A Distributed Space-Time Trellis Coding Approach for Multi-Terminal Relay Networks

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Abstract—Cooperative relaying is a promising alternative for conventional mobile communications systems as it is able to increase coverage and throughput of these systems. Due to practical constraints most cooperative relaying protocols employ half-duplex relays which implies the need for an increased data rate on the individual links and causes a significant performance loss at higher rates.

This paper presents a novel cooperative relaying approach exploiting existing large scale space diversity as well as an additional coding gain by employing a spatial duplex approach. The scheme utilizes a network of alternatingly sending and receiving relay terminals where only one relay node is transmitting at a particular time. This spatial duplexing approach is used to implement a distributed Space-Time Trellis Code (STTC) for virtual antenna arrays.

In this work, the protocol is applied to a convolutionally coded system. Due to the serial concatenation of an inner STTC and an outer convolutional code, the destination is able to use well-known turbo-decoding algorithms to reduce the frame error rate of the system. The assessment of the approach is done by comparing it with conventional relaying, a time/frequency duplex cooperative relaying protocol and direct transmission.

I. INTRODUCTION

Relaying was originally proposed in [1] and emerged during the last years as a promising alternative to conventional mobile communications systems using a direct transmission between the source and destination terminals. The conventional relaying approach employs a system with intermediate nodes (the relay nodes) which decode and retransmit the source message. Although conventional relaying is a promising approach to exploit energy savings due to nonlinear pathloss, it still suffers from signal fading on the individual links. One way to alleviate these effects is to exploit existing diversity, i.e., time, frequency and space diversity. To take advantage of existing time and frequency diversity one should use coding, e.g., convolutional coding [2]. To exploit space diversity one could use physical antenna arrays [3] employing MIMO coding schemes such as Space-Time Trellis Codes (STTC) [4].

Beside the utilization of small-scale spatial diversity with physical antenna arrays one can also take advantage of large-scale spatial diversity using virtual antenna arrays. In [5], [6] cooperative relaying was introduced as a special form of virtual antenna arrays. Compared to conventional relaying it utilizes both the direct source-destination as well as the relay-destination link. This first proposal was later extended in [7] which introduced the orthogonality constraint, i.e., a relay terminal can not transmit and receive at the same time on the same resource. The usage of such half-duplex relays implies that the individual links have to support higher data rates compared to direct transmission to ensure that the end-to-end data rate of both is the same. Due to this data rate increase, most cooperative relaying protocols proposed so far offer their benefits at low rates.

In this work we introduce a novel cooperative relaying protocol for convolutionally coded systems which alleviates the previous effects by using a Distributed Space Division Duplexing (DSDD) approach based on a time/frequency duplexing between the employed relay nodes. This protocol implements a distributed STTC based on a network of relay terminals. Due to the serial concatenation of an inner STTC and an outer convolutional code we are further able to use well-known turbo-decoding algorithms to improve the coding advantage of the analyzed system.

The rest of this paper is structured as follows. In Section II we present all assumptions on the system as well as the channel model. We proceed in Section III with the presentation of a distributed STTC for coded systems and its performance evaluation in Section IV. Finally, we discuss our conclusions and possible extensions of the proposal in Section V.

II. SYSTEM AND CHANNEL MODEL

For the sake of simplicity and to keep our performance evaluation comprehensive we describe the protocol for an OFDM based system (although the proposed protocol is not restricted to such a system). The system consists of a source node $s$ which transmits an encoded and OFDM modulated codeword frame $x_s[n]$ to a destination node $d$ and to $N-1$ currently receiving relay nodes, with $N \geq 2$. Furthermore, one relay node concurrently transmits the frame $x_r[n]$ based on the previously received $N-1$ superimposed frames $x_s[n]$ and $x_r[n]$, respectively. The currently transmitting relay is chosen in a circular manner so that each relay terminal is able to observe $N-1$ consecutive frames. Fig. 1 shows an example deployment for $N = 4$ relay nodes.

