Visually tracking of robots in uncalibrated environments

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Abstract

This paper presents a new adaptive controller for visual tracking control of a robot manipulator in 3D general motion with a fixed camera whose intrinsic and extrinsic parameters are uncalibrated. In addition to camera parameters, the feature positions are also assumed unknown. Based on the fact that the unknown parameters appear linearly in the closed-loop dynamics of the system if the depth-independent interaction matrix is adopted to map the image errors onto the joint space of the manipulator, a new adaptive algorithm was developed to estimate the unknown parameters online. With a full consideration of dynamic responses of the robot manipulator, the Lyapunov method is employed to prove asymptotic convergence of the image errors. Simulation and experiment results are used to demonstrate the performance of the proposed approach.

1. Introduction

Visual servos is an approach of controlling motion of a robot using visual feedback signals from a vision system. This has been extensively investigated since early 1990s [1–4]. Two configurations are possible to set up a vision system [5–11]. First, the camera can be mounted at the end-effector of the robot, called eye-in-hand system. Second, the camera can be fixed at the position near the robot, called fixed camera system. Furthermore, two kinds of strategies: image-based control and position-based control depend on the feedback information. In this paper, we address a problem of controlling a set of feature points on a robot to trace desired trajectories on the image plane with a fixed camera. This problem falls into the category of the image-based control in the fixed camera setup.

In implementation of robot visual servo controller, one of the most tedious but important tasks is to calibrate the intrinsic and extrinsic parameters of the camera. To avoid camera calibration, extensive efforts have been made to controller design using uncalibrated visual signals. Dixon [12] designed a controller with uncertainty in the kinematics and dynamic models. The adaptive controllers proposed by Astolfi et al. [13] and Kelly [14] can cope with unknown camera parameters when the manipulators moves in planes. Hu et al. [15] proposed a homography-based visual servo controller invariant to changes in the camera parameters. In our early work [16,17], we proposed an adaptive controller to cope with 3-D image-based position control of feature points. The underlying idea was the proposals of the depth-independent image Jacobian (or interaction) matrix and a new adaptive algorithm. However, the methods mentioned above (including ours) work for position control only and do not apply to trajectory tracking of feature points.

Papanikolopoulos and Khosla [18] was among the first group of people who addressed image-based trajectory tracking and developed a visual tracking algorithm by estimating the depth of the target. Hsu and Aquino [19] proposed an adaptive visual tracking controller for a planar robot manipulator. Zergeroglu et al. [20] designed a tracking controller for planar robot with uncertain parameters. The controller developed by Hashimoto et al. [21,22] incorporated the nonlinear robot dynamics in controller design and employed a motion estimator to estimate motion of an object in a plane. Parra-Vega and Fierro-Rojas [23] proposed sliding PID uncalibrated visual servoing for finite-time tracking of planar robots. We [24] also developed a controller for dynamic visual tracking of feature points when the camera parameters are not calibrated. However, the approach in [24] has several limitations. First, the adaptive rule [31] in [24] includes real depth information, which cannot be measured. Although in experiments, the convergence can be achieved if estimated depth information was used instead, the system still may out of control if the depth changed in a large range. Second, the control gains should satisfy the inequality (40), which can only guarantee the semi-global stability of the closed-loop system. To achieve global rigorous stability, a new controller for this problem need be further developed.

Based on the above observations, this paper presents a new controller for controlling a number of feature points on a robot
manipulator to trace desired trajectories specified on the image plane of a fixed camera by extending our previous work [24]. In addition to camera intrinsic and extrinsic parameters, the feature positions are also assumed unknown. The depth-independent interaction matrix, proposed in our early work [17], is adopted to map the trajectory errors of the image plane onto the joint space of the robot manipulator. The depth-independent image Jacobian matrix enables us not to estimate the depth of the feature point and to linearize the closed-loop dynamics by the unknown parameters. We developed a new adaptive algorithm to estimate the unknown parameters on-line. A potential force is introduced to drive the estimated parameters away from the values that result in singular depth-independent interaction matrix. With a full consideration of dynamic responses of the robot manipulator, we employ the Lyapunov method to prove asymptotic convergence of the image errors. To verify the proposed method, we have implemented the controller on 3 degrees of freedom (DOF) robot manipulator. The simulation and experiment results demonstrated good convergence of the image errors.

