Modeling Capacitated Location-Allocation Problem with Fuzzy Demands

Jian Zhou
Department of Industrial Engineering, Tsinghua University, Beijing 100084, China

Baoding Liu*
Department of Mathematical Sciences, Tsinghua University, Beijing 100084, China

Abstract

In order to model capacitated location-allocation problem with fuzzy demands, three types of fuzzy programming models — fuzzy expected cost minimization model, fuzzy α-cost minimization model, and credibility maximization model — are proposed according to different decision criteria. For solving these models, some hybrid intelligent algorithms are also designed. Finally, several numerical experiments are presented to illustrate the efficiency of the proposed algorithms.

Keywords: Location-allocation problem, fuzzy set, fuzzy programming, genetic algorithm

1 Introduction

The terms location-allocation (LA), facility location, facility layout and multi-Weber problem are often used interchangeably. In this paper, we adopt the first one. LA problem is to locate a set of new facilities such that the transportation cost from facilities to customers is minimized. For its general practical application backgrounds, LA problem has been studied by many researchers since Cooper [13] proposed it for the first time.

LA problem was studied detailedly in Gen and Cheng [17][18], where all kinds of cases were discussed. At first, let us examine uncapacitated LA problem (see [6][13][35]) in which the capacities of facilities are limitless. In this case, it is easy to know that each customer should be supplied by the nearest facility. Gradually, more researchers investigated capacitated LA problem (see [1][5][16]),

* Corresponding author. Tel.: +86-10-62787724. Fax: +86-10-62781785. *E-mail address*: liu@tsinghua.edu.cn (B. Liu).

where the capacities of facilities are limited. Most of work is done for the deterministic case. And many models and algorithms were presented in the literature [21][23][24][36][37].

In practice, many parameters, for example, demands of customers, are stochastic rather than deterministic. Some stochastic programming models have been proposed for stochastic LA problem in [9][35][39][42]. Especially, in Zhou and Liu [43], the expected cost minimization model, α-cost minimization model and probability maximization model were proposed for capacitated location-allocation problem with stochastic demands.

Although the assumption that demands of customers are stochastic has been adopted and tallied with the facts in widespread cases, it is not reasonable in a vast range of situations. For many cases, the estimations of probability distributions for demands of customers are not easy due to the lack of data. Instead, expert opinion is used to provide their estimations. This calls for the incorporation of fuzzy set theory into LA problem. In the past decades, there are many people who have brought fuzzy theory into facility location problem. For example, in Bhattacharya *et al.* [3][4] new facilities are considered to be located under multiple fuzzy criteria, and a fuzzy goal programming approach has been developed to deal with the problems. In Canós *et al.* [8], a fuzzy set of constraints is introduced into the classical p-median problem, and the decision is made which provides significantly lower costs by leaving a part of the demand uncovered. Also Chen and Wei [12], Darzentas [14], Rao and Saraswati [38] have discussed various facility location problems by fuzzy logic methods. However, all the parameters in these problems are deterministic, and fuzzy theory is only used to solve the classical mathematical programming effectively. Different from the above-mentioned papers, this paper will assume that the demands of customers are fuzzy variables, and will give some new fuzzy programming models for LA problem.

The paper is organized as follows. In Section 2, we review the concepts of possibility space, credibility and expected value operator of fuzzy variable briefly. Section 3 gives the problem description of LA problem and proves a theorem. And then we formulate the capacitated LA problem as fuzzy expected cost minimization model, fuzzy α-cost minimization model, and credibility maximization model in Section 4 according to different criteria. For computing the uncertain functions in the above models, Section 5 designs some fuzzy simulations for LA problem. Some hybrid intelligent algorithms are presented in Section 6 to solve these fuzzy models efficiently. Finally, Section 7 provides some numerical examples to illustrate the performance and the effectiveness of the proposed algorithms.

2 Preliminaries

Since its introduction in 1965 by Zadeh, fuzzy set theory has been well developed and applied in a wide variety of real problems. In the following, we briefly review the concepts of possibility space, credibility of fuzzy event and expected value operator of fuzzy variable.

Let Θ be a nonempty set, $\mathcal{P}(\Theta)$ the power set of Θ, and Pos a possibility measure. Then the triplet $(\Theta, \mathcal{P}(\Theta), \text{Pos})$ is called a possibility space. A fuzzy variable variable is defined as a function from a possibility space $(\Theta, \mathcal{P}(\Theta), \text{Pos})$ to the set of real numbers.

Suppose that ξ is a fuzzy variable with membership function μ. Then the possibility, necessity, and credibility of a fuzzy event $\{\xi \geq r\}$ can be defined by

$$\begin{aligned}
&\text{Pos}\{\xi \geq r\} = \sup_{u \geq r} \mu(u), \\
&\text{Nec}\{\xi \geq r\} = 1 - \sup_{u < r} \mu(u), \\
&\text{Cr}\{\xi \geq r\} = \frac{1}{2}\left(\text{Pos}\{\xi \geq r\} + \text{Nec}\{\xi \geq r\}\right),
\end{aligned} \tag{1}$$

respectively. Note that a fuzzy event may fail even though its possibility achieves 1, and hold even though its necessity is 0. However, the fuzzy event must hold if its credibility is 1, and fail if its credibility is 0.

There are many ways to define a mean value for fuzzy variables (see [7][15][19][22][41]). Here we use the definition of expected value operator by Liu and Liu [32] as

$$E[\xi] = \int_0^{+\infty} \text{Cr}\{\xi \geq r\}\boldsymbol{d}r - \int_{-\infty}^{0} \text{Cr}\{\xi \leq r\}\boldsymbol{d}r \tag{2}$$

provided that at least one of the above two integrals is finite. Especially, if ξ is a nonnegative fuzzy variable, then

$$E[\xi] = \int_0^{+\infty} \text{Cr}\{\xi \geq r\}\, \boldsymbol{d}r.$$

For detailed expositions, the interested reader may consult the books [33][34].

3 Problem Description

A fuzzy capacitated LA problem may be described as follows: there are m customers whose fixed locations and fuzzy demands are known, and the objective is to locate n new facilities to supply demands of customers optimally so that the total transportation cost is minimized, where the capacities of facilities $i, i = 1, 2, \cdots, n$ are limited. In order to model this problem, the following indices, parameters, and decision variables are used:

$i = 1, 2, \cdots, n$: index of facilities;

$j = 1, 2, \cdots, m$: index of customers;

(a_j, b_j) : location of customer j, $1 \leq j \leq m$;

ξ_j : fuzzy demand of customer j, $1 \leq j \leq m$;

s_i : capacity of facility i, $1 \leq i \leq n$;

(x_i, y_i) : decision variable representing the location of facility i, $1 \leq i \leq n$;

z_{ij} : quantity supplied to customer j by facility i after the fuzzy demands are realized, $1 \leq i \leq n$, $1 \leq j \leq m$.

For convenience, we denote $\boldsymbol{\xi} = (\xi_1, \xi_2, \cdots, \xi_m)$ and

$$(\boldsymbol{x}, \boldsymbol{y}) = \begin{pmatrix} x_1 & y_1 \\ x_2 & y_2 \\ \cdots & \cdots \\ x_n & y_n \end{pmatrix}, \quad \boldsymbol{z} = \begin{pmatrix} z_{11} & z_{12} & \cdots & z_{1m} \\ z_{21} & z_{22} & \cdots & z_{2m} \\ \cdots & \cdots & \cdots & \cdots \\ z_{n1} & z_{n2} & \cdots & z_{nm} \end{pmatrix}.$$

Here we will consider the LA problem with assumptions that the path between any customer and facility is connected and unit transportation cost is proportionate of the quantity supplied and the travel distance. And facility i is permitted to be located within a certain region $R_k = \{(\boldsymbol{x}, \boldsymbol{y}) | g_k(\boldsymbol{x}, \boldsymbol{y}) \leq 0\}$, $k = 1, 2, \cdots, p$, respectively since there are possibly some lakes, existing buildings or other obstacles in the locating area.

