Transmit Beamforming for MISO Frequency-Selective Channels with Per-Antenna Power Constraint and Limited-Rate Feedback

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Abstract—In this paper, we consider transmit beamforming (BF) design under per-antenna power constraint (PPC) for MISO frequency selective channels. Both cyclic prefixed (CP) single carriers (SC) and orthogonal frequency-division multiplexing (OFDM) systems are investigated. Since there is no closed-form expression for the optimal PPC BF coefficients and the optimization tools employed to find the PPC solution are usually complicated, we here propose three computationally more effective suboptimal solutions to minimize the arithmetic mean of the effective error probabilities (ARITH-EEP). Moreover, we address the issue of limited-rate feedback and a simple codebook design method for frequency selective channels is proposed. Unlike the conventional methods where the codebook is designed for the BF vectors directly, we here construct codebooks for the amplitude and phase of the time domain channel state information (CSI) to reduce the rate of feedback. Further, the non-uniform feedback strategy is investigated. Simulation results show that the proposed suboptimal solutions have much lower complexities than the convex optimization based PPC solutions. The effectiveness of the proposed uniform and non-uniform limited-rate feedback methods is also demonstrated via simulations.

Index Terms—Transmit beamforming, cyclic prefix, MISO, frequency-selective channels, limited-rate feedback

I. INTRODUCTION

Transmit beamforming (BF) has attracted much attention recently. By using a partial or full knowledge of channel state information (CSI) at the transmitter, transmit BF can greatly enhance energy efficiency in wireless communications [1]. Optimal BF design under the total power constraint (TPC) has been studied for Multiple-input-multiple-output (MIMO) and MIMO OFDM systems (see e.g. [2]–[6] and references therein). However, TPC may not be practically appropriate from the high-power amplifier design perspective since the powers allocated to different antennas may considerably vary over time [7]. Moreover, in the context of cooperative BF, where transmitters are not collocated, TPC is not desirable since it may drain the energy of some nodes too quickly, thus reducing the network lifetime of distributed BF systems. Therefore, designing BF with the per-antenna power constraint (PPC) is well motivated. This has been considered in [7]–[9] for single user MIMO systems and in [10], [11] for multiuser MIMO systems. However, only flat fading channels were considered in those works. Using OFDM modulation or SC frequency-domain equalization (FDE) schemes, existing PPC algorithms can be straightforwardly extended to frequency selective channels. However, optimality of these conventional PPC methods is not guaranteed.

Reformulating the PPC optimization problem as a semidefinite programming (SDP) problem, a class of semidefinite relaxation (SDR) algorithms, e.g. CVX [12] or SeDuMi [13], can be adapted to solve the optimization problem [14]. However, the complexity of these methods is increased significantly with the number of subcarriers. This drawback may make the SDR algorithms inappropriate for multi-antenna OFDM systems. In [15], we investigated the BF design for cyclic prefixed single carrier (SC) and OFDM transmissions over frequency selective channels. Optimal TPC solutions were given as closed-form expressions and an interior-point method was adopted to find the PPC solutions. However, solving the Karush-Kuhn-Tucker (KKT) system in the PPC optimization requires a large scale matrix inversion which may be too cumbersome as the number of transmitters or/and the number of subcarriers increases. Moreover, the Logarithmic barrier function added to approximate the implicit constraint in the interior-point method decreases the accuracy of optimization, especially for OFDM systems at high SNR, since the value of the original cost function is much smaller than that of the added Logarithmic barrier function. In this paper, to reduce the design complexity under PPC, we first consider independent frequency BF designs for the different antennas, then extend the cyclic algorithm (CA) studied in [7] to frequency selective channels and finally propose a normalization method. All these suboptimal methods can be obtained in closed-form and have lower complexity compared to the interior-point method in [15].

Considering the fact that perfect knowledge of the CSI at the transmitter is unrealistic, we extend our design of transmit BF to cater for the limited-rate feedback channel. Many effective feedback methods for BF systems have been studied in [7], [9], [16], [17] and references therein. However, most of the existing feedback designs only consider frequency flat-fading channels.
Although those feedback design techniques can be easily extended to frequency-selective channels by employing OFDM as discussed in [18], the feedback requirements generally grow proportionally with the number of active subcarriers. To reduce the feedback, interpolation based feedback design was proposed in [19]. However, since the power was allocated equally across all subcarriers in [19], the proposed spherical interpolator cannot be used in our scenario. Another way to keep a low feedback rate is applying a cluster based BF as studied in [20], where a single BF vector is used for a cluster of subcarriers. However, the performance of this method deteriorates when the channel becomes more frequency-selective or the cluster size is increased. Unlike existing low rate feedback methods, in this paper, we formulate the problem as a vector quantization (VQ) of the time domain CSI instead of the BF vectors themselves. The idea of time domain CSI feedback can also be found in [21] for MIMO OFDM eigenbeam-space division multiplexing systems. However, no detail of feedback design was discussed in [21]. Similar time domain CSI quantization was studied in [22] for multiuser MIMO broadcast systems, where the complex channel is decoupled into real and imaginary parts. However, the quantization method adopted in [22] is scalar based and a sign bit is needed for the real and imaginary parts. In this paper, we divide the time domain CSI into two parts, i.e. amplitude and phase, and design two codebooks by employing the generalized Lloyd algorithm (GLA). After receiving the feedback information, the transmitter first recovers the quantized time domain CSI and then calculates the BF vectors by employing our PPC BF design algorithms. Since errors due phase and amplitude quantizations have different effects on performance, non-uniform feedback design is also investigated in this paper. Simulation results show that the proposed BF design algorithms combined with our limited-rate feedback method can have a close-ideal performance.

