Performance Analysis of S-MAC Protocol under Unsaturated Conditions
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Abstract—In this letter, we present a new analytical model that accurately evaluates the throughput, service delay and energy consumption of S-MAC protocol. Our model takes into account the impact of several factors together, including periodic listen and sleep cycle, various incoming traffic loads, the backoff mechanism in S-MAC protocol, the queueing behavior at the MAC layer, and the non-independent nature of service delay distributions of nodes. Simulations show that our analytical results are highly accurate.

Index Terms—Wireless sensor networks, S-MAC protocol, unsaturated traffic conditions, performance analysis.

I. INTRODUCTION

As an emerging technology, wireless sensor networking has a wide range of potential applications such as environment monitoring, homeland security and digital warfare. Such a network consists of a great number of sensing devices equipped with limited power resources. Thus, a critical concern is the efficient use of limited energy while satisfying the QoS requirement.

From the energy efficiency standpoint, a simple but effective solution is to place the nodes in sleep mode. Many MAC protocols designed for wireless sensor networks have adopted this idea. The well-known Sensor-MAC (S-MAC) protocol [1] is one of them. It reduces energy consumption by introducing a periodic listen and sleep cycle into IEEE 802.11 distributed coordination function (DCF). Recently, many analytical models have been proposed to analyze the impact of sleeping nodes on network performance. However, only a few works focus on modeling the S-MAC protocol [2,3]. In [2], the authors study the trade-off between energy consumption and average service delay. Nevertheless, they use the analytical result for IEEE 802.11 DCF under saturated conditions [4] to compute the contention delay under unsaturated conditions. The model presented in [3] provides an analysis of energy consumption, but it fails to consider the cancellation of backoff timer when a node goes into sleep and the important fact that the probabilities of different nodes having no packets in their transmission queues are not independent of each other.

In this letter, we provide an analytical model that accurately evaluates the performance of S-MAC protocol under unsaturated conditions. The proposed model is composed of four sub-models, namely 1) a node behavior model that studies the backoff mechanism in S-MAC protocol, 2) a service delay model that characterizes the service delay distribution, 3) an M/G/1/∞ model that reflects the queueing dynamics under unsaturated conditions, and 4) a cycle probability model that considers the non-independent nature of service delay distributions of nodes and captures the behavior of the network.

II. ANALYTICAL MODEL FOR S-MAC PROTOCOL

A. Node Behavior Model

In this study, we consider a single-hop wireless sensor network which consists of \( n \) identical sensor nodes. For each node, we use a stochastic process \( b(t) \) to represent the change of its backoff timer. According to S-MAC protocol, if a node fails to capture the channel in a cycle, it will cancel its backoff counter before sleep and extract a new one when the next cycle comes. Thus, the stochastic process \( b(t) \) can be modeled with the discrete-time Markov chain shown in Fig. 1. And for

\[
P\{k | i[k]\} = p/W \\
P\{k - 1 | k\} = p/W + 1 - p \\
P\{0 | 0\} = 1/W
\]

where \( W \) is the constant contention window size and \( p \) is the probability that the tagged node fails to capture the channel.

The latter probability \( p \) equals to the probability that at least one of the remaining nodes transmits a packet.

Let \( b_k \) be the stationary distribution of this Markov chain. We have

\[
b_k = \begin{cases} 
(1 - p)b_{k+1} + b_{W-1} & 1 \leq k \leq W - 2 \\
\left(b_0 + p \sum_{i=1}^{W-1} b_i\right) \frac{1}{W} & k = W - 1
\end{cases}
\]

Using the normalization condition, we obtain the closed-form solutions of this Markov chain. Thus the transmission probability \( p_t \) can be expressed as

\[
p_t = b_0 = \frac{p[1 - (1 - p)^W]}{pW - (1 - p)[1 - (1 - p)^W]}
\]

In addition to the relation between \( p \) and \( p_t \) described in (3), \( p_t \) can also be calculated through [5]

\[
p = 1 - \frac{1}{(1 - (1 - p_t)p_1)^n - 1}
\]
where \( p_0 \) is the probability that the transmission queue of a node is empty. By solving (3) and (4), we can calculate \( p \) and \( p_i \) with a given \( p_0 \).

