DYNAMIC PRICING & LEARNING IN ELECTRICITY MARKETS
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INTRODUCTION

Power markets in South America, Norway and New Zealand are predominantly hydro-based.

Pricing rules for available water under centralized dispatch were well developed (using stochastic dynamic programming techniques).

The opportunity cost of increasing current water usage thereby expanding current hydro generation is that less water is available for future electricity production.

In the new market structure it is necessary to model the strategic use of water.

Price, essentially, is the control variable influencing the water levels and future profits available to hydro generators.
EXAMPLE

Two identical generators with reservoir and production capacity equal to 1. Demand equals 1. Both generators receive inflows according to random variable $w$, where:

$$w = \begin{cases} 
1 & \text{with probability } q \\
0 & 1 - q 
\end{cases}$$

We posit a symmetric equilibrium.

Let $V_{x,y}$ denote firm $i$’s value function for the state $(x, y)$, where $x \in \{0, 1\}$ denotes firm $i$’s reservoir level, and $y \in \{0, 1\}$ denotes its rival’s reservoir level.

Letting $\beta \in (0, 1)$ represent the discount factor, firm $i$’s value function for the state $(0, 0)$ can be expressed as follows:

$$V_{0,0} = \beta [(1 - q)V_{0,0} + qV_{1,1}]. \quad (1)$$
Next, consider the state, \((0, 1)\):
\[
V_{0,1} = \beta [(1 - q)V_{0,0} + qV_{1,1}] .
\] (2)

Next, consider the state, \((1, 0)\).
\[
V_{1,0} = c^* + \beta [(1 - q)V_{0,0} + qV_{1,1}] .
\] (3)

Note that, by subtracting equation (1) (or (2)) from equation (3), it follows that:
\[
V_{1,0} = c^* + V_{0,0} = c^* + V_{0,1} .
\] (4)

Lastly, consider the “competitive” state, \((1, 1)\). Firm \(i\)’s value from withholding is as follows:
\[
(p_{1,1} - \varepsilon) + \beta [(1 - q)V_{0,1} + qV_{1,1}] .
\]
Alternatively, firm $i$ can bid $\varepsilon$ above $p_{1,1}$.

$$\beta [(1 - q)V_{1,0} + qV_{1,1}].$$

In equilibrium,

$$p_{1,1}^* + \beta [(1 - q)V_{0,1} + qV_{1,1}] = \beta [(1 - q)V_{1,0} + qV_{1,1}].$$

Rearranging equation (5), we obtain:

$$p_{1,1}^* = \beta (1 - q)(V_{1,0} - V_{0,1}).$$

Recognizing that $V_{1,0} - V_{0,1} = c^*$ from equation (4), we derive the following equation:

$$p_{1,1}^* = \beta (1 - q)c^*.$$
A GENERAL FORMULATION

Let $K_i$ be a positive integer that denotes the storage capacity of the reservoir owned by player $i \in \{1, 2, 3, \ldots, n\}$.

The state space is $S = \prod_i \{0, 1, 2, \ldots, K_i\}$.

Player $i$’s production capacity is denoted by $x_i$ (also a positive integer) in each period. We shall assume that players can only produce electricity whenever the water stored exceeds productive capacity.

In each period, demand for electricity is denoted by $D$ (a positive integer) and its assumed to be perfectly inelastic with respect to price.

Bids are linear and can take any value in $\mathcal{P} = [0, c^*]$ where $c^*$ the maximum bid allowed or price cap imposed by the regulator.

