

U Network protocol
scalability via a
topological Kadanoff
transformation **B**

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Context

- Dynamic routing protocols in networks
 - need to adapt to current network state
 - BUT, flow of network state information is over the same communication channels as data traffic
- Adaptive routing protocols
 - are prone to instabilities
 - do not scale well to large networks

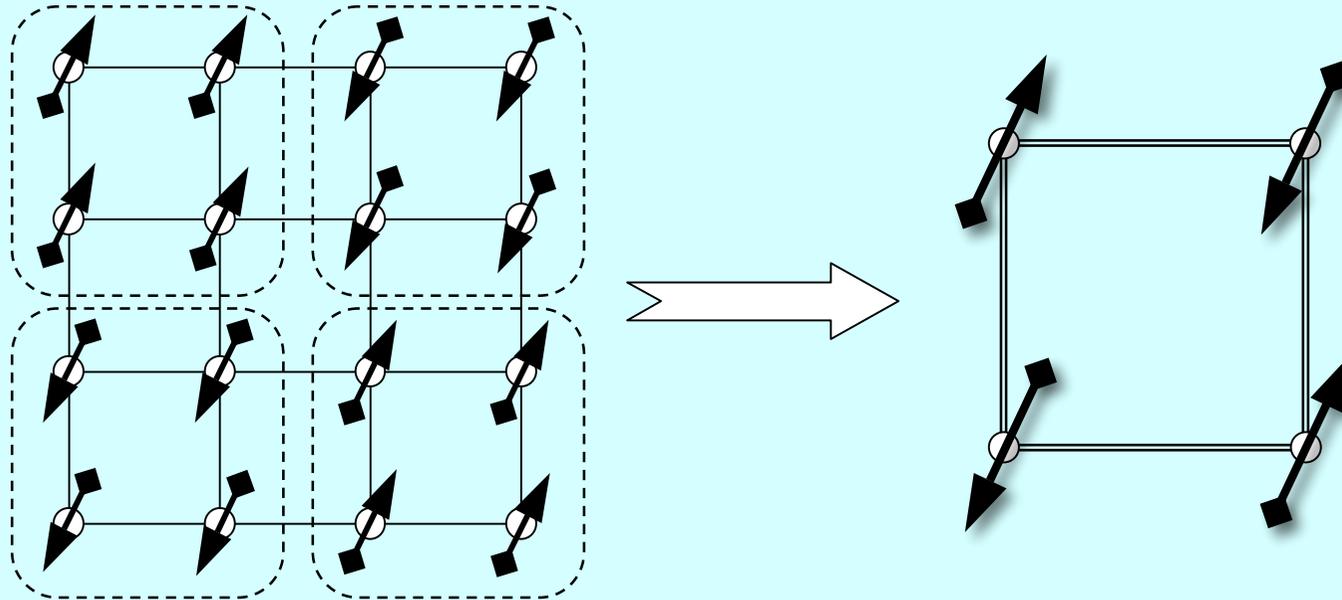
Introduction

- “Scalability” – no unique or rigorous definition
 - Hardware scalability = monotonic increase in performance with increased resources
 - Protocol scalability = acceptable (at best sub-linear) growth with increasing number of network nodes of:
 - communication & control overhead bandwidth
 - computational complexity in making forwarding decisions
 - address space & associated size of routing tables

Introduction

- Renormalisation group – in Physics
 - Natural hierarchy of scales of interactions between increasingly larger coarse grained “blocks” governing the system properties at each scale
 - Interactions must be such that the effective interaction type between blocks is preserved at all levels of coarse-graining (i.e. all scales)

Introduction



Real space RG:

Group spins in blocks; integrate over internal degrees of freedom; new equivalent spins populate new lattice, but interaction between them remains unchanged

Introduction

- Notions of scalability and natural hierarchy of scales not as disparate as might appear naively
 - A hierarchy of scales can be used as a framework to achieve scalability
 - Employing a hierarchy of topological scales (hierarchical clustering) has long been recognised as the key to achieve network addressing scalability [Kleinrock & Kamoun, 1977]
 - A natural hierarchy of scales can be augmented it with routing protocol rules, yielding a new class of routing protocols with built-in desirable scalability properties