We note that time-domain (TD) transmit BF has been investigated in different contexts before, e.g. [23] and [6] for MIMO and MIMO OFDM systems, respectively. Since the BF vectors are designed in the time-domain, the feedback requirement can be reduced compared to conventional frequency-domain BF. However, as shown in [6], the low feedback rate is achieved at the cost of decreased performance. Moreover, the low rate TD BF vectors can only be obtained by numerical methods. Further, to the best of our knowledge, PPC has not been considered for TD BF design. In this paper, we combine the benefits of frequency-domain BF and time-domain CSI feedback. Hence, both good performance and low feedback rate properties are preserved. In particular, we will show that our feedback method works well under PPC condition. It is also worth pointing out that random BF is not applicable to our single user scenario; random BF was shown in [24] and references therein to achieve maximum capacity with limited-rate feedback in the multiuser context.

The rest of this paper is organized as follows. Section II presents the system model of CP-based transmit BF. The optimal TPC and three suboptimal PPC BF solutions are given for the infinite-accuracy feedback scenario in Section III. The limited-rate feedback design is considered in Section IV and verified via simulation in Section V. Finally, some conclusions are drawn in Section VI.

II. SYSTEM MODEL

Consider a transmit BF system with \( M \) antennas at the transmitter and a single antenna at the receiver as shown in Fig. 1. Such a scenario may occur in down-link cellular transmissions and cooperative communication. To simplify receiver complexity, we consider transmissions with a cyclic prefix (CP) and frequency domain BF. We also assume that the length of CP is larger than the maximum channel delay-spread \( L \). After removing the CP, the received signal at the receiver can be written as

\[
y = \sum_{i=1}^{M} H_i \theta_i s + v
\]  
(1)

where \( s = [s(0), \ldots, s(N-1)]^T \) is the data vector; \( H_i \) is an \((N \times N)\) circulant matrix whose first column is \([h_i(0), \ldots, h_i(L-1), 0, \ldots, 0]^T\). \( \theta_i \) is a BF matrix and \( v \) is an additive white Gaussian noise (AWGN) vector with correlation matrix \( \sigma_v^2 I_N \) with \( I_N \) denoting the \((N \times N)\) identity matrix. After equalization, the obtained signal can be expressed as

\[
r = G y = G \sum_{i=1}^{M} H_i \theta_i s + G v
\]  
(2)

where \( G \) denotes the equalization matrix.

A. OFDM Systems

The BF matrix in eq. (1) for OFDM systems can be expressed as

\[
\theta_i = F^H W_i
\]  
(3)

where \( F = \frac{1}{\sqrt{N}} \left[ e^{-j2\pi mn/N} \right]_{m,n=0}^{N-1} \) is the DFT matrix and \( W_i = \text{diag}\{w_i(0), \ldots, w_i(N-1)\} \). Since \( H_i \) is circulant, it can be written as \( H_i = F^H H_i F \), where \( H_i = \text{diag}\{H_i(0), \ldots, H_i(N-1)\} \) with \( H_i(k) = \sum_{l=0}^{L-1} h_i(l) e^{-j2\pi kl/N} \).

For zero-forcing (ZF) equalization, \( G \) is given by

\[
G = D F
\]  
(4)

where \( D = \sum_{i=1}^{M} H_i W_i \) and \((\cdot)^d\) denotes the pseudo-inverse operation. The decision-point SNR of the \( k \)th subcarrier is given by

\[
\gamma_{\text{oldm}}(k) = \frac{\sigma^2}{\sigma_p^2} \left| \sum_{i=1}^{M} H_i(k) w_i(k) \right|^2
\]  
(5)
where \( \sigma_s^2 = E \{|s(k)|^2\} \) with \( E \{\cdot\} \) denoting the statistical expectation operator.