Now, let us examine the decision variables. Recall that in a deterministic LA problem, $(\boldsymbol{x}, \boldsymbol{y})$ and $\boldsymbol{z}$ are decision variables, and $\boldsymbol{z}$ is decided along with $(\boldsymbol{x}, \boldsymbol{y})$. However, in a LA problem with fuzzy demands, the decision $\boldsymbol{z}$ will be made every period after the fuzzy demands are realized.

For each $\theta \in \Theta$, the value $\xi_j(\theta)$ is a realization of ξ_j for each j. We say that an allocation $\boldsymbol{z}$ is feasible if $\boldsymbol{z}$ is in the feasible allocation set

$$Z(\theta) = \left\{ \boldsymbol{z} \;\middle|\; \begin{array}{l} \sum_{i=1}^{n} z_{ij} = \xi_j(\theta), \quad j = 1, 2, \cdots, m \\ \sum_{j=1}^{m} z_{ij} \leq s_i, \quad i = 1, 2, \cdots, n \\ z_{ij} \geq 0, \quad i = 1, 2, \cdots, n, \quad j = 1, 2, \cdots, m \end{array} \right\}. \tag{3}$$

Obviously, it is possible that $Z(\theta) = \emptyset$ for some θ.

For each fixed $(\boldsymbol{x}, \boldsymbol{y})$, we should determine the optimal allocation $\boldsymbol{z}^*$ for each $\theta \in \Theta$ in order to minimize the transportation cost $C(\boldsymbol{x}, \boldsymbol{y}|\theta)$, where

$$C(\boldsymbol{x}, \boldsymbol{y} \mid \theta) = \min_{\boldsymbol{z} \in Z(\theta)} \sum_{i=1}^{n} \sum_{j=1}^{m} z_{ij} \sqrt{(x_i - a_j)^2 + (y_i - b_j)^2}. \tag{4}$$

If $Z(\theta) = \emptyset$, then the demands of some customers are impossible to be met, and the right-hand side of (4) is meaningless. As a penalty, we define

$$C(\boldsymbol{x}, \boldsymbol{y} \mid \theta) = \sum_{j=1}^{m} \xi_j(\theta) \max_{1 \le i \le n} \sqrt{(x_i - a_j)^2 + (y_i - b_j)^2}. \tag{5}$$

Let us examine the properties of the minimal transportation cost $C(\boldsymbol{x}, \boldsymbol{y}|\theta)$.

Theorem 1 *Let $\theta_1, \theta_2 \in \Theta$ be given. If $\xi_j(\theta_1) \le \xi_j(\theta_2), j = 1, 2, \cdots, m$, then we have*

$$C(\boldsymbol{x}, \boldsymbol{y} \mid \theta_1) \le C(\boldsymbol{x}, \boldsymbol{y} \mid \theta_2).$$

Proof: If $Z(\theta_1) = \emptyset$, then we have $\sum_{i=1}^{n} s_i < \sum_{j=1}^{m} \xi_j(\theta_1)$. Since $\xi_j(\theta_1) \le \xi_j(\theta_2), j = 1, 2, \cdots, m$, it is apparent that $\sum_{i=1}^{n} s_i < \sum_{j=1}^{m} \xi_j(\theta_2)$, which means $Z(\theta_2) = \emptyset$. It follows from equation (5) that

$$C(\boldsymbol{x}, \boldsymbol{y} \mid \theta_k) = \sum_{j=1}^{m} \xi_j(\theta_k) \max_{1 \le i \le n} \sqrt{(x_i - a_j)^2 + (y_i - b_j)^2}$$

for $k = 1, 2$ which may deduce that $C(\boldsymbol{x}, \boldsymbol{y}|\theta_1) \le C(\boldsymbol{x}, \boldsymbol{y}|\theta_2)$.

If $Z(\theta_1) \neq \emptyset$ and $Z(\theta_2) = \emptyset$, it follows from equations (4), (5) and $\xi_j(\theta_1) \le \xi_j(\theta_2), j = 1, 2, \cdots, m$ that

$$\begin{aligned}
C(\boldsymbol{x}, \boldsymbol{y} \mid \theta_1) &= \min_{\boldsymbol{z} \in Z(\theta_1)} \sum_{i=1}^{n} \sum_{j=1}^{m} z_{ij} \sqrt{(x_i - a_j)^2 + (y_i - b_j)^2} \\
&\le \min_{\boldsymbol{z} \in Z(\theta_1)} \sum_{j=1}^{m} \max_{1 \le i \le n} \sqrt{(x_i - a_i)^2 + (y_j - b_j)^2} \left(\sum_{i=1}^{n} z_{ij} \right) \\
&= \sum_{j=1}^{m} \xi_j(\theta_1) \max_{1 \le i \le n} \sqrt{(x_i - a_i)^2 + (y_j - b_j)^2} \\
&\le \sum_{j=1}^{m} \xi_j(\theta_2) \max_{1 \le i \le n} \sqrt{(x_i - a_i)^2 + (y_j - b_j)^2} \\
&= C(\boldsymbol{x}, \boldsymbol{y} \mid \theta_2).
\end{aligned}$$

If $Z(\theta_1) \neq \emptyset$ and $Z(\theta_2) \neq \emptyset$, in order to prove $C(\boldsymbol{x}, \boldsymbol{y}|\theta_1) \le C(\boldsymbol{x}, \boldsymbol{y}|\theta_2)$, we only need to prove that, for any $\boldsymbol{z}^{(2)} \in Z(\theta_2)$, there exists $\boldsymbol{z}^{(1)} \in Z(\theta_1)$ such that

$$\sum_{i=1}^{n} \sum_{j=1}^{m} z_{ij}^{(1)} \sqrt{(x_i - a_j)^2 + (y_i - b_j)^2} \le \sum_{i=1}^{n} \sum_{j=1}^{m} z_{ij}^{(2)} \sqrt{(x_i - a_j)^2 + (y_i - b_j)^2}. \tag{6}$$

In fact, for any $\boldsymbol{z}^{(2)} \in Z(\theta_2)$, $\boldsymbol{z}^{(1)}$ may be obtained by the following process,

Step 1. $z_{kt}^{(1)} \leftarrow z_{kt}^{(2)}$ for $k = 1, 2, \cdots, n, t = 1, 2, \cdots, m$, respectively.

Step 2. Set $j = 1$.

Step 3. Set $i = 1$ and $\Delta = \xi_j(\theta_2) - \xi_j(\theta_1)$.

Step 4. If $z_{ij}^{(1)} \geq \Delta$, then $z_{ij}^{(1)} \leftarrow z_{ij}^{(1)} - \Delta$ and go to Step 6. Otherwise $\Delta \leftarrow \Delta - z_{ij}^{(1)}$ and let $z_{ij}^{(1)} = 0$.

Step 5. If $i < n$, then $i \leftarrow i + 1$ and go to Step 4.

Step 6. If $j < m$, then $j \leftarrow j + 1$ and go to Step 3.

Step 7. Stop.