Key idea

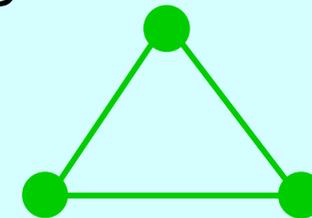
- RG allows description of interactions between distant parts of system through knowledge of interaction at micro level
- Construct topological objects and associated interactions such that interactions are preserved at all levels of topological hierarchy

“RG blocks” for networks (1)

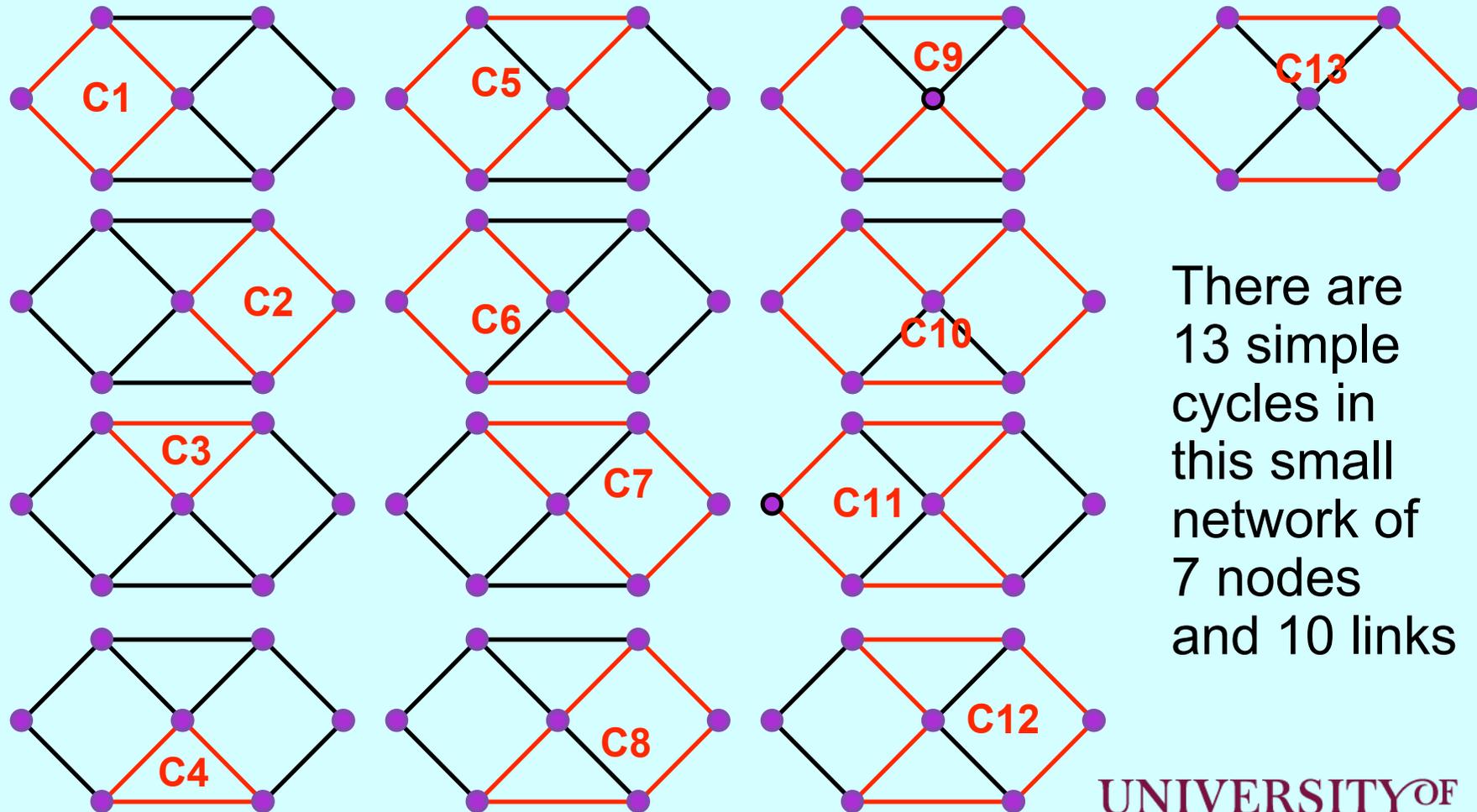
- Nodes (basic block) 
- Links (‘interaction’) 
- Not very good as building blocks as we need *extrinsic* criteria to the network topology in order to group nodes into blocks (clustering)
- What then?