The MMSE equalization matrix \( G \) can be written as
\[
G = D^H \left( DD^H + \frac{\sigma_s^2}{\sigma_d^2} I_N \right)^{-1} F
\]  
(6)

Since ZF and MMSE equalizations have the same performance for OFDM when data are drawn from constant amplitude constellations, we omit the analysis of MMSE equalization for OFDM in what follows.

B. Single carrier systems

The BF matrix in eq. (1) for SC systems can be expressed as
\[
\theta_i = F^H W_i F
\]  
(7)

where \( W_i = \text{diag}\{w_i(0), \ldots, w_i(N-1)\} \). The ZF matrix \( G \) can be written as
\[
G = F^H D^H F
\]  
(8)

The decision-point SNR of the \( k \)th data symbol is given by
\[
\gamma_{\text{sc}}(k) = \frac{N\sigma_s^2}{\sigma_d^2 \sum_{k=0}^{N-1} |H(k)w_i(k)|^2}
\]  
(9)

For MMSE equalization, matrix \( G \) can be written as
\[
G = F^H D^H \left( DD^H + \frac{\sigma_s^2}{\sigma_d^2} I_N \right)^{-1} F
\]  
(10)

As studied in [15], the ZF equalization has almost the same performance as MMSE equalization for optimal transmit BF systems when the number of transmit antennas is larger than one. Hence, we also omit the analysis of MMSE equalization for SC in this paper. The results obtained later can be easily extended to MMSE equalization.

III. Perfect CSI-based Beamforming

In this section, we assume that the transmitter has perfect knowledge of CSI. The BF matrices are designed first under the TPC, and then under the PPC. Among various design criteria discussed in [2], the minimization of the arithmetic mean of the effective error probability (ARITH-EEP) was argued to be the best criterion in terms of the average bit error ratio (BER). We also adopt ARITH-EEP as our design criterion instead of using the signal-to-noise ratio (SNR) adopted in [7]. The ARITH-EEP is defined as
\[
P_{\text{eff}} = \frac{1}{N} \sum_{n=0}^{N-1} P_{\epsilon}(n)
\]  
(11)

where \( P_{\epsilon}(n) \) is the probability of error associated with the \( n \)th data symbol (subcarrier). Under the AWGN assumption, \( P_{\epsilon}(n) \) can be expressed as \( P_{\epsilon}(n) = Q(\sqrt{\alpha_n \gamma(n)}) \), where \( Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-u^2/2} du \) is the Q-function, \( \alpha_n \) depends on the modulation scheme applied to each data (subcarrier) and \( \gamma(n) \) is the decision-point SNR. As pointed out in [2], it is too cumbersome to solve the minimization problem (11) directly. An approximated solution can be found by applying the Chernoff upper bound to \( P_{\epsilon}(k) \). The Chernoff upper bound is \( Q(x) \leq e^{-x^2/2} \).

A. Total power constraint

We assume that the subcarriers (data symbols) use the same modulation scheme and \( \sigma_s^2 = 1 \). Using the Chernoff upper bound, the minimization problem under TPC can be approximated as
\[
\text{minimize} : f(w) := \frac{1}{N} \sum_{k=0}^{N-1} e^{-\alpha_k \gamma(k)/2}
\]  
(12)

subject to : \( \sum_{i=1}^{M} \sum_{k=0}^{N-1} |w_i(k)|^2 \leq \mathbb{P}_t \)  
(13)

where \( \mathbb{P}_t \) denotes the total power allocated to \( M \) transmit antennas, \( w = [w_1^T, \ldots, w_M^T]^T \) and \( w_i = [w_i(0), \ldots, w_i(N - 1)]^T \). It is easy to see that the constraint should be met with equality at the optimum BF coefficients. Thus, we consider the equality constraint for the TPC scenario.

1) OFDM systems: Replacing \( \gamma(k) \) in eq. (12) with \( \gamma_{\text{ofdm}}(k) \) and using the Lagrange multipliers method, the optimal BF coefficients for OFDM systems are found to be
\[
w_{1}(k)_{\text{ofdm}} = e^{j\angle H_1(k)} \sqrt{P_t(k)_{\text{ofdm}}}
\]  
(14)

where \( \angle a \in (-\pi, \pi) \) denotes the phase of \( a \) and
\[
P_{1}(k)_{\text{ofdm}} = \frac{|H_1(k)|^2}{\beta t^2(k)} \left( \ln t(k) - \frac{\sum_{k=0}^{N-1} \ln t(k)}{t(k)} - \beta \frac{\mathbb{P}_t}{t(k)} \right)^{+}
\]  
(15)

where \( t(k) = \sum_{i=1}^{M} |H_i(k)|^2 \), \( \beta = \alpha_k \sigma_s^2/(2\sigma_d^2) \) and \( (\cdot)^+ \) denotes the operation \( \max\{x, 0\} \). Similar results can be found in [3].