This work differs from our early work [24] in the following three aspects. First and the most important, the previous work can only guarantee semi-global stability, but the global stability of the system can be achieved in this paper. Second, the adaptive algorithms used are different. Roughly estimated depth information is needed in previous work, while the adaptive algorithm in this paper only based on measure information. Third, this approach also copes with the unknown feature positions. The contribution of this paper used are different. Roughly estimated depth information is needed in previous work, while the adaptive algorithm in this paper only based on measure information. Third, this approach also copes with the unknown feature positions. The contribution of this paper

2. Camera and robot model

2.1. Problem definition

In this work, we consider a fixed camera set-up (Fig. 1), in which a camera is placed near a robot manipulator to monitor its motion. There are k feature points marked on the robot manipulator, which are being traced by the vision system. Suppose that the camera is a pin-hole camera with perspective projection and no distortion. Assume that the feature points positions as well as camera intrinsic parameters and the extrinsic parameters, i.e. the homogeneous transform matrix between the vision frame and the robot base frame, are not calibrated. The problem addressed is defined as follows:

**Problem 1.** Given desired time-varying trajectories of the k feature points on the camera image plane, design a proper joint input for the robot manipulator such that the feature points asymptotically track the desired trajectories in uncalibrated environments.

![Fig. 1. A fixed camera setup for visual tracking.](image)

2.2. Kinematics

Denote the joint angle of the manipulator by a $n \times 1$ vector $\mathbf{q}(t)$, where $n$ is the number of the joints of the manipulator. Denote the perspective projection matrix of the camera by $\mathbf{M} \in \mathbb{R}^{3 \times 4}$. Note that this matrix depends on both the intrinsic and extrinsic parameters. Denote the position of the tracked point $i$ in the end-effector frame as $\mathbf{r}_i$, which is unknown. Denote the image coordinates of feature point $i$ on the image plane by $\mathbf{y}_i(t) = (u_i, v_i)^T$ and its homogeneous coordinates with respect to the robot base frame by a $4 \times 1$ vector $\mathbf{x}_i(t)$, then

$$\mathbf{x}_i(t) = \mathbf{R}(\mathbf{q}(t))\mathbf{r}_i + \mathbf{p}(\mathbf{q}(t))$$

(1)

where $\mathbf{R}(\mathbf{q}(t))$ is the 3 by 3 rotation matrix and $\mathbf{p}(\mathbf{q}(t))$ denotes the 3 by 1 translational vector from the end-effector frame to the robot base frame. Let $\mathbf{J}(\mathbf{q}(t)) \in \mathbb{R}^{2 \times 6}$ denote the manipulator Jacobian matrix, mapping from the joint velocity to Cartesian velocity including translational and angular velocity. From the forward kinematics of the manipulator, we obtain

$$\mathbf{x}_i(t) = (\mathbf{I}_{3 \times 3} - sk(\mathbf{R}(\mathbf{q}(t))\mathbf{r}_i))\mathbf{J}(\mathbf{q}(t))\mathbf{q}(t)$$

(2)

where $sk$ is a matrix operator and $sk(\mathbf{x})$ with vector $\mathbf{x} = [x_1, x_2, x_3]^T$ can be written as a matrix form

$$sk(\mathbf{x}) = \begin{pmatrix}
0 & -x_3 & x_2 \\
x_3 & 0 & -x_1 \\
-x_2 & x_1 & 0
\end{pmatrix}$$