“RG blocks” for networks (2)

- Framework has to be capable of describing network connectivity in full (i.e. network path diversity)
- Need to consider fundamental topological units embodying notion of path diversity
- Basic “atomic” block: **cycle** of nodes
 - Simplest path diversity topological unit
 - Paths either lie on part of network which is a tree (no diversity) or can be expressed as an arc on some cycle
- “Interaction” ?



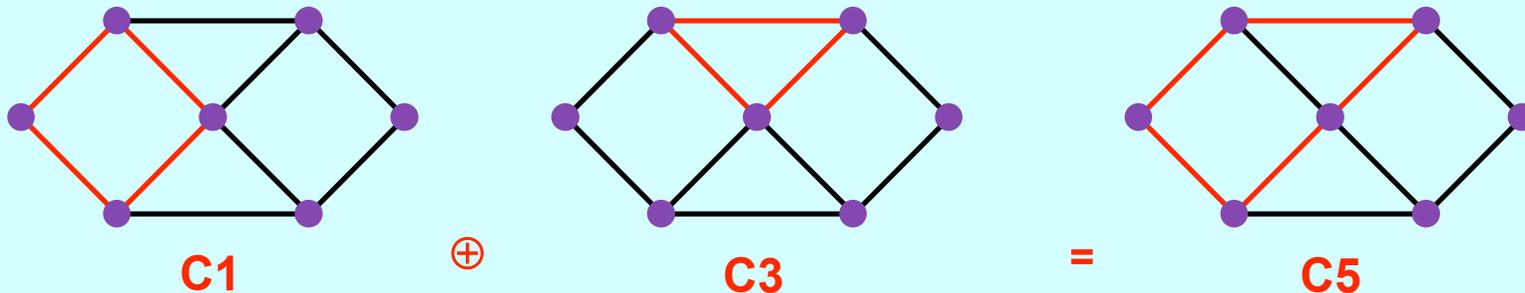
“RG blocks” for networks (2)



There are 13 simple cycles in this small network of 7 nodes and 10 links

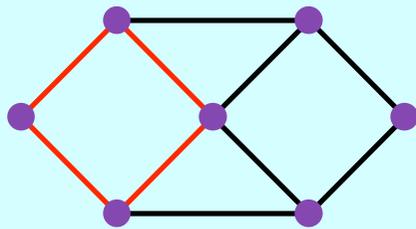
“RG blocks” for networks (2)

- Cycle space, and an operator on cycles, yields a rigorous notion of independence for cycles
- Need maximal set of **independent** cycles to generate all the remaining ones, i.e. a **basis set**
- The symmetric difference of two cycles (Boolean XOR operation on their edges) is the operator



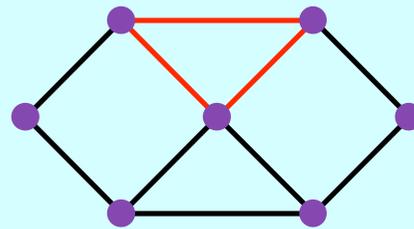
“RG blocks” for networks (2)

- Two types of diversity unit in graphs
 - Basic blocks: Cycles
 - “Interactions”: Diverse cycle connections
 - Operation is in **link space**
 - Boolean AND operation on the sets of cycle edges