Given bids $b_i$ by players, the spot price for electricity $p^*(b)$ (where $b = (b_1, b_2, \ldots, b_n) \in \prod_i \mathcal{P}$) is set to equal the marginal generator’s bid.
Letting $b_{[k]}$ denote the $k$—minimum price bid and $[k] \in \{1, 2, 3, \ldots, n\}$, the associated player’s index, the spot price can be formally expressed as:

$$p^*(b) = b_{[m^*]}$$

where:

$$m^* = \arg\min\{k : \sum_{i=1}^{k} x[i] \geq D\}$$

Player’s $i$ demand as a function of the bids and the state of the reservoirs $s = (s_1, s_2, \ldots, s_n) \in S$ in the system is:

$$D_i(b; s) = \begin{cases} 
\tilde{x} & b_i = p^*(b) \quad \text{and} \quad s_i \geq x^* \\
x_i & b_i < p^*(b) \quad \text{and} \quad s_i \geq x_i \\
0 & \text{otherwise}
\end{cases}$$

where $x^* = \sum_{i=1}^{m^*} X[i] - D$
Moreover, since marginal costs are assumed to be negligible, the immediate resulting payoffs are of the form:

\[ r_i(b; s) = p^*(b) \cdot D_i(b; s) \]

The evolution of the reservoir is governed by a first-order stochastic difference equation:

\[ s'_i = \min[s_i - D_i(b; s) + \xi_i, K_i] \]

where \( \xi_i \) is a (non-negative, integer valued) random variable.

To simplify notation in what follows we shall refer to the probability of reaching state \( s' \) from state \( s \) after players bid \( b \), as \( f(s'; b, s) \).

Finally, we assume a discount factor \( \beta \in (0, 1) \).
Stationary Markovian Pricing Strategies.

For every player $i$, a pure Markovian pricing strategy is denoted by $\pi_i : S \mapsto \mathcal{P}$.

A Markovian strategy combination $\pi = (\pi_1, \pi_2, ..., \pi_n)$ is a vector of Markovian pricing strategies, one entry for each player.

$\Pi$ is the set of all (pure) Markovian strategy combinations.

Value function associated to strategy $\pi$:

$$Q_\pi^i(s) = r_i(\pi(s); s) + \beta \sum_{s' \in S} Q_\pi^i(s') \cdot f(s'; \pi(s), s)$$
A strategy combination $\pi^*$ is a stationary MPE iff for every player $i$ and every state $s \in S$:

$$Q_{\pi_i}^*(s) \geq Q_{(\pi_i, \pi_{-i}^*)}(s)$$

where $(\pi_i, \pi_{-i}^*)$ is the strategy combination with player $i$ bidding according to $\pi_i$ (instead of $\pi_i^*$).
CHARACTERIZATION OF MPE
Let $M_i \subset S$ be defined as follows:

$$M_i = \{ s \in S \mid x_i < s_i \}$$

In the following discussion we restrict our attention to stationary Markovian pricing strategies $\pi$ that have the following property:

$$s \in M_i \implies \pi_i(s) = c^*$$
Valuation

For \( s \in M_i^c \) and \( b \in \mathcal{P} \) we have:

\[
V_i^\pi(s; b) = p^*((b, \pi_{-i}(s)) \cdot D_i(b, \pi_{-i}(s)) + \beta \sum_{s' \in S} V_i^\pi(s') \cdot f(s'; (b, \pi_{-i}(s)), s) \quad (1)
\]

\[
V_i^\pi(s) = \sup_{b \in \mathcal{P}} [V_i^\pi(s; b)] \quad (2)
\]

where \((b, \pi_{-i}(s))\) stands for the strategy combination that equals \( \pi \) except at state \( s \) where player \( i \) bids \( b \).

Equation (1) determines the value of bidding \( b \) today assuming players will play according to \( \pi_{-i} \).

Equation (2) summarizes the value of today’s best decision.
To Release or Not to Release? (that is the question)

If \( s \in M_c^i \) and
\[
V_i^{\pi}(s; c^*) \geq V_i^{\pi}(s; b)
\]
for any \( b \in \mathcal{P} \), then player \( i \) will opt to withhold capacity. In other words, \( c^* \) is a solution to problem (2) and we say that player \( i \) is supra-marginal. Similarly if:
\[
V_i^{\pi}(s; 0) \geq V_i^{\pi}(s; b)
\]
for any \( b \in \mathcal{P} \), the inverse situation, player \( i \) would find that selling today at price \( p^*(\pi(s)) \) is optimal. In other words, bidding zero is a solution to problem (2) and we say that player \( i \) is infra-marginal.

