Kinematic and Kinetostatic Subsystem Design for Articulated Legged Wheel Robot

by

YIN CHI CHEN

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Department of Mechanical and Aerospace Engineering
State University of New York at Buffalo
Buffalo, New York 14260
Abstract

In this project, the systematic approach of kinematic synthesis and kinetostatic synthesis for articulated wheel subsystem is developed. Four bar and adjustable four bar mechanism is selected as candidate design. We formulated and derived a precision point synthesis-based systematic approach to generate both convention four bar/adjustable four bar kinematic solution for trajectory tracking problem. Sequentially, using principle of virtual work, kinetostatic synthesis is employed to generate required spring constant. The design is to achieve, (i) the greatest motion-ranges between wheel axle and chassis (ii) reducing required spring assistance by actuation.
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Chapter 1. Introduction

In past decade, considerable interest and research are dedicated on wheeled locomotion systems. The wheeled locomotion systems provide significant advantages such as high mobility, maneuverability and, which offer partial desired features for the rough/unprepared terrain application. However, it remains a great challenge the wheel only system for unprepared terrain; since, a hybrid articulated legged wheel system is being introduced, which is capable for obstacle surmounting. Therefore, this particular system, combining both advantages of wheeled and legged system, enhance locomotion system to having abilities of high-mobility and obstacle surmounting for uneven terrain. Such rough terrain adapting system has diverse application from military, planetary exploration, to disaster environment [1].

The articulated leg-wheel designs under consideration consist of multiple lower pair joints (revolute/prismatic) between the wheel and the chassis. Numerous variants of the articulated leg-wheel system design are possible depending upon the type, number, sequencing and nature of actuation (active/passive) of the joint.

There are less interest are focuses on developing a systematic approach to design legged wheel subsystem, thus, two types of articulated subsystem are derived and demonstrated in this project, and also evaluate this two design after two criteria:

I. achieve the greatest motion-ranges between wheel axle and chassis

II. reducing required spring assistance by actuation
1.1 Literature review: Articulated Wheel Vehicle

Articulated Wheeled Vehicles (AWVs) are a class of wheeled locomotion systems where the chassis is connected to a set of ground-contact wheels via actively- or passively-controlled articulations, which can regulate wheel placement with respect to chassis during locomotion.

Different types of examples range from the Mars Rover [2] and Shrimp [3] with rocker bogie suspensions, Nomad [4] with articulated frames; to systems like the WorkPartner [5] and ALDURO [6] with powered legs and active/passive wheels. In [7], a history of articulated-wheeled vehicles leading up to current day is investigated, then a systematic screw-theoretic framework to analyze two-dimensional articulated vehicles was presented. Then, in [8], Alamdari et al. extended the screw theory formulation on three dimensional articulated wheeled vehicles with 16-DOF, and also on the ground vehicles in [9] to realize the significant advantages of this class of vehicles including stability, obstacle surmounting as well as robustness evaluation of this type of articulated mechanisms. In [10], Alamdari et al. focused on the reconfiguration of the highly-redundant articulated vehicle on the uneven terrains to enhance the contact kinematics i.e. reducing the slippage and improving the traction forces at the wheel ground interfaces in a systematic way. The traversal capabilities of this type of vehicles by virtue of the reconfigurability within articulated structure can be realized only at the price of increased actuation-based equilibration to support the gravitational support. Therefore, the simultaneous reduction of the overall actuation remains one of the critical challenges in such AWVs. In [11], the static balancing of six degree-of-freedom articulated wheeled vehicles with multiple leg-wheel subsystem has been addressed. In this study, elastic elements such as springs
are employed in conjunction with parallelogram linkages to achieve the static balancing. Almadari et al. moved one step further to examine kinetostatic optimization of candidate articulated leg-wheel subsystem designs based on the adjustable four-bar mechanism to enhance locomotion capabilities of land-based vehicles. They were trying to achieve the greatest motion-ranges between wheel axle and chassis while reducing the overall actuation requirements by spring assist, and also with active-structural control, actively change subsystem parameters during the terrain traversal [12].

1.2 Candidate Mechanism of Legged Wheel subsystem

In order to track complex trajectory to surmounting obstacle, lower pair joints and lower degree of freedom are the objective features of desired mechanism. In past, coupled serial chain and four bar based systemic design for articulated legged wheel subsystem for obstacle surmounting has been developed[13,14]. Four bar is one of the simplest parallel mechanism provide one degree on freedom with highly potential to track desired trajectory. In addition, such design allowed tacking 1 D.O.F curve passively, which can provide benefit such as avoid using actuator, simplify the control requirement, and saving potential energy waste. This type of design is shown in Fig1.1 (a)

However, some of terrain contains highly complexity, than using convention four bar mechanism will encounter difficulty to overcome obstacle. Therefore, in order to enhance terrain accommodation, we extend the previous frame work to proposing an adjustable four bar with active parameters control to adjust subsystem during motion. There are three types of adjustable
mechanism are developed and demonstrated. The idea of adjustable four bar mechanism is adding a prismatic joint on one of the four linkages. The benefit of this type mechanism is provide more flexibility than the convention four bar without increasing too much actuator, control, but it still have the advantage of convention four bar mechanism design. Three different type of adjustable four bar mechanism are adapted, including

Type I. Adjustable Crank Link Fig1.1 (b), Type II Adjustable Follower Link Fig1.1 (c), Type III. Adjustable Coupler Link Fig1.1 (d)

![Figure 1.1 Illustration of Four bar/ Adjustable Four bar](image)

(a) Convention Four bar Mechanism  (b) Adjustable Crank Four bar Mechanism  
(c) Adjustable Follower Four bar Mechanism  (d) Adjustable coupler Four bar Mechanism
1.2 Project Organization

The project’s organization is as follows:

In chapter 2 we present background review of existence legged wheel robotic system. Furthermore, introduce the concept of kinematic and kinetostatic optimization-based design, and provide the simple approach to determine desired trajectory and torque profile.

In chapter 3 we demonstrate kinematic synthesis approach of convention four bar using precision point synthesis and optimization combined method. Moreover, we also develop the derivation process for kinetostatic synthesis and optimization.

In chapter 4 we demonstrate kinematic synthesis approach of three different type of adjustable four bar (Adjustable Crank Link, Type II Adjustable Follower Link, Type III. Adjustable Coupler) using precision point synthesis and optimization combined method. Also, we develop the derivation process for kinetostatic synthesis and optimization.

Finally, in chapter 5 we will summarized and discuss the results and outline the future work.
2. Background

2.1 Articulated Legged Wheel System

Articulated Legged Wheel System is one particular type of AWV (Articulated Wheel Vehicle). By integrating robotic leg to wheeled system, overcoming the constraint that most wheeled rover can only perform on flat and regular terrain. High cross-country ability and maneuverability are the major requirements for autonomous mobile robots intended for operation on natural terrain. Plenty of research has dedicated in legged wheel platform design for terrestrial and extra-terrestrial application. However, among these work, almost no effort was focus to develop a systematic method for legged wheel subsystem design.

Mars Rover [2], as shown in Figure 2.1, is using a system called a rocker bogie. The rocker-bogie, uses a two wheeled rocker arm on a passive pivot attached to a main bogie that is connected differentially to the main bogie on the other side. The body of the rover is attached to the differential so it is suspended at an angle that is the average of the two sides. The ride is further smoothened by the rocker, which only passes on a portion of a wheel’s displacement to the main bogie.