Note that $\sum_{i=1}^{n} z_{ij}^{(2)} = \xi_j(\theta_2) \geq \xi_j(\theta_2) - \xi_j(\theta_1)$, which ensures that $z_{ij}^{(1)} \geq \Delta$ may hold for a certain j $(1 \leq j \leq m)$ when $i \leq n$ in Step 4. Now, let us prove that $\boldsymbol{z}^{(1)} \in Z(\theta_1)$. It follows from Step 4 that $z_{ij}^{(1)} \geq 0$, $z_{ij}^{(1)} \leq z_{ij}^{(2)}$ for $i = 1, 2, \cdots, n, j = 1, 2, \cdots, m$, and $\sum_{i=1}^{n} z_{ij}^{(1)} = \xi_j(\theta_2) - (\xi_j(\theta_2) - \xi_j(\theta_1)) = \xi_j(\theta_1)$, $j = 1, 2, \cdots, m$. Then for $\boldsymbol{z}^{(2)} \in Z(\theta_2)$, we may deduce that $s_i \geq \sum_{j=1}^{m} z_{ij}^{(2)} \geq \sum_{j=1}^{m} z_{ij}^{(1)}$ for $i = 1, 2, \cdots, n$. Therefore, we have $\boldsymbol{z}^{(1)} \in Z(\theta_1)$. The inequality (6) may be deduced from $z_{ij}^{(1)} \leq z_{ij}^{(2)}$, $i = 1, 2, \cdots, n, j = 1, 2, \cdots, m$ directly. Then the theorem is proved.

Remark 1: Theorem 1 implies that the minimal total transportation cost will decrease with the reduction of demands of customers, which is actually a very natural result. In the following section, Theorem 1 will be used to prove some useful properties of objective functions.

If $Z(\theta) \neq \emptyset$ for $\theta \in \Theta$, in order to obtain $C(\boldsymbol{x}, \boldsymbol{y}|\theta)$, we must solve the following optimization subproblem,

$$\begin{cases} \min \sum_{i=1}^{n} \sum_{j=1}^{m} z_{ij} \sqrt{(x_i - a_j)^2 + (y_i - b_j)^2} \\ \text{subject to:} \\ \qquad \sum_{i=1}^{n} z_{ij} = \xi_j(\theta), \quad j = 1, 2, \cdots, m \\ \qquad \sum_{j=1}^{m} z_{ij} \leq s_i, \quad i = 1, 2, \cdots, n \\ \qquad z_{ij} \geq 0, \quad i = 1, 2, \cdots, n, \quad j = 1, 2, \cdots, m. \end{cases} \tag{7}$$

Note that the parameters x_i, y_i and $\xi_j(\theta)$ are fixed real numbers for $i = 1, 2, \cdots, n, j = 1, 2, \cdots, m$, which implies that (7) is a linear programming problem, whose optimal objective value is $C(\boldsymbol{x}, \boldsymbol{y}|\theta)$.

4 Fuzzy LA Models

In this section, we will present a theoretic framework of fuzzy programming for capacitated LA problem by providing three types of fuzzy programming models according to different criteria. Simultaneously, some relevant mathematical theorems are proved.

4.1 Fuzzy expected cost minimization model

The first type of fuzzy programming is the so-called expected value model (EVM), which is initiated by Liu and Liu [32] and widely used to model practical problems with fuzzy factors. Since the demands of customers are fuzzy variables, the conveying cost $C(\boldsymbol{x},\boldsymbol{y}|\theta)$ is also a fuzzy variable. In order to evaluate the location design, we use its expected cost

$$E[C(\boldsymbol{x},\boldsymbol{y}|\theta)] = \int_0^{+\infty} \mathrm{Cr}\{\theta \in \Theta \bigm| C(\boldsymbol{x},\boldsymbol{y}|\theta) \geq r\}\boldsymbol{d}r.$$

In order to find the decision $(\boldsymbol{x},\boldsymbol{y})$ with the minimal expected cost, we have the following expected cost minimization model for LA problem,

$$\begin{cases} \displaystyle\min_{\boldsymbol{x},\boldsymbol{y}} \int_0^{+\infty} \mathrm{Cr}\left\{\theta \in \Theta \bigm| \ C(\boldsymbol{x},\boldsymbol{y}|\theta) \geq r\right\} \mathrm{d}r \\ \text{subject to:} \\ \qquad g_k(\boldsymbol{x},\boldsymbol{y}) \leq 0, \ \ k = 1,2,\cdots,p \end{cases} \tag{8}$$

where $g_k(\boldsymbol{x},\boldsymbol{y}) \leq 0, \ k = 1,2,\cdots,p$ are the potential region of locations of new facilities, and $C(\boldsymbol{x},\boldsymbol{y}|\theta)$ is defined by (4) and (5).

In order to discuss the property of the objective function of model (8), a definition is given as follows.

Definition 1 *Let $\boldsymbol{\xi}_1 = (\xi_1^1,\xi_2^1,\cdots,\xi_n^1)$ and $\boldsymbol{\xi}_2 = (\xi_1^2,\xi_2^2,\cdots,\xi_n^2)$ be two fuzzy vectors defined on the possibility space $(\Theta,\mathcal{P}(\Theta),\mathrm{Pos})$. We say $\boldsymbol{\xi}_1 \leq \boldsymbol{\xi}_2$ if and only if $\xi_i^1(\theta) \leq \xi_i^2(\theta), i = 1,2,\cdots,n$ for all $\theta \in \Theta$.*

Theorem 2 *Let e_1 and e_2 be the optimal objective values of model (8) when the fuzzy demands are $\boldsymbol{\xi}_1$ and $\boldsymbol{\xi}_2$, respectively. If $\boldsymbol{\xi}_1 \leq \boldsymbol{\xi}_2$, then $e_1 \leq e_2$.*

Proof: Denote the minimal costs with fuzzy demands $\boldsymbol{\xi}_i$ by $C_i(\boldsymbol{x},\boldsymbol{y}|\theta)$, $i = 1,2$, respectively. It follows directly from Theorem 1 and Definition 1 that

$$C_1(\boldsymbol{x},\boldsymbol{y} \bigm| \theta) \leq C_2(\boldsymbol{x},\boldsymbol{y} \bigm| \theta), \quad \forall\theta \in \Theta \tag{9}$$

which implies that $\{\theta \in \Theta \mid C_1(\boldsymbol{x}, \boldsymbol{y}|\theta) \geq r\} \subseteq \{\theta \in \Theta \mid C_2(\boldsymbol{x}, \boldsymbol{y}|\theta) \geq r\}$ for any positive number r. Since the credibility measure Cr is an increasing set function, we get

$$\mathrm{Cr}\{\theta \in \Theta \mid C_1(\boldsymbol{x}, \boldsymbol{y}|\theta) \geq r\} \leq \mathrm{Cr}\{\theta \in \Theta \mid C_2(\boldsymbol{x}, \boldsymbol{y}|\theta) \geq r\}, \ \forall r \geq 0$$

which may deduce that

$$\int_0^{+\infty} \mathrm{Cr}\{\theta \in \Theta \mid C_1(\boldsymbol{x}, \boldsymbol{y}|\theta) \geq r\}\mathrm{d}r \leq \int_0^{+\infty} \mathrm{Cr}\{\theta \in \Theta \mid C_2(\boldsymbol{x}, \boldsymbol{y}|\theta) \geq r\}\mathrm{d}r. \tag{10}$$

Thus $e_1 \leq e_2$.

Remark 2: Theorem 2 implies that the minimal expected cost decreases with the reduction of fuzzy demands of customers, which is similar to the result of deterministic LA problem in Theorem 1. Thus according to Theorem 2, we can say that the expected cost minimization model (8) is a reasonable modeling method for fuzzy LA problem.

4.2 Fuzzy α-cost minimization model

Chance-constrained programming (CCP) is also applied to solve the practical optimization problems with the requirement that the chance constraints should hold with at least some given confidence levels provided as an appropriate safety margin by the decision-maker. In Zhou and Liu [42][43], CCP was applied into stochastic uncapacitated and capacitated LA problem, respectively to meet such a type of requirement. A framework of fuzzy CCP has been presented in Liu and Iwamura [26][27] and Liu [28]. In a fuzzy LA problem, the decision maker may just want to obtain the optimization goals with some fuzzy constraints holding at least some confidence levels. In this case, an α-cost minimization model will be established based on a new concept of α-cost as follows.

