A framework for the classification and prioritization of arrival and departure routes in Multi-Airport Systems Terminal Manoeuvring Areas

Stavros Sidiropoulos,[1] Dr. Arnab Majumdar,[2] Dr. Ke Han,[3] Dr. Wolfgang Schuster[4] and Prof. Washington Y. Ochieng[5]
Imperial College London, SW7 2AZ, UK

Typically major cities (London, New York, Tokyo) are served by several airports effectively creating a Multi-Airport System or Metroplex. The operations of the Metroplex airports are highly dependent on one another, which renders their efficient management difficult. This paper proposes a framework for the prioritization of arrival and departure routes in Multi-Airport Systems Terminal Manoeuvring Areas. The framework consists of three components. The first component presents a new procedure for clustering arrival and departure flights into dynamic routes based on their temporal and spatial distributions through the identification of the important traffic flow patterns throughout the day of operations. The second component is a novel Analytic Hierarchy Process model for the prioritization of the dynamic routes, accounting for a set of quantitative and qualitative characteristics important for Multi-Airport Systems operations. The third component is a priority-based model for the facility location of the optimal terminal waypoints (fixes), which accounts for the derived priorities of each dynamic route, while meeting the required separation distances. The proposed Analytic Hierarchy Process model characteristics are validated by subject matter experts. The developed framework is applied to the London Metroplex case study.

Nomenclature

A-CDM	= *Airport Collaborative Decision Making*
AHP	= *Analytic Hierarchy Process*
ANSP	= *Air Navigation Service Provider*
ATCo	= *Air Traffic Controllers*
ATS	= *Air Traffic Services*
ATM	= *Air Traffic Management*
C.I.	= *Consistency Index*
C.R.	= *Consistency Ratio*
CONOPs	= *Concept of Operations*
RNAV	=*Area Navigation*
RNP	= *Required Navigation Performance*
DAC	= *Dynamic Airspace Configuration*
FCFS	= *First-Come-First-Served*
GNSS	= *Global Navigation Satelite Systems*
ICAO	=*International Civil Aviation Organization*
MAS	= *Multi-Airport System*
MCDM	= *Multi-Criteria Decision Making*
NextGen	= *Next Generation Air Transportation System*
OFP	= *Operational Flight Plan*
SBT	= *Shared Business Trajectory*
SESAR	= *Single European Sky ATM Research*
SID	= *Standard Instrument Departure Route*

[1] PhD student, Department of Civil and Environmental Engineering, Skempton Building, AIAA Member.
[2] Lecturer, Department of Civil and Environmental Engineering, Skempton Building.
[3] Lecturer, Department of Civil and Environmental Engineering, Skempton Building.
[4] Research Associate, Department of Civil and Environmental Engineering, Skempton Building, AIAA Member.
[5] Professor, Department of Civil and Environmental Engineering, Skempton Building.

STAR = *Standard Instrument Arrival Route*
TMA = *Terminal Manoevring Area*

I. Introduction

THE continuous increase in demand for air travel throughout the last century has resulted in the emergence of secondary airports in the proximity of large metropolitan areas. [1] These are in addition to one or more primary airports serving a given area, forming a Multi-Airport System (MAS) or "Metroplex" (e.g. London, New York, Tokyo). As a result of the close proximity between the Metroplex airports, their operations are interdependent, making their efficient management challenging. Due to the lack of effective central coordination for the operations in such systems, the current practice is for Air Traffic Controllers (ATCo) to allocate traffic in an ad-hoc manner based upon experience, which results in a sub-optimal utilization of the potential capacity of the system. [2]

There have been a few attempts to optimize MAS TMA operations through airspace redesign consultations. [3-5] However, in most cases they comprise of ad-hoc measures for specific airport systems (eg. the VFR corridors in NY and LA basin Metroplex), resulting in a sub-optimal solution for the system of airports. The most comprehensive approach for the analysis and optimization of Metroplex operations is presented in Ref. 2. A set of concepts is proposed for Metroplex operations, evaluated on the basis of their spatial and temporal impacts on the system. Consequently, the authors identify the concept of scheduling flights as the most effective solution for improving current Metroplex operations. The drawback of this approach is that the proposed route geometry is developed manually and in an ad-hoc fashion based on specific airport systems, thus failing to capture the full complexity of diverse Metroplex systems.

In this paper, the route geometry is examined in a systematic manner, independently of the location of existing waypoints (designing airspace from scratch), with the aim of producing a generic model for MAS TMA airspace design for the system of airports, rather than developing a set of fragmented solutions implemented for individual airports or systems. To facilitate better utilization of the current infrastructure to meet the future arrival and departure demand, a new Concept of Operations (CONOPs) is required for MAS TMA. This CONOPS departs from the traditional, ad-hoc First-Come / First-Served (FCFS) service policy that handles individual aircraft, by moving towards the clustering of aircraft that share common characteristics (demand direction, destination/ arrival airports) into dynamic arrival and departure routes over specified time periods that correspond to the dynamic demand ("*dynamic route" policy*).

The key contribution of this paper is the development of a framework aimed to enhance decision making for the planning of MAS operations. The developed framework explores the classification and prioritization of arrival and departure routes in MAS TMAs with the direct involvement of the decision makers in the planning of operations. The proposed solution supports the Single European Sky Air Traffic Management Research (SESAR) and Next Generation Air Transportation System (NextGen) concepts of Dynamic Airspace Configuration, 4D trajectories, Arrival Manager (AMAN) and Departure Manager (DMAN). The steps of the analysis are summarized in Figure 1.

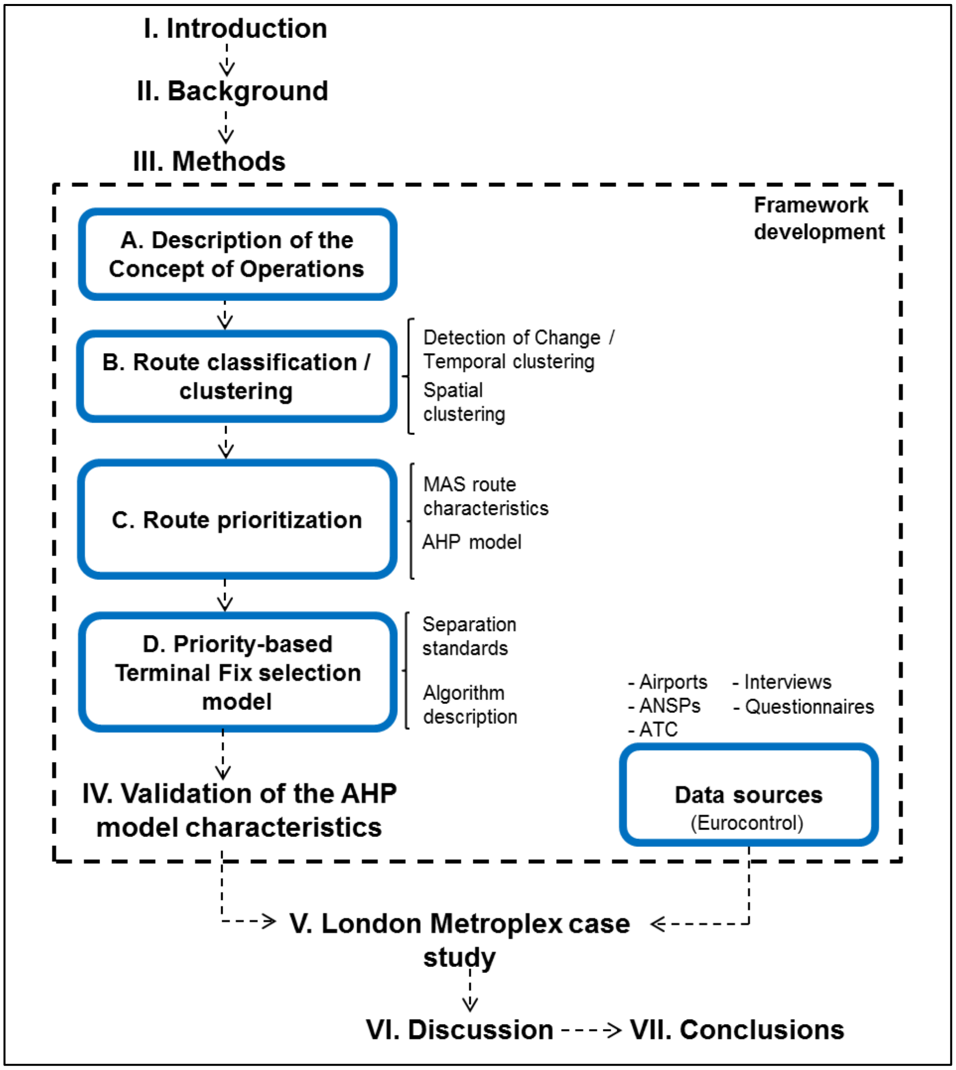

Figure 1 Analysis framework

II. Background

MAS are one of the most significant regional level scaling mechanisms that have allowed the air transportation system to adapt in order to meet demand. [6] The effective management of such systems is a key requirement by all the relevant stakeholders, namely the airports, Air Navigation Service Providers (ANSPs), ATC and airspace users (especially the airlines). Currently, operations in MAS are *loosely coordinated*, [2] resulting in a number of system inefficiencies that affect the stakeholders. The lack of effective Airport Collaborative Decision Making (A-CDM) leads to the airports within the MAS to compete for the use of the common airspace resources. [1] The ANSPs are required to provide the necessary structures that will ensure an efficient service for both airports and airspace users. However, a lack of effective network planning imposes operational constraints to the latter. Airlines face the problem of minimizing their operational costs while providing an efficient service to the passengers and conforming to these constraints. For these reasons, the burden of accommodating the air traffic demand in MAS falls to ATCo, who due to capacity constraints and increased controller workload often resort to the vectoring of flights and the extensive use of holding stacks (mainly in Europe and to a lesser extent in the U.S.), both of which highlight the inefficiency of the current system structure to effectively manage MAS operations.

To improve operations in Metroplex areas, the airspace route structures should be planned not individually for each airport but cooperatively for the system of airports. [2] To this end, the solution should be aligned with the current notion for the modernization of the ATM system and move towards more dynamic, 4-D trajectory-based operations, as described in the SESAR and NextGen programmes. [7,8] There have been a few methods in the literature that adopt the concept of Dynamic Airspace Configuration for terminal area operations. Ref. 9 developed a model for TMA Dynamic Airspace Configuration (DAC) under uncertain weather conditions that focuses on single-airport systems. Experimental results for Chicago O'Hare International airport are promising and indicate increased airspace capacity through the dynamic planning of arrival and departure routes accounting for the effect of convective weather. [9] Ref. 10,11 propose a model for optimal routing in single-airport systems combining DAC with an extended terminal airspace design that further allows for additional airspace and flexibility to enable severe weather avoidance planning. Finally, Ref. 12 propose a trajectory clustering technique for the design of routes in Metroplex systems, following a graph-based approach for defining nominal terminal area routes using historical flight track data. Even though this approach can accurately depict current operations, it is biased by the existing Standard Instrument Departure Routes (SIDs) and Standard Instrument Arrival Route (STARs) (which are dependent on existing navaid infrastructure) and the ATCo ad-hoc decisions, both of which are reflected in the data, thus failing to obtain an optimal route geometry for the Metroplex.