![Figure 2.1](image-url)
SHRIMP [3], as shown in Figure, has passive suspension including one front wheel, one trailing wheel, front wheel having a four link suspension with a spring and two middle wheels on each side connected with links. The robot can overcome the obstacle so, that the front wheel after the collision with an obstacle starts raising and steeping away. The SHRIMP’s passive mobility is because of the parallel architecture of the front fork and of the bogies. The bogie design of SHRIMP ensures that the instantaneous center of rotation is always situated under the wheel axis.

NOMAD [4], as shown in Figure 2.3, uses steerable 4 wheels and the steering angles of the wheels on each side are linked with a 4 bar mechanism and a lead screw. In order to distribute
the normal forces on the wheels, NOMAD has two floating side frames called bogies. Each bogie is a structure that supports and deploys two wheels (left or right). By allowing the side frames to pivot on a central axle, the wheels can conform to uneven terrain and maintain even ground pressure. NOMAD features individual propulsion drive units that reside inside the wheel. This is unlike typical all-terrain vehicles, which have a central drive unit that distributes power to each of the wheels.

![Figure 2.4](a) Roller-Walker and (b) Leg Mechanism of Roller-Walker

The Roller-Walker [7], as shown in Figure 2.4, realizes wheeled locomotion by roller-skating using passive wheels. Roller-Walker is a hybrid robot with legs and wheels which has good efficiency by wheel running on a flat area and good ground adaptability by leg type propulsion on a irregular areas. Wheels used in this robot are passive without a driving device. They act as the foot sole in walking mode and wheels for roller-skating in wheel running mode.
2.2 Kinematic and Kinetostatic Design
In this section, our overview design approach and procedure will be discussed. We provide an intuitive criterion to determine the desired kinematic profile (trajectory) and objective kinetostatic profile (torque profile). First, we set up a set of precision points and desired trajectory as problem statement. Using optimization-based precision point approach solve for the trajectory tracking (in order to determine the configuration and dimension of all linkages). Subsequently, we use an extended kinetostatic formulation to determine parameters that also match desired force-specification (determine the required torsion spring preload and spring constants).

2.2.1 Kinematic Profile Design using Precision Point
Kinematic Profile plays a core rule in the surmounting obstacle task, since the trajectory represent the moving curve of wheel fit to obstacle shape and it also determine maximum height potential of this design. Therefore, poor selection may results in the fact that the design mechanism can’t overcome the obstacle or incapable to attach the surface. According to the ‘Uniform Building Codes’ (Figure 2.5), it defined maximum riser height is 8 1/4 inches. This utilizes this step size to select desired precision point and trajectory.

![Figure 2.5 403.21. Uniform Construction Code](image)
The maximum riser height is 8 1/4 inches. There may be no more than a 3/8 inch variation in riser height within a flight of stairs. The riser height is to be measured vertically between leading edges of the adjacent treads.
Two typical motion curves are considered in this project, one is if consider the chassis of entire system are static, which means that the vehicle is not moving, there are no relative motion between obstacle and chassis, the mechanism have to overcome obstacle only by generated motion and no extra motion is adding. For this type design, three precision points are $p_1=[22.0, -15.0]$, $p_2=[22.0, -7]$, $p_3=[26.0, 1.0]$ and desired trajectory are exactly following desired obstacle. This is illustrates in Figure 2.6.

Another design is assume chassis have a constant speed moving forward, this will generate relative motion between chassis and obstacle. In this case, chassis will provide a constant normal force to compress mechanism assist wheel to generate enough traction force to attach the surface. By adapting this design, $p_1=[22.0, -15.0]$, $p_2=[20, -7]$, $p_3=[26.0, 1.0]$ are the selective points for moving chassis, and desired motion is shown in figure 2.7.
2.2.2 Kinetostatic Profile Design

Applied loads at the end-effector of a leg-wheel design

In order to determine the external loads that would be applied on the end-effector of this linkage, we consider the free-body-diagrams of the final link, the wheel and the ground. Assuming this wheel model is simple pure rolling model, no slipping, sliding, and friction are considered. Figure 2.8 illustrates the schematic of an articulated subsystem attaching a wheel to the chassis.

![Figure 2.8 Free-Body-Diagrams of the Final Link, the Wheel and the Ground](image)

The overall weight of chassis (divided by number of legs) and the weight of the links in the articulation create the normal force $F_g$ applied at the axle. We assume a ground slope of $\alpha$ and denote the ground, wheel and axle as bodies- j, i, k respectively. Assuming $F_{ij}$ as force of body-i act on body-j and $F_{ji}$ as force of body-j act on body i, the force that wheel exerts on axle and force that body-j exerts on the body-k is

$$
\begin{bmatrix}
F_{ik,x} \\
F_{ik,y}
\end{bmatrix} = -
\begin{bmatrix}
\cos \alpha & -\sin \alpha \\
\sin \alpha & \cos \alpha
\end{bmatrix}
\begin{bmatrix}
-M_{\text{motor}} \\
-R \\
-m_{\text{wheel}} \cdot g
\end{bmatrix}
$$

(2-1)

Which

- $M_{\text{motor}}$: torque from wheel motor
- $R$: wheel radius
- $m_{\text{wheel}}$: wheel mass
- $g$: gravity
Equation (3-1) shows that the terrain interaction forces, at the end-effector are intimately dependent upon the slope of the ground. By adapting this concept, a desired torque profile can be formulated. Based on the result of desired trajectory, we used the gradient of ground (the slope change of ground) as the slope $\alpha$ to generate $F_{ik,x}$ and $F_{ik,y}$ and utilize these external force $F_x, F_y$ to generate the external load (detail refer to chapter3 kinetostatic synthesis). Thus, this approach is capable to provide a variant torque profile that can adapt to specific terrain.

\[
T_{Ext} = \sum F_x \delta p_x + \sum F_y \delta p_y + \sum M_z \delta \phi + \sum F_{Weight} \delta y_g
\]

\[
= -F_x(Z_2 \sin(\theta_2) \delta \theta_2 + Z_6 \sin(\theta_3 + \phi)) C_1 \delta \theta_2
\]

\[
+F_y(Z_2 \cos(\theta_2) \delta \theta_2 + Z_6 \cos(\theta_3 + \phi)) C_1 \delta \theta_2 + M_z C_1 \delta \theta_2 + \sum F_{Weight} \delta y_g
\]

Figure 2.9  Free Body Diagram of Four bar-Based legged Wheel Design and External Load Equation

Figure 2.9 illustrate the detail component for every point in the desired trajectory, than sum every point’s external load, the results are the desired torque profile. Following Figure2.10 demonstrate two type desire torque profile relate to the static chassis and moving chassis respectively. As the results, the torque profiles are depend on the motion curve shape, which provide a flexible and customized method for specific design of different objective.

Figure 2.10  (a) Desired Torque Profile respect to Fixed Chassis (b) Desired Torque Profile respect to Moving
2.2.3 Optimization-Based Design Approach of Kinematic and Kinetostatic Synthesis

There are many choices of various mechanism configuration and mechanism design parameters that need to be made in order to design an articulated leg-wheel system that is capable of surmounting an obstacle using minimal actuation forces while supporting the weight of the chassis. Given a set of task specifications and the type of mechanism, an optimization problem relating the parameters of the device to the set of desired specifications can be formulated and solved. Specifically, we match the desired specifications on end-effector positions and forces. Other types of design specifications can now also be explored. However, the resulting solution mechanism satisfies all these desired specifications only in the least squares sense without guaranteeing exact satisfaction of any specification.