It is worth pointing out that a phase rotation sequence, which must be the same across the antennas, can be added without altering the BF performance, if necessary; i.e. using \( e^{j\angle H_1^*(k)+\phi(k)} \) instead of \( e^{j\angle H_1^*(k)} \) in eq. (14). However, the phase sequence must be known to the receiver. This extra phase rotation may be introduced to reduce the peak-to-average power ratio (PAPR), which may be high for OFDM.

2) SC systems: Since the decision-point SNR for SC systems is the same for all data symbols, the optimization problem eq. (12) can be simplified as
\[
\text{minimize} : \sum_{k=0}^{N-1} \frac{1}{|\sum_{i=1}^{M} H_i(k)w_i(k)|^2}
\]  
subject to (13). Applying the Lagrange multipliers method, the optimal BF coefficients can be written as
\[
w_{1}(k)_{\text{sc}} = e^{j\angle H_1^*(k)} \sqrt{P_t(k)_{\text{sc}}}
\]  
(16)

and
\[
P_{1}(k)_{\text{sc}} = \frac{|H_1(k)|^2}{t(k)} \left( \frac{\mathbb{P}_t}{\sqrt{t(k)} \sum_{k=0}^{N-1} \frac{1}{\sqrt{t(k)}}} \right)
\]  
(17)
B. Per-antenna power constraint

Using the Chernoff upper bound, the minimization problem under PPC can be approximated as

\[
\begin{align*}
\text{minimize : } & f(w) := \frac{1}{N} \sum_{k=0}^{N-1} e^{-\alpha_k \gamma(k)/2} \\
\text{subject to : } & \sum_{k=0}^{N-1} |w_i(k)|^2 \leq P_i, \ i = 1, \cdots, M
\end{align*}
\]

(18)

(19)

where \(P_i\) is the maximum transmit power available for the \(i\)-th transmit antenna. Similar to the TPC case, the constraints in the above optimization problem should be met with equality at the optimum solution. A possible approach to solve the SDP is to employ semi-definite relaxation (SDR) followed by a randomization technique to obtain the optimal PPC solution. Standard convex optimization tools, e.g. CVX and SeDuMi could be used for this purpose. Although globally optimal solutions could be found, solving the SDP problem requires a complexity close to \(O((MN)^2)\), which may be unpractical for online calculation. Details on the application of SDR techniques to beamforming can be found in [14]. An alternative interior-point-method-based solution of the PPC BF design problem can be found in [15]. However, the Logarithmic barrier function added to make sure that the \(|w_i(k)|^2\)'s are positive reduces the accuracy of the optimization. Moreover, large scale matrix inversion required for solving the KKT systems has a high computational complexity. These drawbacks make the interior-point method proposed in [15] unattractive to a practical system. Next, we propose three low complexity suboptimal solutions for the PPC optimization problem.

1) Isolated Antennas Algorithm (IAA): A direct and simple way to preserve the PPC is to perform frequency BF design for each transmit antenna independently. Hence, following the same derivations for the TPC case with \(M = 1\), the IAA suboptimal solution for the PPC problem can be expressed as

\[
\tilde{w}_i = e^{j\angle H_i^*(k)} \sqrt{\tilde{P}_i(k)}
\]

(20)

and

\[
\tilde{P}_i(k)_{\text{ofdm}} = \frac{P_i P_i^*(k)_{\text{ofdm}}}{\sum_{k=0}^{N-1} P_i^*(k)_{\text{ofdm}}} \quad \tilde{P}_i(k)_{\text{sc}} = \frac{|H_i(k)|}{\sum_{k=0}^{N-1} |H_i(k)|} \quad (21)
\]

\[
(22)
\]

where \(t_i(k) = |H_i(k)|^2\) and

\[
P_i^*(k)_{\text{ofdm}} = \frac{1}{\beta t_i(k)} \left( \ln t_i(k) - \frac{\sum_{k=0}^{N-1} \ln t_i(k) - \beta P_i}{\sum_{k=0}^{N-1} \frac{1}{t_i(k)}} \right) +
\]

It is worth pointing out that the normalization operation in eq. (21) is necessary since the \(|\cdot|\) operation in \(P_i^*(k)_{\text{ofdm}}\) will break the constraint \(\sum_{k=0}^{N-1} P_i^*(k)_{\text{ofdm}} = P_i\) for some particular channel realizations at low \(\sigma_f^2/\sigma_w^2\).