Under the perspective projection model, the coordinates are related by

$$\mathbf{y}_i(t) = \frac{1}{z_i(t)} \mathbf{P} \mathbf{x}_i(t)$$

(3)

where $\mathbf{P} \in \mathbb{R}^{2 \times 4}$ is the matrix consisting of the first two rows of the perspective projection matrix $\mathbf{M}$; $z_i(t)$ denotes the depth of the feature point $i$ with respect to the camera frame. By differentiating (3), we obtain

$$\dot{\mathbf{y}}_i(t) = \frac{1}{z_i(t)} \mathbf{A}_i(t)(\mathbf{I}_{3 \times 3} - sk(\mathbf{R}(\mathbf{q}(t))\mathbf{r}_i))\mathbf{J}(\mathbf{q}(t))\mathbf{q}(t)$$

(4)

where the matrix $\mathbf{A}_i(t)$ is called depth-independent interaction matrix in [17]. And $\mathbf{A}_i(t)(\mathbf{I}_{3 \times 3} - sk(\mathbf{R}(\mathbf{q}(t))\mathbf{r}_i))\mathbf{J}(\mathbf{q}(t))$ is called depth-independent image Jacobian matrix. Define by $\mathbf{Q}(t)$ the combined depth-independent image Jacobian matrix:

$$\mathbf{Q}(t) = \begin{bmatrix}
\mathbf{A}_1(\mathbf{y}_1(t))(\mathbf{I}_{3 \times 3} - sk(\mathbf{R}(\mathbf{q}(t))\mathbf{r}_1)) \\
\mathbf{A}_2(\mathbf{y}_2(t))(\mathbf{I}_{3 \times 3} - sk(\mathbf{R}(\mathbf{q}(t))\mathbf{r}_2)) \\
\vdots \\
\mathbf{A}_k(\mathbf{y}_k(t))(\mathbf{I}_{3 \times 3} - sk(\mathbf{R}(\mathbf{q}(t))\mathbf{r}_k))
\end{bmatrix} \mathbf{J}(\mathbf{q}(t))$$

(5)

The dimension of the matrix $\mathbf{Q}(t)$ is $2k \times n$. From (4) and (5),

\begin{align*}
\dot{z}_1(t) & = \frac{z_1(t)}{z_1(t)} \mathbf{y}_1(t) \\
\dot{z}_2(t) & = \frac{z_2(t)}{z_2(t)} \mathbf{y}_2(t) \\
\vdots \\
\dot{z}_k(t) & = \frac{z_k(t)}{z_k(t)} \mathbf{y}_k(t)
\end{align*}

(6)

It is important to note [26]:

**Property 1.** For any $4 \times 1$ vector $\mathbf{p}$, the product $\mathbf{A}_i(t)(\mathbf{I}_{3 \times 3} - sk(\mathbf{R}(\mathbf{q}(t))\mathbf{r}_i))\mathbf{p}$ can be written in the following linear form:

$$\mathbf{A}_i(t)(\mathbf{I}_{3 \times 3} - sk(\mathbf{R}(\mathbf{q}(t))\mathbf{r}_i))\mathbf{p} = \mathbf{D}_i(\mathbf{p}, y_i(t))\theta$$

(7)

where $\mathbf{D}_i(\mathbf{p}, y_i(t))$ is a regression matrix without depending on the intrinsic and extrinsic parameters of the camera and feature points positions. Vector $\theta$ includes all the combined unknown parameters.
2.3. Robot dynamics

The dynamic equation of a manipulator has the form:

\[ H(q(t)) \dot{q}(t) + \left[ \frac{1}{2} H(q(t)) + C(q(t), \dot{q}(t)) \right] q + g(q(t)) = u(t) \]  

(8)

where \( C(q(t), \dot{q}(t)) \) is a skew-symmetric matrix. And for any vector \( s \), we have

\[ s^T C(q(t)) \cdot \dot{q}(t) = 0. \]

3. Adaptive image-based visual tracking

This section proposes a new controller that forces the feature points to trace their desired trajectories while estimating the unknown parameters on-line.