Greater structure is added to this problem by employing Precision Point Synthesis (PPS). The requirement to match specifications exactly at precision points creates constraints between the various parameters of the mechanism which aids the final selection of the parameters of the designed device. The design constraint equations, themselves, are created from the equations of loop-closure for end-effector position specifications or by application of the principle of virtual work for end-effector force specifications as explored in the thesis. [14,15,16,17]
An optimization scheme over the different candidate mechanisms yields the parameters of the optimal mechanism which satisfies the design specifications exactly at the selected precision points and in the least-squares sense elsewhere. This procedure of Kinematic and Kinetostatic Optimization is established by [14,16]. It is illustrate in Figure 2.11. Moreover, this Design approach will applied on convention four bar/adjustable four bar mechanism in the following chapters.
3. Design of Legged Wheel system using Four-Bar mechanism

In this Chapter, we examining the basic four bar Mechanism as legged Wheel system candidate. Four-bar mechanism is the simplest parallel mechanism which provides one degree of freedom motion can possibly let wheel climb on the vertical obstacle, or stairs. Kinematic synthesis generates potential four bar mechanism to track the desire trajectory. Static synthesis can generate desire torsion spring preload configuration and spring constants provide enough traction force, apply wheel a vertical direction force, support wheel to overcome the obstacle.

3.1 Four-bar Mechanism

Kinematic formulation

\[ Z_2 \cos \theta_2 + Z_3 \cos \theta_3 = Z_4 \cos \theta_4 + Z_1 \cos \theta_1 \]

\[ Z_2 \sin \theta_2 + Z_3 \sin \theta_3 = Z_4 \sin \theta_4 + Z_1 \sin \theta_1 \]

(3.1)
3.2 Kinematic Synthesis
Path is defined as the control of a point in the plane such that it follows some prescribed path.

Tracking desire path is which end-effector, in this case, coupler link to pass through a set of desired output points. It is common for the timing of the arrival of the coupler point at particular locations along the path to be defined. In order to generate desire Path, the precision points, prescribed for successive location of the output link in the plane, are also the essential components to synthesize desire mechanism configuration. Adopting analytical kinematic synthesis approach [18], close-form dyadic was used as kinematic constraint equations.

Although Synthesis for two or three precision points are relatively straightforward, Four or more precision points require solving nonlinear equations, in this design approach, only linear precision synthesis is considered. Two precision and three precision point synthesis method will demonstrate as following section:

![Figure 3.2: Four-bar Synthesis Model for (a) Two Dyads (b) Three Dyads](image-url)
Three precision points synthesis Formulation

Problem statement: Given precision points \( p_1, p_2, p_3 \)

Design variables \( \alpha_2, \alpha'_2, \beta, \beta', \alpha_4, \alpha'_4 \) are the relative angle for second and third configuration, refer to Fig3.2, which are select as free choice. Adopt specific fixed pivot location method, \( \alpha_2, \alpha'_2, \alpha_4, \alpha'_4 \) can be generate by providing \( \beta, \beta' \), position of ground pivot [19], The loop-closure equations of Four bar mechanisms are:

\[
\begin{align*}
Z_2 + Z_6 &= p_1 \quad \quad Z_4 + Z_5 = p_1 \\
Z_2' + Z_6' &= p_2 \quad \quad Z_4' + Z_5' = p_2 \\
Z_2'' + Z_6'' &= p_3 \quad \quad Z_4'' + Z_5'' = p_3 \\
\end{align*}
\]  

(3-2)

According to Euler formula, these close form equation can be simplify as

\[
\begin{align*}
Z_2 (e^{i\alpha_2} - 1) + Z_6 (e^{i\beta} - 1) &= p_2 - p_1 \\
Z_2 (e^{i\alpha'_2} - 1) + Z_6 (e^{i\beta'} - 1) &= p_3 - p_1 \\
Z_4 (e^{i\alpha_4} - 1) + Z_5 (e^{i\beta} - 1) &= p_2 - p_1 \\
Z_4 (e^{i\alpha'_4} - 1) + Z_5 (e^{i\beta'} - 1) &= p_3 - p_1 \\
\end{align*}
\]  

(3-3)
Rearrange in matrix form:

\[
\begin{bmatrix}
    p_2 - p_1 \\
    p_3 - p_1 \\
    p_2 - p_1 \\
    p_3 - p_1
\end{bmatrix} =
\begin{bmatrix}
    e^{i\alpha_2} - 1 & e^{i\beta} - 1 & 0 & 0 \\
    e^{i\alpha_2'} - 1 & e^{i\beta'} - 1 & 0 & 0 \\
    0 & 0 & e^{i\alpha_4} - 1 & e^{i\beta} - 1 \\
    0 & 0 & e^{i\alpha_4'} - 1 & e^{i\beta'} - 1
\end{bmatrix}
\begin{bmatrix}
    Z_2 \\
    Z_6 \\
    Z_4 \\
    Z_5
\end{bmatrix}
\]

Which $Z_2, Z_6, Z_4,$ and $Z_5$ are the solutions need to be solved, thus,

\[
\begin{bmatrix}
    Z_2 \\
    Z_6 \\
    Z_4 \\
    Z_5
\end{bmatrix}
= \begin{bmatrix}
    e^{i\alpha_2} - 1 & e^{i\beta} - 1 & 0 & 0 \\
    e^{i\alpha_2'} - 1 & e^{i\beta'} - 1 & 0 & 0 \\
    0 & 0 & e^{i\alpha_4} - 1 & e^{i\beta} - 1 \\
    0 & 0 & e^{i\alpha_4'} - 1 & e^{i\beta'} - 1
\end{bmatrix}^{-1}
\begin{bmatrix}
    p_2 - p_1 \\
    p_3 - p_1 \\
    p_2 - p_1 \\
    p_3 - p_1
\end{bmatrix}
\]

Solutions $Z_2, Z_6, Z_4,$ and $Z_5$ are complex number which provide configuration results, including linkage length and orientation. Note that not every input can have reasonable results. If input is not well considered, it may generate ill-condition matrix and singularities.

### 3.3 Kinematic synthesis optimization

Since different design variables for kinematic synthesis will generate countless results. Among all these possible results, we are seeking an optimized design which can eliminate the discrepancy between the desire and actual curve. The differences between desired and actual curve is defined as ‘structure error’. Two type of desired curves are demonstrate as following figure. Three precision points are used, which are $p_1=[22.0, -15.0], p_2=[22.0, -7], p_3=[26.0, 1.0]$ for fixed chassis and $p_1=[22.0, -15.0], p_2=[20, -7], p_3=[26.0, 1.0]$ for moving chassis.
The objective function of this optimization problem is stated as:

\[
\text{Min} \sum_{k=1}^{N} \left( (P_{xk} - Q_{xk})^2 + (P_{yk} - Q_{yk})^2 \right)
\]  

Subject to

\[
\begin{bmatrix}
Z_2 \\
Z_6 \\
Z_4 \\
Z_5
\end{bmatrix} = \begin{bmatrix}
e^{i\alpha_2} - 1 & e^{i\beta} - 1 & 0 & 0 \\
e^{i\alpha_2'} - 1 & e^{i\beta'} - 1 & 0 & 0 \\
0 & 0 & e^{i\alpha_4} - 1 & e^{i\beta} - 1 \\
0 & 0 & e^{i\alpha_4'} - 1 & e^{i\beta'} - 1
\end{bmatrix}^{-1}
\begin{bmatrix}
p_2 - p_1 \\
p_3 - p_1 \\
p_2 - p_1 \\
p_3 - p_1
\end{bmatrix}
\]

\[
\alpha_{2\min} < \alpha_2 < \alpha_{2\max} \quad \quad \alpha'_{2\min} < \alpha'_2 < \alpha'_{2\max}
\]

\[
\alpha_{4\min} < \alpha_4 < \alpha_{4\max} \quad \quad \alpha'_{4\min} < \alpha'_4 < \alpha'_{4\max}
\]

\[
\beta_{\min} < \beta < \beta_{\max} \quad \quad \beta'_{\min} < \beta' < \beta'_{\max} \quad \quad Z_{i\min} < Z_i < Z_{i\max}
\]
DV are all design variables $\alpha_2, \alpha'_2, \beta, \beta', \alpha_4, \alpha'_4$, and the structure Error between the desire curve position $(P_{xk}, P_{yk})$ and the actual curve position $(Q_{xk}, Q_{yk})$. Both desired and actual curve should be divided into same length per segment or point to compare the structure error.