In order to avoid incorporating current system constraints that are an implicit characteristic of historical data into the solution, updated dynamic demand data should be considered instead. Airlines use internal optimization systems in order to calculate the optimal routes for their operations. The main objectives of this optimization are to minimize fuel, cost and distance (time) tracks. [13] The cost of each of these objectives is a function of several factors such as airspace charges, compliance of the estimated time of arrival to specific waypoints and to the destination (in other words punctuality, which receives the highest priority during tactical decision making), weather conditions, airspace configuration and route availability, all of which play an important role for the finalization of the trajectory planning. [13] There is a high level of uncertainty associated with these factors, which can result in the airlines having to revise their flight plans and submitting the Shared Business Trajectories (SBT) up to three hours prior to operations. To this end, the dynamic nature of the ATM system prevents the airlines from flying repetitive routes even for the same O-D pairs. [13] This causes the air traffic flows in and out of the MAS to vary over the spatial and temporal dimensions and adds to the notion that the current static airspace structures are inadequate for accommodating the dynamic nature of the air traffic demand. The International Civil Aviation Organization (ICAO) has taken a number of initiatives that attempt to tackle this issue by revising the currently static flight planning procedures (protocols) to reflect the need for more flexibility for the airspace users, while at the same time ensuring transparency for the other stakeholders. [14]

In line with these initiatives, the purpose of this paper is to use updated airline trajectory data (from the SBT) as the input and analyze the air traffic demand for MAS airports across the spatial and temporal dimensions. The required data can be obtained from the up-to-date airline Operational Flight Plan (OFP), instead of the Air Traffic Services (ATS) flight plan that is currently transmitted to the ANSPs. The OFP data are more detailed and include additional flight information. [13] Current system structures fail to make use of the available dynamic system information of the OFP data for the planning of routes, resulting in a sub-optimal performance of the system. In light of these current system inefficiencies, the framework proposed in this paper introduces a new CONOPs that makes use of this information to enable the effective planning and management of MAS operations.

III. Methods

The subsequent sections describe the steps followed in the framework, starting with the proposed CONOPS and the introduction of the dynamic route service policy. This is followed by the development of a route classification model and an AHP model for the prioritization of the dynamic routes. Finally, a priority-based model is proposed for the allocation of the derived dynamic routes to terminal fixes in the TMA.

A. Concept of Operations – Dynamic route service policy

The schedule of operations for each airport in the MAS varies during the day, both temporally and spatially. The directional demand of different airports also fluctuates and can be divided into demand for specific arrival and departure routes. The static route structures currently used for the arrival and departure operations of the airports comprising the MAS (SIDs, STARs) do not account for the variations in demand. This transfers to ATCo the problem of managing in-bound and out-bound MAS airport traffic and providing de-confliction and separation between aircraft. Normally, ATCo resort to the implementation of ad-hoc measures for the mitigation of potential trajectory conflicts on a FCFS basis. This results in increased airspace complexity and controller workload, with the consequence of reduced airspace capacity.

In the case of MAS, there are additional airspace constraints to be met, as the operations of one airport are directly affected by the operations of the other airports in the system. To avoid further complexity, controllers assign flights to the existing set of SIDs and STARs on a FCFS basis. However, during periods of high demand the vectoring of flights and the extensive use of holding stacks is inevitable and results in an inefficient allocation of the airspace resources. To overcome this limitation, a different concept is needed for the design of arrival and departure routes in MAS TMAs, that departs from the traditional FCFS policy. This concept should consider the system of airports as a whole and incorporate the dynamic variations in the demand for the different airports in the route planning process. Therefore, the model proposed in this paper considers as initial input the SBTs of the airspace users. Significant traffic flow patterns are identified based on the variation in the spatial and temporal distribution of flights in the MAS throughout a day of operations. The flights that share common spatial and temporal characteristics are then clustered together creating a set of dynamic routes with specific demand characteristics. These are: the time period during which the dynamic routes are active and the location where they intercept the MAS TMA. To capture these characteristics, a dynamic route is defined in this paper as: "*a set of flights that are part of a significant traffic flow pattern and share similar spatial and temporal characteristics*". Each dynamic route refers only to one of the MAS airports and consists of either arriving or departing flights to/from that particular airport.

The TMA intercepts of different dynamic routes during the same time period of operations may overlap. For this reason, there is a need to prioritize the dynamic routes that belong to the same time period of operations. Each dynamic route is characterized by a set of attributes that are important to MAS operations, such as the type of operation (being an arrival route or a departure route), the aircraft flows and others, which are described and analyzed in detail in section C. These characteristics are important to the Network Manager and are currently accounted for by the ATCo when they prioritize individual flights in order to assign them to specific SIDs and STARs. In order to incorporate these characteristics, an Analytic Hierarchy Process (AHP) model is developed, allowing the local network manager to influence the planning of routes, while accounting for the dynamic operational status of the system. The AHP model is presented in section C. The final location of the terminal arrival and departure fixes is determined on the basis of the AHP derived priorities, while maintaining the required separation distances between adjacent waypoints to ensure safe operations. A priority-based terminal fix selection model is developed and presented in section D.

The dynamic route service policy allows for operations in MAS to achieve a high level of efficiency. It further enhances the FCFS concept, which can be applied for flights that are already assigned to specific dynamic routes. ATCo can either handle the generated traffic on a FCFS basis, or operations can be further optimized by the scheduling of flights along each dynamic route. The dynamic route service policy aims to enable the planning of operations at the strategic (a few days prior to flight) and pre-tactical (up to 3 hrs prior to flight) levels.

Finally, the TMA model that is the best fit for MAS is considered. Currently, the TMA extends up to 50 nm horizontally in most airport systems. In addition, all interactions between neighboring air traffic facilities are governed by Letters of Agreement (LOAs), [10] which determine the horizontal and vertical boundaries for the hand-off of air traffic flows. This adds to the static nature of the airspace structures and limits any flexibility for ad-hoc routings and the smooth transition from en-route to TMA airspace. Ref. 10,11 propose the extension of the TMA to include the transitional portion of airspace between the en-route and the TMA phases of the flight. The TMA is proposed to be extended horizontally up to 100 nm from the airport reaching the Top of Descent (TOD) (for arrivals), thus allowing greater flexibility in the management of air traffic flows, especially in the case of arriving traffic, while also reducing the need for LOAs. [10] Finally, in most airport systems a four corner post configuration is used, whereby arrival and departure fixes alternate between one another, while ensuring 2-D separation between arrival and departure flows. [9] To capture these requirements, a unified 2-D TMA model is assumed for the MAS in this paper, with its center located at the centroid of the MAS airports and with a radius of 150 km.

B. Route classification / clustering

The MAS TMA airspace is characterised by high-density operations [2,7,15,16] and the location where flights enter and exit the TMA varies according to their spatial and temporal distribution. However, aircraft trajectories share origin and destination directions (TMA entry-exit points and MAS airports). This enhances the notion of clustering the aircraft that share common spatial and temporal characteristics into "*dynamic routes*" to be active over specific time windows of operations. Figure 2 presents the distribution of flights for the London Metroplex over one day of operations. *Theta* is the azimuth indicating the direction of entry/exit to/from the TMA and *Time* is the time of entry/exit for each flight:

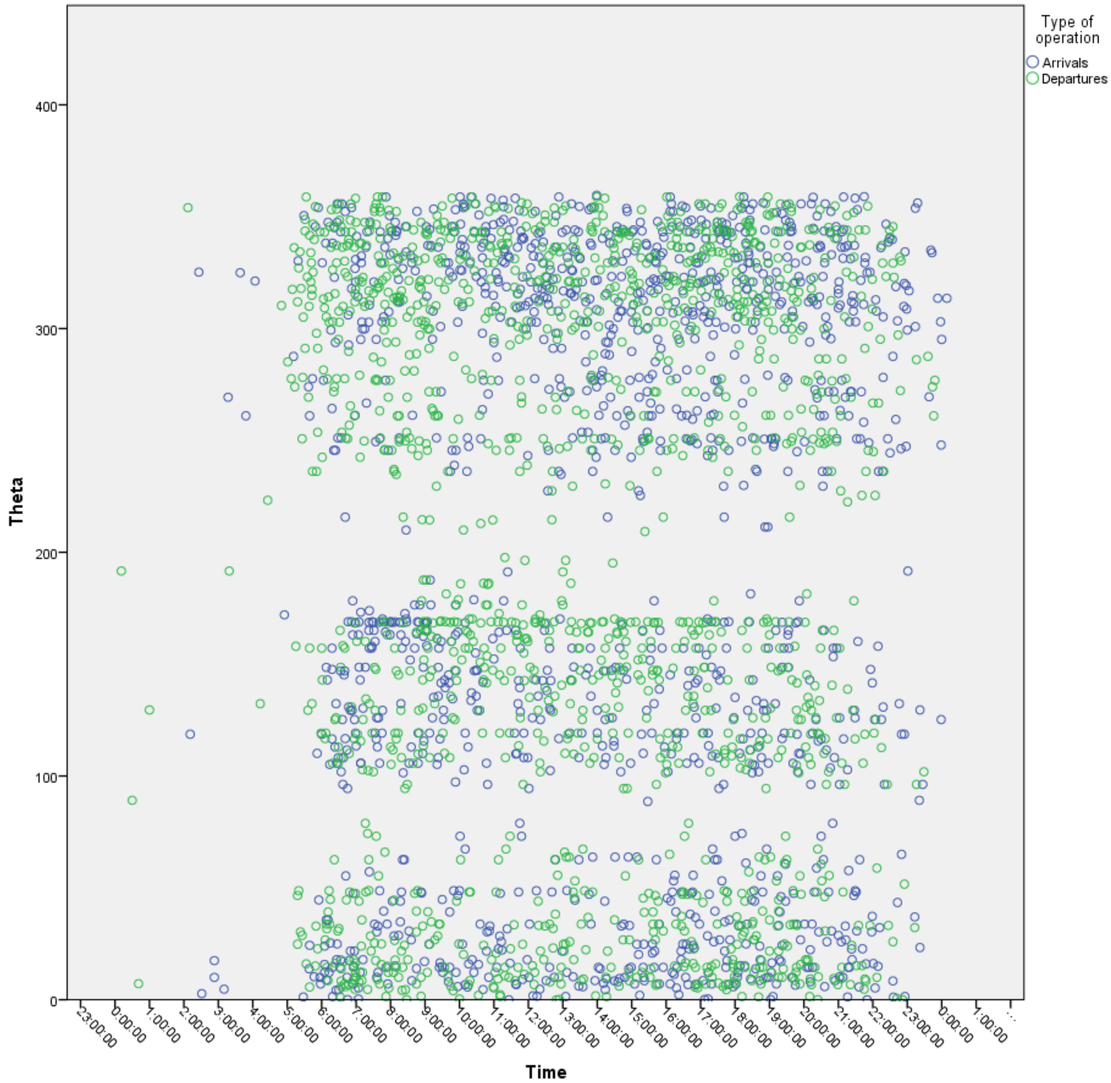

Figure 2 Spatial and temporal distribution of flights for the London MAS

From the scatter plot in Figure 2, it is evident that distinct clusters over both dimensions cannot be identified to encompass useful information about specific traffic flow patterns in the operations. This is due to the fact that the temporal distribution of flights is almost continuous, whilst some *gaps* can be identified in the spatial dimension, for zones of the TMA that are less busy. Flights tend to arrive or depart to/from the TMA in streams, the timing of which fluctuates during the day. This results in certain directions being busier than others. To manage the incoming and outgoing aircraft flows efficiently, this change in the distribution of demand should be accounted for. The dynamic routes are derived by a sequential process that consists of two steps; the first is the temporal clustering of flights that incorporates the significant changes in the spatial and temporal distribution of flights throughout the day. The second step is the spatial clustering of the flights that occur during specific time-windows of operations, as discussed in the following sections.