3.1. Definition of nominal references

Denote the desired trajectory of the \( i \)th feature point on the image plane by \( \hat{y}_d(i, t) \), \( \hat{y}(i, t) \), \( \hat{y}_d(t) \), where \( \hat{y}_d(t) \) and \( \hat{y}(t) \) represent the time-varying desired position, velocity and acceleration, respectively. For convenience, introduce the following nominal reference:

\[ \hat{y}_d(t) = y_d(t) - \lambda_i \Delta y_i(t) \]

(9)

where \( \lambda_i \) is a positive constant. The error of the feature point to the defined nominal reference is given by

\[ s_y(i, t) = \hat{y}(i, t) - y_d(i, t) - \lambda_i \Delta y_i(t) \]

(10)

Note that the image errors \( \Delta y_i(t) \) and \( \Delta \dot{y}_i(t) \) are convergent to zero if the error vector \( s_y(i, t) \) is convergent to zero.

Taking Eq. (6) into account, we introduce the following nominal reference using the estimated parameters to map the image errors onto the robot manipulator joint space:

\[ \dot{q}_e(i, t) = Q^+(t) \begin{bmatrix} \dot{\hat{y}}_1(i, t) \\ \dot{\hat{y}}_2(i, t) \\ \vdots \\ \dot{\hat{y}}_n(i, t) \end{bmatrix} \]

(11)

where a hat mean the symbol is calculated based on estimated parameter \( \theta(t) \). And

\[ \dot{Q}^+(t) = (Q^T(t) \dot{Q}(t))^{-1} Q^T(t) \]

(12)

Then the error vector in the joint space is given by

\[ s_y(i, t) = \dot{q}(i, t) - \dot{q}_e(i, t) \]

(13)

3.2. Estimation of the unknown parameters

The proposed controller employs an on-line algorithm to estimate the unknown camera parameters. The adaptive algorithm is so designed that the three objectives are satisfied: (a) the nonlinear regression term in the closed loop dynamics is canceled, (b) the rank 3 condition of the estimated projection matrix is satisfied so that the pseudo-inverse \( Q^+(t) \) is computable, and (c) the estimated projection errors of the feature points are minimized on-line.

For convenience, define the following vector:

\[ s_y(t) = \begin{bmatrix} \dot{\hat{y}}_1(t) \\ \dot{\hat{y}}_2(t) \\ \vdots \\ \dot{\hat{y}}_n(t) \end{bmatrix} \]

(14)

First, to derive the regression term, we consider

\[ s_y(t) = Q^T(t) K \dot{y}_s(t) = q^T(t) Q^T(t) K \dot{y}_s(t) - \begin{bmatrix} \dot{z}_1(t) y_1(t) \\ \dot{z}_2(t) y_2(t) \\ \vdots \\ \dot{z}_n(t) y_n(t) \end{bmatrix}^T \]

\[ Q(t) (Q^T(t) Q(t))^{-1} Q(t) K \dot{y}_s(t) = q^T(t) Q^T(t) K \dot{y}_s(t) - \begin{bmatrix} \dot{z}_1(t) y_1(t) \\ \dot{z}_2(t) y_2(t) \\ \vdots \\ \dot{z}_n(t) y_n(t) \end{bmatrix} \]

(15)

where \( K \) is a positive definite matrix.

By introducing the identity matrix \( Q^T(t) Q(t) (Q^T(t) Q(t))^{-1} \) in above equation, we have

\[ s_y(t) Q^T(t) K \dot{y}_s(t) = q^T(t) Q^T(t) Q(t) (Q^T(t) Q(t))^{-1} Q(t) K \dot{y}_s(t) \]

\[ = q^T(t) (Q^T(t) Q(t))^{-1} Q(t) K \dot{y}_s(t) - \begin{bmatrix} \dot{z}_1(t) y_1(t) \\ \dot{z}_2(t) y_2(t) \\ \vdots \\ \dot{z}_n(t) y_n(t) \end{bmatrix} \]

(16)

Note that

\[ q^T(t) (Q^T(t) Q(t))^{-1} Q(t) = q^T(t) (Q^T(t) Q(t))^{-1} (Q^T(t) Q(t))^{-1} Q(t) K \dot{y}_s(t) \]

\[ = q^T(t) (Q^T(t) Q(t))^{-1} (Q^T(t) Q(t))^{-1} K \dot{y}_s(t) \]