3.4 Examples

Case1. Fixed Chassis

Figure 3.4 (a) Synthesis results of Fixed Chassis  (b) Simulation  (c) Desire and actual trajectory

In this case, we assume chassis are fixed and static, so the desire trajectory will be exactly the objective obstacle shape. We can observe that the generated curve is mostly smooth except some jerk and the second precision point doesn’t be passed in the input angle range, but as kinematics aspect, it is still an acceptable result. However, it will have some effects on static synthesis; the consequence will be discussed in the kinetostatic part.
Case 2. Moving Chassis

Figure 3.5 (a) Synthesis results of Moving Chassis (b) Simulation (c) Desire and actual trajectory

Now if we assume the chassis are moving, as an observer on chassis, desire trajectory is smooth continuous motion. As the result above, compare with the fixed chassis, the generated curve from four-bar mechanism is more capable tracking continuous smooth trajectory. In addition, the linkages sizes are lesser and more reasonable for the legged wheel system application.
3.5 Static Synthesis of Four Bar Mechanism

The force interaction between end-effector and terrain is a critical factor will affect the performance if the mechanism is capable to surmounting obstacles. The kinetostatic synthesis procedure developed in [14,15,17] is used to aid the selection of the optimal configuration and required minimum torques to support these external loads by static equilibration. In addition, torsional springs at each joint to optimize the peak actuator torques by a judicious combination of static equilibration design and spring assists was considered.

By using the principle of virtual work, being an equilibrium system, the total virtual work done by external force in virtual displacement must to be zero. The reaction torque is determined by the summation of external torque and the spring torque; such statement is written as:

\[ T_{\text{reaction}} = T_{\text{external}} + T_{\text{spring}} \]
\[ \delta W = T_{\text{reaction}} \delta \alpha_2 + T_{\text{Ext}} \delta p + \sum T_{\text{Spring}} \delta \sigma = 0 \quad (3-6) \]

Where
\[ T_{\text{Ext}} = \sum F_x \delta p_x + \sum F_y \delta p_y + \sum M_z \delta \phi + \sum F_{\text{Weight}} \delta y_g \quad (3-7) \]

Deriving the virtual displacement from loop closure equation of four bar mechanism
\[ Z_2 \cos \theta_2 + Z_3 \cos \theta_3 = Z_1 \cos \theta_1 + Z_4 \cos \theta_4 \quad (3-8) \]
\[ Z_2 \sin \theta_2 + Z_3 \sin \theta_3 = Z_1 \sin \theta_1 + Z_4 \sin \theta_4 \]

Take derivative respect to each angles, \( \theta_1 \) is constant, we get
\[ \begin{bmatrix} Z_2 \sin \theta_2 \\ Z_2 \cos \theta_2 \end{bmatrix} \delta \theta_2 + \begin{bmatrix} Z_3 \sin \theta_3 \\ Z_3 \cos \theta_3 \end{bmatrix} \delta \theta_3 = \begin{bmatrix} Z_4 \sin \theta_4 \\ Z_4 \cos \theta_4 \end{bmatrix} \delta \theta_4 \]
\[ \begin{bmatrix} \delta \theta_3 \\ \delta \theta_4 \end{bmatrix} = \begin{bmatrix} -Z_3 \sin \theta_3 & Z_4 \sin \theta_4 \\ -Z_3 \cos \theta_3 & Z_4 \cos \theta_4 \end{bmatrix}^{-1} \begin{bmatrix} Z_2 \sin \theta_2 \\ Z_2 \cos \theta_2 \end{bmatrix} \delta \theta_2 \quad (3-9) \]

So we can represent \( \delta \theta_3 \) and \( \delta \theta_4 \) in terms of \( \delta \theta_2 \)
\[ \begin{bmatrix} \delta \theta_3 \\ \delta \theta_4 \end{bmatrix} = \begin{bmatrix} Z_2 (\sin \theta_4 \cos \theta_2 - \cos \theta_4 \sin \theta_2) \\ Z_3 (\sin \theta_3 \cos \theta_4 - \cos \theta_3 \sin \theta_4) \\ Z_2 (\sin \theta_3 \cos \theta_2 - \cos \theta_3 \sin \theta_2) \\ Z_4 (\sin \theta_3 \cos \theta_4 - \cos \theta_3 \sin \theta_4) \end{bmatrix} \delta \theta_2 \]
\[ \begin{bmatrix} \delta \theta_3 \\ \delta \theta_4 \end{bmatrix} = \begin{bmatrix} Z_2 \sin (\theta_4 - \theta_2) \\ Z_2 \sin (\theta_3 - \theta_4) \\ Z_2 \sin (\theta_3 - \theta_2) \\ Z_4 \sin (\theta_3 - \theta_4) \end{bmatrix} \delta \theta_2 \quad (3-10) \]
Since on the same rigid body, end-effector orientation \( \phi = \theta_6 = \theta_3 + \varphi \), \( \varphi \) is constant,

\[ \delta \theta_6 = \delta \theta_3 \]

\[
\begin{bmatrix}
\delta \theta_2 \\
\delta \theta_3 \\
\delta \theta_4
\end{bmatrix}
= \begin{bmatrix}
1 \\
z_2 \sin (\theta_2-\theta_3) \\
z_3 \sin (\theta_3-\theta_4) \\
z_4 \sin (\theta_3-\theta_4)
\end{bmatrix}
\begin{bmatrix}
\delta \theta_2 \\
\delta \theta_3 \\
\delta \theta_4
\end{bmatrix}
= \begin{bmatrix}1 \\
C1 \\
C2
\end{bmatrix}
\delta \theta_2
\tag{3-11}
\]

And the relationship between \( \alpha_2 \), \( \beta \), \( \alpha_4 \), and \( \theta_2, \theta_3, \theta_4 \) are

\[
\begin{align*}
\theta_2 &= \theta_{2\text{initial}} + \alpha_2 & \delta \theta_2 &= \delta \alpha_2 \\
\theta_3 &= \theta_{3\text{initial}} + \beta & \delta \theta_3 &= \delta \beta \\
\theta_4 &= \theta_{4\text{initial}} + \alpha_4 & \delta \theta_4 &= \delta \alpha_4
\end{align*}
\tag{3-12}
\]

The external torque is be state as

\[
T_{\text{Ext}} = \sum F_x \delta p_x + \sum F_y \delta p_y + \sum M_z \delta \phi + \sum F_{\text{Weight}} \delta y_g
\]

\[
= -F_x (Z_2 \sin(\theta_2) \delta \theta_2 + Z_6 \sin(\theta_3 + \varphi)) C1 \delta \theta_2 + F_y (Z_2 \cos(\theta_2) \delta \theta_2 + Z_6 \cos(\theta_3 + \varphi)) C1 \delta \theta_2
\]

\[+ M_z C1 \delta \theta_2 + \sum F_{\text{Weight}} \delta y_g \]
\tag{3-13}

Which weight (of the body) torque is:

\[
\sum F_{\text{Weight}} \delta y_g = m_2 g \frac{Z_2}{2} \cos(\theta_2 + \alpha_2) \delta \theta_2 + m_3 g (Z_2 \cos(\theta_2 + \alpha_2) + \frac{Z_6}{2} \cos(\theta_3 + \varphi) C1 \delta \theta_2 + m_4 g \frac{Z_4}{2} \cos(\theta_4 + \alpha_4) C2 \delta \theta_2
\]
\tag{3-14}