The source transmission is structured as follows: Each source frame $x_s[n]$ consists of $W$ information bits which are at first encoded using a convolutional code (CC) and afterwards interleaved using a random interleaver. These encoded bits are...
modulated and divided into $B$ blocks of $M$ symbols $u_{s}^{(m)}[b]$ with $0 \leq m \leq M - 1$ and $nB \leq b \leq nB + B - 1$ where $n$ denotes the current frame index. Now we use a STTC coder to map each symbol $u_{s}^{(m)}[b]$ to an output symbol $X_{s}^{(m)}[b]$ which is further processed by the OFDM modulator. The frequency domain output of the modulator can be represented by the vector

$$X_{s}[b] = [X_{s}^{(0)}[b], X_{s}^{(1)}[b], \ldots, X_{s}^{(M-1)}[b]].$$

Applying the IDFT on $X_{s}[b]$ we can model the output of the OFDM modulator by its discrete time domain signal

$$x_{s}[b] = [x_{s}^{(0)}[b], x_{s}^{(1)}[b], \ldots, x_{s}^{(M-1)}[b]]$$

with

$$x_{s}^{(k)}[b] = \sum_{l=0}^{M-1} X_{s}^{(l)}[b] \exp \left( j \frac{2\pi}{M} kl \right).$$

We can further denote the $n$-th frame by the $M \times B$ matrix

$$x_{s}[n] = [x_{s}^{T}[nB], x_{s}^{T}[nB+1], \ldots, x_{s}^{T}[nB+B-1]],$$

where the superscript $T$ denotes the transpose. Assuming a system with a sufficiently long cyclic prefix the codeword frame received at node $j$ originating from node $i$ can be modeled by

$$y_{j}[b] = x_{s}[b] \circ \tilde{h}_{i,j}[b] + \tilde{n}_{j}[b],$$

where $\circ$ denotes circular convolution. The vector $\tilde{h}_{i,j}[b]$ of length $L$ models an $L$-tap Rayleigh fading channel and $\tilde{n}_{j}[b]$ denotes a $M$ dimensional vector modeling the receiver noise. Each fading channel tap $h_{i,j}^{(l)}[b]$, $0 \leq l \leq L - 1$, as well as each $n_{j}^{(m)}[b]$, $0 \leq m \leq M - 1$, is a zero-mean, mutually-independent, circularly symmetric, complex Gaussian random variable with variance $\kappa_{i}\sigma_{j}^{2}/2L$ and $N_{0}/2$ real dimension, respectively. The parameter $\kappa_{i}$ denotes the fraction of the overall power spent for the transmission from node $i$ and $\sigma_{j}^{2}$ denotes the mean pathloss between nodes $i$ and $j$. As we are not interested in any large scale fading effects, i.e., pathloss and shadowing, we normalize all variances $\sigma_{j}^{2}$ to the source destination link $\sigma_{s,d}^{2}$ (which implies that $\sigma_{s,d}^{2} = 1$). We assume in our analysis that perfect channel state information (CSI) about $\tilde{h}_{i,j}[b]$ is only available at the respective receiver $j$. Furthermore, the system does not provide any feedback information to any node.

### III. Description of a Distributed STTC Approach

Tarokh et al. proposed in [4] Space-Time Trellis Codes (STTC) which are a class of coding schemes applied on MIMO systems. In this section we describe at first STTC for physical antenna arrays before we show how and under which constraints they can be applied to virtual antenna arrays.

#### A. Space-Time Trellis Codes for physical antenna arrays

A STTC can be described as a state machine with a set of possible input and output symbols, a set of states and a set of state transitions. The set of input and output symbols is defined by $\mathcal{A}$ with $|\mathcal{A}|$ elements. Furthermore, if we assume that the STTC has a memory of $K$, the current state of the STTC can be represented by $s[k] \in \mathcal{S}$ with $|\mathcal{S}| \leq |\mathcal{A}|^{K}$. Using the set of states $\mathcal{S}$ and the symbol alphabet $\mathcal{A}$ we characterize a STTC by the two projections

$$f_{1} : \mathcal{A} \times \mathcal{S} \rightarrow \mathcal{A}_{\text{out}} \subseteq \mathcal{A}^{N_{T_{r}}},$$