(17)

By submitting eq. (17) into eq. (16), we have

\[ s_y(t) Q^T(t) K \dot{y}_s(t) = S_y(t) (Q(t) (Q^T(t) Q(t))^{-1} Q(t) K \dot{y}_s(t) \]

\[ - \begin{bmatrix} \dot{z}_1(t) y_1(t) \\ \dot{z}_2(t) y_2(t) \\ \vdots \\ \dot{z}_n(t) y_n(t) \end{bmatrix} \]

(18)

From Property 1, the last term in eq. (18) can be represented as a linear form of the estimation errors of the parameters, and hence eq. (18) can be re-written as

\[ s_y(t) Q^T(t) K \dot{y}_s(t) = S_y(t) (Q(t) (Q^T(t) Q(t))^{-1} Q(t) K \dot{y}_s(t) \]

\[ - \Delta \theta(t) Y (q(t), \dot{q}(t), \ddot{q}(t), \theta(t)) \dot{y}_s(t) \]

(19)

Next, to guarantee the existence of the pseudo-inverse of \( Q(t) \), the potential force \( \Delta \theta(t) Y (q(t), \dot{q}(t), \ddot{q}(t), \theta(t)) \dot{y}_s(t) \) introduced in [24] was adopted here.

Finally, we consider the error between the real image coordinates of the feature point and those calculated using the estimated projection matrix for the feature point \( i \):

\[ e_i(t) = \dot{z}_i(t) y_1(t) - P(t) x_i(t) \]

(20)

By combining with Eqs. (1) and (3), the above equation can be re-written as follows:
\[ e_i(t) = \dot{z}_i(t) y_i(t) - P(t)(R(q(t)) \dot{r}_i(t) + p(q(t))) = y_i(t)(\ddot{z}_i(t) - \dot{z}_i(t)) - (P(t) - P(y_i(t))) + (P(t)R(q(t)) \dot{r}_i(t) - P(t)R(q(t)) \dot{r}_i(t)) \]  

(21)

From Property 1, Eq. (21) can be represented in a linear form of the parameter estimation errors:

\[ e_i(t) = W(x_i(t), y_i(t)) \Delta \theta(t) \]  

(22)

where \( W(x_i(t), y_i(t)) \) does not depend on the parameters. Based on the above discussions, the following adaptive rule is proposed for updating the estimation of the parameters:

\[ \frac{d}{dt} \dot{\theta}(t) = -\Gamma^{-1} \left\{ Y(q(t), y_i(t), \theta(t))(\dot{S}_q(t) - Q^T(t)K_5 \dot{S}_y(t)) \right\} + \sum_{j=1}^{k} W^T(x_j(t), y_j(t)) B_1 e_j(t) + B_2 || q_j(t) ||^2 \frac{\partial U_j(\dot{\theta}(t))}{\partial \theta(t)} \]  

(23)

where \( \Gamma, B_1 \) and \( B_2 \) are positive-definite and diagonal gain matrices. Note that the first regressive terms come from the Slotine-Li [25] algorithm.

Remark 1. It should be noted that the adaptive algorithm in this paper only based on measure information and initial estimated parameters. However, the adaptive rule [31] in [24] includes real depth information, which cannot be measured.

3.3 Controller design

Based on the nominal references defined above, we propose the following controller:

\[ \tau = H(q(t)) \dot{q}_i(t) + \left[ \frac{1}{2} H(q(t)) + C(q(t), \dot{q}_i(t)) \right] \dot{q}_i(t) + g(q(t)) - \left( K_1 + (K_2 + \| \dot{\theta}(t)B_2 \|) \frac{\partial U_j(\dot{\theta}(t))}{\partial \theta(t)} \right) S_q(t) - Q^T(t)K_5 \dot{S}_y(t) \]  

(24)

where \( K_i \) (\( i = 1, 2, 3 \)) are positive-definite gain matrices. The first three terms are to cancel the inertia forces, the nonlinear centrifugal and Coriolis forces, and the gravitational force, respectively. The fourth term represents a feedback in the joint space. The last term is the image error feedback.