\( \sigma_i \) are angular extension of each spring, refer to Fig.
\[\sigma_2 = \theta_{2\text{initi}} + \alpha_2 - \Omega_2\]
\[\sigma_3 = (\theta_{3\text{initi}} + \beta - \Omega_3) - (\theta_{2\text{initi}} + \alpha_2 - \Omega_2)\]
\[\sigma_4 = \theta_{4\text{initi}} + \alpha_4 - \Omega_4\]
\[\sigma_5 = (\theta_{5\text{initi}} + \beta - \Omega_5) - (\theta_{4\text{initi}} + \alpha_4 - \Omega_4)\]

\[\sigma_2, \sigma_3, \sigma_4, \sigma_5\] Can be express in terms of their corresponding virtual displacement \(\alpha_2, \beta, \alpha_4,\)

\[\frac{\partial \sigma_2}{\partial \alpha_2} = 1\]
\[\frac{\partial \sigma_3}{\partial \alpha_2} \frac{\partial \beta}{\partial \alpha_2} = C_1 \frac{\partial \sigma_2}{\partial \alpha_2} = C_1 - 1\]
\[\frac{\partial \sigma_4}{\partial \alpha_2} \frac{\partial \alpha_4}{\partial \alpha_2} = C_2 \frac{\partial \sigma_2}{\partial \alpha_2} = C_2\]
\[\frac{\partial \sigma_5}{\partial \alpha_2} \frac{\partial \beta}{\partial \alpha_2} = C_2 = C_1 - C_1,\]
\[C_1 = \frac{Z_2 \sin(\theta_2 - \theta_3)}{Z_3 \sin(\theta_3 - \theta_4)},\quad C_2 = \frac{Z_2 \sin(\theta_2 - \theta_3)}{Z_4 \sin(\theta_3 - \theta_4)}\]

The equilibrium equation of virtual works is

\[T_{\text{reaction}} + \sum F_x \frac{\partial p_x}{\partial \alpha_2} + \sum F_y \frac{\partial p_y}{\partial \alpha_2} + \sum M_z \frac{\partial \phi}{\partial \alpha_2} + \sum F_{\text{weight}} \frac{\partial y_g}{\partial \alpha_2}\]

\[-(k_2 \sigma_2 \frac{\partial \sigma_2}{\partial \alpha_2} + k_3 \sigma_3 \frac{\partial \sigma_2}{\partial \alpha_2} + k_4 \sigma_4 \frac{\partial \sigma_2}{\partial \alpha_2} + k_4 \sigma_4 \frac{\partial \sigma_4}{\partial \alpha_2} + k_5 \sigma_5 \frac{\partial \sigma_5}{\partial \alpha_2}) \partial \alpha_2 = 0\] (3-17)

Substitute the relationship in equation 3.13, we can get:

\[-F_x(Z_2 \sin(\theta_2) + Z_6 \sin(\theta_3 + \varphi)) C_1 + F_y(Z_2 \cos(\theta_2) + Z_6 \cos(\theta_3 + \varphi)) C_1 +\]

\[+ M_2 C_1 + m_2 g Z_2 \cos(\theta_2 + \alpha_2) + m_3 g Z_2 \cos(\theta_2 + \alpha_2) + \frac{Z_6}{2} \cos(\theta_3 + \varphi) C_1 +\]

\[m_4 g \frac{Z_4}{2} \cos(\theta_4 + \alpha_4) C_2 - (k_2 \sigma_2 \frac{\partial \sigma_2}{\partial \alpha_2} + k_3 \sigma_3 \frac{\partial \sigma_2}{\partial \alpha_2} + k_4 \sigma_4 \frac{\partial \sigma_2}{\partial \alpha_2} + k_4 \sigma_4 \frac{\partial \sigma_4}{\partial \alpha_2} + k_5 \sigma_5 \frac{\partial \sigma_5}{\partial \alpha_2}) \partial \alpha_2 \] (3-18)
The sum of virtual work done by the external force reaction torque at joint and spring forces is equal to zero, which meant reaction torque at driving joint is

\[
T_{reaction} + T_{Ext} + T_{Spring} = -T_{joint} + T_{Ext} + T_{Spring} = 0 \tag{3-19}
\]

\(T_{Joint}\) will be the balance torque that comparing the desire torque profile, but before we get to that point, additional spring torque need to be arrange in terms of the relative angular motion \(\alpha_2, \beta, \alpha_4\) as a linear equation. Although \(\beta, \alpha_4\) does not have any linear relationship with \(\alpha_2\), \(T_{Spring}\) is still possible to formulate in a linear equation with spring constants\((k_i)\) and preload\((\Omega_i)\). The derivation is demonstrated as follows:

\[
T_{Spring} \delta \sigma = -k_2 \sigma_2 \frac{\partial \sigma_2}{\partial \alpha_2} - k_3 \sigma_3 \frac{\partial \sigma_3}{\partial \alpha_2} - k_4 \sigma_4 \frac{\partial \sigma_4}{\partial \alpha_2} - k_5 \sigma_5 \frac{\partial \sigma_5}{\partial \alpha_2}
\]

\[
= -k_2 (\theta_2_{initial} + \alpha_2 - \Omega_2) \frac{\partial \sigma_2}{\partial \alpha_2} - k_3 ((\theta_3_{initial} + \beta - \Omega_3) - (\theta_2_{initial} + \alpha_2 - \Omega_2)) \frac{\partial \sigma_3}{\partial \alpha_2}
\]

\[
- k_4 (\theta_4_{initial} + \alpha_4 - \Omega_4) \frac{\partial \sigma_4}{\partial \alpha_2} - k_5 ((\theta_5_{initial} + \beta - \Omega_5) - (\theta_4_{initial} + \alpha_4 - \Omega_4)) \frac{\partial \sigma_5}{\partial \alpha_2} \tag{3-20}
\]

Rearrange the spring torque in the linear form

\[
T_{Spring} = a\alpha_2 + b\alpha_4 + c\beta + d \tag{3-21}
\]
\[-k_2 + k_3(C1 - 1)\alpha_2
\]
\[+(-k_4C2 + k_5(C1 - C2))\alpha_4
\]
\[+(-k_3(C1 - 1) + k_5(C1 - C2))\beta
\]
\[(-k_2(\theta_{2\text{\,initial}} - \Omega_2)) - k_3(C1 - 1)((\theta_{3\text{\,initial}} - \Omega_3) - (\theta_{2\text{\,initial}} - \Omega_2))
\]
\[(-k_4C2(\theta_{4\text{\,initial}} - \Omega_4)) - k_5(C1 - C2)((\theta_{5\text{\,initial}} - \Omega_5) - (\theta_{4\text{\,initial}} - \Omega_4)) \quad (3-22)
\]

This formulation gives designer benefit that only needs to solve for linear equation by matrix.