1. *Detection of change – Temporal clustering*

The objective of this step is to detect changes in the traffic flow patterns on the boundary of the TMA throughout a day of operations and group the flights into distinct time-windows that reflect these spatio-temporal changes in demand. The output clusters will allow for the changes in the spatial distribution of flights to be inferred in order to obtain significant changes in the temporal patterns. There is a distinct category of problems in the literature related to change detection. The general problem of the off-line estimation of the change time can be expressed as: [17]

Let y_k, $1 \le k \le n$ be a sequence of observed random variables with conditional density $d_\theta(\mathrm{y}_k \left| y_{k-1},...,y_1 \right.)$. The unknown change time t_0 is assumed to be such that $1 \le t_0 \le n$. The conditional density parameter θ before t_0 is constant and equal to θ_0. After the change, the parameter becomes θ_1. The unknown change time can be estimated from the observations $y_1,...,y_n$, where $1 \le n \le \infty$. The information about θ_0 and θ_1 can be used for the estimation of t_0 subject to their availability. The unknown is the time t_0. The detection is performed by a stopping rule of the general form:
$t_0 = \inf \{ n : g_n(\mathrm{y}_1,...,\mathrm{y}_n) \ge \lambda \}$, where λ is a threshold, and $g_n, n \ge 1$ is a family of n-dimenional functions. The problem that arises is the selection of the threshold value λ.

In order to select an appropriate value for λ, the period of operations needs to be discretized into a number of time-steps t_s that captures changes in demand which are significant in relation to the size of the TMA zone that is being examined. This leads to the need for the division of the TMA boundary into a number of zones with size z_s. The size of the time and zone steps is dependent on the selection of threshold λ. The threshold value should reflect a significant change in both the TMA zone that is being examined and the TMA as a whole. This creates the need for two distinct threshold values T_1 and T_2, associated with a specific time step and zone step. The first threshold value T_1 reflects a significant change in the individual TMA zone, while the second, T_2, accounts for a change in the "*system status*" in all TMA zones.

For this purpose, a set of values is initially selected for the four variables (t_s, z_s, T_1, T_2) through trial and error. The boundary conditions for the selection of the zone and time step sizes should be such that they capture the smallest change in demand that is considered to be significant. A set of proposed values for the time and zone step sizes is presented in Table 1. Another indicator for the most appropriate selection of the threshold values is the resulting number of temporal clusters N, which is the number of time periods of operations throughout the day. The most appropriate threshold values can then be identified with the consultation of SMEs who can assess the values within each set along with the resulting number of temporal clusters. Table 1 summarizes a set of several threshold values for different time and zone steps, together with the resulting number of temporal clusters. The function g_n, where $n \ge 1$, that is used as a measure for the threshold values represents the "*absolute rate of change*" $|\Delta N_{i,j}|$ in the number of flights that arrive or depart to/from the TMA at a specific zone j, during a specific time window i.

Table 1 Relationship between selected time step t_s, zone step z_s, threshold values T_1, T_2 and resulting number of temporal clusters N

	Zone step								
	10^o			20^o			30^o		
Time step	**T1**	**T2**	**N**	**T1**	**T2**	**N**	**T1**	**T2**	**N**
30 min	3	8	15	4	6	9	4	6	4
30 min	2	15	14	4	5	15	3	6	18

For this application of the model, a change detection/temporal clustering algorithm was developed. The following parameters were used:

- A time step t_s of 30 minutes is selected to divide a full day of operations into n ($n=48$) time periods.
- A zone step z_s of 10^o is selected to divide the TMA into m ($m=36$) zones.

Let F be the set of all the flights that enter and exit the MAS TMA throughout one full day of operations. For each time period $t_i \in T$, each zone $z_j \in Z$ is populated with the flights $f_k \in F$ that enter or exit the TMA at the specific zone during that period. The number of flights during each time period and in each zone are summarized in a cell transmission model [18], where each cell "$Status_{ij}$" indicates the number of flights that enter or exit zone j during time period i.

Let CT be the set of temporal clusters $\{CT_1, CT_2, \dots, CT_n\}$. The duration of all temporal clusters is initially 30 minutes and the elements of each cluster are all the elements $f_k \in t_i$.

Let CZ be the set of zone clusters $\{CZ_1, CZ_2, \ldots, CZ_m\}$, where the size of all zone clusters is initially 10^o and the elements of each zone cluster are all the elements $f_k \in z_j$. The rate of change in the total number of flights is calculated for each zone, between the consecutive time periods, as:

$$\left|\Delta N_{\mathrm{p},j}\right| = \left|N_{i,j} - N_{i-1,j}\right| \tag{1}$$

where

$\left|\Delta N_{\mathrm{p},j}\right|$ is the rate of change where $p = n-1$ for zone j between time periods i and $i-1$

$N_{i,j}$ is the number of flights in zone j during time period i

$N_{i-1,j}$ is the number of fligths in zone j during time period $i-1$

In order to identify a significant change in the number of flights in each zone over a specific time period, a threshold value T_1 is selected. The threshold value is to be determined by an SME, as described above. If the rate of change is greater than the threshold value T_1, then a significant change in the demand for the specific zone has taken place and the binary variable index $ChangeSign_{p,j}$ is assigned values:

$$ChangeSign_{\mathrm{p},j} = \begin{cases} 1, & \left|\Delta N_{\mathrm{p},j}\right| \geq T_1 \\ 0, & otherwise \end{cases} \tag{2}$$

To identify a significant change in the "*System Status*" between two consecutive time periods, a second threshold value T_2 is set, to be compared with the number of zones of the TMA where a significant change has taken place. If this number is greater or equal to the threshold value T_2, then the "*System Status*" has changed significantly from time period $i-1$ to time period i.

$$counter_p = \sum_{j=1}^{m} ChangeSign_{\mathrm{p},j} \tag{3}$$

A binary variable index $TimeSign_j$ is compared to $counter_p$ and is assigned values:

$$TimeSign_p = \begin{cases} 1, & counter_p \geq T_2 \\ 0, & otherwise \end{cases} \tag{4}$$

Two consecutive temporal clusters should be merged when there is no significant change in the "*System Status*", indicated by $TimeSign_p = 0$. A $MustLink$ index is used, indicating the unique cluster ID. The final temporal clusters are created such as:

$$TimeSign_p = \begin{cases} 1, & \begin{cases} MustLink(i-1) = q \\ MustLink(i) = q \end{cases} \\ 0, & \begin{cases} MustLink(i-1) = q \\ MustLink(i) = q+1 \end{cases} \end{cases} \tag{5}$$

where
q is the temporal cluster id, initially equal to 1
i is the time period index

The assignment of individual flights to their final temporal clusters can be summarized as:

a. Let CT be the set of temporal clusters $\{CT_1, CT_2, \ldots, CT_n\}$, where the duration of all temporal clusters is initially 30 minutes.
b. For each instance CT_i in CT: Perform *TimeSign* check: if there is no significant change in the number of flights between two consecutive time periods, then assign CT_i to the previous cluster CT_{i-1}, else assign CT_i to the next temporal cluster
c. Return the final temporal clusters partition CT_f.

2. Spatial clustering

In the previous step, the temporal clusters are determined while taking into account changes in the spatial and temporal distribution of flights. The next step is to identify the significant traffic flows within the derived temporal clusters for the airports of the MAS in order to identify the "*dynamic routes*" for each airport. Each "*dynamic route*" should consist of a number of flights that share the following characteristics:

- Type of operation, whether it is an arrival or a departure route
- MAS airport
- Direction (location) of entry/exit to or from the TMA

Regarding the first two characteristics, the flights within each temporal cluster (time period of operations) can be grouped into arrivals and departures for each of the MAS aiports. Once this categorization is complete, the resulting flight groups are clustered according to the direction they enter or exit the TMA using a K-means clustering algorithm, such that flights can only group with flights that belong to the same type of operation (arrival or departure) and fly to/from the same MAS airport. The K-means algorithm is a center-based algorithm and is selected here as the resulting cluster centers will later be used as the dynamic route intercepts with the TMA boundary. The K-means algorithm is briefly described as: given a set of observations $(x_1, x_2, \ldots, x_n)$, where each observation is a d-dimensional real vector, K-means clustering partitions the n observations into $K \le n$ sets $S=\{S_1, S_2, \ldots, S_k\}$ with the objective to minimize the within-cluster sum of squares:

$$\arg\min_{S} \sum_{i=1}^{k} \sum_{x \in S_i} \left\| x - \mu_i \right\|^2$$, where μ_i is the mean of points in S_i

The K-means algorithm is implemented as follows: [20]

1. Choose K initial cluster centers $c_1(1), c_2(1), \ldots, c_K(1)$
2. At the k-th iterative step, assign the samples $\{x\}$ among the K clusters using the relation $x \in C_j(k)$ if $\left\| x - c_j(k) \right\| < \left\| x - c_i(k) \right\|$, $\forall i = 1,2,\ldots,K; \quad i \ne j$, where $C_j(k)$ denotes the set of samples whose cluster center is $c_j(k)$.
3. Compute the new cluster centers $c_j(\mathrm{k}+1), \quad j = 1,2,\ldots,K$, so as to minimize the sum of the squared distances from all points in $C_j(k)$ to the new cluster center. The measure that minimizes this is the sample mean of $C_j(k)$ and the new cluster center is $c_j(\mathrm{k}+1) = \frac{1}{N_j} \sum_{x \in C_j(k)} x, \quad j = 1,2,\ldots,\mathrm{K}$, where N_j is the number of samples in $C_j(k)$.
4. Check if $c_j(k+1) = c_j(k)$ for $j = 1,2,\ldots,K$ then the algorithm has converged. Otherwise, go to step 2.

One problem that arises when using the K-means algorithm (or some other unsupervised learning algorithm) is how to select an appropriate number of clusters *K*. A common heuristic method used to identify the optimal number of clusters is the "*elbow*" method. The "*elbow*" method examines the relationship between the within-cluster dispersion W_k and the selected number of clusters. W_k (or the error measurement) decreases monotonically up to a point where this decrease levels off. This point forms an "*elbow*", which indicates the optimal number of clusters [21]. Ref. 21 formalized this heuristic technique and developed the "*gap statistic*" to evaluate the optimal number of clusters. According to this approach, the optimal number of clusters occurs at the solution with the largest local or global gap value within a tolerance range. The gap statistic is used in this paper to evaluate and obtain the optimal number of spatial clusters.

The gap value is calculated as: $Gap_n(k) = E_n^*\{\log(\mathrm{W}_k)\} - \log(\mathrm{W}_k)$, where *n* is the sample size, *K* is the number of clusters to be evaluated and W_k is the within-cluster dispersion measurement $W_k = \sum_{r=1}^{k} \frac{1}{2n_r} D_r$, where n_r is the number of data points in each cluster *r*, and D_r is the sum of the pairwise distances for all data that belong to cluster *r*. The expected value $E_n^*\{\log(\mathrm{W}_k)\}$ is determined by Monte-Carlo sampling from a reference distribution (the reference data are generated from a uniform distribution over a box aligned with the principal components of the data matrix) and $\log(W_k)$ is computed from the sample data. The search method being used for the evaluation of the optimal number of clusters is the Global Maximum Standard Error (GMSE). The selected number of clusters is the smallest one to satisfy: $Gap(K) \geq Gap_{\max} - SE(\text{Gap}_{\max})$, where K is the number of clusters, $Gap(K)$ is the gap value for each clustering solution with K clusters, Gap_{max} is the largest gap value and $SE(Gap_{max})$ is the standard error for the largest gap value. Once the optimal number of clusters are evaluated under the gap value criterion, the total number of clusters should be such that the minimum separation distances between terminal fixes on the TMA are met. This only allows for a maximum number of clusters that correspond to the maximum number of terminal waypoints $N_{\max} = \frac{2\pi R}{D_{\min}}$. N_{max} is the maximum number of spatial clusters, R is the TMA radius and D_{min} is the minimum separation distance between two adjacent terminal fixes.

To ensure that this condition is met, the following constraint is added to the spatial clustering algorithm: $\sum_{a=1}^{M}\sum_{i=1}^{I}(Arr_{aj} + Dep_{aj}) \leq N_{\max}$, where Arr_{aj} is the total number of arrivals at airport α during period I, Dep_{aj} is the total number of departures at airport α during period i, I is the total number of time periods and M is the total number of MAS airports. An example of the resulting dynamic routes is presented in Figure 3, while the route classification process is summarized in Figure 4.