By substituting the control law into the robot dynamics, we obtain the following closed loop dynamics:

\[ \dot{S}_q(t) = H(q(t)) S_q(t) + \left[ \frac{1}{2} H(q(t)) + C(q(t), \dot{q}_i(t)) \right] \dot{S}_q(t) = - \left( K_1 + (K_2 + \| \dot{\theta}(t)B_2 \|) \frac{\partial U_j(\dot{\theta}(t))}{\partial \theta(t)} \right) S_q(t) - Q^T(t)K_5 \dot{S}_y(t) \]  

(25)

3.4 Stability analysis

We here analyze the stability of the robot manipulator under the control of the proposed controller and adaptive algorithm. For simplicity, we assume that the feature points are visible during the motion so that their depths with respect to the camera frame are always positive. Following is the main result of this paper:

**Theorem 1.** Under the control of controller (24) and the adaptive rule (23), the feature points are convergent on the image plane in the following way:

\[ \lim_{t \to \infty} Q^T(t)K_5 \dot{S}_y(t) = 0 \]  

(26)

**Proof.** Introduce the following non-negative function:

\[ V(t) = \frac{1}{2} \left\{ S_q^T(t)H(q(t))S_q(t) + \Delta \theta^T(t) \Delta \dot{\theta}(t) \right\} \]  

(27)

Multiplying the \( Q^T(t) \) from the left to the closed loop dynamics (25) results in

\[ S_q^T(t)H(q(t))S_q(t) + \frac{1}{2} S_q^T(t)H(q(t))S_q(t) = -S_q^T(t) \left( K_1 + (K_2 + \| \dot{\theta}(t)B_2 \|) \frac{\partial U_j(\dot{\theta}(t))}{\partial \theta(t)} \right) S_q(t) - S_q^T(t)Q^T(t)K_5 \dot{S}_y(t) \]  

(28)

From eq. (18), we have

\[ S_q^T(t)H(q(t))S_q(t) + \frac{1}{2} S_q^T(t)H(q(t))S_q(t) = -S_q^T(t) \left( K_1 + (K_2 + \| \dot{\theta}(t)B_2 \|) \frac{\partial U_j(\dot{\theta}(t))}{\partial \theta(t)} \right) S_q(t) - \Delta \theta^T(t) \left( Y(q(t), y_i(t), \dot{\theta}(t)) \right) \dot{S}_y(t) \]  

(29)

By multiplying the \( \Delta \theta^T(t) \) from the left to the adaptive rule (23), we obtain

\[ \Delta \dot{\theta}^T(t) \Gamma \Delta \dot{\theta}(t) = -\Delta \dot{\theta}^T(t) \dot{Y}(q(t), y_i(t), \dot{\theta}(t)) \dot{S}_y(t) - \sum_{j=1}^{k} \Delta \dot{\theta}^T(t)W^T(x_j(t), y_j(t))B_1W(x_j(t), y_j(t)) \Delta \theta(t) \]  

(30)

By combining the Eqs. (28)–(30), we have

\[ V(t) \leq -S_q^T(t) (K_1 \dot{S}_q(t) - S_q^T(t)Q^T(t)Q^T(t)Q^T(t)K_5 \dot{S}_y(t)) \]  

(31)

It should be noted that

\[ K_1 + \| \dot{\theta}(t)B_2 \| - \| \Delta \dot{\theta}^T(t)B_2 \| \geq K_2 - \| \dot{\theta}(t)B_2 \| \]  

(32)

Above condition can be easily satisfied by properly select a gain \( K_2 \) such that

\[ K_2 \geq \max \| \dot{\theta} \| \zeta I \]  

(33)

where \( \zeta \) is the maximum eigenvalue of gain matrix \( B_2 \). I is the identity matrix. max \( \| \dot{\theta} \| \) is the maximum value of the real parameters, which can be roughly estimated.