Definition 2 *The α-cost of a fuzzy LA problem is defined as* $\min\{C^0 \mid \mathrm{Cr}\{C(\boldsymbol{x}, \boldsymbol{y}|\theta) \leq C^0\} \geq \alpha\}$, *where α is the predetermined confidence level.*

In order to minimize the α-cost of a fuzzy LA problem, the following so-called α-cost minimization model is obtained,

$$\begin{cases} \min\limits_{\boldsymbol{x}, \boldsymbol{y}} C^0 \\ \text{subject to:} \\ \qquad \mathrm{Cr}\left\{\theta \in \Theta \ \middle| \ C(\boldsymbol{x}, \boldsymbol{y}|\theta) \leq C^0\right\} \geq \alpha \\ \qquad g_k(\boldsymbol{x}, \boldsymbol{y}) \leq 0, \ \ k = 1, 2, \cdots, p \end{cases} \tag{11}$$

where C^0 is the α-cost, and $C(\boldsymbol{x}, \boldsymbol{y}|\theta)$ is defined by (4) and (5).

Theorem 3 *Let c_1 and c_2 be the optimal objective values of model (11) when the fuzzy demands are $\boldsymbol{\xi}_1$ and $\boldsymbol{\xi}_2$, respectively. If $\boldsymbol{\xi}_1 \le \boldsymbol{\xi}_2$, then $c_1 \le c_2$.*

Proof: Denote the minimal costs with fuzzy demands $\boldsymbol{\xi}_i$ by $C_i(\boldsymbol{x}, \boldsymbol{y}|\theta)$, $i = 1, 2$, respectively. It follows from Theorem 1 and Definition 1 that

$$C_1(\boldsymbol{x}, \boldsymbol{y} \mid \theta) \le C_2(\boldsymbol{x}, \boldsymbol{y} \mid \theta), \quad \forall \theta \in \Theta$$

which may deduce that

$$\left\{C^0 \mid \mathrm{Cr}\{\theta \in \Theta \mid C_2(\boldsymbol{x}, \boldsymbol{y}|\theta) \le C^0\} \ge \alpha\right\} \subseteq \left\{C^0 \mid \mathrm{Cr}\{\theta \in \Theta \mid C_1(\boldsymbol{x}, \boldsymbol{y}|\theta) \le C^0\} \ge \alpha\right\}.$$

Thus we obtain

$$\inf\left\{C^0 \mid \mathrm{Cr}\{\theta \in \Theta \mid C_1(\boldsymbol{x}, \boldsymbol{y}|\theta) \le C^0\} \ge \alpha\right\} \le \inf\left\{C^0 \mid \mathrm{Cr}\{\theta \in \Theta \mid C_2(\boldsymbol{x}, \boldsymbol{y}|\theta) \le C^0\} \ge \alpha\right\}$$

for any $(\boldsymbol{x}, \boldsymbol{y})$. It follows from

$$c_1 = \min_{\boldsymbol{x}, \boldsymbol{y}} \inf\left\{C^0 \mid \mathrm{Cr}\{\theta \in \Theta \mid C_1(\boldsymbol{x}, \boldsymbol{y}|\theta) \le C^0\} \ge \alpha\right\}$$

and

$$c_2 = \min_{\boldsymbol{x}, \boldsymbol{y}} \inf\left\{C^0 \mid \mathrm{Cr}\{\theta \in \Theta \mid C_2(\boldsymbol{x}, \boldsymbol{y}|\theta) \le C^0\} \ge \alpha\right\}$$

that $c_1 \le c_2$.

Remark 3: Theorem 3 says that the minimal α-cost decreases with the reduction of fuzzy demands of customers. Theorem 3 is indeed a useful result which implies that it is reasonable to formulate the fuzzy capacitated LA problem as the α-cost minimization model (11).

4.3 Credibility maximization model

Sometimes the decision maker may hope that the total transportation cost does not exceed a given level $\overline{C}$ for the sake of economy. That is, the credibility of fuzzy event $\{C(\boldsymbol{x}, \boldsymbol{y}|\theta) \le \overline{C}\}$ should be maximized. In order to formulate this type of decision system, Liu [29][30] provided a theoretical framework of fuzzy dependent-chance programming (DCP), where the underlying philosophy is based on selecting the decision with maximal credibility to meet the event. From this view, we can present a credibility maximization model for LA problem as follows,

$$\begin{cases} \max\limits_{\boldsymbol{x},\boldsymbol{y}} \ \mathrm{Cr}\left\{\theta \in \Theta \,\middle|\, C(\boldsymbol{x},\boldsymbol{y}|\theta) \le \overline{C}\right\} \\ \text{subject to:} \\ \qquad g_k(\boldsymbol{x},\boldsymbol{y}) \le 0, \quad k = 1, 2, \cdots, p \end{cases} \tag{12}$$

where $\overline{C}$ is a given level of the total transportation cost, and $C(\boldsymbol{x},\boldsymbol{y}|\theta)$ is defined by (4) and (5).

Similar property can be obtained for the objective function of model (12) as follows.

Theorem 4 *Let p_1 and p_2 be the optimal objective values of model (12) when the fuzzy demands are $\boldsymbol{\xi}_1$ and $\boldsymbol{\xi}_2$, respectively. If $\boldsymbol{\xi}_1 \le \boldsymbol{\xi}_2$, then $p_1 \ge p_2$.*

Proof: Denote the minimal costs with fuzzy demands $\boldsymbol{\xi}_i$ by $C_i(\boldsymbol{x},\boldsymbol{y}|\theta)$, $i = 1, 2$, respectively. It follows from Theorem 1 and Definition 1 that

$$C_1(\boldsymbol{x},\boldsymbol{y} \mid \theta) \le C_2(\boldsymbol{x},\boldsymbol{y} \mid \theta), \quad \forall \theta \in \Theta.$$

Since

$$\{\theta \in \Theta \mid C_2(\boldsymbol{x},\boldsymbol{y} \mid \theta) \le \overline{C}\} \subseteq \{\theta \in \Theta \mid C_1(\boldsymbol{x},\boldsymbol{y} \mid \theta) \le \overline{C}\},$$

we have

$$\mathrm{Cr}\{\theta \in \Theta \mid C_2(\boldsymbol{x},\boldsymbol{y} \mid \theta) \le \overline{C}\} \le \mathrm{Cr}\{\theta \in \Theta \mid C_1(\boldsymbol{x},\boldsymbol{y} \mid \theta) \le \overline{C}\}$$

which may deduce that

$$\max_{\boldsymbol{x},\boldsymbol{y}} \mathrm{Cr}\{\theta \in \Theta \mid C_2(\boldsymbol{x},\boldsymbol{y} \mid \theta) \le \overline{C}\} \le \max_{\boldsymbol{x},\boldsymbol{y}} \mathrm{Cr}\{\theta \in \Theta \mid C_1(\boldsymbol{x},\boldsymbol{y} \mid \theta) \le \overline{C}\}.$$

That is, $p_2 \le p_1$.

Remark 4: According to Theorem 4, the credibility that the total transportation cost does not exceed a given cost level will increase when the fuzzy demands of customers decrease, which implies that the minimal conveying cost decreases too. As a result, we say that it is reasonable to formulate the fuzzy capacitated LA problem as the credibility maximization model (12).

5 Fuzzy Simulations

By *uncertain functions* we mean the functions with fuzzy parameters. In the above models in Section 4, there exist several uncertain functions with fuzzy variables. Due to the complexity, we design some fuzzy simulations to calculate these uncertain functions according to the concepts of expected value and credibility measure of fuzzy variable in this section.