2) Extended Cyclic Algorithm (ECA): The main idea behind the CA in [7] is that \(|w_i(k)|^2\)'s are set to be equal. Inspired by this idea, in our scenario, we make the assumption that

\[
|w_i(k)|^2 = \frac{P_i}{P_1} |w_1(k)|^2, \ i = 2, \cdots, M
\]

where \(\{w_1(k)\}_{k=0}^{N-1}\) are taken as referential coefficients. Using the fact that \(\sum_{k=0}^{M-1} |H_i(k)w_i(k)|^2 \leq \left(\sum_{k=0}^{M-1} |H_i(k)| |w_i(k)|^2\right)^2\), where equality is achieved only when \(\angle w_i(k) = \angle H_i^*(k) + \phi(k)\), the PPC optimization problem can be re-written as

\[
\begin{align*}
\text{minimize : } & \frac{1}{N} \sum_{k=0}^{N-1} e^{-\alpha_k \gamma(k)/2} \\
\text{subject to : } & \sum_{k=0}^{N-1} p(k) = P_i
\end{align*}
\]

(23)

and

\[
\gamma_{\text{ofdm}}(k) = \frac{\sigma_f^2}{\sigma_w^2} \gamma(k) \quad \gamma_{\text{sc}}(k) = \frac{N_o \sigma_f^2}{\sigma_w^2} \sum_{k=0}^{N-1} \frac{1}{p(k) |H_i(k)|}
\]

where \(p(k) = |w_1(k)|^2\) and

\[
\epsilon(k) = \left( \sum_{i=1}^{M} \frac{P_i}{P_1} |H_i(k)| \right)^2
\]

Employing the Lagrange multipliers method, the solution of eq. (23) for OFDM and SC systems can be derived as

\[
p_{\text{ofdm}}(k) = \frac{\ln (\epsilon(k))}{\beta \epsilon(k)} - \frac{\sum_{k=0}^{N-1} \ln (\epsilon(k)) - \beta P_1}{\beta \epsilon(k) \sum_{k=0}^{N-1} \frac{1}{\epsilon(k)}}
\]

\[
p_{\text{sc}}(k) = \frac{1}{\sqrt{\epsilon(k)} \sum_{k=0}^{N-1} \frac{1}{\epsilon(k)}}
\]

Thus, the ECA suboptimal solution for the PPC problem can be expressed as

\[
\tilde{w}_i(k)_{\text{ofdm}} = e^{j\angle H_i^*(k)} \sqrt{\frac{P_i}{P_1} (p_{\text{ofdm}}(k))_+}
\]

\[
\tilde{w}_i(k)_{\text{sc}} = e^{j\angle H_i^*(k)} \sqrt{\frac{P_i}{P_1} p_{\text{sc}}(k)}
\]

(24)

(25)

3) Normalization-Based Algorithm (NBA): To satisfy the constraint (19), another effective way to solve the PPC problem is normalizing the TPC solution directly. Thus, the NBA-based suboptimal PPC solution can be obtained by the following steps:

Step 1) perform TPC optimization to find the optimal \(P_i(k)\) for OFDM and SC systems from eq. (15) and (17) respectively.

Step 2) normalize the \(P_i(k)\) to satisfy constraint (19) as

\[
\tilde{P}_i(k) = \rho_i P_i(k)
\]

(26)

where

\[
\rho_i = \frac{P_i}{\sum_{k=0}^{N-1} P_i(k)}
\]
Beamforming

In OFDM, the number of subcarriers is, in practical systems, generally much larger than the size of the BF vectors, as illustrated in Fig. 2. This is motivated by the fact that the amplitude (phase) part of CSI is quantized and phase (amplitude) part is assumed to be known perfectly at the transmitter, whereas ‘Am’ (‘Ph’) indicates that only the amplitude (phase) part of CSI is quantized; whereas ‘Am+Ph’ indicates that both the amplitude and phase parts of CSI are quantized. The simulation results will quantify the gaps in performance between the proposed three solutions.

IV. LIMITED-RATE FEEDBACK BEAMFORMING

In the previous section, we designed the BF vectors under the assumption that the transmitter has perfect knowledge of CSI. However, this assumption is unrealistic in a practical system.

As shown in [7], vector quantization (VQ) can reduce the feedback overhead and provide better performance than scalar quantization. Intuitively, VQ methods studied in [7, 16, 17] for flat-fading channels can be extended to frequency-selective channels by designing codebooks for BF coefficients on a subcarrier basis, i.e. performing VQ for \( \{ w_i(k) \} \) with \( k = 0, \ldots, N - 1 \). However, the feedback requirements of this method grow proportionally with the number of subcarriers and it is a big challenge to preserve the PPC condition.

To obtain a low-rate feedback, we propose to vector-quantize the time-domain CSI instead of quantizing the BF vectors, as illustrated in Fig. 2. This is motivated by the fact that the size of the BF vectors is generally much larger than the length of the channel impulse response (CIR); for example in OFDM, the number of subcarriers is, in practical systems, generally larger than four times the length of the effective discrete-time CIR.