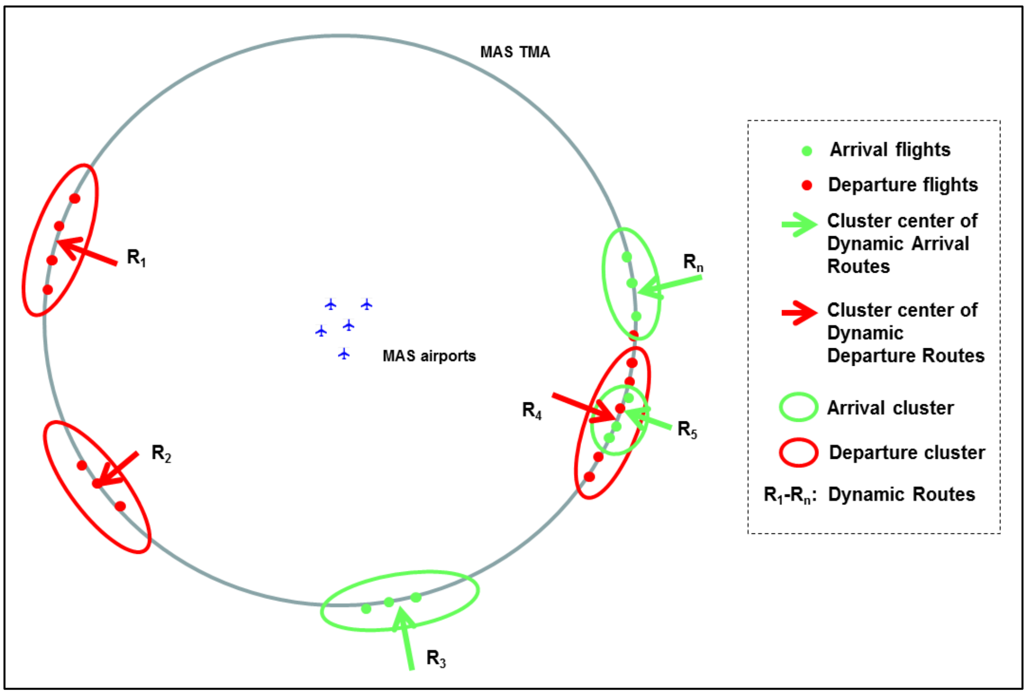

Figure 3 Dynamic routes for arrivals and departures

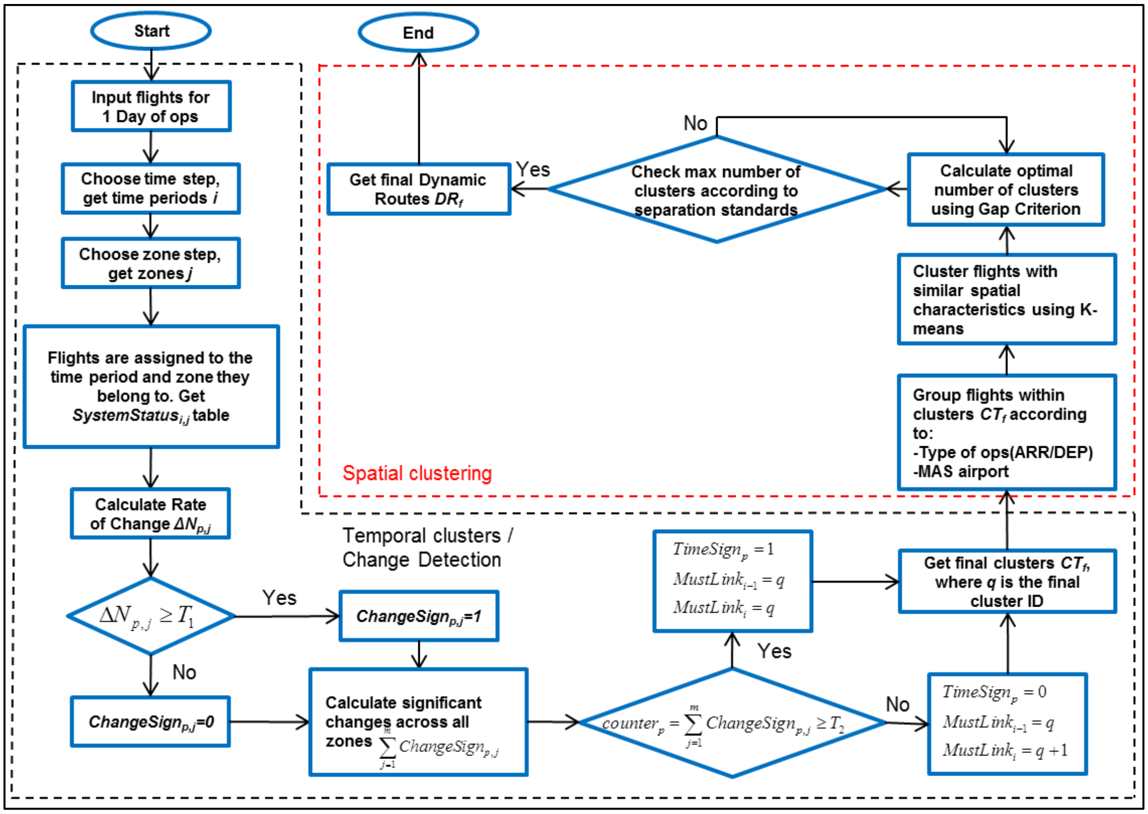

Figure 4 Route classification/clustering flow chart

C. Route prioritization

The obtained "*dynamic routes*" contain the demand for the arrival and departure routes for the MAS airports during each time window of operations (see temporal clustering results in B). The cluster center for each "*dynamic route*" is assumed to be the point where all flights that belong to that cluster should enter or exit the TMA and also coincides with the *desired (optimal)* terminal fix for the route. However, during a specific *time window*, two or more "*dynamic routes*" may act antagonistically for the same entry/exit location on the TMA. This creates the need to prioritize the "*dynamic routes*" during each *time window*. For the prioritization process to be effective, the model should capture the most important operational aspects in MAS. Previous analysis carried out by the authors identifies a set of both qualitative and quantitative characteristics related to MAS operations [1]. Based on these characteristics, an AHP model is developed here to enable the route prioritization.

The AHP is a Multi-Criteria Decision Making (MCDM) technique that enables reducing complex decisions through a series of pair-wise comparisons between criteria that are considered important to achieve a specific goal. [22] It is capable of incorporating both quantitative and qualitative information in the decision making process whilst the weights, or priorities for the several factors that affect the final decision are derived from expert knowledge. These characteristics make the AHP a powerful tool and an adequate approach for enhancing the decision-making process in MAS. The AHP has been used in several design problems including routing. Sattayaprasert developed an AHP model for optimal hazmat routing based on route prioritization. [23] In the model, several aspects that are important to hazmat routing are considered, such as the risk of carriage unit explosion, the risk of road accidents as well as the consequences of an accident, together with additional operational and spatial characteristics that correspond to the examined route alternatives. [23] The AHP has also been used in ATM, for example Ref. 24 developed an AHP framework to analyze a new CONOPs for implementing the Business Trajectory as introduced by SESAR. This concept is based on the agreement of stakeholders on specific 4-D intervals, known as *target windows*, that describe the flight trajectory and are included in the airline *contract of objective*. [24] In the model the *target windows / contract of objective* concept is compared to the *business as usual* scenario for the three main stakeholders involved: the airlines, the ANSP and the airports. [24] Finally, Ref. 25 use the AHP to derive weights for flight trajectory clusters used for the creation of a proposed trunk route network in China, based on the concept of *tube design*.

The AHP model that is developed here for the route prioritization in MAS is presented in Figure 5:

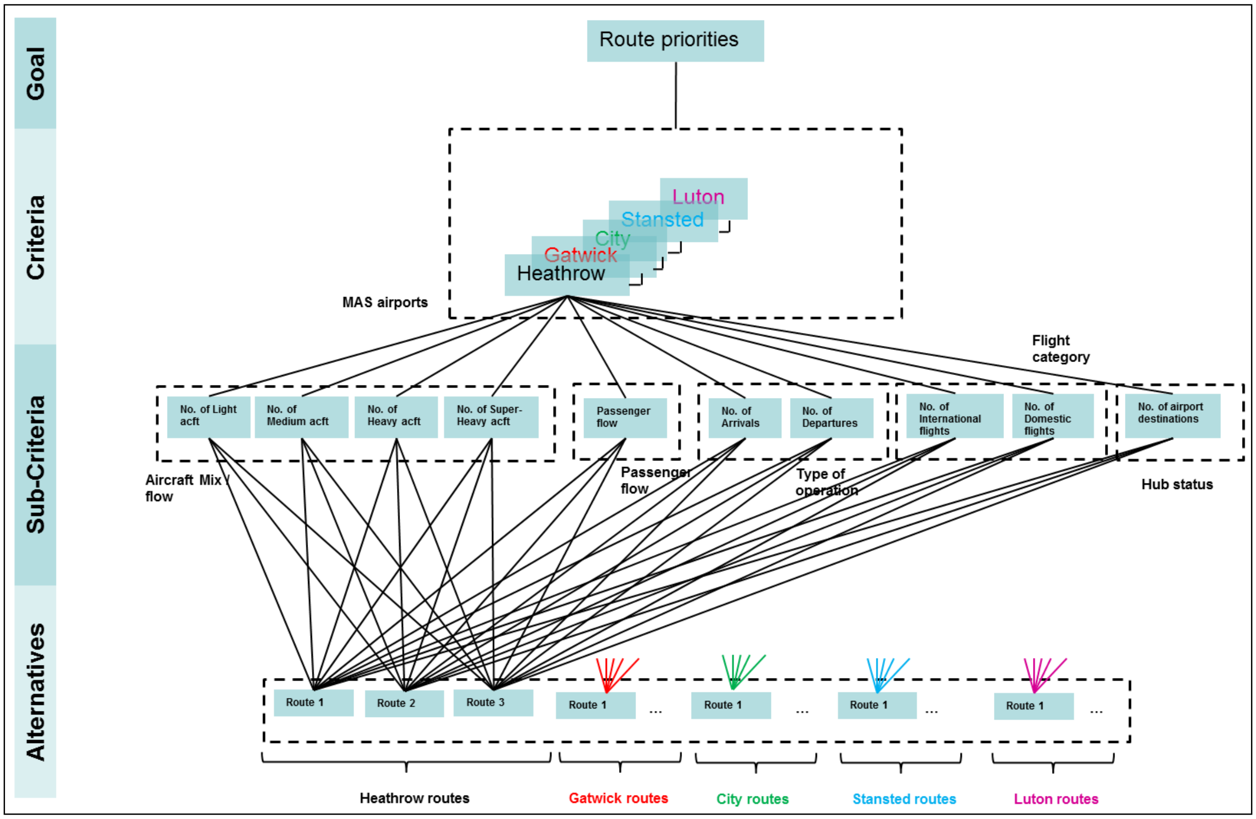

Figure 5 AHP model

The AHP structure can be decomposed into the following steps: [22]

1. Define the objective of the decision process
2. Decompose the objective into elements, based on their common characteristics
3. Organize the elements into levels according to their relative importance in a top-down scheme

The nodes in the model represent the elements, while the links that connect the nodes indicate the interdependencies between them. The highest level in the hierarchy is the goal level. The goal in this model is to derive the higher priority route from the set of "*dynamic routes*" that was obtained in the previous step, as well as the relative importance of the other routes. The second level of the hierarchy, comprises the main criteria for prioritizing the "*dynamic routes*". This level of criteria consists of the different MAS airports and enables the decision maker to give different priorites to routes according to the airport they fly to or from. Alternatively, all airports may be considered of equal importance. The next level consists of the sub-criteria for each airport. In this level, there is a range of sub-criteria to examine. Firstly, the "*aircraft mix*" refers to the number of aircraft of each of four wake turbulence categories (light, medium, heavy, super-heavy). Secondly, the route specific sub-criteria include the "*passenger flow*", which is the total number of passengers on each route. Thirdly, the "*type of operation*" determines whether the route is an arrival or a departure route. This parameter can either take binary values of 0 and 1, or alternatively, the "*arrival*" node can be linked only to arrival routes from the lower level of alternatives, while the "*departures*" node can be linked only to departure routes. Fourthly, the "*category*" refers to the number of flights on each route that are international or domestic. Finally, the "*airport status*" includes the number of connections that each airport serves, which allows the decision maker to give priority to routes that fly to airports that have a more important hub role in the network. The last level of the hierarchy consists of the "*dynamic route alternatives*". The dynamic route alternatives only connect to the sub-criteria that belong to the "*specific airport*" (criteria level) that they fly to or from (see Figure 6).