### 3.6 Kinetostatic Optimization

The ultimate goal is of this formulation is solve for torsion spring constant and preload configuration. In previous derivation, spring constant and preload are all collect in \(a, b, c, d\) four variables. Now, if using four static precision point process, four position \(\alpha_2, i\) and its corresponding external torque and desire torque is needed.

\[
\begin{bmatrix}
\alpha_{2,1} & \alpha_{4}(\alpha_{2,1}) & \beta(\alpha_{2,1}) & 1 \\
\alpha_{2,2} & \alpha_{4}(\alpha_{2,2}) & \beta(\alpha_{2,2}) & 1 \\
\alpha_{2,3} & \alpha_{4}(\alpha_{2,3}) & \beta(\alpha_{2,3}) & 1 \\
\alpha_{2,4} & \alpha_{4}(\alpha_{2,4}) & \beta(\alpha_{2,4}) & 1 \\
\end{bmatrix}^{-1}
\begin{bmatrix}
T_{\text{desire}}(\alpha_{2,1}) - T_{\text{Ext}}(\alpha_{2,1}) \\
T_{\text{desire}}(\alpha_{2,2}) - T_{\text{Ext}}(\alpha_{2,2}) \\
T_{\text{desire}}(\alpha_{2,3}) - T_{\text{Ext}}(\alpha_{2,3}) \\
T_{\text{desire}}(\alpha_{2,4}) - T_{\text{Ext}}(\alpha_{2,4}) \\
\end{bmatrix}
\quad (3-23)
\]

Then, using the results to solve for \(k_2, k_3, k_4, k_5\)
\[
\begin{bmatrix}
k_2 \\
k_3 \\
k_4 \\
k_5 \\
\end{bmatrix}
= \begin{bmatrix}
-1 & (C_1 - 1) & 0 & 0 \\
0 & 0 & -C_1 & (C_1 - C_2) \\
0 & -(C_1 - 1) & 0 & (C_1 - C_2) \\
-(\theta_{2_{\text{initial}}} - \Omega_2) & -((C_3_{\text{initial}} - \Omega_3) - (\theta_{2_{\text{initial}}} - \Omega_2)) & C_2(\theta_{4_{\text{initial}}} - \Omega_4) & -(C_1 - C_2)((\theta_{5_{\text{initial}}} - \Omega_5) - (\theta_{4_{\text{initial}}} - \Omega_4)) \\
\end{bmatrix}^{-1} \begin{bmatrix}
a \\
b \\
c \\
d \\
\end{bmatrix}
\]

Or implement Kinetostatic Optimization approach

\[
\text{Min} \sum_{i=1}^{N} ((T_{\text{desired}} - T_{\text{actual}})^2)
\]

(3-24)

Which \( T_{\text{actual}} = T_{\text{Ext}} + T_{\text{Spring}} = T_{\text{Ext}} + a\alpha_2 + b\alpha_4 + c\beta + d \)

The desire torque file is the designed required torque will capable to cross the obstacle. Thus, the process of optimization is to eliminating the differences between desire torque and actual torque by adjusting spring constant and preload. In other words, it is to change the internal torque generate by torsion spring to achieve the desire torque in order have the capability to cross obstacle.
3.7 Examples:

Case 1 Fixed Chassis

(a) Initial External Torque and Desire Torque
(b) Balanced Torque and Desire Torque
(c) Synthesized Spring Constant

<table>
<thead>
<tr>
<th>$k_2$</th>
<th>$k_3$</th>
<th>$k_4$</th>
<th>$k_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>29.8497</td>
<td>44.1172</td>
<td>21.6462</td>
<td>31.6115</td>
</tr>
</tbody>
</table>

Figure 3.7 (a) Initial External Torque and Desire Torque (b) Balanced Torque and Desire Torque

(c) Synthesized Spring Constant
**Case.2 Moving Chassis**

![Graph showing external torque and desire torque](image1)

![Graph showing balanced torque and desire torque](image2)

![Table showing spring constants](image3)

**Figure 3.8 (a) Initial External Torque and Desire Torque (b) Balanced Torque and Desire Torque (c) Synthesized Spring Constant**

As we mentioned previously in kinematic synthesis part, the consequence of that design is that even after spring balancing, during the jerk motion, it will still affect the entire mechanism, which might cause unstable or damage. Also, if we compare the difference between Fixed chassis and moving chassis, fixed chassis require larger spring constant which demonstrate that the normal force provide by moving chassis assist the legged wheel system to cross obstacle.
4. Design Mechanism of Legged Wheel System using Adjustable Four-bar Mechanism

In this chapter we will introduce an adjustable four bar mechanism, also known as ‘reconfigurable/programmable’ four bar mechanism. The idea adjustable four bar mechanism is adding a prismatic joint on one of the four linkages. The benefit of this type mechanism is provide more flexibility than the convention four bar without increasing too much actuator, control, but it still have the advantage of convention four bar mechanism design. A analytical precision point method was developed by J.F. McGovern[20] and T. Chuenchom[21] derived a five precision points adjustable dyad synthesis method. In the following sections, we will introduce the kinematics for adjustable four bars, and kinematic synthesis, static synthesis for legged wheel sub system based design. We do not consider the adjustable ground link due the requirement of this type of application.

4.1 Adjustable Four bar Mechanism:

\[
\begin{align*}
    kZ_2 \cos \theta_2 + Z_3 \cos \theta_3 &= Z_4 \cos \theta_4 + Z_1 \cos \theta_1 \\
    kZ_2 \sin \theta_2 + Z_3 \sin \theta_3 &= Z_4 \sin \theta_4 + Z_1 \sin \theta_1
\end{align*}
\] (4-1)

In the above Kinematics loop-closure equation, stretch ratio ‘\(k\)’ is being introduced to represent the extension or reduction of particular linkage. In this example, we make an assumption that crank ‘\(Z_2\)’ is the adjustable link.

Figure 4.1. Adjustable four-bar mechanism
4.2 Kinematic synthesis

Similarly, the analytical close-form precision point approach for adjustable four bar can synthesize desired linkage dimension by using up to five points [18]. However, using four or more points will increase the complexity for the problem involving nonlinear equation. Considering the application of this mechanism, three points will be sufficient for our problem.

Three Precision Points Synthesis Formulation for Adjustable Four bar

Type I. Loop-Closure Equation of Adjustable Crank

\[
\begin{align*}
\lambda_2 Z_2 (e^{i\alpha_2} - 1) + Z_6 (e^{i\beta} - 1) &= p_2 - p_1 \\
\lambda_3 Z_2 (e^{i\alpha_2'} - 1) + Z_6 (e^{i\beta'} - 1) &= p_3 - p_1 \\
Z_4 (e^{i\alpha_4} - 1) + Z_5 (e^{i\beta} - 1) &= p_2 - p_1 \\
Z_4 (e^{i\alpha_4'} - 1) + Z_5 (e^{i\beta'} - 1) &= p_3 - p_1
\end{align*}
\]

Three point Synthesis of Adjustable Crank

\[
\begin{bmatrix}
Z_2 \\
Z_6 \\
Z_4 \\
Z_5
\end{bmatrix} =
\begin{bmatrix}
\lambda_2 e^{i\alpha_2} - 1 & e^{i\beta} - 1 & 0 & 0 \\
\lambda_3 e^{i\alpha_2'} - 1 & e^{i\beta'} - 1 & 0 & 0 \\
0 & 0 & e^{i\alpha_4} - 1 & e^{i\beta} - 1 \\
0 & 0 & e^{i\alpha_4'} - 1 & e^{i\beta'} - 1
\end{bmatrix}^{-1}
\begin{bmatrix}
p_2 - p_1 \\
p_3 - p_1 \\
p_2 - p_1 \\
p_3 - p_1
\end{bmatrix}
\]

