Efficient Constellation Design Based on Improved Non-dominated Sorting Genetic Algorithm-II

Tengyue Mao, Zhengquan Xu, Rui Hou

Abstract—Constellation design is a typical multiple peaks, multiple valleys and non-linear multi-objective optimization problem. How to design satellite constellation is one of the key sectors of research in the aerospace field. In this paper, in order to improve the global convergence and diversity performance of traditional constellation optimization algorithm, multi-parent arithmetic crossover and SBX crossover operator of NSGA-II are used to improve searching capability of this algorithm. Meanwhile, Gaussian mutation and Cauchy mutation, with diversity of population, make the algorithm get better behaviors in convergence and diversity of finding solutions. Based on the methods, an improvement NSGA-II is presented to design constellation in the paper. The algorithm uses fixed length chromosome representation. Real coding is adopted for that the problem has both integer continuous variables. Combining the coverage assessment criterions, an orbit parameters optimization framework based on non-dominated sorting genetic algorithm (NSGA-II) was proposed. This method is applied to a detailed example, and result shows that a group of Pareto solutions with good spread can be achieved, which gives strong support to constellation scheme determination.

Index Terms—Multi-objective Optimization, Genetic Algorithm, Non-dominated Sorting Genetic Algorithm, Satellite Constellation

I. INTRODUCTION

LEO satellite constellation has played important practical role in our country and the prospect of its application is promising. LEO regional satellite constellation has important practical significance and wide application prospects in our country. The goal of constellation design goal is to find a set of constellation parameters, and enable constellations formed by these parameters in designated coverage area not only to obtain optimal constellation performance, but also to save cost in system construction and long-term maintenance costs of constellation [1, 2]. Satellite constellation design includes selecting satellite type (satellite altitude, inclination, eccentricity, etc.), and determining the number of satellites and configurations in the orbital plane so as to achieve the best navigation, positioning and timing.

In recent decades, foreign countries have undertaken a number of optimized designs based on satellite application system of constellation. Some researchers like Castile have discussed how to optimize coverage rate of constellations with the aid of elliptical orbit, and Eric has studied how to optimize the Earth observation of constellations with the help of the improved genetic algorithm. Results showed that compared with the traditional constellation “walker”, optimized design has improved the average response time of constellations. Through simulated annealing, researchers like Mallory have discussed the optimization of orbital parameters that are applied to GEO constellation to interfere survey. Researchers like Munson have explored strategy selection of constellation launch. Munson first established an alternative database of vehicles, and then combined with the number of constellation, orbital dynamics and available constraints so that a mathematical model of launch strategy selection was established and branch-and-bound algorithm was employed for solutions. Wang made use of analytical method to study how to reduce mutual interferences among the satellites in non-geostationary orbit (NGSO) constellations. In comparison with the method based on numerical simulation, the advantage of...
analytical method lies in its less computation cost. Researchers like Nizar introduced lifecycle cost estimate of a constellation and an optimization tool—SCORPIO, which with the aid of historical data of hundreds of items in the past three decades, employed regression analysis to establish a model of cost analysis, and made use of the lumped parameter optimization to predict cost and risk of aerospace project. Researchers like Asvial have explored how to use the genetic algorithm to undertake multi-day optimization for NGSO constellation. The goal of optimization is to use the minimum satellites to optimize angle shift among satellites and orbital inclination angle.

In China, Liu Huijie researched constellation optimal design and performance analysis of regional satellite positioning system, proposed constellation design method of two-step optimization and conducted optimal design and performance analysis of various constellations like MEO, IGSO, GEO / MEO. Li Zan studied constellation of satellite mobile communication system and inter-satellite link design, with the help of analytical method built the relationship between inter-satellite transmission power and beam width under the vibration, and established an information exchange routing table of constellation which is composed of eight stars. Starting from large system cybernetics, Xiang Kaiheng has explored maintenance of absolute and relative constellation positioning, pointed out that the improvement direction of maintenance strategy of constellation positioning is hierarchical control strategy relative positioning maintains, and according to sub-level coordinated control theory proposed hierarchy control strategy walker-6 constellation positioning maintains, including deciding the algorithm of referred constellation based on optimization principles. Zhang Fan studied optimization design of optical remote sensing small satellite constellation, selected the overall program optimally through the fuzzy multi-objective optimization, and with the help of two-tier multi-disciplinary and multi-objective optimization founded overall optimization model of constellation in which cost, performance, efficiency-cost ratio and reliability have been taken seriously and designed optimally.

The design of regional coverage constellation which involves a number of feature points and optimized indexes is a typical multi-objective optimization problem. The key of design lies in the mutual trade-offs between the structural deployment and performance of the entire constellation. Always based on the overall objectives and constraints, its analysis and design use the lowest cost to meet these requirements and constraints. Global constellation coverage problem has mature design methods, but until now there is yet lack of general and classic design for regional coverage constellation design [3, 4]. In recent years, genetic algorithm has been introduced to the constellation design.