### B. Service Delay Model

Before analyzing the service delay distribution, we first calculate the successful transmission probability of a node in a cycle, denoted as \( P_{STC} \), and the probability that a successfully transmitting node has a backoff counter initially set to \( i \), denoted as \( P\{BC = i|STC\} \). In order to successfully transmit in a cycle, a node has to first decrease its backoff counter to zero, and then ensure that there are no other transmissions on the channel when it starts to transmit. So the probability that a node with a backoff counter initialized to \( i \) will successfully transmit in a cycle is

\[
P\{STC|BC = i\} = (1 - p)^{i+1}
\]

Thus we have

\[
P_{STC} = \sum_{i=0}^{W-1} P\{BC = i\} P\{STC|BC = i\}
\]

where \( P\{BC = i\} = 1/W \). Thus

\[
P\{BC = i|STC\} = \frac{P\{STC, BC = i\}}{P_{STC}}
\]

Service delay is defined as the time from the moment the packet is at the head of the queue, ready to be transmitted, until an acknowledgement for this packet is received. For simplicity, we assume that the time spent for synchronization is negligible. Therefore, the service delay can be calculated as the sum of the following delay components: 1) delay due to the lost transmission opportunity caused by sleeping, denoted as \( t_1 \); 2) delay due to the node’s failure to successfully transmit in a cycle, denoted as \( t_2 \); 3) delay due to the backoff procedure in the successfully transmitting cycle, denoted as \( t_3 \); 4) delay due to the transmission, denoted as \( t_4 \). On account of the slotted operation of the protocol, the packet service delay can be viewed as an integer multiple of a time slot. In the rest of this letter, \( F(z) \) is denoted as the probability generating function (PGF) of delay \( f \) (e.g. \( T_1(z) \) is the PGF of delay \( t_1 \)).

Under unsaturated conditions, the distribution of \( t_1 \) depends on whether the next packet to be served is \( (a) \) from the transmission queue or \( (b) \) a newly arrived one. Case \( (a) \) occurs with probability \( 1 - p_0 \) and case \( (b) \) occurs with probability \( p_0 \).

In case \( (a) \), \( t_1 \) is the time from the previous packet being successfully transmitted until the beginning of the next cycle, so

\[
T_1'(z) = \sum_{i=0}^{W-1} P\{BC = i|STC\} \cdot z^{1/2} (T_{cl}-T_s-i) \sigma
\]

where \( T_{cl} \) is the duration of a cycle, \( T_s \) is the average transmission time and \( \sigma \) is the duration of a time slot. Since the duration of listen time is set to be a constant value in S-MAC protocol, \( T_{cl} \) is determined by duty cycle (dc), which is the portion of listen time in a cycle. In case \( (b) \), \( t_1 \) is the time from the arrival of the new packet to the beginning of the next cycle. Assuming a random packet arrival time, \( T_1(z) \) in case \( (b) \) is

\[
T_1(z) = \sum_{i=1}^{T_{cl}/\sigma} (T_{cl}/\sigma)^{-1} \cdot z^{i}
\]

Hence,

\[
T_1(z) = (1 - p_0) T_1'(z) + p_0 T_1''(z)
\]

With the probabilities \( P_{STC} \) and \( P\{BC = i|STC\} \), \( T_2(z) \), \( T_3(z) \) and \( T_4(z) \) can be calculated as follows

\[
T_2(z) = \sum_{k=0}^{\infty} \left( 1 - P_{STC} \right)^k P_{STC} \cdot z^{kT_{cl}/\sigma}
\]

\[
T_3(z) = \sum_{i=0}^{W-1} P\{BC = i|STC\} \cdot z^i
\]

\[
T_4(z) = z^{T_s/\sigma}
\]

The PGF of total service delay can thus be expressed as

\[
T_d(z) = T_1(z)T_2(z)T_3(z)T_4(z)
\]

By inverting the generating function \( T_d(z) \), we obtain the service delay distribution [6]. And the average service delay can be easily obtained by differentiating (14)

\[
E[t_d] = (P_{STC}^{-1} - p_0/2)T_{cl} + p_0T_{bc} + p_0T_s
\]

where

\[
T_{bc} = \frac{(1-p)(1-pW(1-p)^{W-1} - (1-p)^W)}{p[1-(1-p)^W] \sigma}
\]

### C. M/G/1/∞ Queueing Model

We assume that at each node, packet arrivals are Poisson with rate \( \lambda \) (in packet per second). Based on the M/G/1/∞ model, we can calculate \( p_0 \) through

\[
p_0 = 1 - p_0 = 1 - \lambda E[t_d]
\]

Since \( E[t_d] \) can be expressed as a function of \( p_0 \), we can calculate \( p_0 \) with a given \( \lambda \) using numerical techniques.