(4-2)
Type II. Loop-Closure Equation of Adjustable Coupler Link

\[ Z_2 (e^{i\alpha_2} - 1) + \kappa_2 Z_6 (e^{i\beta} - 1) = p_2 - p_1 \]

\[ Z_2 (e^{i\alpha_2'} - 1) + \kappa_3 Z_6 (e^{i\beta'} - 1) = p_3 - p_1 \]

\[ Z_4 (e^{i\alpha_4} - 1) - Z_3 (e^{i\beta} - 1) + \kappa_2 Z_6 (e^{i\beta} - 1) = p_2 - p_1 \]

\[ Z_4 (e^{i\alpha_4'} - 1) - Z_3 (e^{i\beta'} - 1) + \kappa_2 Z_6 (e^{i\beta'} - 1) = p_3 - p_1 \]

Three point Synthesis of Adjustable Coupler

\[
\begin{bmatrix}
Z_2 \\
Z_6 \\
Z_4 \\
Z_3
\end{bmatrix} =
\begin{bmatrix}
e^{i\alpha_2 - 1} & \kappa_2 (e^{i\beta} - 1) & 0 & 0 \\
e^{i\alpha_2'} - 1 & \kappa_3 (e^{i\beta'} - 1) & 0 & 0 \\
0 & \kappa_2 (e^{i\beta} - 1) & e^{i\alpha_4 - 1} & -(e^{i\beta} - 1) \\
0 & \kappa_3 (e^{i\beta'} - 1) & e^{i\alpha_4'} - 1 & -(e^{i\beta'} - 1)
\end{bmatrix}^{-1} \times \begin{bmatrix}
p_2 - p_1 \\
p_3 - p_1 \\
p_2 - p_1 \\
p_3 - p_1
\end{bmatrix} \tag{4-3}
\]

Type III. Loop-Closure Equation of Adjustable Follower Link

\[ Z_2 (e^{i\alpha_2} - 1) + Z_6 (e^{i\beta} - 1) = p_2 - p_1 \]

\[ Z_2 (e^{i\alpha_2'} - 1) + Z_6 (e^{i\beta'} - 1) = p_3 - p_1 \]

\[ \eta_2 Z_4 (e^{i\alpha_4} - 1) + Z_5 (e^{i\beta} - 1) = p_2 - p_1 \]

\[ \eta_3 Z_4 (e^{i\alpha_4'} - 1) + Z_5 (e^{i\beta'} - 1) = p_3 - p_1 \]

Three point Synthesis of Adjustable follower
\[
\begin{bmatrix}
Z_2 \\
Z_6 \\
Z_4 \\
Z_5
\end{bmatrix}
= \begin{bmatrix}
\lambda_2 e^{i \alpha_2} - 1 & e^{i \beta} - 1 & 0 & 0 \\
\lambda_2' e^{i \alpha_2'} - 1 & e^{i \beta'} - 1 & 0 & 0 \\
0 & 0 & \eta_2 e^{i \alpha_4} - 1 & e^{i \beta} - 1 \\
0 & 0 & \eta_3' e^{i \alpha_4'} - 1 & e^{i \beta'} - 1
\end{bmatrix}^{-1} \times \begin{bmatrix}
p_2 - p_1 \\
p_3 - p_1 \\
p_2 - p_1 \\
p_3 - p_1
\end{bmatrix}
\]

(4-4)

\(\lambda_{i}, \kappa_{i}, \eta_{i}\) are the stretch ratio of the \(i_{th}\) precision points of adjustable crank, coupler, and follower respectively. \(\alpha_{2}, \alpha_{2}', \beta, \beta', \alpha_{4}, \alpha_{4}'\) are the relative angle for second and third configuration.

### 4.3 Kinematic synthesis optimization:

Similar as four bar kinematic synthesis optimization, we are suing the same three precision points and desired trajectory. The objective function is eliminate the structure Error between the desire curve position \((P_{xk}, P_{yk})\) and the actual curve position \((Q_{xk}, Q_{yk})\), and the design variables are \(\alpha_2, \alpha_2', \beta, \beta', \alpha_4, \alpha_4', k_2, k_3\),

\[
\min_{DV} \sum_{k=1}^{N} ((P_{xk} - Q_{xk})^2 + (P_{yk} - Q_{yk})^2)
\]

(4-5)

Subject to

Different constraints for different adjustable type, using adjustable crank as example
\[ \alpha_{2\min} < \alpha_2 < \alpha_{2\max} \quad \alpha_{2\min}' < \alpha_2' < \alpha_{2\max}' \]
\[ \alpha_{4\min} < \alpha_4 < \alpha_{4\max} \quad \alpha_{4\min}' < \alpha_4' < \alpha_{4\max}' \]
\[ \beta_{\min} < \beta < \beta_{\max} \quad \beta_{\min}' < \beta' < \beta_{\max}' \quad Z_{i\min} < Z_i < Z_{i\max} \]
\[ \lambda_{2\min} < \lambda_2 < \lambda_{2\max} \quad \lambda_{3\min} < \lambda_3 < \lambda_{3\max} \]

4.4 Examples:

**Case 1. Fixed Chassis for Adjustable Crank**

Figure 4.3  (a) Three Points Synthesis Results  
(b) Simulation  
(c) Profile of Crank Angle and Crank Stretch Ratio  
(d) Desire and actual trajectory
Case 2. Moving Chassis for Adjustable Crank

![Diagram](image)

Figure 4.4  (a) Three Points Synthesis Results  (b) Simulation  (c) Profile of Crank Angle and Crank Stretch Ratio  (d) Desire and actual trajectory

Case 3. Fixed Chassis for Adjustable Follower

![Diagram](image)
Case 4. Moving Chassis for Adjustable Follower

Figure 4.5 (a) Three Points Synthesis Results  
(b) Simulation  
(c) Profile of Crank Angle and Follower Stretch Ratio  
(d) Desire and actual trajectory

Figure 4.6 (a) Three Points Synthesis Results  
(b) Simulation  
(c) Profile of Crank Angle and Follower Stretch Ratio  
(d) Desire and actual trajectory
Above examples illustrate the synthesized configuration, kinematic simulation, angle and stretch ratio profile, and discrepancy of least square results. According to the results, first, all adjustable mechanism generated a relative smooth and continuous motion. Compare to conventional four bar Figure3.4(c), in fixed chassis condition, adjustable mechanism can preventing such jerk motion and also reduce the reaction force applied on ground link/chassis. In

Figure 4.7 (a)Three Points Synthesis Results  (b) Simulation  
(c) Profile of Crank Angle and Coupler Stretch Ratio  (d) Desire and actual trajectory
addition, convention four bar required greater dimension to achieve such objective. Therefore, we can conclude that in a static environment, adjustable four bar provide greater potential for surmounting obstacles.

Secondary, in moving chassis case, both convention and adjustable four bar generate acceptable trajectory. However, it is a benefit if no actuators are applied; thus, a well-designed four bar mechanism still sustain an ideal design in practice.

Third, seeking an optimum adjustable mechanism depends on different purpose and requirement. In the legged wheel application, adjustable has a relative complicated kinematic synthesis formulation, which result in difficulty to generate desired design. Adjustable crank and adjustable follower both provide smooth curve and reasonable linkage dimension. However, comparing the stretch ratio profile, adjustable crank has a homotonic performance, also, synchronizing angular motor and linear actuator will be more realistic using adjustable crank in practice.
4.5 Static Synthesis of Adjustable Four Bar Mechanism

Similar as the static synthesis of conventional four bar mechanism, by applying principle of virtual work, desired preload and spring constants can be generated from solving the static equilibrium. Same desired torque profile is implemented for adjustable four bar mechanism. However, due to an extra prismatic joint is introduced, virtual displacement will separate to rotational and translational component; furthermore, different type of adjustable mechanism has different virtual displacement which result in $T_{Ext}$. The derivation of will be demonstrated as following section.