The amplitude and phase parts of the time-domain CSI are quantized separately. Let \( g(l) = [h_1(l), \ldots, h_M(l)]^T \) and \( \eta(l) = [\Delta h_1(l), \ldots, \Delta h_M(l)]^T \) for \( l = 0, \ldots, L - 1 \). The codebooks of \( g(l) \) and \( \eta(l) \) and \( N(l) \), can be constructed off-line by employing GLA. Taking \( g(0) \) as an example, the codebook of \( g(0) \), \( g(0) := \{ g_1(0), g_2(0), \ldots, g_{N_C}(0) \} \) with \( N_C = 2^B \) denoting the number of codewords and \( B \) denoting the number of feedback bits, can be constructed as follows.

1) Generate a training set \( \{ g_1(0), g_2(0), \ldots, g_{Q}(0) \} \) for a sufficiently large number, \( Q \), of channel realizations and initialize the codebook as \( \{ g_0(0) = g_i(0) \} \).
2) Nearest neighborhood condition (NNC): for \( k = 1, \ldots, Q \), assign the \( k \)-th training element \( g_k(0) \) to the \( i \)-th region if it is closest to the \( i \)-th codeword among all \( N_C \) codewords, i.e.

\[
R_i(0) = \{ g_k(0) \mid \| g_k(0) - g_i(0) \|^2 \leq \| g_k(0) - g_j(0) \|^2, \forall j \neq i \} \tag{28}
\]

where \( \{ R_i(0) \}_{i=1}^{N_C} \) is the partition set for the \( i \)-th codebook \( g_i(0) \).
3) Centroid condition (CC): for a given partition \( R_i(0) \), update the codewords \( g_i(0) \) as

\[
\hat{g}_i(0) = \text{avg}\{ g_k(0) \mid g_k(0) \in R_i(0) \} \tag{29}
\]

where \( \text{avg}\{ x \mid x \in A \} \) is the arithmetic mean of all vectors \( x \) in \( A \).
4) Perform NNC and CC iteratively until no further improvement is observed.

It is worth pointing out that the periodic property of the phase should also be taken into account when designing the codebook for \( \eta(l) \) or selecting the codeword from \( N(l) \). For example, the distance between 0.9\( \pi \) and \(-0.8\pi\) is smaller than that between \(-0.1\pi \) and \(-0.8\pi\). It is worth noting that less than 2\( L \) codebooks may be required by exploiting the statistical property of each channel tap, but in order to make the design general, we employ 2\( L \) codebooks in our limited-rate feedback method.

With the same 2\( L \) codebooks maintained at the transmitter and the receiver, the latter first chooses the optimal codeword in the codebook as:

\[
g^*(l) = \arg \min_{g(l) \in G(l)} \| g(l) - \hat{g}(l) \|^2 \tag{30}
\]
\[
\eta^*(l) = \arg \min_{\eta(l) \in \hat{N}(l)} \| \eta(l) - \hat{\eta}(l) \|^2 \tag{31}
\]

for \( l = 0, \ldots, L - 1 \). Then the receiver feeds back the indices of \( \{ g^*(l) \}_{l=0}^{L-1} \) and \( \{ \eta^*(l) \}_{l=0}^{L-1} \) to the transmitter, which requires 2\( LB \) bits. Based on the quantized CSI, the transmitter designs the BF coefficients under PPC by employing the methods introduced previously.

To evaluate the performance of VQ with feedback rate \( B \), a useful indicator is the loss in the SNR. This quantity is defined as

\[
\Delta \gamma(B) = \frac{\gamma(VQ)}{\gamma(\text{ideal})} \tag{32}
\]

where \( \gamma(\text{ideal}) \) and \( \gamma(VQ) \) are the decision-point SNR averaged over the subcarriers with infinite and finite rate feedbacks, respectively. Note that \( \gamma(VQ) \) can be obtained after computing the BF coefficients from the quantized CSI.

Fig. 3 shows \( \Delta \gamma(B) \) versus the number of feedback bits \( B \) for SC and OFDM systems under TPC, where ‘Am+Ph’ indicates that both the amplitude and phase parts of CSI are quantized; whereas ‘Am’ (‘Ph’) indicates that only the amplitude (phase) part of CSI is quantized and phase (amplitude) part is assumed to be known perfectly at the transmitter. For illustration purposes, \( M, N \) and \( L \) are set to 2, 64 and 8, respectively, in Fig. 3. We also assume that the channel taps \( h_i(l) \) are uncorrelated zero-mean complex Gaussian random variables with exponential power delay profile.
Nevertheless, since both $B$ cause different SNR losses in SC and OFDM systems.