$$f_{2} : \mathcal{A} \times \mathcal{S} \rightarrow \mathcal{S}.$$  

Projection $f_{1}$ defines the mapping of an input symbol $u[k] \in \mathcal{A}$ to $N_{T_{r}}$ output symbols depending on the current state $s[k]$. As we consider in this work only deployments with one relay transmitting at a particular time it follows that $N_{T_{r}} = 2$ for all setups. The second projection $f_{2}$ uniquely defines the follow-up state $s[k+1]$ depending on the current state and the input symbol. One way to represent a STTC is to use a trellis graph where each state has exactly $|\mathcal{A}|$ leaving branches which represent the state transitions depending on the $|\mathcal{A}|$ possible input symbols. Fig. 2 shows an example for a STTC with a symbol alphabet consisting of $|\mathcal{A}| = 4$ elements, a memory of $K = 3$ and $N_{T_{r}} = 2$ output symbols. There exists a comprehensive literature on various algorithms to decode a STTC encoded message. For our analysis in Section IV we choose the BCJR algorithm [8] which represents the Maximum A Posteriori Probability (MAP) decoder.

#### B. A distributed STTC approach for coded systems

In [9] a Space Division Duplex (SDD) approach was analyzed where two alternatingly sending and receiving relays ensure that the rate on the individual links needs not to be increased. We take up this idea and apply the previously
Our proposed scheme Simple DSDD is structured as follows. At first the source uses the current STTC to map each encoded and modulated symbol $u_s^{(m)}[b]$ to $N_T$, output symbols of which the first one is assigned to $X_s^{(m)}[b]$. The assignment is done in such a way that an independent STTC is applied on each subcarrier of the $B$ input blocks over $F$ consecutive frames. Note, that the STTC should not be implemented over equal subcarriers of consecutive OFDM symbols which is later discussed and reasoned in more detail. The current source frame $x_s[n]$ is now transmitted to the destination node as well as the $N-1$ currently receiving relays. Using the outer convolutional code the relay nodes try to decode the source message and compute reliability information in terms of soft-values (LLR-values) on the input and output bits of the convolutional coder.

Concurrently to the source transmission one relay node is transmitting the frame $x_r[n]$ according to the currently employed STTC, i.e., source and relay transmit the output symbols such that both form a virtual antenna array using the described STTC per symbol. Due to the usage of an outer convolutional code and an inner STTC we inherently implement a distributed system with serially concatenated codes offering a space diversity gain as well as an additional coding advantage. On the other hand, the concurrent transmission by the relay nodes leads to an increased level of interference at the relays which might result in an increased number of symbol errors. Therefore, the relay nodes work in an adaptive manner, i.e., depending on the soft-information on the source symbol $X_s^{(m)}[b]$ the currently transmitting relay node decides whether it transmits in the next frame the corresponding relay symbol or remains silent. This adaptive transmission ensures that decoding errors at the relay are not propagated. In our analysis we require a minimum LLR-value of 8 per output bit of the convolutional coder.

As the destination is aware of both the employed outer convolutional code as well as the inner STTC it is able to use well-known algorithms for turbo-decoding of serially concatenated codes [10]. Since we concentrate in our work on the proposed distributed STTC approach we do not explain the turbo-decoding principle in more detail and refer to the comprehensive literature on this topic.

The half-duplex constraint on the relay nodes implies that the relay nodes can only keep track of the last $N-1$ source symbols. Therefore, in order to be able to implement our protocol the STTC has to satisfy the following constraint: The current state of the STTC per subcarrier $s^{(m)}[b]$ needs to be uniquely definable by the last $N-1$ source frames $x_s[n-1], \ldots, x_s[n-N+1]$. The necessity to know the current state per subcarrier is implied by the fact that the relay output depends (only) on the current state. Therefore, the relay node must know the current STTC states to transmit the correct symbols according to the STTC. Furthermore, due to the fact that the relay nodes cannot observe the input symbols it must be able to determine the STTC states only by observing the source transmissions.

In our setup we deploy each STTC on a frame basis, i.e., we have per frame $BM$ independent STTC. If the STTC is applied on an OFDM symbol basis, i.e., $M$ independent STTC per frame, it is not possible to use the outer convolutional code at the relay nodes. Consider for instance a setup with $B > 1$ OFDM symbols per frame and let the relay receive the first OFDM symbol. Using the received OFDM symbol the relay must determine the current STTC states on all $M$ subcarriers of the OFDM symbol. The relay is now transmitting the second OFDM symbol and due to the employed random interleaver it is not able to exploit the outer code for the decoding of the first OFDM symbol. Therefore, the STTC should not be applied on an OFDM-symbol basis but must be applied on a frame basis. Actually this constraint can be weakened if the relay does not use the outer convolutional code to decode the source frame which on the other hand would lead to a significantly higher FER at the relay node.