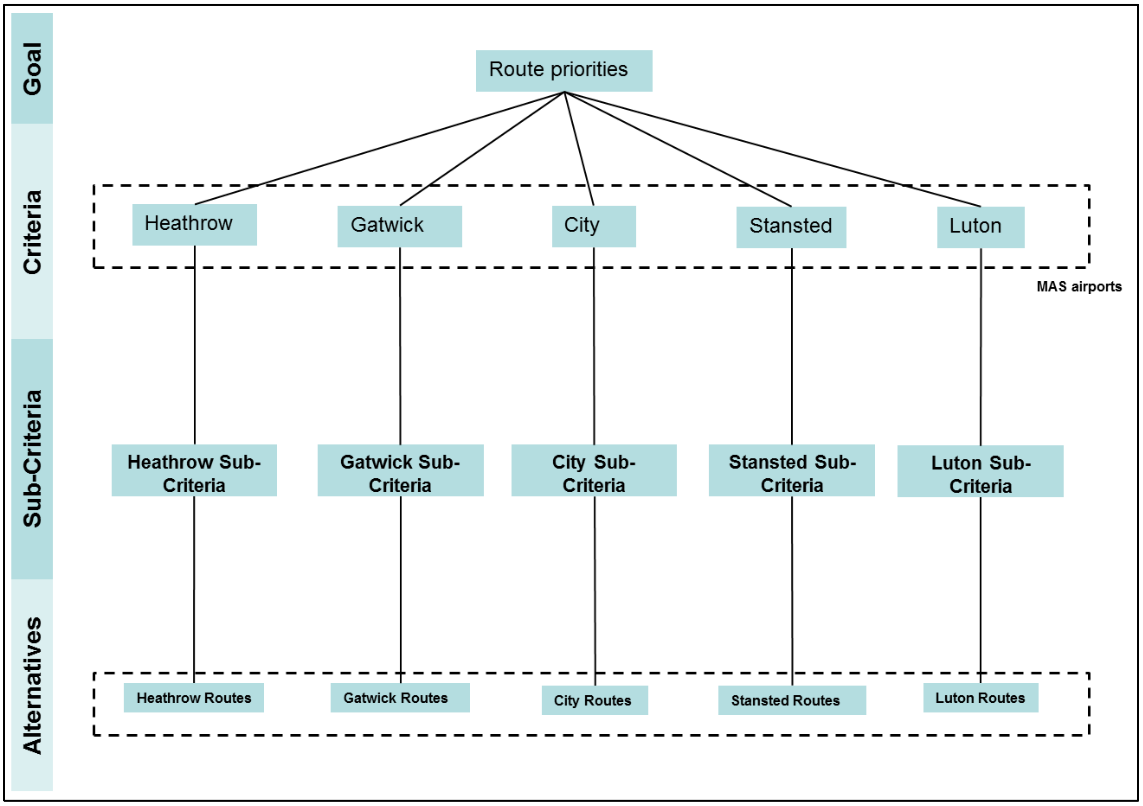

Figure 6 AHP model connections

The criteria used here and the AHP model structure aim to allow the decision maker to include in the route prioritization a range of actions that are often desirable during operations in MAS. For example, the decision maker may wish to prioritize arrival or departure operations to/from a specific MAS airport, international flights, routes that accommodate many super-heavy aircraft operations, maximize passenger flow, or give priority to specific airports over others. The elements of the hierarchy are linked in such a way as to allow for airport comparisons to be made first and then continue to lower level comparisons for the particular characteristics of the routes of each airport. In case there is a requirement for all airports to have the same priority, this becomes feasible by assigning equal importance at the *criteria* level. To use the AHP model, the decision maker needs to pairwise compare the elements at each level with respect to the higher element in the hierarchy to which they are linked. The most common scale used to evaluate the relative importance (or priority) of the elements is the Saaty scale [22] (see Table 2):

Table 2 Saaty's scale for pairwise comparisons *(adopted from Kyriakidis)*

Intensity of importance	Definition	Explanation
1	Equal importance	Two factors contribute equally to the objective
2	Weak	Compromise between values 1 & 3
3	Moderate importance	Experience and judgment slightly favour one factor over another
4	Moderate plus	Compromise between values 3 & 5
5	Strong importance	Experience and judgment strongly favour one factor over another
6	Strong plus	Compromise between values 5 & 7
7	Very strong or demonstrated importance	A factor is favoured very strongly over another; its dominance demonstrated in practice
8	Very very strong	Compromise between values 7 & 9
9	Extreme importance	The evidence favouring one factor over another is of highest possible order of affirmation

The developed AHP model combines both the qualitative and quantitative characteristics that affect operations in MAS. The decision maker is called to conduct the pairwise comparisons for all the levels of the hierarchy down to the sub-criteria level (see Figure 6). Once the comparisons are complete, the quantitative characteristics of the elements that comprise the alternatives (the "*dynamic routes*") are used. The relative weights (eigenvectors) of the examined factors are then calculated based on the eigenvalue approach. The mathematical formulation is as follows: $Aw = \lambda_{\max} w, \quad e^T w = 1$, where A is the positive reciprocal pairwise comparison matrix, w are the ratio-scale priority weights, λ_{max} is the principal eigenvalue of the matrix A and e^T=1,1,…,1 is a unit row vector. [26] The output, which is the priority vector corresponds to the main eigenvector of the comparison matrix. Consequently, the decision maker's judgements need to be evaluated for their consistency. The consistency of the matrix is addressed by calculating the Consistency Index: $C.I. = \frac{\lambda_{\max} - \mathrm{n}}{n-1}$, where λ_{max} is the principal eigenvalue of the pairwise square matrix and n is the order of the matrix. The $C.I.$ is then compared to the Consistency Ratio $C.R. = \frac{C.I.}{R.I.}$ where $R.I.$ is the Random Index generated by Saaty for a very large sample of randomly populated matrices. In order for the assessment to be considered complete, the $C.R.$ should take values ≤ 0.10 (both $C.I.$ and $C.R.$ are unitless). Otherwise, the decision maker should reevaluate their assessment to improve consistency. [22] The values for the $R.I.$ are presented in Table 3.

Table 3 Saaty's Random Index values *(adopted from Kyriakidis)*

The Random Index										
n	1	2	3	4	5	6	7	8	9	10
RI	0	0	0.58	0.90	1.12	1.24	1.32	1.41	1.45	1.49

D. Priority-based terminal fix selection model

The previous sections have presented methods for the derivation of the "d*ynamic routes*" together with their associated priorities. The next step is to assign each "*dynamic route*" to a specific terminal fix. The problem that arises here is related to the minimum separation distances between adjacent terminal fixes that need to be met in order to ensure safe operations, which can be phrased as: "*only one dynamic route can be assigned to each terminal fix location during each time period of operations, while the minimum separation distances between neighboring terminal fixes must be kept*". This is recognized to be a *facility location problem*. [27] There have been a few approaches to solve such problems, with the most popular ones following a linear programming formulation. [9] The primary disadvantage of employing such an approach here is that it complicates the procedure, effectively acting as a *black box* that provides a result based on an input, disallowing potential substantial parameter-tuning in the process. A second concern is that the computational complexity of the available fundamental linear programming algorithms is not strongly polynomial (i.e. they have very unfavouring worst-case scenario behavior).

Historically, the design of TMA airspace was based on "conventional" navigation that relied heavily on the location of ground-based beacons. This currently poses an additional constraint to the design of SIDs and STARs. Recent improvements in navigation technology, especially the use of Global Navigation Satellite Systems (GNSS) can be implemented within procedures such as area navigation (RNAV) that allow aircraft to navigate independently of the radar location with satisfactory location accuracy (e.g. PRNAV: 1nm, RNP: 0.3 nm). [28] This allows for the redesign of terminal airspace independently of the location of navaids, constrained only by the aircraft manoeuvring capabilities. Linear programming formulations require a finite set of facility locations, which in this case would be the constrained by the location of navigation infrastructure terminal fixes currently being used. To tackle the facility location problem, a geometric approach is proposed here instead.

The unimpeded "optimal" (in terms of the output of the airline trajectory optimization) entry/exit points are considered initially for the facility location of each dynamic route. A check is performed to ensure that neighboring dynamic routes do not overlap with each other, while at the same time the separation standards are met. In the case that the previous two conditions do not hold, then the entry/exit point for each dynamic route should be displaced and the new location is equal to: $\bar{x} = x_0 + \Delta x$, where $\bar{x}$ is the new location for each dynamic route, x_0 is the cluster center of the initial dynamic routes and Δx is the required displacement calculated based on the priorities derived in the previous section to meet the minimum separation distance condition. The displacement Δx for each dynamic route is proportional to the overlapping distance and the inverse priorities of the involved routes. The displacements Δx_i and Δx_{i+1} for two neighboring dynamic routes are calculated based on the inverse distance weighted method (for $n=1$):

$$z_j = \frac{\sum_{i=1} \frac{z_i}{d_{ij}^n}}{\sum_{i=1} \frac{1}{d_{ij}^n}} \text{, as:} \tag{6}$$

$$\Delta x_i = \frac{(\mathrm{bz} - \mathrm{d}_{\mathrm{i,i+1}})\frac{1}{p_i}}{P_{\mathrm{i,i+1}}},\ \Delta x_{i+1} = \frac{(\mathrm{bz} - \mathrm{d}_{\mathrm{i,i+1}})\frac{1}{p_{i+1}}}{P_{\mathrm{i,i+1}}}, \tag{7}$$

where the condition $d_{\mathrm{i,i+1}} \geq bz$ must hold; bz is the minimum separation distance (bufferzone), $d_{i,i+1}$ is the distance between the unimpeded "optimal" entry/exit points of two neighboring "*dynamic routes*", p_i is the priority for *dynamic route i*, p_{i+1} is the priority for *dynamic route i+1* and $P_{i,i+1}$ is the sum of the inverse priorities p_i and p_{i+1} calculated as: $P_{\mathrm{i,i+1}} = \frac{1}{p_i} + \frac{1}{p_{i+1}}$. Let DR_1 and DR_2 be two dynamic routes with overlapping initial terminal fix locations x_0^1 , x_0^2 and priorities P_1 and P_2, with $P_1>P_2$. The dynamic routes will be displaced according to Eq. 6 and Eq.7, with DR_1 (the dynamic route with the higher priority) having a smaller displacement than DR_2 (the dynamic route with the lower priority), as shown in Figure 7.