The channel frequency responses, the CSI quantization errors will be deduced that, for a given number of total feedback bits, better performance can be achieved by allocating the number of feedback bits non-uniformly between the phase and amplitude parts can be obtained by solving the following optimization problem.

$$\{B^p_0, B^p_1\} = \arg \min_{B_a + B_p = 2B, B_a > 0, B_p > 0} E\{f(w')\}$$  \hspace{1cm} (33)

where $w'$ indicates the BF coefficients associated with the VQ CSI; $B_a$ and $B_p$ denote the number of feedback bits for the amplitude and phase parts, respectively. Unfortunately, it is untractable to solve the above optimization problem analytically since it is difficult to get a closed-form expression for the expectation of the cost function $f(w')$ with respect to the channel. Nevertheless, since both $B_a$ and $B_p$ are discrete values, we can evolve numerical methods to solve the above optimization problem effectively.

In the above optimization problem, one may also allocate the number of feedback bits non-uniformly among channel taps. However, we have found in the simulation that this can only improve performance slightly. Thus, to keep the codebook design task of low complexity, we only consider uniform feedback design among channel taps in this paper.

V. SIMULATION RESULTS

In this section, the bit error rate performance for various BF designs is evaluated. Unless stated otherwise, we consider an uncoded system. The total number of data (subcarriers) in each data block is $N = 64$ and the data symbols are drawn from QPSK constellations. The length of the channel impulse response (CIR) $L$ is set to 8. We assume that the channel taps, the $h_i(l)$’s are uncorrelated zero-mean complex Gaussian random variables. The exponential power delay profile introduced in Section IV is considered. Moreover, the channels for different transmit antennas are assumed mutually uncorrelated and known perfectly at the receiver. The total transmit power $P_t$ is set to 128 for $M = 2$ and 192 for $M = 3$, respectively.

A. Perfect CSI at transmitters

We here assume that the CSI is known perfectly at the transmitter and that the power is allocated equally among the
transmit antennas, i.e. PPC with $P_i = 64$. The averaged (over the channels) bit error rates (BER) of the different BF designs for the two and three transmit antennas (or relay nodes) cases are shown in Fig. 4 and Fig. 5 respectively. In the figures, CVX refers to the PPC solution obtained by employing CVX tools and CON refers to the convex optimization based solution obtained in [15] at the 10-th iteration. The iterative convex optimization in [15] is initialized by setting $|w_i(k)|^2 = \frac{P_i}{N}$. From both Fig. 4 and Fig. 5, we see that the optimal SC TPC BF scheme outperforms the optimal OFDM TPC BF scheme at moderate to high SNR values. This can be ascribed to the fact that the inverse DFT operation at the SC receiver spreads the noise contributions corresponding to all subcarriers [25]. Moreover, we can see that the CVX method can achieve performance close to that of the PPC solution and that NBA only suffers a negligible performance loss compared to the CVX method for both SC and OFDM systems. As we described in Section III.B, due to the imprecise approximation introduced by the Logarithmic barrier function, the CON method fails to achieve optimal PPC performance, particularly for OFDM systems. For SC systems, ECA is outperformed by CON in the two transmit antennas case, whereas for the three transmit antenna case, the BERs of the two methods are close to each other. For OFDM systems, both ECA and NBA outperform CON significantly for moderate to high values of $\frac{\sigma_s^2}{\sigma_y^2}$. It is seen that the IAA solution has several dBs performance loss compared to the ECA and NBA solutions. Since the $(\cdot)^+$ operation may affect the performance of the IAA scheme at low $\frac{\sigma_s^2}{\sigma_y^2}$, the performance of IAA for OFDM is lower than that of IAA for SC at all values of $\frac{\sigma_s^2}{\sigma_y^2}$ considered.

Now, we consider an unequal PPC scenario, where $P_1 = 24$ and $P_2 = 104$ for the two transmit antennas case, and $P_1 = 24, P_2 = 64$ and $P_3 = 104$ for the three transmit antennas case. The BER simulation results are shown in Fig. 6 and Fig. 7, respectively. We can see that the PPC solutions based on the CVX method still can provide the best performance. Moreover, it can be seen that ECA and NBA outperform IAA for both two and three transmit antennas cases. For SC systems, CON outperforms ECA and NBA for the two transmit antennas case and the reverse is true in the case of three transmit antennas. For OFDM systems, like the equal PPC scenarios, CON is outperformed by both ECA and NBA. Further, we can find that ECA outperforms NBA at moderate to high SNR values.

Fig. 8 shows the BER performance when using two transmit antennas and convolutional codes with rate 0.5. Similar conclusions as from Fig. 4 can be drawn from Fig. 8, i.e. the order from best to worst method is TPC, CVX, NBA, CON, ECA and IAA for SC systems, and is TPC, CVX, NBA, ECA, CON and IAA for OFDM systems, for most SNR values. The IAA scheme for OFDM systems outperforms the CON scheme in the high SNR regime. For the case of three transmit antenna systems and unequal PPC, the same conclusions drawn from the uncoded system case are drawn from the simulations of the convolutional coded system and thus the results are not shown here.