From (31), the \( V(t) \) is non-positive and the function \( V(t) \) never increases its value so that it is upper bounded. From the definition (27), bounded \( V(t) \) directly implies that the joint velocity, the image errors, and the estimation errors are all bounded. Then, we can claim the boundedness of the \( S_q(t) \) from the closed-loop
dynamics (25) and that of \( \dot{\theta}(t) \) from the adaptive algorithm (23). Therefore, the joint velocity \( \dot{\mathbf{q}}(t) \) and the estimated parameters are uniformly continuous. Therefore, from the Barbalat’s Lemma, we can conclude that

\[
\lim_{t \to \infty} \mathbf{Q}^T(t) \mathbf{K}_1 \dot{\mathbf{s}}_q(t) = 0 \quad (34)
\]
\[
\lim_{t \to \infty} \mathbf{s}_q(t) = 0 \quad (35)
\]
\[
\lim_{t \to \infty} \mathbf{W}(\mathbf{x}_i(t), \mathbf{y}_i(t)) \Delta \dot{\theta}(t) = 0 \quad (36)
\]

From (34), the image errors are convergent to zero when the number \( n \) of degrees of freedom of the manipulator are larger than or equal to 2, or when the number \( n \) of degrees of freedom of the manipulator are larger than or equal to 6. The proof is similar to [24].

**Remark 2.** It should be noted that the control gains have to satisfy the inequality condition (40) in previous paper [24], which requires the initial states of the system and may cause the system unstable. In this paper, the condition for selecting \( \mathbf{K}_2 \) can be easily satisfied.

### 4. Simulations

In this section, the performance of the proposed image-based controller was demonstrated by simulations. The simulations were conducted on a 3 DOF manipulator with one feature point as shown in Fig. 1. The physical parameters of the robot are set as \( l_1 = 0.24 \) m, \( l_2 = 0.43 \) m, \( l_3 = 0.09 \) m, \( m_1 = 17.4 \) kg, \( m_2 = 4.8 \) kg. The real camera intrinsic parameters are \( a_u = 800, a_v = 900, u_0 = 300, v_0 = 400 \).

In the first simulation, the desired trajectory of the feature point is to trace the following circular trajectory:

\[
\mathbf{y}_d(t) = \begin{pmatrix} 30 \sin t + 390 \\ -30 \cos t + 500 \end{pmatrix} \text{pixel} 
\]

The control gains are \( \mathbf{K}_1 = 2 \times 10^{-3}, \mathbf{K}_2 = 0.001, \mathbf{K}_3 = 7.5 \times 10^{-4} \), \( \Gamma = 2 \times 10^{6}, \mathbf{B}_1 = 0.02, \mathbf{B}_2 = 1 \times 10^{-5} \). The transformation matrix of the base frame with respect to the vision frame is

\[
\mathbf{T} = \begin{bmatrix} 0 & 1 & 0 & 0.2 \\ 0.0896 & 0.996 & 0.2 \\ 0.996 & 0.0896 & -3 \\ 0 & 0 & 0 & 1 \end{bmatrix} 
\]

The initial estimated target position with respect to the end-effector frame is \( \mathbf{x}(0) = (0.002, -0.003, 0.005)^T \) m. The real one is \( \mathbf{x} = (0.003, -0.005, 0.007)^T \) m.

As shown in Figs. 2 and 3, the image feature points asymptotically converge to the desired ones. The results confirmed the con-
vergence of the image error to zero under control of the proposed method.

In the second simulation, Random noise (Fig. 4) was added to the image position feedback. The trajectory and image errors of the feature points on the image plane are demonstrated in Figs. 5 and 6. This simulation indicated that it is possible to tracking the image trajectory even with noise. The controller may out of work if the noise is too big. However, low pass filter could be introduced to avoid such situation in real application.