In addition, by applying the Pollaczek-Khinchin formula, we can derive the average packet delay \( T \) including average service delay and queuing delay.

### D. Throughput

The throughput \( S \) can be computed via Little’s Law as [7]

\[
S = \frac{E[N]E[P]}{E[t_d]}\]

where \( E[P] \) is the average amount of payload bits transmitted in a successful transmission, and \( E[N] \) represents the average number of competing nodes, which is given by

\[
E[N] = n(1-p_0)
\]

### E. Cycle Probability Model

We characterize a cycle by the event that happens within it: an idle cycle, a successful transmission cycle or a collision cycle. In an idle cycle, all nodes have no packet to transmit. In a successful transmission cycle, only one of the nodes which have packets awaiting transmission wins the channel and transmits successfully. In a collision cycle, more than one node selects the same backoff counter and causes an RTS collision. Define \( P_{st} \), \( P_{cl} \) and \( P_{col-m} \) to be the occurrence probabilities of an idle cycle, a successful transmission cycle and a collision cycle caused by \( m \) nodes respectively.
Since the collisions of packets have a significant impact on the time nodes spend in delivering a packet, the service delay distributions of nodes are not independent of each other and so are the probabilities $p_m$, (although they have the same value). Consequently, we cannot use a method similar to [3] to obtain the cycle probabilities. Here we present a new method to solve this problem.

According to the definition of the unified throughput, the throughput $S$ can also be expressed as

$$S = \frac{P_{st} E[P]}{T_{td}}$$

(20)

Thus probability $P_{st}$ can be computed by substituting (18) into (20). Moreover, $P_{st}$ and $P_{col-m}$ can be expressed as a function of $P_{id}$, $p_0$ and $p_1$, so we can derive the probabilities $P_{id}$ and $P_{col-m}$ by solving the following equations

$$P_{st} = (1 - P_{id})(1 - p_0)p_1[1 - (1 - p_0)p_1]^{n-1}$$

$$P_{col-m} = (1 - P_{id})\left(\frac{n}{m}\right)[(1-p_0)p_1]^m[1-(1-p_0)p_1]^{n-m}$$

(21)

(22)

where $1-[(1-(1-p_0)p_1]^n$ is the probability that at least one node in the network will transmit in a given time slot.

F. Energy Consumption

The average energy consumption of the network during a cycle can be calculated using the obtained cycle probabilities

$$E = P_{id}E_{id} + P_{st}E_{st} + \sum_{m=2}^{n} P_{col-m}E_{col-m}$$

(23)

where $E_{id}$, $E_{st}$ and $E_{col-m}$ are the average network energy consumptions in an idle cycle, a successful transmission cycle and a collision cycle respectively.

III. SIMULATION AND NUMERICAL RESULTS

Our analytical model is verified by ns-2 simulator (version 2.31). Fig. 2 compares the predicted and simulated cycle probabilities against traffic load. Fig. 3 shows predicted and simulated throughput, average service delay and average energy consumption against traffic load.

From Fig. 3, it can be observed that throughput increases linearly with the packet arrival rate before the node becomes saturated. This characteristic is obvious if we substitute equations (17) and (19) into (18), and we can further calculate that the slope is $n \cdot E[P]$. It also can be observed that lower traffic load induces larger energy consumption. This is because the nodes spend more energy in idle cycles. In idle cycles, every node has to sense the channel throughout the predetermined listen duration, so a lot of energy is wasted in idle listening. However, in a successful transmission cycle or a collision cycle, nodes can go to sleep upon hearing the RTS packets from other nodes.

IV. CONCLUSION

We provide an analytical model to compute the throughput, service delay, and energy consumption of S-MAC protocol in single-hop networks under unsaturated conditions. Simulation results show that our model accurately evaluates the performance of S-MAC protocol. It serves as a guide for designers in determining the dominating protocol parameter according to different traffic conditions.

REFERENCES