5.1 Fuzzy simulation for credibility

The first type of uncertain function is

$$U_1 : (\boldsymbol{x}, \boldsymbol{y}) \to \mathrm{Cr}\left\{\theta \in \Theta \;\middle|\; C(\boldsymbol{x}, \boldsymbol{y}|\theta) \le \overline{C}\right\} \tag{13}$$

which is the credibility of fuzzy event $\{C(\boldsymbol{x}, \boldsymbol{y}|\theta) \le \overline{C}\}$. According to the concept of credibility given in Section 2, we design a fuzzy simulation to compute it as follows,

Step 1. Randomly generate θ_k from Θ such that $\mathrm{Pos}\{\theta_k\} \ge \epsilon$ for $k = 1, 2, \cdots, M$, where ϵ is a sufficiently small number and M is a large number.

Step 2. For $\boldsymbol{\xi}(\theta_k)$, solve the linear programming (7) by the network simplex algorithm and denote the optimal objective value by $c_k, k = 1, 2, \cdots, M$, respectively.

Step 3. Set $v_k = \mathrm{Pos}\{\theta_k\}$ for $k = 1, 2, \cdots, M$.

Step 4. Return L as the credibility calculated, where

$$L = \frac{1}{2}\left(\max_{1\le k\le M}\{v_k \mid c_k \le \overline{C}\} + \min_{1\le k\le M}\{1 - v_k \mid c_k > \overline{C}\}\right).$$

In Step 2 of the above process, for each given location $(\boldsymbol{x}, \boldsymbol{y})$ of new facilities and the realized demand $\boldsymbol{\xi}(\theta_k)$, we should determine the optimal allocations to minimize the total transportation cost. In other words, we must solve the linear programming (7), which is virtually a linear *transportation problem*. Thus the *network simplex algorithm* is used to obtain the optimal solution. Similar process can be found in [43]. Interested readers may also consult Bazaraa *et al.* [2] for detailed expositions.

5.2 Fuzzy simulation for α-cost

The second type of uncertain function is the α-cost

$$U_2 : (\boldsymbol{x}, \boldsymbol{y}) \to \min\left\{C^0 \;\middle|\; \mathrm{Cr}\left\{\theta \in \Theta \;\middle|\; C(\boldsymbol{x}, \boldsymbol{y}|\theta) \le C^0\right\} \ge \alpha\right\}. \tag{14}$$

In order to estimate it, we need to find the minimal value C^0 such that $\mathrm{Cr}\left\{\theta \in \Theta | C(\boldsymbol{x}, \boldsymbol{y}|\theta) \le C^0\right\} \ge \alpha$. The fuzzy simulation for calculating U_2 can be summarized as follows,

Step 1. Generate θ_k from Θ such that $\mathrm{Pos}\{\theta_k\} \geq \epsilon$ for $k = 1, 2, \cdots, M$, where ϵ is a sufficiently small number and M is a large number.

Step 2. For $\boldsymbol{\xi}(\theta_k)$, solve the linear programming (7) by the network simplex algorithm and denote the optimal objective value by $c_k, k = 1, 2, \cdots, M$, respectively.

Step 3. Find the maximal value r such that $L(r) \geq \alpha$ holds, where

$$L(r) = \frac{1}{2}\left(\max_{1\leq k\leq M}\{v_k \mid c_k \geq r\} + \min_{1\leq k\leq M}\{1 - v_k \mid c_k < r\}\right).$$

Step 4. Return r.

5.3 Fuzzy simulation for expected cost

The last type of uncertain function is

$$U_3 : (\boldsymbol{x}, \boldsymbol{y}) \to \int_0^{+\infty} \mathrm{Cr}\left\{\theta \in \Theta \;\middle|\; C(\boldsymbol{x}, \boldsymbol{y}|\theta) \geq r\right\} \mathrm{d}r \tag{15}$$

which is actually the expected cost. According to the concept of expected value of fuzzy variable given in Section 2, the fuzzy simulation for estimating U_3 can be run as follows,

Step 1. Set $e = 0$.

Step 2. Generate θ_k from Θ such that $\mathrm{Pos}\{\theta_k\} \geq \epsilon$ for $k = 1, 2, \cdots, M$, where ϵ is a sufficiently small number and M is a large number.

Step 3. For $\boldsymbol{\xi}(\theta_k)$, solve the linear programming (7) by the network simplex algorithm and denote the optimal objective value by $c_k, k = 1, 2, \cdots, M$, respectively.

Step 4. Set $a = c_1 \wedge c_2 \wedge \cdots \wedge c_M$ and $b = c_1 \vee c_2 \vee \cdots \vee c_M$.

Step 5. Randomly generate r from $[a, b]$.

Step 6. $e \leftarrow e + \mathrm{Cr}\{\theta_k \mid c_k \geq r\}$.

Step 7. Repeat the fifth to sixth steps for M times.

Step 8. Return $a + e \cdot (b - a)/M$.

6 Hybrid Intelligent Algorithm

Several hybrid intelligent algorithms have been designed for solving LA problem with stochastic demands in [43], which integrate the network simplex algorithm, stochastic simulation and genetic algorithm (GA). In this section, some similar algorithms will be presented by replacing stochastic simulations with fuzzy simulations. Now let us introduce the hybrid intelligent algorithms for fuzzy LA problem detailedly in the following way.

6.1 Representation structure

In the fuzzy LA problem, we use a nonnegative vector $V = (x_1, y_1, x_2, y_2, \cdots, x_n, y_n)$ as a chromosome to represent a location of new facilities, where (x_i, y_i) is the location of facility i, $i = 1, 2, \cdots, n$, respectively.

6.2 Initialization process

As the initialization process, we randomly generate *pop_size* chromosome from the potential region $\{(\boldsymbol{x}, \boldsymbol{y}) | g_k(\boldsymbol{x}, \boldsymbol{y}) \leq 0,\ k = 1, 2, \cdots, p\}$ uniformly, denoted by $V_1, V_2, \cdots, V_{pop\_size}$.

6.3 Crossover operation

The crossover operation is used to renew the chromosomes $V_k, k = 1, 2, \cdots, pop\_size$ with the probability P_c. In order to determine the parents for crossover operation, we repeat the following process from $k = 1$ to *pop_size*: generating a random real number r from the interval $[0, 1]$, the chromosome V_k' will be selected as a parent provided that $r < P_c$. Then we group the selected parents $V_1', V_2', V_3', \cdots$ to the pairs $(V_1', V_2'), (V_3', V_4'), \cdots$ Without loss of generality, let us illustrate the crossover operator on each pair by (V_1', V_2'). At first, we generate a random number λ from the open interval $(0, 1)$, then the crossover operator on V_1' and V_2' will produce two children X and Y as follows:

$$X = \lambda \cdot V_1' + (1 - \lambda) \cdot V_2', \qquad Y = (1 - \lambda) \cdot V_1' + \lambda \cdot V_2'. \tag{16}$$

If both children are in the region $\{(\boldsymbol{x}, \boldsymbol{y}) | g_k(\boldsymbol{x}, \boldsymbol{y}) \leq 0,\ k = 1, 2, \cdots, p\}$, then we replace the parents with them. If not, we keep the feasible one if it exists, and then redo the crossover operator by regenerating another real number from $(0, 1)$ until two feasible children are obtained. In this case, we only replace the parents with the feasible children. Finally, *pop_size* new chromosomes $V_k, k = 1, 2, \cdots, pop\_size$ are obtained.