Genetic algorithm is a self-organization and adaptive artificial intelligence technology that simulates the process and mechanism of natural evolution, solve optimization and search problems. Because it is the evolution and operation of the whole groups, the genetic algorithm focuses on a collection of individuals, while the non-inferior solution of the multi-objective optimization problem is also a collection. This feature of genetic algorithm shows that genetic algorithm this feature is ideal for multi-objective optimization problem [5, 6, 7]. This paper mainly researches non-dominated sorting genetic algorithm and its improved algorithm NSGA-II, the algorithm which develops faster and has better effects of optimization.

With the development of space technology, constellation which is made of multi-satellites has played an increasingly important role in the many fields such as communications and navigation. Since the requirement of the constellation coverage performance is the main basis for design, this paper explores the constellation coverage, and then uses the improving INSGA-II algorithm to achieve the design of satellite constellation. The result shows that a group of Pareto solutions with good spread can be achieved, which gives strong support to constellation scheme determination. Your goal is to simulate the usual appearance of papers in a Journal of the Academy Publisher. We are requesting that you follow these guidelines as closely as possible.

II. SYSTEM MODEL

A. System Description

In order to meet the diversity and stability of the constellation constitutes, the orbital elements of the satellite constellation can be restricted as follows: 1) the satellite constellations should have the same orbital shape, that is to say, the same eccentricity and semi-major axis; 2) the angle of perigee satellites at the same orbital plane is the same; 3) the orbital inclination and the ascending node of right ascension at different orbital plane is preferably determined; 4) the relative position of satellites in the same orbit is preferably determined.

Suppose constellation has N orbital planes in total, with which the number of satellites is \( Q_j (j = 1, 2, \cdots, N) \), the total number of satellites in the constellation is \( T = \sum_{j=1}^{N} Q_j \). According to the above constraints, from the optimization According to the above constraints, the parameters including \( i_j, \omega_j, \Omega_j, M_{0j} (j = 1, 2, \cdots, N) \), need optimizing from the angle of the optimization. The model shown in “Fig. 1” is a simplified schema of satellite constellation. Although simplified, its universe still exists which is the basis of simulation and optimization of the subsequent constellation.

B. LEO satellite constellation parameters

LEO satellite constellation constrains orbital altitude and geometry of satellites. When the orbital inclination is not to 64.43° or 116.57° due to the impact of earth's gravity model items, the elliptical orbit will produce the drift of an arch point. Elliptical orbit is suitable for the high-latitude regions. At present, LEO constellation usually employs a circular orbit, such as the Globalstar system, Iridium system, Orbcomm system and so on. The
system choose circular orbit, eccentricity is e = 0. At this time, argument of perigee is \( \omega = 0 \).

When orbit altitude of LEO satellite is below 2 000 km, the height of LEO satellite not only affects the coverage performance, but also the life expectancy of the system, communication performance and so on. When orbit altitude of LEO satellite is below 700 km, the satellite is greatly influenced by air drag and the corrosion of oxygen atom and is short-lived. When the height of ranges from 1 500 to 5 000 km, there exists Van Allen belt which has highly destructive results on the stars so that satellite orbit should avoid this height. In addition, from the perspective of expectation, it is expected that track of sub-satellite point of satellite have periodic repetition so as to analyze constellation coverage easily. Based on the above mentioned, satellite will have relatively large coverage area on the ground if an orbital altitude is 1 450 km and located below the edge of Van Allen belt. Meantime, the satellite with this orbital altitude has 2-day return cycle, and the number of return cycle is 25 laps. So the orbital parameters can be determined: semi-major axis \( a = (1450 + R_e) \text{ km} \).

III. PROPOSED EFFICIENT CONSTELLATION DESIGN

To great degree, NSGA-II has improved NSGA's shortcomings, but the performances of the adopted SBX (Simulated Binary Crossover) crossover and mutation operators are relatively weak so that it limits the search performance of the algorithm to some extent, making diversity and convergence of population unsatisfactory and the distribution of solution not ideal when solving high dimension [8]. Based on the shortcomings of NSGA-II, we propose an improved NSGA-II algorithm -- NSGA-II algorithm.

A. The Improvement of Crossover Operator
a. SBX Operator

NSGA-II has used SBX crossover operator. SBX operator simulates the process of binary crossover operator and executes cross-operation on real-coded paternal individuals. To put it differently, in view of given random intersections, exchange the parts of paternal individual on both sides of intersections and enable relevant information model of paternal chromosomes to be protected in the offspring.