5. Experiments

To verify the performance of the proposed controller, we have implemented the controller in a 3 DOF robot manipulator in the Chinese University of Hong Kong. Fig. 7 shows the experiment set-up system. The robot manipulator has three revolute joints driven by Maxon brushed DC motors. The gear ratios at the three joints are 480:49, 12:1, and 384:49, respectively. The moment inertia about its vertical axis of the first link of the manipulator is 0.005 kg m², the masses of the second and third links are 0.167 kg, and 0.1 kg, respectively. The lengths of the second and third links are, 0.145 m and 0.1285 m, respectively. High-precision encoders with a resolution of 2000 pulses/turn are used to measure the joint angles. The joint velocities are obtained by differentiating the joint angles. A Ptgrey camera connecting to an IEEE 1394 card installed in a PC with Intel Pentium IV CPU acquires the video signal with 120 fps frame rate. The sampling time used in the experiments is 13 ms.

In the experiment, we mark a feature point on the end-effector of the robot manipulator. The desired trajectory of the feature point is to trace the following circular trajectory:
The control gains used are \( C_0 = 0.0002 \), \( K_1 = 20 \), \( K_2 = 0.02 \), \( K_3 = 0.0015 \), \( B_1 = 0.1 \) and \( B_2 = 0.0001 \). The initial estimation of the camera extrinsic parameters is \( \mathbf{T} = \begin{bmatrix} 1 & 0 & 0 & 0.6 \\ 0 & 0 & -1 & -0.1 \\ 0 & 1 & 0 & 0.15 \\ 0 & 0 & 0 & 1 \end{bmatrix} \). The real values of the intrinsic parameters are \( a_x = 1806 \) pixels, \( a_y = 1812 \) pixels, \( u_0 = 282 \) pixels and \( v_0 = 249 \) pixels. The initial estimations are \( \hat{a}_x(0) = 900 \) pixels, \( \hat{a}_y(0) = 900 \) pixels, \( \hat{u}_0(t) = 120 \) pixels and \( \hat{v}_0(t) = 120 \) pixels. The trajectory of the feature point and the position errors on the image plane are shown in Fig. 8, which confirmed expected asymptotic convergence of the trajectory error to the zero under the control of the proposed method. Fig. 9 plots the 3-D trajectory of the end-effector. In the second experiment, we selected three feature points on the third link. The control gains used are \( K_1 = 5 \), \( K_2 = 0.1 \), \( K_3 = 0.005 \), \( \Gamma^{-1} = 0.00002 \), \( B_1 = 0.1 \), \( B_2 = 0.0001 \). The trajectory of the feature point and the position errors of the feature points are plotted in Figs. 10 and 11, which validated the asymptotic stability of the proposed controller. Fig. 12 plot the profile of the first 11 estimated parameters.

6. Conclusions

This paper proposed a new adaptive controller for trajectory tracking of a number of feature points on a robot manipulator using an uncalibrated fixed camera by extending our previous work [24]. In addition to camera intrinsic and extrinsic parameters, the feature positions are also assumed unknown. The depth-independent interaction matrix is adopted to map the trajectory errors of the image plane onto the joint space of the robot manipulator. A novel adaptive algorithm has been developed to estimate the unknown parameters on-line. A potential function is introduced to guarantee the existence of the pseudo inverse of the estimated depth-independent interaction matrix. It is rigorously proved by the Lyapunov method based on the nonlinear robot dynamics that the trajectories of the feature points are asymptotically convergent to the desired ones. Experimental results validated the proposed method.

Acknowledgments

This work was supported in part by Specialized Research Fund for the Doctoral Program of Higher Education of China under Grants 20100073120020 and 20100073110018, in part by Shanghai Municipal Natural Science Foundation under Grant 11ZR1418400, in part by the Hong Kong Research Grants Council under Grants 41707, 415110 and 415011, in part by the Natural Science Foundation of China under Projects 61105095, 60334010, 60475029, 60775062 and 60934006, in part by the Program for New Century Excellent Talents in University under Grant NCET-07-0538, in part by the State Key Laboratory of Robotics and System (HIT), in part by the Research Councils UK under UK-China Science Bridge Grant No. EP/G042594/1.

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