6.4 Mutation operation

Now we update the chromosomes $V_k, k = 1, 2, \cdots, pop\_size$ by mutation operation with the probability P_m. Similar to the process of selecting parents for crossover operation, we repeat the following steps from $k = 1$ to *pop_size*: generating a random real number r from the interval $[0, 1]$, the chromosome V_k will be selected as a parent provided that $r < P_m$. For each selected parent $V' = (x'_1, y'_1, \cdots, x'_n, y'_n)$, we mutate it in the following way. At first, we choose a mutation direction $\boldsymbol{d}$ in $\Re^{2n}$ randomly. If

$$X = V' + M \cdot \boldsymbol{d} \tag{17}$$

is not feasible for the region constraints, then we set M as a random number between 0 and M until it is feasible, where M is an appropriate large positive number. If the above process cannot find a feasible solution in a predetermined number of iterations, then we set $M = 0$. Anyway, we replace the parent V' with its child X. Then *pop_size* new chromosomes may be generated, and we still denote them by $V_k, k = 1, 2, \cdots, pop\_size$.

6.5 Evaluation function

We first calculate the objective values for all chromosomes $V_k, k = 1, 2, \cdots, pop\_size$ by the designed fuzzy simulations, where the network simplex algorithm is used to solve the linear programming (7). According to the objective values, an order relationship among these chromosomes is presented to rearrange them from good to bad. Then, the fitness of each chromosome can be computed by some evaluation function. Here, the rank-based evaluation function is defined as

$$Eval(V_k) = a(1-a)^{k-1}, \ k = 1, 2, \cdots, pop\_size \tag{18}$$

where the chromosomes $V_1, V_2, \cdots, V_{pop\_size}$ are assumed to have been rearranged from good to bad according to their objective values and $a \in (0, 1)$ is a parameter in the genetic system.

6.6 Selection process

Generally, we select the chromosomes for a new population based on spinning the roulette wheel characterized by the fitness of all chromosomes for *pop_size* times to generate a new generation to update the chromosomes. First, we compute the cumulative probability q_i for each chromosome V_k, where $q_0 = 0$ and

$$q_i = \sum_{j=1}^{k} Eval(V_k), k = 1, 2, \cdots, pop\_size.$$

Second, we randomly generate a number r in $(0, q_{pop\_size}]$. Then we select the chromosome V_i such that $q_{i-1} < r \le q_i$. Repeating the above steps *pop_size* times, we can obtain *pop_size* copies of chromosome to be a new generation of chromosomes.

After repeating crossover operation, mutation operation and selection process for a given number of cycles, we report the best chromosome $V^* = (\boldsymbol{x}^*, \boldsymbol{y}^*)$ as the optimal location for new facilities.

7 Numerical Examples

In order to illustrate the effectiveness of the proposed algorithms, we give some numerical examples that are performed on a personal computer.

Suppose that there are 20 customers whose locations and demands are given in Table 1, and there are 4 facilities to be located whose capacities $s_i\,(i = 1, 2, 3, 4)$ are 80, 90, 100 and 100, respectively, where all demands are assumed to be trapezoidal fuzzy numbers. We also assume that the potential region for locating new facilities is a rectangle area described by $\{0 \le x_i \le 100, 0 \le y_i \le 100, i = 1, 2, 3, 4\}$.

Table 1: Locations and demands of 20 customers

Customer j	(a_j, b_j)	ξ_j	Customer j	(a_j, b_j)	ξ_j
1	(28, 42)	(14,15,16,17)	11	(14, 78)	(13,15,16,17)
2	(18, 50)	(13,14,16,18)	12	(90, 36)	(11,14,15,17)
3	(74, 34)	(12,14,15,16)	13	(78, 20)	(13,15,17,18)
4	(74, 6)	(17,18,19,20)	14	(24, 52)	(11,13,14,16)
5	(70, 18)	(21,23,24,26)	15	(54, 6)	(12,14,15,16)
6	(72, 98)	(24,25,26,28)	16	(62, 60)	(13,14,15,17)
7	(60, 50)	(13,14,15,16)	17	(98, 14)	(12,14,16,18)
8	(36, 40)	(12,14,16,17)	18	(36, 58)	(12,13,14,15)
9	(12, 4)	(13,15,16,17)	19	(38, 88)	(12,13,15,16)
10	(18, 20)	(22,24,26,28)	20	(32, 54)	(24,26,27,29)

Example 1. If we want to minimize the expected transportation cost, then we have the following

expected cost minimization model,

$$\begin{cases} \min\limits_{\boldsymbol{x},\boldsymbol{y}} \displaystyle\int_0^{+\infty} \mathrm{Cr}\left\{\theta \in \Theta \;\middle|\; C(\boldsymbol{x},\boldsymbol{y}|\theta) \geq r\right\} \mathrm{d}r \\ \text{subject to:} \\ \qquad 0 \leq x_i \leq 100, \quad i = 1,2,3,4 \\ \qquad 0 \leq y_i \leq 100, \quad i = 1,2,3,4 \end{cases} \tag{19}$$

where

$$C(\boldsymbol{x},\boldsymbol{y} \mid \theta) = \begin{cases} \min\limits_{\boldsymbol{z} \in Z(\theta)} \sum\limits_{i=1}^{4} \sum\limits_{j=1}^{20} z_{ij} \sqrt{(x_i - a_j)^2 + (y_i - b_j)^2}, & \text{if } Z(\theta) \neq \emptyset \\ \sum\limits_{j=1}^{20} \max\limits_{1 \leq i \leq 4} \xi_j(\theta) \sqrt{(x_i - a_j)^2 + (y_i - b_j)^2}, & \text{otherwise} \end{cases} \tag{20}$$

and

$$Z(\theta) = \left\{ \boldsymbol{z} \;\middle|\; \begin{array}{l} \sum\limits_{i=1}^{4} z_{ij} = \xi_j(\theta), \quad j = 1,2,\cdots,20 \\ \sum\limits_{j=1}^{20} z_{ij} \leq s_i, \quad i = 1,2,3,4 \\ z_{ij} \geq 0, \quad i = 1,2,3,4, \quad j = 1,2,\cdots,20 \end{array} \right\}. \tag{21}$$

In order to solve model (19), the hybrid intelligent algorithm proposed in Section 6 has been run with 10000 cycles in fuzzy simulations. Adopting different environment parameters of GA and letting GA terminate after 1000 cyclic repetitions of the above steps, we obtain the computational results shown in Table 2, where P_c and P_m are the probabilities of crossover and mutation, respectively, a is the parameter in the evaluation function, and "cost" is the minimal expected cost.

In Table 2, we compare solutions when different parameters are taken with the same generations as a stopping rule. It appears that all the minimal expected costs differ little from each other. In order to account for it, we present a parameter, called the percent error, i.e., (actual value $-$ optimal value)/optimal value $\times$ 100%, where optimal value is the least one of all the ten costs obtained. The last column named by "error" in Table 2 is just this parameter. It follows from Table 2 that the maximal percent error does not exceed 0.92% when different parameters are selected, which implies that the hybrid intelligent algorithm is robust to the parameter settings and effective to solve model (19).