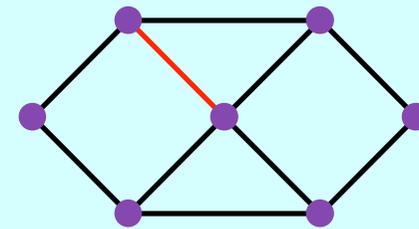
C1

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C3

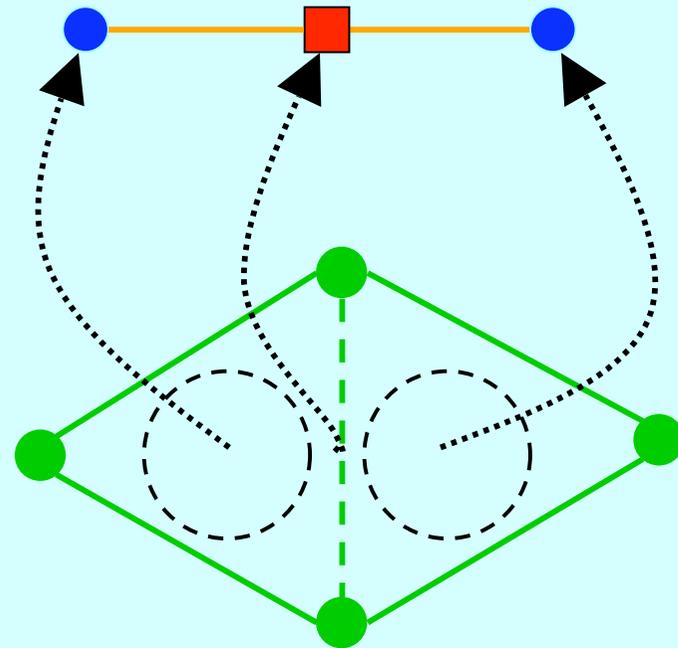
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E3