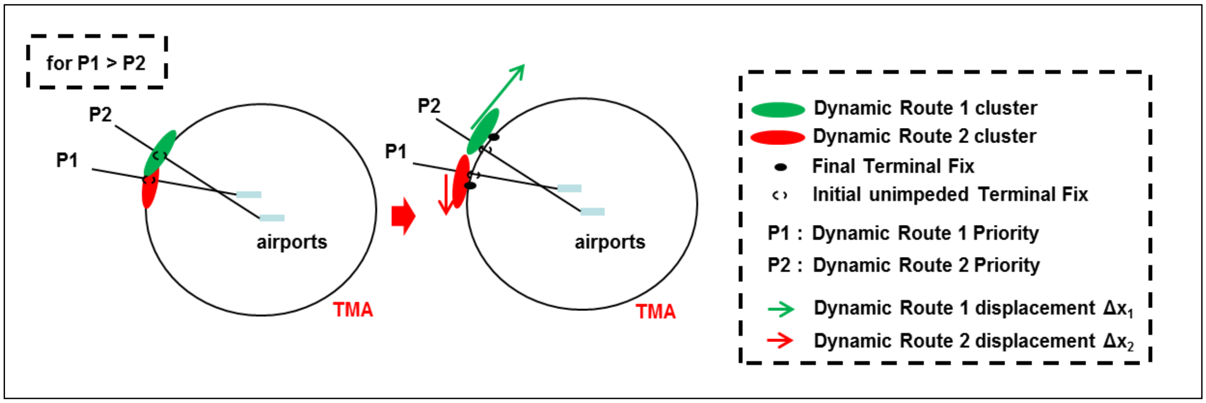

Figure 7 Priority-based facility location of dynamic routes

Each terminal fix can be described by a set of polar coordinates *(R, Az)* where *R* is the radius centered in the TMA centroid and *Az* is the azimuth. The new azimuth (terminal fix location on the TMA) for each dynamic route terminal fix is calculated as a function of its initial azimuth Az_0:

MoveOnCircle function:

$$Az = Az_0 + m \cdot \theta \text{, where } m = \begin{cases} 1, & clockwise \\ -1, & counterclockwise \end{cases} \text{ and } \theta = \arccos\left(\frac{1-\Delta x_i^2}{2R^2}\right) \tag{8}$$

The above described procedure is repeated for the allocation of all dynamic route terminal fixes. To ensure that there is no overlap between neighboring fixes, the distance between them should meet the condition $d_{1,2} \leq bz$, where *bz* is the minimum separation distance (bufferzone). For this purpose, an iterative rearrangement method was developed. The method examines all points in sequential pairs in a *rearrangement cycle*, repositioning a pair according to the aforementioned principle if the pair distance violates the minimum separation and then repeating the cycle all over. In specific, a direction is chosen (clockwise or counterclockwise) and points are examined in pairs, i.e. Pt_1 with Pt_2, Pt_2 with Pt_3, Pt_3 with Pt_4 and so on. After the *rearrangement cycle* is over, the procedure is repeated, as long as at least one change has been made within that cycle. The procedure stops right after a full *rearrangement cycle* runs without reallocating any of the points.

The set of initial unimpeded terminal fixes implicitly entails a segmentation of the TMA. However, the minimum separation distance also implies a standard symmetrical segmentation pattern, dividing the circle into a set of sectors of specific central angles, with each sector destined to contain at most one terminal fix. The problem then is to rearrange the initial unimpeded terminal fixes in order for the result to be compatible with this set of sectors “containers-of-one”. Clearly, this formulation poses a straightforward limitation in the maximum number of terminal fixes, as a sector may contain two terminal fixes at the same time, which would render the problem unsolvable. Thus, appropriate limits are enforced in both the minimum separation distance (see *bufferzone*) and the number of terminal fixes (see Eq. 6 and section B). However, this problem does not cause serious concern for the solution, due to the TMA circle length being many orders of magnitude larger than the minimum separation distance. This ensures a more than adequate margin for the total number of distance-compatible final terminal fixes.

IV. Validation of the AHP model characteristics

The hierarchy proposed in the model is developed based on the literature on airport and MAS operations and the developed criteria are validated by subject matter experts (SMEs). The SMEs represent different stakeholders (Air Traffic Controllers (ATC), Air Navigation Service Providers (ANSP), Airport Authorities, Civil Aviation Organizations) and academic institutions, namely:

- **ATC**: Maastricht
- **ANSPs**: NATS (the UK ANSP), AENA (the Spanish ANSP)
- **Metroplex airport authorities**: London, Paris, Frankfurt
- **Civil Aviation Organizations**: Eurocontrol

- **Academic Institutions**: Imperial College London, Universidad Politecnica Madrid

The validation process was based on questionnaires and semi-structured face-to-face interviews. The questionnaires that were used addressed the various issues of Metroplex operations. Their structure aimed to allow for the SMEs to identify inefficiencies in unique Metroplex systems. Conducting the interviews face-to-face enabled the interviewees to highlight additional areas that are important to MAS operations and may not have been covered initially in the questionnaires. The identified characteristics were combined into the AHP model presented in section C.

V. London Metroplex case study

A. Data description

The data used for the application of the framework are the completed aircraft trajectories of one typical day of operations (Thursday, 12 Jun 2013) in Europe (undisrupted by extreme weather conditions or special demand characteristics) and were provided by Eurocontrol. Each trajectory is described by the FLIGHT_ID corresponding to each flight, the sequence of waypoints and the 4-D coordinates at each waypoint (longitude, latitude, altitude, time). In the case where flights were vectored no waypoint was specified and the 4-D coordinates were recorded assigned a “No-Point” ID. The initial dataset included all European flights and it had to be filtered to extract the trajectories that fly to/from the London MAS airports.

B. Clustering results

The great circle paths were considered as the desired routes for all flights and the time and direction of entry/exit to the TMA is illustrated in Figure 8. The temporal clustering algorithm is applied to the London MAS for the operations of Thursday, 12 June 2013 and for threshold values of T_1=3 and T_2=8. This results in N=15 time periods of operations throughout the day, (see Figure 8 and Table 4). The influence of the early U.S. flights arriving to Heathrow in the formation of the temporal clusters can be seen in Figure 8, during the period 07:31-09:31. Also, the constant traffic flow pattern identified for the first period of operations (00:10-05:40) reflects the night flying restrictions for the London airports (see Table 4).

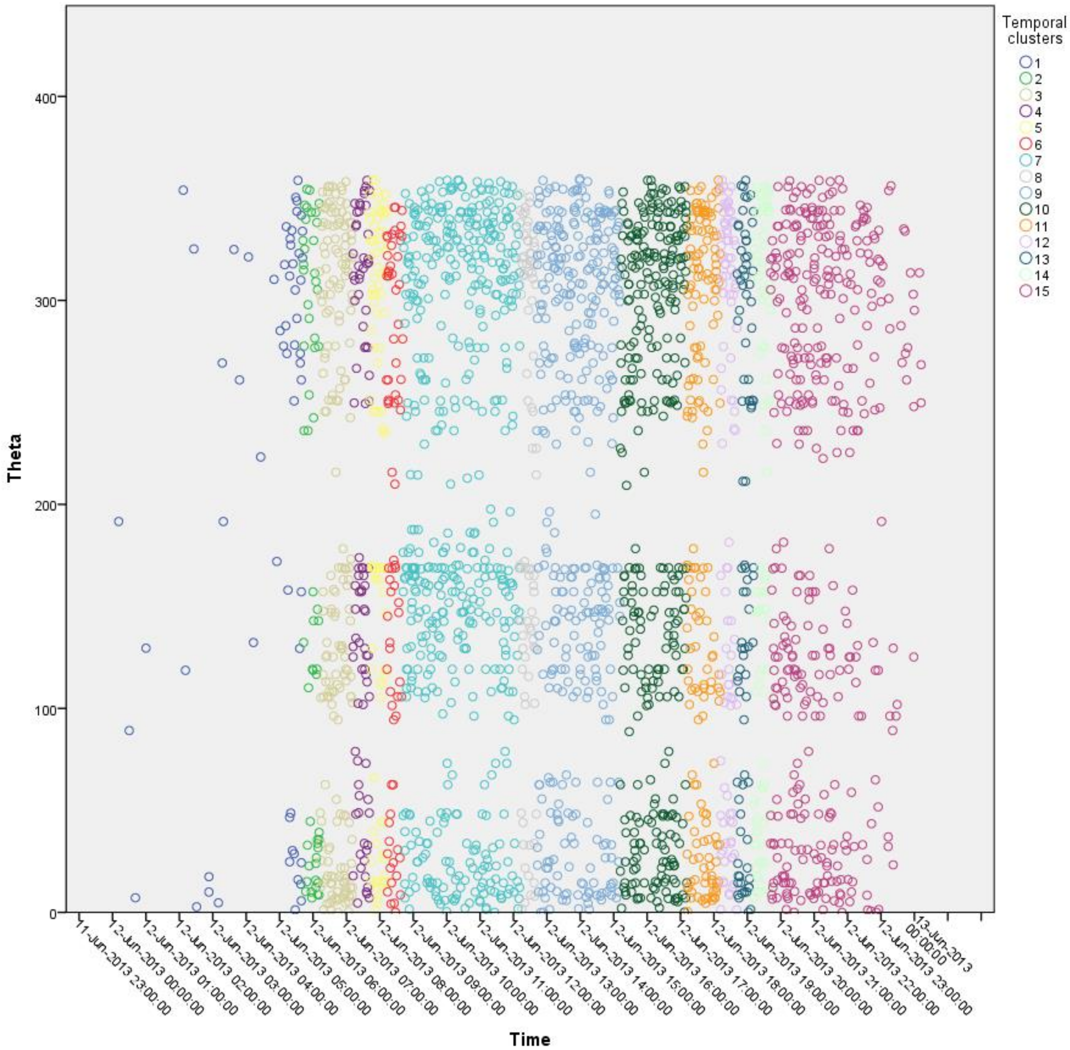

Figure 8 Temporal clustering results

Table 4 Temporal clustering results: Time windows of operations

Time Period	Start Time	End Time	Duration (hrs)
1	12/06/2013 00:11	12/06/2013 05:41	5.5
2	12/06/2013 05:42	12/06/2013 06:11	0.5
3	12/06/2013 06:12	12/06/2013 07:11	1
4	12/06/2013 07:12	12/06/2013 07:41	0.5
5	12/06/2013 07:42	12/06/2013 08:11	0.5
6	12/06/2013 08:12	12/06/2013 08:41	0.5
7	12/06/2013 08:42	12/06/2013 12:11	3.5
8	12/06/2013 12:12	12/06/2013 12:41	0.5
9	12/06/2013 12:42	12/06/2013 15:11	2.5
10	12/06/2013 15:12	12/06/2013 17:11	2
11	12/06/2013 17:12	12/06/2013 18:11	1
12	12/06/2013 18:12	12/06/2013 18:41	0.5
13	12/06/2013 18:42	12/06/2013 19:11	0.5
14	12/06/2013 19:12	12/06/2013 19:41	0.5
15	12/06/2013 19:42	13/06/2013 00:11	4.5

Once the temporal clusters are defined, the spatial clustering algorithm identifies the dynamic routes for each of the 15 time periods. The 7th time window was selected to display the spatial clustering, as it represents a significant traffic flow pattern during the peak period. The dynamic routes for this period are identified as described in section 5 and the results are presented in Figure 9 (for illustration purposes only Heathrow dynamic routes are shown).