Example 2. If we want to minimize the 0.9-cost, we have the following 0.9-cost minimization model,

Table 2: Comparison Solutions of Example 1

	pop_size	P_c	P_m	a	optimal location	cost	error
1	40	0.3	0.1	0.08	(16.89, 18.35), (53.27, 81.05) (75.99, 20.74), (31.57, 51.93)	5344	0.75%
2	40	0.1	0.2	0.05	(17.34, 19.98), (52.33, 80.03) (75.77, 21.44), (31.04, 52.55)	5342	0.71%
3	40	0.1	0.3	0.10	(18.33, 20.85), (51.35, 81.46) (75.62, 21.26), (31.43, 50.09)	5353	0.92%
4	40	0.2	0.2	0.05	(18.42, 18.15), (53.25, 79.29) (76.26, 21.92), (30.20, 51.25)	5332	0.53%
5	40	0.3	0.2	0.10	(18.23, 18.92), (53.94, 81.83) (75.06, 21.33), (32.12, 53.14)	5314	0.19%
6	50	0.2	0.2	0.10	(17.09, 20.26), (52.19, 81.22) (77.49, 20.24), (31.45, 52.83)	5312	0.15%
7	50	0.1	0.2	0.05	(18.14, 18.38), (52.26, 81.09) (76.93, 21.03), (30.34, 53.03)	5314	0.19%
8	50	0.1	0.3	0.10	(18.42, 18.15), (53.25, 79.29) (76.26, 21.92), (30.20, 51.25)	5332	0.53%
9	50	0.3	0.1	0.08	(17.96, 19.35), (52.37, 82.27) (77.32, 19.72), (30.93, 52.01)	5308	0.07%
10	50	0.3	0.2	0.05	(17.73, 19.18), (52.63, 80.86) (76.56, 20.24), (30.96, 52.63)	5304	0.00%

$$\begin{cases} \min\limits_{\boldsymbol{x},\boldsymbol{y}} C^0 \\ \text{subject to:} \\ \qquad \text{Cr}\left\{\theta \in \Theta \;\middle|\; C(\boldsymbol{x},\boldsymbol{y}|\theta) \le C^0\right\} \ge 0.9 \\ \qquad 0 \le x_i \le 100, \;\; i = 1,2,3,4 \\ \qquad 0 \le y_i \le 100, \;\; i = 1,2,3,4 \end{cases} \tag{22}$$

where $C(\boldsymbol{x},\boldsymbol{y}|\theta)$ is given by (20) and (21).

A numerical study was also carried out to compare the solutions obtained by running the hybrid intelligent algorithm with various parameters, and all the computational results are showed in Table 3, where "cost" is the minimal 0.9-cost, and the cycle number in the fuzzy simulation is 10000.

Solutions with different parameters of GA are compared on the basis of equivalent 1000 generations. In order to measure the differentia of each other, the percent error is proposed and showed in Table 3 as "error". It follows from Table 3 that the maximal percent error does not exceed 1.10% when different parameters are selected. Thus the hybrid intelligent algorithm is also robust to the parameter settings and effective to solve model (22).

Example 3. In order to maximize the credibility that the total conveying cost does not exceed

Table 3: Comparison Solutions of Example 2

	pop_size	P_c	P_m	a	optimal location	cost	error
1	40	0.3	0.1	0.08	(18.34, 19.20), (74.52, 18.99) (29.16, 52.88), (64.12, 63.58)	5659	0.64%
2	40	0.1	0.2	0.05	(19.11, 18.92), (77.01, 19.62) (29.98, 51.03), (60.33, 59.21)	5662	0.69%
3	40	0.1	0.3	0.10	(18.99, 20.15), (75.94, 17.12) (29.34, 53.56), (62.33, 62.36)	5685	1.10%
4	40	0.2	0.2	0.05	(21.54, 20.98), (78.14, 18.61) (28.24, 52.01), (62.36, 62.25)	5679	0.99%
5	40	0.3	0.2	0.10	(19.02, 18.34), (77.63, 19.44) (29.15, 53.88), (62.03, 62.72)	5640	0.30%
6	50	0.2	0.2	0.10	(19.06, 18.26), (74.96, 17.14) (29.26, 53.84), (63.04, 61.22)	5644	0.37%
7	50	0.1	0.2	0.05	(19.23, 19.91), (75.62, 19.35) (28.28, 51.02), (62.98, 61.37)	5634	0.20%
8	50	0.1	0.3	0.10	(19.15, 18.76), (75.34, 19.54) (28.95, 53.91), (60.02, 62.46)	5629	0.11%
9	50	0.3	0.1	0.08	(19.90, 18.88), (75.25, 18.97) (28.12, 53.34), (61.87, 60.24)	5639	0.28%
10	50	0.3	0.2	0.05	(19.49, 19.22), (76.11, 18.25) (28.14, 52.76), (61.98, 60.96)	5623	0.00%

5800, we have a credibility maximization model for LA problem as follows,

$$\begin{cases} \max\limits_{\boldsymbol{x},\boldsymbol{y}} \ \mathrm{Cr}\left\{\theta \in \Theta \;\middle|\; C(\boldsymbol{x},\boldsymbol{y}|\theta) \le 5800\right\} \\ \text{subject to:} \\ \qquad 0 \le x_i \le 100, \quad i = 1,2,3,4 \\ \qquad 0 \le y_i \le 100, \quad i = 1,2,3,4 \end{cases} \tag{23}$$

where $C(\boldsymbol{x},\boldsymbol{y}|\theta)$ is defined by (20) and (21).

The hybrid intelligent algorithm is run with 10000 cycles in simulations and 1000 generations in GA. Different environment parameters of GA are taken, and corresponding solutions are given in Table 4, where “cre” is the maximal credibility.

Similarly, we run the hybrid intelligent algorithm for ten times with different parameters of GA on the basis of equivalent generations. In order to measure the differentia between these results, “error”, i.e., the percent error, is calculated and given in Table 4. From these computational results, we see that the maximal percent error does not exceed 1.38% when different parameters are chosen. Therefore, the hybrid intelligent algorithm is also robust to the parameter settings and effective to solve model (23).

Table 4: Comparison Solutions of Example 3

	pop_size	P_c	P_m	a	optimal location	cre	error
1	40	0.3	0.1	0.08	(19.51, 20.88), (74.02, 17.25) (26.01, 54.29), (63.25, 66.31)	0.9233	1.38%
2	40	0.1	0.2	0.05	(18.78, 18.14), (76.42, 19.80) (26.19, 54.42), (61.20, 65.92)	0.9292	0.75%
3	40	0.1	0.3	0.10	(19.23, 20.03), (76.81, 19.44) (29.19, 54.02), (63.05, 65.26)	0.9238	1.32%
4	40	0.2	0.2	0.05	(18.93, 18.04), (76.66, 18.62) (28.83, 54.87), (62.56, 65.11)	0.9301	0.65%
5	40	0.3	0.2	0.10	(19.14, 18.42), (76.05, 18.54) (27.47, 53.92), (62.48, 65.19)	0.9299	0.67%
6	50	0.2	0.2	0.10	(19.33, 18.02), (76.21, 19.98) (28.12, 52.00), (62.78, 65.86)	0.9325	0.39%
7	50	0.1	0.2	0.05	(17.29, 20.93), (76.61, 20.28) (28.91, 54.38), (60.73, 65.01)	0.9352	0.10%
8	50	0.1	0.3	0.10	(17.80, 19.71), (76.93, 19.05) (27.69, 53.29), (60.10, 64.22)	0.9301	0.65%
9	50	0.3	0.1	0.08	(18.89, 20.02), (76.91, 18.27) (27.91, 53.72), (62.39, 65.88)	0.9344	0.19%
10	50	0.3	0.2	0.05	(18.12, 19.83), (75.93, 19.14) (27.31, 53.09), (61.66, 64.57)	0.9362	0.00%

8 Conclusions

In this paper, capacitated location-allocation problem is investigated in fuzzy environments, where demands of customers are assumed to be fuzzy variables. We proposed three types of fuzzy programming model — fuzzy expected cost minimization model, fuzzy α-cost minimization model, and credibility maximization model — to formulate the LA problem with fuzzy demands for the first time. And some theorems are proved which imply that the formulations suggested are reasonable. For solving these models efficiently, some fuzzy simulations are designed and embedded into genetic algorithm together with the network simplex algorithm to produce some hybrid intelligent algorithms. Finally, some numerical experiments are given to show the effectiveness of the proposed algorithms.

Acknowledgments

This work was supported by National Natural Science Foundation of China (No.60425309), and Specialized Research Fund for the Doctoral Program of Higher Education (No.20020003009).