The following is the process in which t substitute’s individual \( x_i^{(1,t)} \), \( x_i^{(2,t)} \) and produces \( t+1 \) to take place of \( x_i^{(1,t+1)} \), \( x_i^{(2,t+1)} \).

1) Take a random number:
\[
\mu_i \in [0,1)
\]

2) Calculate evenly distributed factors:
\[
\beta_i = \begin{cases} 
\frac{\mu_i}{\mu_i} & \text{if } \mu_i \leq 0.5 \\
\frac{1-\mu_i}{1-\mu_i} & \text{otherwise}
\end{cases}
\]

According to the following probability density function, make the area ranging from 0 to \( \beta_i \), below the curve of the probability density equal to \( \mu_i \) so that \( \beta_i \) can be calculated.

3) \( P(\beta_i) = \begin{cases} 
0.5(\eta+1)\beta_i^\eta & \beta_i \leq 0.5 \\
0.5(\eta+1)\beta_i^{\eta-1} & \beta_i > 0.5
\end{cases} \)

Here the parameter \( \eta \) is a non-negative real number and defined by itself when being used, which is called cross-distribution index. As value of \( \eta \) becomes greater, there is more probability of generating solutions close to paternal individuals. On the contrary, smaller value may generate individual far from paternal individual. The above equation can be used to calculate the value of \( \beta_i \), as in (4).

4) \( \beta_{qi} = \begin{cases} 
\frac{1}{(2\mu_i)^{1\eta}} & \mu_i \leq 0.5 \\
\frac{1}{(1-\mu_i)^{1\eta}} & \mu_i > 0.5
\end{cases} \)

5) Calculate offspring individual:
\[
x_{i}^{(1,t+1)} = 0.5[(1 + \beta_{pi})x_i^{(1,t)} + (1 - \beta_{pi})x_i^{(2,t)}] \\
x_{i}^{(2,t+1)} = 0.5[(1 - \beta_{pi})x_i^{(1,t)} + (1 + \beta_{pi})x_i^{(2,t)}]
\]

b. Crossover Operator of Multi-Parent Arithmetic

In this paper, we introduce crossover operator of multi-parent arithmetic into NSGA-II. The crossover way of multi-parent arithmetic is as follows: m individuals are involved in every crossover and m new individuals are obtained. The operating method of getting multi-parent arithmetic crossover through the principle of two individuals’ crossover is as follows. Corresponding to selected m paternal individuals \( x_i^m (1 \leq i \leq m) \), every time extract m random numbers \( p_1, p_2, \ldots, p_m \) from the even distribution between 0 and 1. \( p_m \) is generated m numbers
so that generated m new individuals should be: 
$$x_i^{(n+1)} = \frac{1}{m} \sum_{j=1}^{m} p(x_j)/m, i = 1, 2, \ldots, m.$$ 

Since the number that evenly distributed random sequence $p_1, p_2, \ldots, p_m$ may extract is infinitive, the number that generated new individuals may extract is also infinitive.

b. The Improvement of Mutation Operator

a. Gaussian Mutation

Mutation operator mainly brings new genes to population in order to restore diversity of individual that is lost due to selecting functions of operators.

1) Gaussian mutation: introduce Gaussian mutation into NSGA-II.

$$\phi(x) = \frac{1}{\sqrt{2\pi} \sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}, -\infty < x < +\infty$$ \hspace{1cm} (6)

C. The Introduction of SA

SA (Simulated Annealing) is an optimization algorithm that simulates the process in which metal materials anneal. This algorithm may not only obtain design variables randomly, but also accept the solution mechanism which is worse than the current solution. As a result, it is likely to jump out of local optimum. Statistical mechanics shows that below temperature $T$, the probability of an atom in the state of I belongs to Boltzmann probability distribution:

$$P[E = E(i)] = \frac{1}{Z(T)} \exp(-\frac{E(i)}{k_B T}).$$ \hspace{1cm} (10)

In this equation, $E(i)$ is the energy of atoms at the state of $i$, $E$ is the possible value. $k_B > 0$ is Boltzmann constant. $Z(T)$ is the normalization factor of probability distribution:

$$Z(T) = \sum_{i=0}^{\infty} \exp(-\frac{E(i)}{k_B T}).$$ \hspace{1cm} (11)

SA method is a single-point search method. In order to simulate the state of metal objects under some temperature, the algorithm selected a number of points to simulate the different energy states in which metal atoms in the current temperature may be. In reality, algorithm employs Metropolis sampling to complete the extraction process. In this algorithm the new values of design variables can be obtained in the way the current value plus a random number. It accepted both the better new value of the objective function and the worse value through probability $p$.

$$p = \exp(-f(j) - f(i)) < \xi.$$ \hspace{1cm} (12)

Where $f(j)$ is target value of new state, $f(i)$ is current target value, and current value is less than new value. $\xi$ is a random number between 0 and 1. SA can be regarded as using Metropolis sampling iteratively at different temperatures and ultimately finding the optimal solution. Generate random number. If the probability is less than Metropolis probability, places accept solutions whose crowding distances are relatively small through a certain probability. Conversely, solutions whose crowding distances are large are accepted.