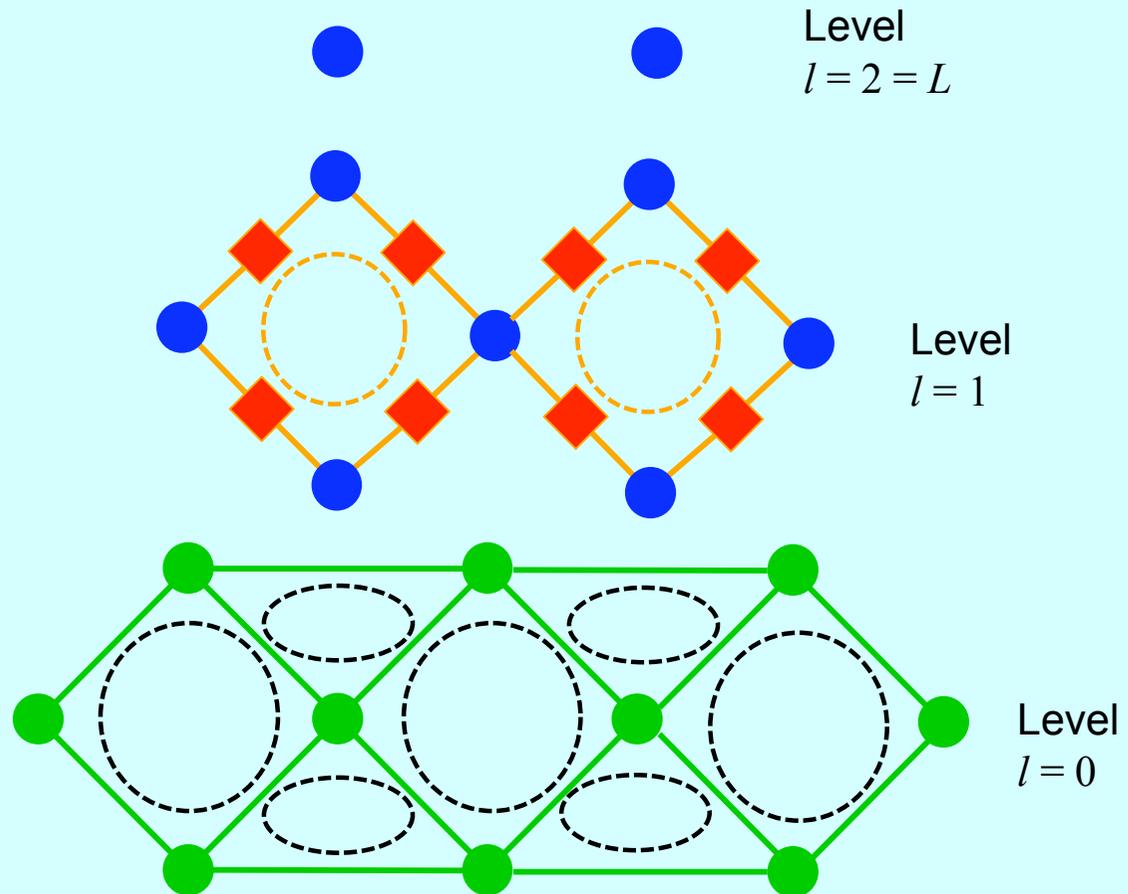
Kadanoff transformation

- Abstraction:
 - Cycles/blocks into nodes of first type
 - Diverse cycle connections/interactions into nodes of second type

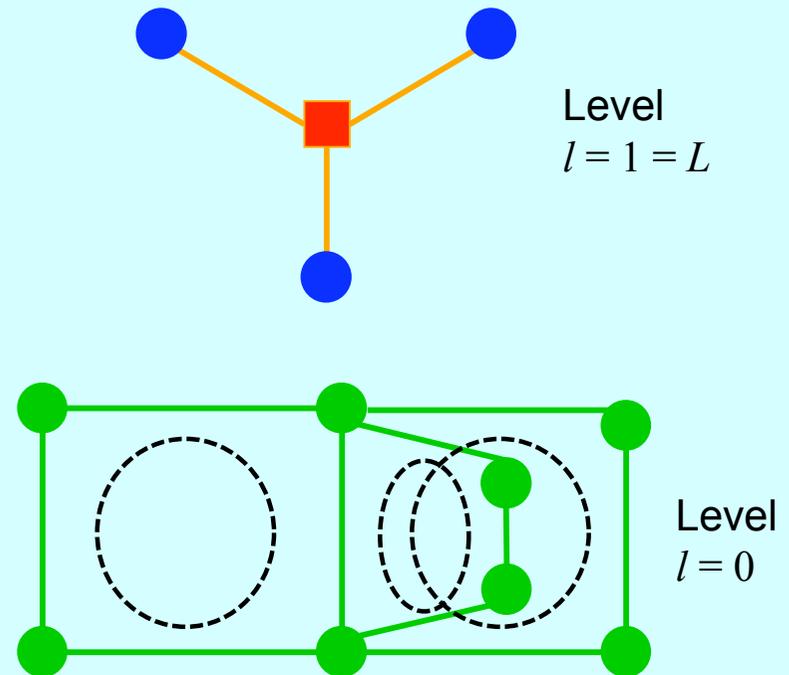
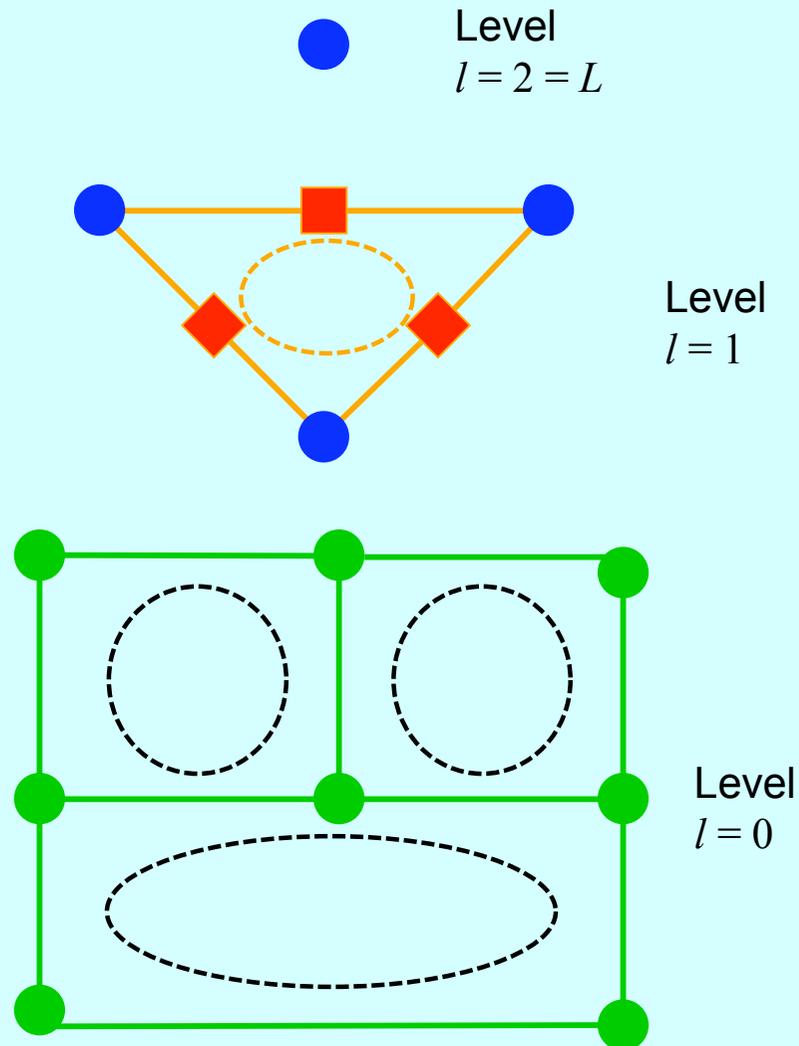