In Fig. 2 we show an example STTC which satisfies the discussed constraints for $N = 4$ relay nodes. One can see from the figure that each transmitted symbol halves the number of possible states after the transmission. Therefore, after three transmitted frames the trellis state is uniquely defined. To ensure that each relay is able to observe the necessary three consecutive source frames we need to deploy $N = 4$ relay terminals. According to this trellis the relay is now able to transmit the second output symbol to implement the distributed STTC. Due to the usage of more complex STTC we are able to benefit from a coding gain additionally to the exploited large-scale spatial diversity.

In [7] various cooperative relaying protocols based on Decode-And-Forward (DF) were proposed. DF based protocols employ relay nodes which receive the source frames and retransmit the same symbol with a delay of one frame.

![Fig. 2. An example STTC for an alphabet of size $|A| = 4$ and two output symbols taken from [4, Fig. 5]. All lines in this figure represent a state transition from left (state $s[k]$) to right (state $s[k+1]$). Every state is connected with four leaving branches which represent the four possible state transitions which itself are ordered in ascending order, i.e., the $i$-th branch corresponds to the state transition for the $i$-th symbol out of $A$. Additionally, each state is labeled with the four possible pairs of output symbols. Furthermore, the left symbol per pair corresponds to the relay output and the right symbol corresponds to the source output.](image-url)
Consider now our proposal using a DF based approach. This case coincides with Delay Diversity Codes (DDC) as proposed in [11]. In this scheme the STTC mapping for the source output symbols is simplified to $u_{s}^{(m)}[b] = x_{s}^{(m)}[b]$. Furthermore, each relay node only retransmits the $n$-th frame the source frame $x_{i}[n-1]$. In comparison to the more complex STTC this simple scheme can already be implemented using $N = 2$ relay nodes. On the other hand DDC provide a lower coding advantage than the more complex STTC. The impacts of this lower coding advantage and the reduced complexity are examined in the next section which analyzes the frame error rate (FER) of both variants.

IV. EVALUATION OF SIMULATION RESULTS

To evaluate the described protocol we present in this section simulation results for the more complex STTC in Fig. 2 and a DDC assuming a QPSK modulation. Nevertheless, the results are similarly applicable to higher modulation schemes although the actual values might change.

A. Simulation setup

We employ a $[7, 5]$-convolutional code and an OFDM system with $M = 128$ subcarriers per symbol and $B = 2$ OFDM symbols per codeword frame. We further use an equally weighted Rayleigh fading channel with $L = 10$ taps (where $i, j$ is a vector of length $L$). In our setup $F = 20$ consecutive frames are encoded using the STTC and the DDC, respectively. Furthermore, we use a power distribution with $\kappa_s = 0.9$ and $\kappa_r = 0.1$ which was found by empirical studies to minimize the $E_b/N_0$-value at a FER of $10^{-2}$. Besides, this assignment ensures that the overall energy spent in our system is equal to the energy spent by a direct transmission system.

For our evaluation of the proposed protocols we assume that all relays are located at the halfway point between the source and destination node, i.e., if $d_{s,i}$ denotes the distance between nodes $s$ and $i$ it follows that $d_{s,i} = d_{s,j} = d_{s,i}/2$. This implies that under the condition of a pathloss exponent $\alpha = 3$ we have the normalized variances $\sigma_{s,i}^2 = \sigma_{s,j}^2 = 8$. We assess the performance of our proposal in comparison with direct transmission between source and destination and conventional relaying. Additionally, we compare the protocol with Simple Adaptive Decode-and-Forward (AdDF) [12] which is an extension of Selection Relaying [7] and does not utilize any feedback information. Therefore, the source does not retransmit its message if the relay is not able to decode and forward it. We choose this protocol for our evaluation as it would be too much overhead to signalize whether or not a specific subcarrier was retransmitted by the relay node.