IV. SIMULATION RESULTS

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Combined the structure of chromosomes optimized by constellation, feature points of the target area are defined as $F_1(112.25, 36.05), F_2(112.48, 29.94), F_3(112.48, 24.01)$, whose selected coverage performance indicator is coverage rate, requiring the constellation has the largest coverage for three feature points. The whole process of simulation and optimization has taken $J_1$ perturbation into account. Simulation period is one day (24 * 3600s), and simulation step is 30s. Population size is set as 50, and algebra is set 100.
At the end of experiment, we obtain Pareto optimal solutions of a set of low-orbit constellations and coverage rate of three sampling spots. Figures 2 and 3 are respectively working diagram of constellation and track diagram of sub-satellite point of all satellites in constellations during the simulation.

As experimental results in Li Sudan’s literature [9] are calculated by STK [10-13], this paper selects three sets of data Pareto optimal solutions focus on, shown in Table 2, Table 1 shows the experimental results in Li Sudan’s literature Table 3 suggests the experimental results obtained by Li Xiaomeng [14] using improved SPEA2 to solve constellation optimization. Table 4 shows the experimental results obtained by Zheng Wei [15] using OMEA to solve constellation optimization.

Compare the experimental results in Table 1 and Table 2, we can find that the data in this paper is superior to those in Li Sudan’s literature. Li Sudan makes use of NSGA-II algorithm, while we use NSGA-II algorithm (INSGA-II) that is improved in Chapter III of this paper. INSGA-II is mainly improved from diversity of population and equal distribution of solutions, uses hybrid cross and hybrid variability, and introduces simulated annealing algorithm into NSGA-II algorithm so as to constitute a hybrid genetic algorithm.

Experimental data show that the results obtained through improved NSGA-II can better meet the test requirements so as to achieve the expected goals. Obviously, in comparison with the result in Li Sudan’s literature [1], solutions obtained though improved NSGA-II (INSGA-II) are much closer to ideal pareto frontier of constellation optimization. At the same time, through analysis of tractor factors of constellation in Table 2, we can draw the following features:
(1) In the same group of satellite constellation data, the orbital inclinations of two orbital planes get very close to each other.
(2) In the same group of satellite constellation data, right ascension difference between the ascending nodes of two orbital planes is about 180 °.

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<th>Table 3</th>
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Compare the experimental results among Table 2, Table 3 and Table 4, our results are as follows:

(1) The results in Table 4 are 100%, the best results. Table 4 uses OMEA algorithm: it first establish the model through digging distribution rules of population Pareto solutions, then search globally and locally, and making use of orthogonal and key technologies such as k-means and pca in the algorithm. However, OMEA need cluster analysis in evolution of each generation, and spend a large amount of time in the process of stabiling probability model for cluster.

(2) Table 3 uses SPEA2+ algorithm, so optimization results do not reach the global optimum. [1] Probably it is because truncation technology of SPEA2 will get rid of a part of same individuals so that diversity of population decreases. [2] Probably it is because we use Gaussian mutation operator so that search ability of SPEA2 for non-continuous Pareto solutions frontier decreases and falls into local optimum.

(3) Table 2 shows the experiment results calculated by improved NSGA-II algorithm (INSGA-II), and coverage rate of three sampling spots can not reach 100%. The reasons may be as follows: Firstly, when individuals are generated at the time of initialization, groups are not designed, and individuals produced at the time of initialization can not be well distributed in the feasible region, so the efficiency is lower; secondly, the algorithm is mainly improved from diversity of population and equal distribution of solutions. The search performance of population has not been considered comprehensively, and convergence of the algorithm needs to be improved.

V. Conclusion

In this paper, in order to improve the global convergence and diversity performance of traditional constellation optimization algorithm, multi-parent arithmetic crossover and SBX crossover operator of NSGA-II are used to improve searching capability of this algorithm. Meanwhile, Gaussian mutation and Cauchy mutation, with diversity of population, make the algorithm get better behaviors in convergence and diversity of finding solutions. Based on the methods, an improvement NSGA-II is presented to design constellation in the paper. The algorithm uses fixed length chromosome representation. Real coding is adopted for that the problem has both integer continuous variables. Combining the coverage assessment criterions, an orbit parameters optimization framework based on non-dominated sorting genetic algorithm (NSGA-II) was proposed. This method is applied to a detailed example, and result shows a group of Pareto solutions with good spread can be achieved, which gives strong support to constellation scheme determination.

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References


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