References

[1] Badri, M.A., Combining the analytic hierarchy process and goal programming for global facility location-allocation problem, *International Journal of Production Economics*, Vol. 62, No. 3, 237-248, 1999.

[2] Bazaraa, M.S., Jarvis, J.J., and Sherali, H.D., *Linear Programming and Network Flows*, Wiley, New York, 1990.

[3] Bhattacharya, U., Rao, J.R., and Tiwari, R.N., Fuzzy multi-criteria facility location problem, *Fuzzy Sets and Systems*, Vol. 51, 277-287, 1992.

[4] Bhattacharya, U., Rao, J.R., and Tiwari, R.N., Bi-criteria multi facility location problem in fuzzy environment, *Fuzzy Sets and Systems*, Vol. 56, 145-153, 1993.

[5] Bongartz, I., Calamai, P.H., and Conn, A.R., A projection method for l_p norm location-allocation problems, *Mathematical Programming*, Vol. 66, 283-312, 1994.

[6] Brimberg, J., Hansen, P., Mladenovic, N., and Taillard, E.D., Improvements and comparison of heuristics for solving the uncapacitated multisource Weber problem, *Operations Research*, Vol. 48, No. 3, 444-460, 2000.

[7] Campos, L., and Verdegay, J.L., Linear programming problems and ranking of fuzzy numbers, *Fuzzy Sets and Systems*, Vol. 32, 1-11, 1989.

[8] Canós, M.J., Ivorra, C., and Liern, V., An exact algorithm for the fuzzy p-median problem, *Eurean Journal of Operations Research*, Vol. 116, 80-86, 1999.

[9] Carrizosa, E., Conde, E., Munoz-Marquez, M., and Puerto, J., The generalized Weber problem with expected distances, *Rairo-Recherche Operationnelle-Operations Research*, Vol. 29, No. 1, 35-57, 1995.

[10] Carrizosa, E., Munoz-Marquez, M., and Puerto, J., A note on the optimal positioning of service units, *Operations Research*, Vol. 46, No. 1, 155-156, 1998.

[11] Charnes, A., and Cooper, W.W., *Management Models and Industrial Applications of Linear Programming*, Wiley, New York, 1961.

[12] Chen, C.B., and Wei, C.C., An efficient fuzzy MADM method for selecting facility locations, *Journal of Engineering Valuation and Cost Analysis*, Vol. 2, No. 1, 19-32, 1998.

[13] Cooper, L., Location-allocation problems, *Operations Research*, Vol. 11, 331-344, 1963.

[14] Darzentas, J., A discrete location model with fuzzy accessibility measures, *Fuzzy Sets and Systems*, Vol. 23, 149-154, 1987.

[15] Dubois, D., and Prade, H., The mean value of a fuzzy number, *Fuzzy Sets and Systems*, Vol. 24, 279-300, 1987.

[16] Ernst, A.T., and Krishnamoorthy, M., Solution algorithms for the capacitated single allocation hub location problem, *Annals of Operations Research*, Vol. 86, 141-159, 1999.

[17] Gen, M., and Cheng, R., *Genetic Algorithms & Engineering Design*, John Wiley and Sons, New York, 1997.

[18] Gen, M., and Cheng, R., *Genetic Algorithms & Engineering Optimization*, John Wiley and Sons, New York, 2000.

[19] González, A., A study of the ranking function approach through mean values, *Fuzzy Sets and Systems*, Vol. 35, 29-41, 1990.

[20] Hakimi, S., Optimum distribution of switching centers in a communication network and some related graph theoretic problems, *Operations Research*, Vol. 13, 462-475, 1964.

[21] Hansen, P., Jaumard, B., and Taillard, E., Heuristic solution of the multisource Weber problem as a p-Median problem, *Operations Research Letters*, Vol. 22, 55-62, 1998.

[22] Heilpern, S., The expected value of a fuzzy number, *Fuzzy Sets and Systems*, Vol. 47, 81-86, 1992.

[23] Hodey, M., Melachrinoudis, E., and Wu, X., Dynamic expansion and location of an airport: A multiple objective approach, *Transportation Research Part A-Policy & Practice*, Vol. 31A, No. 5, 403-417, 1997.

[24] Kuenne, R.E., and Soland, R.M., Exact and approximate solutions to the multisource Weber problem, *Mathematical Programming*, Vol. 3, 193-209, 1972.

[25] Liu, B., Dependent-chance programming: A class of stochastic programming, *Computers & Mathematics with Applications*, Vol. 34, No. 12, 89-104, 1997.

[26] Liu, B., and Iwamura, K., Chance-constrained programming with fuzzy parameters, *Fuzzy Sets and Systems*, Vol. 94, No. 2, 227-237, 1998.

[27] Liu, B., and Iwamura, K., A note on chance-constrained programming with fuzzy coefficients, *Fuzzy Sets and Systems*, Vol. 100, Nos. 1-3, 229-233, 1998.

[28] Liu, B., Minimax chance-constrained programming models for fuzzy decision systems, *Information Sciences*, Vol. 112, Nos. 1-4, 25-38, 1998.

[29] Liu, B., Dependent-chance programming with fuzzy decisions, *IEEE Transactions on Fuzzy Systems*, Vol. 7, No. 3, 354-360, 1999.

[30] Liu, B., Dependent-chance programming in fuzzy environments, *Fuzzy Sets and Systems*, Vol. 109, No. 1, 95-104, 2000.

[31] Liu, B., and Iwamura, K., Fuzzy programming with fuzzy decisions and fuzzy simulation-based genetic algorithm, *Fuzzy Sets and Systems*, Vol. 122, No. 2, 69-77, 2001.

[32] Liu, B., and Liu, Y.-K., Expected value of fuzzy variable and fuzzy expected value models, *IEEE Transactions on Fuzzy Systems*, Vol. 10, No. 4, 445-450, 2002.

[33] Liu, B., *Theory and Practice of Uncertain Programming*, Physica-Verlag, Heidelberg, 2002.

[34] Liu B., *Uncertainty Theory: An Introduction to its Axiomatic Foundations*, Springer-Verlag, Berlin, 2004.

[35] Logendran, R., and Terrell, M.P., Uncapacitated plant location-allocation problems with price sensitive stochastic demands, *Computers and Operations Research*, Vol. 15, No.2, 189-198, 1988.

[36] Murray, A.T., and Church, R.L., Applying simulated annealing to location-planning models, *Journal of Heuristics*, Vol. 2, No. 1, 31-53, 1996.

[37] Ohlemuller, M., Tabu search for large location-allocation problems, *Journal of the Operational Research Society*, Vol. 48, No. 7, 745-750, 1997.

[38] Rao, J.R., and Saraswati, K., Facility location problem on network under multiple criteria fuzzy set theoretic approach, *International Journal of Systems Science*, Vol. 19, No. 12, 2555-2559, 1988.

[39] Sherali, H.D., and Rizzo, T.P., Unbalanced, capacitated p-median problems on a chain graph with a continuum of link demands, *Networks*, Vol. 21, No. 2, 133-163, 1991.

[40] Yager, R.R., A procedure for ordering fuzzy subsets of the unit interval, *Information Sciences*, Vol. 24, 143-161, 1981.

[41] Yager, R.R., On the evaluation of uncertain courses of action, *Fuzzy Optimization and Decision Making*, Vol. 1, 13-41, 2002.

[42] Zhou, J., Uncapacitated facility layout problem with stochastic demands, *Proceedings of the Sixth National Conference of Operations Research Society of China*, China, 904-911, 2000.

[43] Zhou, J., and Liu, B., New stochastic models for capacitated location-allocation problem, *Computers and Industrial Engineering*, Vol. 45, No. 1, 111-125, 2003.