**B. Limited-rate feedback**

To keep our paper concise, we next only investigate the issue of limited feedback in the case of two transmit antennas and equal PPC. Similar conclusions are obtained for the unequal PPC and/or three transmit antennas scenarios. Fig. 9 shows the BER performance of various numbers of feedback bits ($B = 2, 4, 6, 8$) for the different BF designs under TPC. It can be seen that OFDM systems outperform SC systems when the number of feedback bits is small (e.g., $B = 2, 4$) but the reverse is true for larger numbers of feedback bits (e.g., $B = 8$). The system with 8-bit feedback suffers only 0.5dB performance loss compared to the perfect CSI system. Although the total number of feedback bits is 128 for $B = 8$, the effective number of feedback bits for each subcarrier is only 2! It is also worth pointing out that in order to achieve the close-perfect performance, the number of feedback bits $B$ should increase with the number of transmit antennas. Nevertheless, since the number of feedback bits for the conventional
feedback methods also should be increased to maintain the same performance, the proposed feedback method still has a lower rate compared to conventional feedback methods.

To quantify the gain obtained by non-uniform limited feedback, we define ‘D1 scheme’ as ‘$B_a = B - 2$ and $B_p = B + 2$’, and ‘D2 scheme’ as ‘$B_a = B - 1$ and $B_p = B + 1$’.

Fig. 10 shows the SNR loss of the evaluated non-uniform feedback schemes. We see that ‘D1 scheme’ improves the SNR significantly over the uniform scheme in the case of low feedback rate. However, the gain shrinks as the feedback rate
increases. The ‘D2 scheme’ only provides a negligible gain over ‘D1 scheme’. Similar results can also be seen in Fig. 11. Since ‘D1 scheme’ has almost the same performance as ‘D2 scheme’ but has a lower storage requirement, it should be preferred in a practical system.

The effectiveness of limited-rate feedback can also be verified in Fig. 12 where the PPC solutions and existing low-rate feedback methods are investigated. ‘Interp. LB’ denotes the lower bound of the interpolation-based method proposed in [19]; we use "lower bound" because we here assume an ideal feedback (no quantization error) and the interpolation rate is equal to one, i.e. no interpolation is required at the transmitter. ‘Cluster LB’ refers to the lower bound of cluster based BF systems where the feedback is ideal and the cluster size is set to two, i.e. a cluster only contains two subcarriers. The ‘Interp. LB’ method assumes that the power is equally distributed among the subcarriers [19], and is therefore outperformed by both ‘D1 ECA’ and ‘D1 NBA’ schemes with 8-bit feedback. Compared to ‘Cluster LB’, ‘D1 ECA’ only has a slight loss at low SNR and ‘D1 NBA’ outperforms ‘Cluster LB’ at all considered SNRs.

The performance of IAA shown in Fig. 12 is obtained under the assumption of perfect CSI at the transmitter. We can see that even with ideal CSI, IAA still has several dBs performance loss compared to 8-bit ‘D1 ECA’ and ‘D1 NBA’ schemes. It is also worth pointing out that the interpolated and cluster based BF methods do not satisfy the per-antenna power constraint. Further, more significant gains can be achieved by our proposed schemes when quantization errors are included in the interpolated and cluster based BF methods.
C. Complexity analysis

We briefly compare the complexities of the above BF design algorithms. Considering the number of complex multiplications as a complexity metric, the inversion of an \( n \times n \) matrix requires \( O(n^3) \) operations, and the product of an \( mn \times r \) matrix with a \( r \times n \) matrix requires \( O(mnr) \) operations.

The BF design complexity required by each transmitter is shown in Table I, where \( K_M \) denotes number of iterations in the CON algorithm. Since the proposed suboptimal PPC solutions are given in close-form, they are much more computationally efficient than both the CVX method and the interior-point based solution proposed in [15]. Moreover, since the IAA solution decouples the MISO system into \( M \) SISO systems, the complexity of IAA is \( M \) times lower than those of ECA and NBA. Further, since the codebooks for limited-rate feedback can be designed offline, the complexity of the codebook design is not taken into account.

VI. CONCLUSION

In this paper, we addressed the problem of beamforming (BF) design for cyclic prefixed transmissions over frequency selective channels. Per-antenna power constraint (PPC) and limited-rate feedback were considered. Reduced-complexity PPC-based BF solutions are proposed and it was shown that they incur only a slight performance loss compared to the computationally demanding convex-optimization-based solutions. Further, it was shown that with the proposed non-uniform limited-rate feedback method, the cyclic-prefixed systems with PPC BF can achieve performance close to that obtained with total-power-constraint-based BF even for a small number of feedback bits.

## REFERENCES


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<td>NBA PPC</td>
<td>( O(MN) )</td>
<td>128</td>
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</table>
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