In comparison to already proposed cooperative relaying protocols we use additional resources in form of a relay terminal network. Consider Simple AdDF and conventional relaying: if the number of retransmitting nodes increases both nodes have to further increase the necessary rate per individual link which results in an even worse performance. Another option might be [13] which still suffers from a rate loss but only needs to double the necessary rate per individual link independently of the number of relays. Nevertheless, as we propose an alternative cooperative relaying without rate loss suitable for higher rates one can see that even the space-time coded cooperation as proposed in [13] suffers from a worse performance at higher rates.

To analyze the performance of a distributed STTC we use the simulation results shown in Fig. 3 for the STTC presented in Fig. 2 in comparison to direct transmission, conventional relaying and the time/frequency duplexing cooperative relaying approach Simple AdDF.

B. Performance in the ideal case

At first consider the performance in the ideal case, i.e., the relay always transmits the correct symbols. In this case
the more complex STTC offers an $E_b/N_0$ advantage of about 3.5 dB in comparison to direct transmission whereas the less complex DDC offers an advantage of about 2.5 dB. The higher benefits of the STTC are caused by the higher coding gain due to the higher complexity of the STTC in comparison to the DDC. We can assume that for far lower FER values this advantage will decrease as the coding advantage decreases.

C. Performance in the non-ideal case

In case of non-ideal relays the DDC variant of our proposal does not significantly loose performance for $\sigma_{r, r}^2 = 0$ and $\sigma_{r, r}^2 = 1$, i.e., effects due to decoding errors or the adaptive behavior of the relays are well reduced by the outer code. In comparison to this the STTC looses about 0.3 dB if non-ideal relays are used. This offset is caused by the fact that the relays are required to successfully decode three consecutive symbols. If only one of these symbols is unreliably decoded the relay does not transmit or if one of the three symbols is wrongly decoded the relay transmits an incorrect symbol. The performance gap reduces for higher $E_b/N_0$ values since the relay decodes more reliably and transmits more often the correct symbol.

D. Comparison to time/frequency duplexing protocols

We can further observe that conventional relaying performs significantly worse than direct transmission due to the high number of wrong decisions and the non-adaptive protocol structure. Additionally, we see that our proposal also outperforms the time/frequency duplexing approach Simple AdDF by about 2 dB in case of the STTC and by about 1 dB in case of the DDC. For higher data rates this gap will further increase as the data rates on the individual links is doubled in case of the time/frequency duplexing protocol. Our proposal, by contrast, needs not to increase the data rate and hence still profits from the spatial diversity and coding advantage.

E. Influence of inter-relay interference

Fig. 3 and Fig. 4 show that both protocols have serious problems in scenarios with significantly higher interference; in our analysis we used for the inter-relay interference $\sigma_{r, r}^2 = 10$ which leads to a far higher FER value of about $10^{-1}$ even for an $E_b/N_0$ of 10 dB. In those scenarios we need to increase the number of relay nodes so that the relay terminals are also able to employ a turbo-decoder to decrease the FER and increase the number of submitted symbols. Another problem coming up in these scenarios is that the detector at the relays and the destination assigns higher soft-values to the relay transmission due to the higher path gains. Since the detector has no knowledge about the decoder uncertainty at the relay node these soft-values might be wrong. This problem can be alleviated by using soft-modulated symbols at the relay nodes to express the uncertainty of the decoder. Both problems are still open and will be treated in upcoming work.

V. SUMMARY AND FINAL REMARKS

We presented in our work a novel approach to exploit a network of relay terminals for a cooperative transmission. In comparison to usual cooperative relaying protocols we choose a spatial duplexing approach to support a continuous source transmission. This ensures that the necessary data rate on the individual links needs not to be increased and the protocol can also be used at higher end-to-end data rates. Our approach of using a distributed STTC is able to exploit large scale spatial diversity as well as an additional coding gain.

We have shown that the more complex STTC can provide about 1 dB more performance advantage in comparison to a DDC if ideal relays are employed. Nevertheless, if the relays do not decode perfectly this advantage is reduced. This might be alleviated by using more relay nodes which could also employ a turbo-decoder. In cases where more relay nodes are available it is further possible to implement more complex STTC with more than $N_{r, r} = 2$ output symbols to increase the diversity advantage. Upcoming work should further investigate how the proposed approach is performing in systems with more complex outer codes (convolutional codes with higher memory or turbo-codes).

REFERENCES