Kadanoff transformation

- Iterate previous procedure until network graph is loop-free
- Set of all levels of abstraction called **Logical Network Abridgement (LNA)** of the network



Logical network abridgment



Logical network abridgement

- Cyclomatic number of graph, $\nu = m - n + c$
(m edges, n nodes & c connected components)
gives number of independent cycles
- Set of independent cycle basis is *not* unique
- Choice can be made unique using additional
application-specific criteria (we choose minimal
cycle basis)
- Cycle basis computation is polynomial time:
 $O(mn^3)$ [Horton, 1987] to $O(m^3 + mn^2 \log n)$
[Michail, 2006]

Logical network abridgement

- Procedure always converges, at least for sensible choice of cycle bases, for finite graphs at level L
 - Tree ($m = n - 1, c = 1$): $L = 0$
 - Complete graph K_n ($m = n(n - 1)/2, c = 1$): $L = n - 2$
- L is a natural global measure of path diversity
 - The bigger L for a given n , the more intrinsic path diversity exists in the network

Logical network abridgement

- Every level of abstraction graph conveys summarised path diversity information for lower level
 - Non bi-connected graphs at some level become “disconnected” at the next higher level of abstraction
 - Higher level disconnection is indicative of a diversity “bottleneck”
 - Trees attached to a cycle are logically collapsed into vertex rooted in cycle
- Hierarchical summarisation is not dependent on arbitrary clustering criteria
- Scheme trivially generalises to weighted links

Network path diversity density

- Network path diversity density $D \equiv L/n$
- D is strictly bounded: $0 \leq D \leq 1 - 2/n < 1$
- If $D \approx 0$, network is dominated by trees and a shortest path type protocol is highly scalable and efficient
- If $D \approx 1$, network is very close to fully meshed and random deflection routing is scalable, robust and sufficient

Network path diversity density

- Intermediate D case, $0 < D \ll 1$
 - Shortest path protocol fails to exploit the underlying network diversity and takes time to re-converge if congestion or failures arise
 - Random deflection routing has exponentially small successful data delivery probability as nodes are likely to be separated by many hops
 - To exploit the underlying network diversity a dynamic, adaptive routing protocol is then required

Resilient Recursive Routing

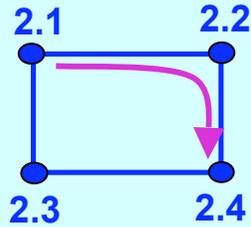
- Augment the LNA with routing rules
- Ensure routing rules can be applied recursively
- Routing rules must exploit path diversity
- “Interaction” now becomes routing protocol operation (more precisely stated later)