the indifference price for player \( i \), \( p_i(s, \pi) \) is the price that equates the
value obtained by releasing today or withholding.

\[ p_i(s, \pi) = \frac{1}{x_i} \beta \sum_{s' \in S} V_{i}^{\pi}(s') \cdot f(s'; (c^*, \pi_{-i}(s)), s) \]

\[ - \frac{1}{x_i} \beta \sum_{s' \in S} V_{i}^{\pi}(s') \cdot f(s'; (0, \pi_{-i}(s)), s) \]

However, \( p_i(s, \pi) \) may not lie in the feasible price set \( P \).
Therefore, one must define a truncated version:

\[ \tilde{p}_i(s, \pi) = \begin{cases} 
0 & \text{if } p_i(s, \pi) \leq 0 \\
 c^* & \text{if } p_i(s, \pi) \geq c^* \\
 p_i(s, \pi) & \text{otherwise} 
\end{cases} \]
To complete our definition of indifference prices, we shall set the indifference price for the marginal bidder (or bidders) as follows:

$$\tilde{p}_i(s, \pi) = \tilde{p}_{[m^*+1]}(s, \pi) - \delta$$

for some $\delta > 0$, where $[m^* + 1]$ is the index associated with first supra-marginal bidder (as defined above) under strategy combination $\pi$ at the given state $s$.

**Theorem 1:** Strategy “always bid your indifference prices” is a Markov Perfect Equilibrium, i.e. if $\pi$ is of the form:

$$\pi_i(s) = \tilde{p}_i(s, \pi)$$

then $\pi$ is a Markov Perfect Equilibrium.
LEARNING DYNAMIC EQUILIBRIUM

We start with a stationary policy

$$\pi^0 : S \rightarrow \{0, \frac{c^*}{M}, 2\frac{c^*}{M}, \ldots c^*\}^n,$$

that maps each state into the bids made by the players in the past. Note that the price interval $[0, c^*]$ is discretized to the set $\mathcal{P} = \{0, \frac{c^*}{M}, 2\frac{c^*}{M}, \ldots, c^*\}$, for some integer $M$.

Each player $i$ has an initial estimate $\hat{V}_i^{(p, \pi^0_{-i})}(s)$ of the value functions for each state $s$, and price $p \in \mathcal{P}$.

Here $(p, \pi^0_{-i})$ denotes the policy where all other players use policy $\pi^0$, but player $i$ uses price $p$ when state $s$ is visited, and $\pi_i(s)$ at every other state.
At stage $k$ of the algorithm, each player $i$ proceeds as follows.

1. Using simulation, the player obtains a raw estimate of the value function $\hat{V}_i^{(p,\pi_{k-i})}(s)$ for each state $s$, and price $p \in \mathcal{P}$.

2. Corrects its estimate using previous estimates, i.e.
   \[
   \tilde{V}_i^{(p,\pi_k)}(s) = (1 - \alpha_k) \cdot \tilde{V}_i^{(p,\pi_{k-1})}(s) + \alpha_k \cdot \hat{V}_i^{(p,\pi_{k-1})}(s),
   \]
   for all states $s$.

3. The estimate of the indifference prices is computed by
   \[
   \hat{p}_i(s, \pi^k) = \left[ \frac{1}{x_i} \cdot [\tilde{V}_i^{(c^*,\pi_{k-i})}(s) - \tilde{V}_i^{(0,\pi_{k-i})}(s)] + p^*((0, \pi_{k-i}(s)) \right]
   \]
   for each state $s$ where $[x]$ is the closest point in $\{0, \frac{c^*}{M}, 2\frac{c^*}{M}, \ldots, c^*\}$ to $x \in [0, c^*]$. 

Computational Tests: Example for $n=2$, $K=2$, $q = 1/3$
Theorem 2: If $\pi \geq \pi'$ then $\tilde{p}_i(s, \pi) \geq \tilde{p}_i(s, \pi')$, for every player $i$ and any given state $s$ then $d(\pi^k, \Pi^*) \to 0$ with probability 1.