Models I-III)