Resilient Recursive Routing

- Fundamental routing algorithm (generic)
 - Ensure loop-free routing around a level 0 cycle
 - Ensure load balancing (or multipath routing) on a level 0 cycle
 - Ensure fast reaction time to cycle failures
- For a level 1 destination iterate fundamental routing algorithm at level 1 as well as level 0
 - Select an arc on the level 1 cycle and then select an arc on the local level 0 cycle, keeping future switching options open & local
- For a level l destination iterate fundamental routing algorithm $l + 1$ times at levels $l, l - 1, \dots, 2, 1, 0$
- Update neighbourhood information at exponentially slower time-scales with increasing l
- At cut nodes and trees revert to deterministic forwarding

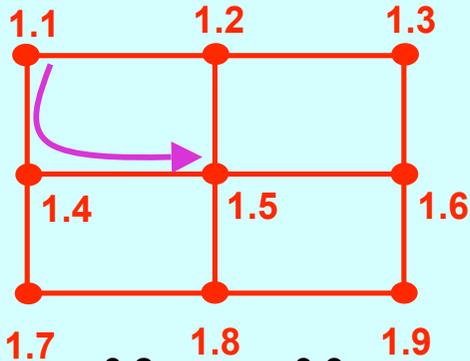
Resilient Recursive Routing

● 3.1

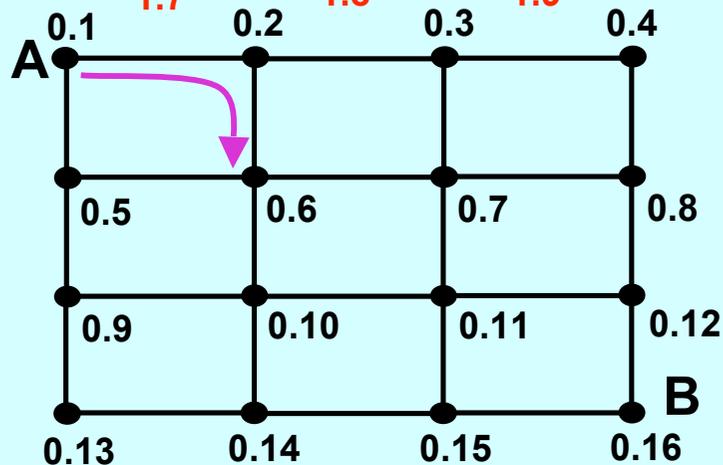


A and B are both on 3.1 → no routing need at level 3

A ∈ 2.1, B ∈ 2.4, choose an arc on cycle, based on 'old' global information



To perform the first hop on chosen arc at level 2, from 2.1 to 2.2, choose an arc on local loop at level 1, based on 'medium age' regional information



To perform the first hop on chosen arc from 1.1 to 1.4, choose an arc on local loop at level 0, based on 'fresh' local information

Resilient Recursive Routing

- Correct operation of R^3 requires that the ensemble of arcs is consistent
- When a lower level arc terminates it needs to be replaced by a newer logically consistent arc locally – actual switching is always done on fresh information
- Higher logical level arcs define coarse-grained topological direction

Resilient Recursive Routing

□ R³ is “scalable” if:

1. Network is efficiently summarisable: $L \ll n$, or equivalently $D \ll 1$
2. Characteristic time-scales of information summarisation correspond to scalable routing overheads:

□ If for a particular network class $L = \log n$ as $n \rightarrow \infty$, then $\tau_l = \tau_0 \cdot b^l$, $l = 0, \dots, L$

Conclusion

- Kadanoff transformation inspired topology abstraction and family of routing protocols
- Natural metric of topological distance is lowest level at which two physical nodes belong to same logical level l logical vertex
- Maximum topological distance L
- Normalised topological “diameter” D determines appropriateness of routing protocol class

Conclusion

- *R^3 routing protocol selection of cycle arcs corresponds to distant “interaction” mechanism between source and destination node pairs*
- Open questions
 - Uniqueness of LNA
 - Computational efficiency of LNA for large graphs
 - Characterisation of LNA for typical families of network graphs
 - Stable routing with incomplete or inconsistent topological information