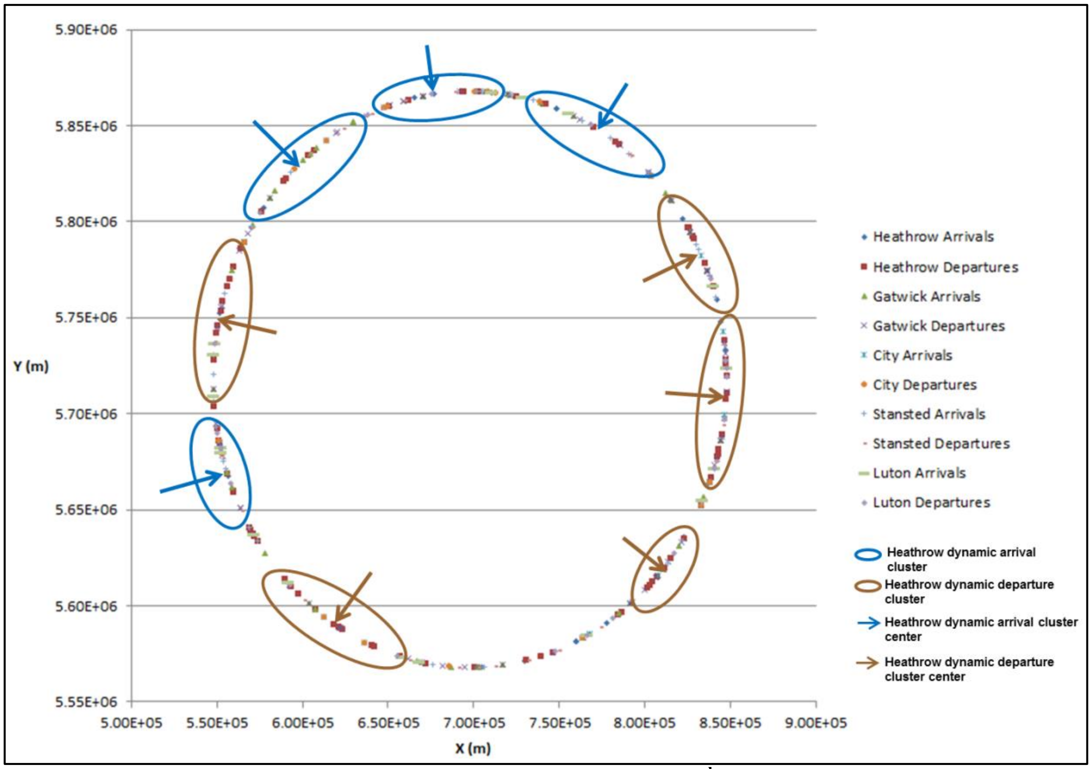

Figure 9 Heathrow dynamic arrival and departure routes for 7th time window of operations

C. AHP model results

The AHP model described in section C is applied here to derive the priorities of the dynamic routes for the 7th period. For this application, all airports were considered of equal priority, while the detailed assessment of the pairwise comparisons between the model criteria are presented in Figure 13 and Figure 14 of the Appendix. The pairwise comparison assessment was checked for consistency with C.R. values for the 2nd and 3rd level subcriteria equal to 0.05 and 0.04 respectively (< 0.10). To illustrate the full capability of the model, a set of appropriate values was assumed for the quantitative characteristics of the dynamic routes based on the total traffic flow data provided by Eurocontrol (see Table 5 and Table 6). The final priorities after the implementation of the AHP model, using the pairwise comparisons for each of the MAS criteria together with the quantitative characteristics of each route are presented in Figure 10. The normalized priorities are presented in the "Normal" column, while the "Ranking", as well as a "Graphic" representation for the route priorities are also available for each dynamic route alternative. Heathrow routes receive higher priorities as was expected due to higher demand characteristics. However, the priority order for the remaining dynamic routes (which are of similar magnitude) does not follow a specific pattern and it reflects the pairwise comparison judgements used for this application of the model (see Figure 13 and Figure 14 of the Appendix).

Table 5 Quantitative characteristics for dynamic routes 1-17

MAS airport		Heathrow							Gatwick								City	
Routes		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
Criteria	**Sub-critreria**																	
Aircraft Mix	No. of Light acft	5	7	4	8	5	5	8	2	0	4	0	1	4	4	0	0	0
	No. of Medium acft	25	30	18	24	30	25	15	8	10	11	8	12	16	9	15	5	4
	No. of Heavy acft	10	7	5	10	3	3	3	1	2	1	2	2	2	2	4	0	1
	No. of Super-Heavy acft	5	3	1	2	4	2	1	0	1	1	0	0	0	0	1	0	0
Passenger flow	Passenger flow	10050	9070	5280	8400	8350	6450	4180	1760	2900	2820	2040	2770	3520	2260	4400	900	1020
Type of ops	Arrival	1	1	1	0	0	0	0	1	1	1	1	0	0	0	0	1	1
	Departure	0	1	0	0	1	1	1	0	0	0	0	1	1	1	1	0	0
Flight category	No. of International flights	20	27	14	24	22	15	20	3	7	7	8	5	11	11	9	3	2
	No. of Domestic flights	25	20	14	20	20	20	7	8	6	10	2	10	11	4	11	2	3
Hub status	No. of airport connections	180	180	180	180	180	180	180	185	185	185	185	185	185	185	185	45	45
	Total flights	45	47	28	44	42	35	27	12	13	17	10	15	22	15	20	5	5

Table 6 Quantitative characteristics for dynamic routes 18-34

MAS airport		City				Stansted					Luton							
Routes		18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34
Criteria	**Sub-critreria**																	
Aircraft Mix	No. of Light acft	0	1	0	0	3	2	1	1	4	3	0	0	0	0	0	0	2
	No. of Medium acft	8	3	11	4	12	10	8	6	7	5	4	4	3	3	4	4	4
	No. of Heavy acft	2	1	0	1	4	6	0	3	0	0	0	0	0	1	0	1	0
	No. of Super-Heavy acft	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
Passenger flow	Passenger flow	2040	850	1980	1020	3890	3620	1450	1990	1300	930	720	720	540	840	720	1020	740
Type of ops	Arrival	1	0	0	0	1	1	0	0	0	1	1	1	1	0	0	0	0
	Departure	0	1	1	1	0	0	1	1	1	0	0	0	0	1	1	1	1
Flight category	No. of International flights	4	3	6	2	11	9	7	7	4	4	1	2	1	2	2	3	4
	No. of Domestic flights	6	2	5	3	9	9	2	3	7	4	3	2	2	2	2	2	2
Hub status	No. of airport connections	45	45	45	45	150	150	150	150	150	100	100	100	100	100	100	100	100
	Total flights	10	5	11	5	20	18	9	10	11	8	4	4	3	4	4	5	6

	Alternatives	Total	Normal	Ideal	Ranking	Graphic
Heathrow	Route1	0.0344	0.1033	1	1	
	Route2	0.0319	0.0958	0.927	2	
	Route3	0.0257	0.0772	0.748	3	
	Route4	0.0205	0.0615	0.595	5	
	Route5	0.0208	0.0625	0.605	4	
	Route6	0.0172	0.0517	0.5	6	
	Route7	0.0143	0.043	0.416	7	
Gatwick	Route8	0.0058	0.0175	0.169	18	
	Route9	0.0093	0.028	0.271	15	
	Route10	0.0093	0.028	0.271	14	
	Route11	0.0064	0.0191	0.184	17	
	Route12	0.0099	0.0298	0.289	12	
	Route13	0.0112	0.0335	0.324	10	
	Route14	0.0098	0.0294	0.285	13	
	Route15	0.0142	0.0425	0.412	9	
City	Route16	0.004	0.012	0.116	25	
	Route17	0.004	0.0121	0.118	24	
	Route18	0.0051	0.0154	0.149	22	
	Route19	0.004	0.0119	0.115	26	
	Route20	0.0052	0.0156	0.151	21	
	Route21	0.004	0.0121	0.118	23	
Stansted	Route22	0.0143	0.0429	0.416	8	
	Route23	0.0106	0.0317	0.307	11	
	Route24	0.0057	0.0171	0.165	20	
	Route25	0.0066	0.0198	0.192	16	
	Route26	0.0058	0.0173	0.167	19	
Luton	Route27	0.0027	0.0082	0.079	31	
	Route28	0.0022	0.0066	0.064	33	
	Route29	0.0022	0.0067	0.065	32	
	Route30	0.002	0.006	0.058	34	
	Route31	0.0037	0.011	0.106	28	
	Route32	0.0031	0.0093	0.09	30	
	Route33	0.0039	0.0117	0.113	27	
	Route34	0.0033	0.0099	0.096	29	

Figure 10 Final AHP route priorities (derived from Super Decisions software)

D. Priority-based selection of terminal fixes for the London MAS

Figure 11 presents the initial optimal (in terms of great circle shortest path) location of the terminal fixes for the dynamic routes of the 7th time period of operations. It is clear from the graph that there is a number of conflicts between neighboring fixes that do not meet the required minimum separation (a minimum separation of 20 km was selected for this application). The conflicting entry/exit points are reallocated based on their initial location and the derived priorities from the previous step, as described in section D. The final arrival and departure fix locations are presented in Figure 12.

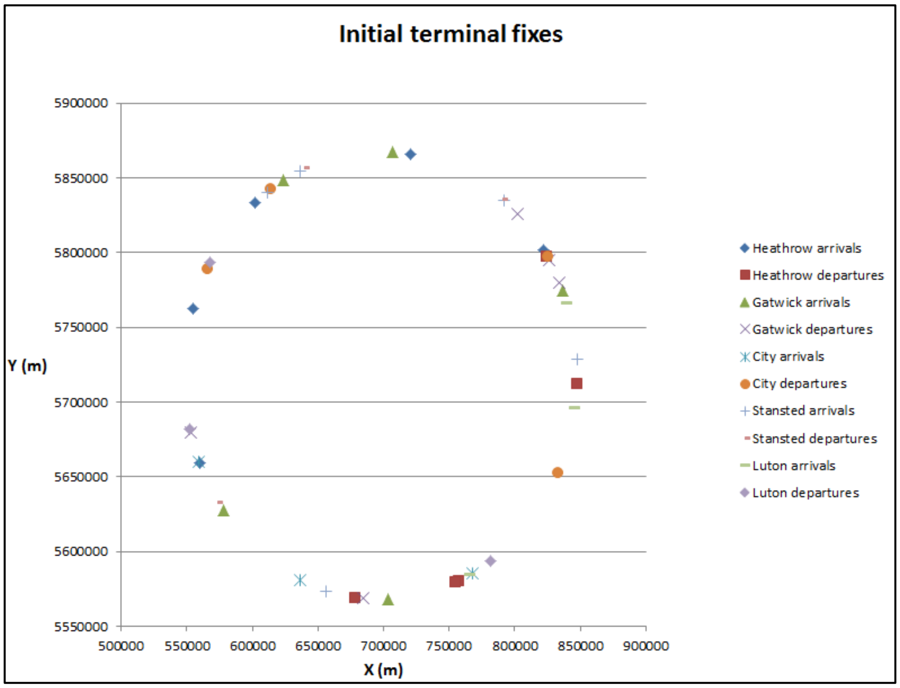

Figure 11 Intitial terminal fixes

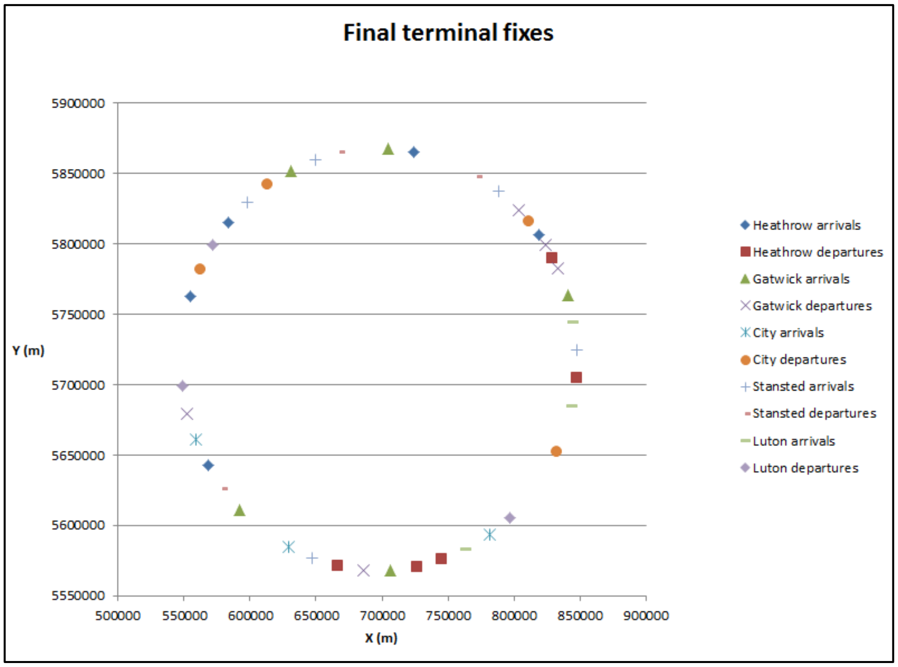

Figure 12 Final terminal fixes

VI. Discussion

The model presented in this paper aims towards the development of a refined CONOPs for MAS and is a proof-of-concept for the enabling of dynamic planning for the TMA airspace in such systems. It can be used in conjunction with existing models proposed in the literature (e.g. for the scheduling of flights) but also with systems that are currently in place at individual airports, such as AMAN and DMAN, while allowing for greater flexibility for decision makers in the planning of operations.

The individual aspects within the presented framework can be further expanded; in specific:

- The current route prioritization model goes as far as the allocation of terminal fixes for arrival and departure routes. Current research being undertaken by the authors explores further enhancements to the model to extend the planning of routes within the TMA and thus develop a holistic routing framework for MAS, while also considering environmental considerations of the operations.
- The AHP model can be revisited for the refinement of additional factors that affect MAS operations and may capture specific characteristics of unique MAS systems, such as Special Use Airspace (SUA) constraints that can in turn be integrated into the model (e.g. the Tokyo MAS TMA airspace is heavily constrained due to the operation of a nearby military airport).
- The AHP model can further be expanded to be used for the prioritization of routes during the en-route phase of the flight along with adequate criteria, especially within the concepts of "*corridors in the sky*" / Dynamic Airspace Super Sectors (DASS) [29] and "*highways in the sky*". [30]

The framework presented here can act as a strong spur work for concepts that are of utmost importance for the future ATM system resilience as described in the SESAR and NextGen programmes, namely DAC, 4-D Trajectory-based operations and AMAN / DMAN, ultimately leading to performance-based operations.

VII. Conclusion

MAS systems comprise a large part of the ATM network and as air traffic demand increases operations in such systems are expected to become more complex. According to the authors, the ad-hoc allocation of air traffic that is currently the rule for most Metroplex areas results in a sub-optimal utilization of the available airspace resources and is deemed to be insufficient to accommodate the future increase in demand. DAC for the planning of the operations in MAS can provide substantial benefit to the ATM system; it can enable decision making at the pre-tactical level and thereby make use of updated demand data to provide more efficient and effective route structures. This is in alignment with the dynamic nature of modern airspace and the needs of the airspace users (the airlines in particular), while maintaining the balance between the different stakeholders of the MAS. Finally, the results from this study show that by using a dynamic route service policy, fewer terminal fixes (varying between 28 and 42 for the different time periods of this application of the model) are required to accommodate the arrival and departure traffic flows compared to the approximately 60 terminal fixes that are currently in place for the SIDs and STARs of the different MAS airports. Along with deriving the adequate number and location of terminal fixes to accommodate the dynamic air traffic demand, individual flights are allocated to specific dynamic routes. This assists in automating and significantly alters the nature of the ATCo with regard to managing air traffic. This should help to reduce the workload of the controllers. The framework presented here ultimately leads to a more efficient use of terminal airspace resources that caters directly for dynamic demand, providing an airspace design that is best fit to the spatial and temporal traffic characteristics.

Appendix

Note 1: For the clustering in the spatial dimension in section B.2 the wrap-around effect needs to be considered for the circle. To this end, the average azimuth is calculated as:

$$\bar{az} = a\tan\left(\frac{\sum_{i=1}^{n}\left(\frac{\sin az_i}{\mathrm{n}}\right)}{\sum_{i=1}^{n}\left(\frac{\cos az_i}{\mathrm{n}}\right)}\right), \tag{9}$$

where *atan* is calculated for the four-quadrant inverse angle.

Note 2: All the coordinates of the framework application to the London Metroplex are in the 30U zone of the UTM projection (Geoid model: WGS84).

Note 3: The pairwise comparisons for the application of the AHP model are presented in Figure 13 and Figure 14:

	Heathrow	Gatwick	City	Stansted	Luton
Heathrow	1	1	1	1	1
Gatwick	1	1	1	1	1
City	1	1	1	1	1
Stansted	1	1	1	1	1
Luton	1	1	1	1	1

Airport pairwise comparisons

	Aircraft Mix	Passenger flow	Type of operation	Flight category	Airport status
Aircraft Mix	1	1/3	1/3	3	3
Passenger flow	3	1	1/3	5	3
Type of operation	3	3	1	5	5
Flight category	1/3	1/5	1/5	1	1
Airport status	1/3	1/3	1/5	1	1

Sub-criteria pairwise comparisons

C.R. = 0.05

Aircraft Mix Sub-criteria pairwise comparisons

	Light	Medium	Heavy	Super-Heavy
Light	1	3	5	7
Medium	1/3	1	1/3	1/5
Heavy	5	3	1	1/3
Super-Heavy	7	5	3	1

C.R. = 0.04

Figure 13 Criteria (airport), sub-criteria and aircraft mix sub-criteria pairwise comparisons

International / Domestic pairwise comparisons for each airport

Heathrow	Int	Dom
Int	1	1/2
Dom	2	1

Gatwick	Int	Dom
Int	1	3
Dom	1/3	1

City	Int	Dom
Int	1	5
Dom	1/5	1

Stansted	Int	Dom
Int	1	1
Dom	1	1

Luton	Int	Dom
Int	1	2
Dom	1/2	1

Arrivals / Departures pairwise comparisons for each airport

Heathrow	ARR	DEP
ARR	1	2
DEP	1/2	1

Gatwick	ARR	DEP
ARR	1	1/2
DEP	2	1

City	ARR	DEP
ARR	1	1
DEP	1	1

Stansted	ARR	DEP
ARR	1	1
DEP	1	1

Luton	ARR	DEP
ARR	1	1/2
DEP	2	1

Figure 14 Individual airport type of operations and flight category pairwise comparisons

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References

1. Sidiropoulos, S., Majumdar, A., Ochieng, W.Y. & Schuster, W., "Levels of Organization in Multi-Airport Systems", *Transportation Research Board 94th Annual Meeting*, Washington, DC, 2015.

2. Clarke, J., Ren, L., McClain, E., Schleicher, D., Timar, S., Saraf, A., Crisp, D., Gutterud, R., Laroza, R. & Thompson, T., "Evaluating Concepts for Metroplex Operations", *AIAA Aviation, Technology, Integration, and Operations Conference*, Fort Worth, Texas, 2010.

3. New York ARTCC, *Kennedy Air Traffic Control Tower, New York TRACON - Kennedy Area Standard Operating Procedures*, 2011.

4. Los Angeles World Airports, *South Airfield Improvement Project (SAIP)*, 2014.

5. NATS, *London Airspace Consultation*, NATS, 2013.

6. Bonnefoy, P., *Scalability of the Air Transportation System and Development of Multi airport Systems: A World Wide Perspective*, Massachussets Institute of Technology, 2008.

7. Eurocontrol, *ATM Masterplan*, 2nd Edition, 2012.

8. FAA, "NextGen Implementation Plan", *Washington, DC, Available: http://www.faa.gov/nextgen/media/ng2011implement ationplan.pdf (accessed March 18, 2011)*. 2010.

9. Pfeil, D.M., *Optimisation of Airport Terminal-Area Air Traffic Operations under Uncertain Weather Conditions*, MIT,2011.

10. Chen, J., Yousefi, A., Krishna, S., Sliney, B. & Smith, P., "Weather avoidance optimal routing for extended terminal airspace in support of Dynamic Airspace Configuration", *Digital Avionics Systems Conference (DASC), IEEE/AIAA 31st*, Williamsburg, VA, 2012, pp. 3A1.

11. Chen, J., Yousefi, A., Krishna, S., Wesely, D., Sliney, B. & Smith, P., "Integrated arrival and departure weather avoidance routing within extended terminal airspace", *Digital Avionics Systems Conference (DASC) IEEE/AIAA 32nd*, East Syracuse, New York, pp. 1A4.

12. Leiden, K. & Atkins, S., "Trajectory clustering for metroplex operations", *Proceedings of 11th AIAA Aviation Technology. Integration, and Oneration (ATIO) Conferenc*, Virginia Beach, VA, 2011.

13. Saturn project, "Future demand management for Europe: Meeting the airspace challenges. A workshop organised by the SESAR Workpackage E Saturn project.", London, 2015.

14. ICAO , *DOC 8643 Aircraft Type Designators*. Available: http://www.icao.int/publications/DOC8643/Pages/default.aspx , 2015.

15. Donaldson, A.D. & Hansman, R.J. 2011, *Improvement of terminal area capacity in the new york airspace*, Massachussets Institute of Technology, 2011.

16. FAA, *Optimization of Airspace Procedures in the Metroplex (OAPM)*. Available: https://www.faa.gov/air_traffic/flight_info/aeronav/procedures/oapm/ [2014.

17. Basseville, M. & Nikiforov, I.V., *Detection of abrupt changes: theory and application,* Prentice Hall Englewood Cliffs, 1993.

18. Daganzo, C.F., "The cell transmission model, part II: network traffic", *Transportation Research Part B: Methodological,* vol. 29, no. 2, pp. 79-93.

19. Tou, J.T. & Gonzalez, R.C., "Pattern recognition principles", 1974.

20. Tibshirani, R., Walther, G. & Hastie, T., "Estimating the number of clusters in a data set via the gap statistic", *Journal of the Royal Statistical Society: Series B (Statistical Methodology),* vol. 63, no. 2, 2001, pp. 411-423.

21. Saaty, T.L., "How to make a decision: the analytic hierarchy process", *European Journal of Operational Research*, vol. 48, no. 1, 1990, pp. 9-26.

22. Sattayaprasert, W., Hanaoka, S., Taneerananon, P., Pradhananga, R., "Creating a Risk-based Network for Hazmat Logistics by Route Prioritization with AHP: Case Study: Gasoline Logistics in Rayong, Thailand", *IATSS research,* vol. 32, no. 1, 2008, pp. 74-87.

23. Castelli, L. & Pellegrini, P., "An AHP analysis of air traffic management with target windows", *Journal of Air Transport Management*, vol. 17, no. 2, 2011, pp. 68-73.

24. Cong, H., Xuejun, Z., Kaiquan, C. & Qingge, P., "Improvements of tube design for trunk routes in China" *, 2011 IEEE International Conference on Vehicular Electronics and Safety (ICVES)* IEEE, Beijing, 2011, pp. 301.

25. Kyriakidis, M., *Developing a human performance railway operational index to enhance safety of railway operations*, Imperial College London, 2013.

26. Frederick, H. & Lieberman, G.J., "Introduction to operations research", *McGraw-Hill, New York,* 1982.

27. ICAO, Doc, I., "9613: Performance based navigation (PBN) manual", *Montréal: International Civil Aviation Organisation, 2008*.

28. Kotecha, P. & Hwang, I., "Optimization Based Tube Network Design for the Next Generation Air Transportation System (NextGen)", *AIAA Guidance, Navigation, and Control Conference*, Chicago, Illinois, 2009, pp. 10.

29. Alipio, J., Castro, P., Kaing, H., Shahid, N., Sherzai, O., Donohue, G. & Grundmann, K., "Dynamic airspace super sectors (DASS) as high-density highways in the sky for a new US air traffic management system", *Systems and Information Engineering Design Symposium, 2003 IEEE*, Charlottesville, VA, 2003, pp. 57.