**Type I. Adjustable Crank Link**

Kinematic Close-Form Equation

\[
\begin{align*}
\lambda Z_2 \cos \theta_2 + Z_3 \cos \theta_3 &= Z_1 \cos \theta_1 + Z_4 \cos \theta_4 \\
\lambda Z_2 \sin \theta_2 + Z_3 \sin \theta_3 &= Z_1 \sin \theta_1 + Z_4 \sin \theta_4
\end{align*}
\]  

(4-6)

rearrange as

\[
\begin{align*}
Z_3 \cos \theta_3 - Z_4 \cos \theta_4 &= -\lambda Z_2 \cos \theta_2 + Z_1 \cos \theta_1 \\
Z_3 \sin \theta_3 - Z_4 \sin \theta_4 &= -\lambda Z_2 \sin \theta_2 + Z_1 \sin \theta_1
\end{align*}
\]

take derivative and arrange dependent virtual displacement ($\delta \beta, \delta \gamma$) to independent virtual displacement in rational component $\delta \alpha$ and linear component $\delta \lambda$

\[
\begin{bmatrix}
-Z_3 \sin(\theta_3 + \beta) & Z_4 \sin(\theta_4 + \gamma) \\
Z_3 \cos(\theta_3 + \beta) & -Z_4 \cos(\theta_4 + \gamma)
\end{bmatrix}
\begin{bmatrix}
\delta \beta \\
\delta \gamma
\end{bmatrix} =
\begin{bmatrix}
\lambda Z_2 \sin(\theta_2 + \alpha) & -Z_2 \cos(\theta_2 + \alpha) \\
-Z_2 \cos(\theta_2 + \alpha) & -Z_2 \sin(\theta_2 + \alpha)
\end{bmatrix}
\begin{bmatrix}
\delta \alpha \\
\delta \lambda
\end{bmatrix}
\]
\[
\begin{bmatrix}
\delta \beta \\
\delta \gamma
\end{bmatrix} = \begin{bmatrix}
-Z_3 \sin(\theta_3 + \beta) & Z_4 \sin(\theta_4 + \gamma) \\
Z_3 \cos(\theta_3 + \beta) & -Z_4 \cos(\theta_4 + \gamma)
\end{bmatrix}^{-1} \begin{bmatrix}
\lambda Z_2 \sin(\theta_2 + \alpha) & -Z_2 \cos(\theta_2 + \alpha) \\
-Z_2 \cos(\theta_2 + \alpha) & -Z_2 \sin(\theta_2 + \alpha)
\end{bmatrix} \begin{bmatrix}
\delta \alpha \\
\delta \lambda
\end{bmatrix}
\]

(4-7)

Simplify the notation as

\[
\begin{bmatrix}
\delta \beta \\
\delta \gamma
\end{bmatrix} = \begin{bmatrix}
C_{11} & C_{12} \\
C_{21} & C_{22}
\end{bmatrix} \begin{bmatrix}
\delta \alpha \\
\delta \lambda
\end{bmatrix}
\]

Using principle of virtual work to derive the total required actuation torque in equilibrium

\[T_{Actuate} = T_{Ext} + T_{Spring},\]

\[\delta W = T_{Actuate} \delta \eta + T_{Ext} \delta p + \sum T_{Spring} \delta \sigma = 0\]  (4-8)

\[T_{Actuate} \delta \eta = T_{joint} \delta \alpha + F_{Adj} \delta k\]  (4-9)

\(T_{joint}\) and \(F_{Adj}\) are the applied actuation torque/force

Where the External load can be formulated as \(F_x, F_y, M_z\), and the weight of each link \(F_{Weight}\)

\[T_{Ext} = \sum F_x \delta p_x + \sum F_y \delta p_y + \sum M_z \delta \phi + \sum F_{Weight} \delta y_g\]  (4-10)

\[\sum F_x \delta x = F_x [A_1 \quad A_2] \begin{bmatrix}
\delta \alpha \\
\delta \lambda
\end{bmatrix}\]  (4-11)

\[\sum F_y \delta y = F_y [A_3 \quad A_4] \begin{bmatrix}
\delta \alpha \\
\delta \lambda
\end{bmatrix}\]  (4-12)

\[\sum M_z \delta \phi = M_z [C_{11} \quad C_{12}] \begin{bmatrix}
\delta \alpha \\
\delta \lambda
\end{bmatrix}\]  (4-13)
\[ \sum F_{\text{Weight}} \delta y_g = m_2 g [A_5 \ A_6] \left[ \frac{\delta \alpha}{\delta \lambda} \right] + m_3 g [A_7 \ A_8] \left[ \frac{\delta \alpha}{\delta \lambda} \right] + m_4 g [A_9 \ A_{10}] \left[ \frac{\delta \alpha}{\delta \lambda} \right] \] (4-14)

where \( A_i, i=\{1,2,3\ldots 10\} \) are express as follows

\[ A_1 = -\lambda Z_2 \sin(\theta_2 + \alpha) - Z_6 \sin(\theta_3 + \beta) C_{11} \quad A_2 = Z_2 \cos(\theta_2 + \alpha) - Z_6 \sin(\theta_3 + \beta) C_{12} \]

\[ A_3 = \lambda Z_2 \cos(\theta_2 + \alpha) + Z_6 \cos(\theta_3 + \beta) C_{11} \quad A_4 = Z_2 \sin(\theta_2 + \alpha) + Z_6 \cos(\theta_3 + \beta) C_{12} \]

\[ A_5 = \lambda \frac{Z_2}{2} \cos(\theta_2 + \alpha) \quad A_6 = \lambda \frac{Z_2}{2} \sin(\theta_2 + \alpha) \]

\[ A_7 = \lambda Z_2 \cos(\theta_2 + \alpha) + \frac{Z_6}{2} \cos(\theta_3 + \beta) C_{11} \quad A_8 = Z_2 \sin(\theta_2 + \alpha) + \frac{Z_6}{2} \sin(\theta_3 + \beta) C_{12} \]

\[ A_9 = \frac{Z_4}{2} \cos(\theta_4 + \gamma) C_{21} \quad A_{10} = \frac{Z_4}{2} \cos(\theta_4 + \gamma) C_{22} \] (4-15)

The linear torsional springs are used at each joint to reduce the required actuator torque on the driving joint. Assume \( \theta_i \) be the absolute initial configuration of each joint, \( \Omega_i \) is the absolute configuration of each joint at which each spring is assembled and \( \alpha_2, \beta, \alpha_4 \) corresponds to the relative displacement of the first link from its initial configuration to the current configuration

\[ \sigma_3 = (\theta_3_{\text{initial}} + \beta - \Omega_3) - (\theta_2_{\text{initial}} + \alpha_2 - \Omega_2) \]
\[ \sigma_4 = \theta_4_{\text{initial}} + \alpha_4 - \Omega_4 \]
\[ \sigma_5 = (\theta_5_{\text{initial}} + \beta - \Omega_5) - (\theta_4_{\text{initial}} + \alpha_4 - \Omega_4) \] (4-16)
Thus, the overall spring torque $T_{spring}$ can be expressed as

$$T_{spring} =$$

$$k_2(\theta_{2\text{initail}} + \alpha_2 - \Omega_2) \begin{bmatrix} 1 \\ 0 \end{bmatrix} + k_3((\theta_{3\text{initail}} + \beta - \Omega_3) - (\theta_{2\text{initail}} + \alpha_2 - \Omega_2)) \begin{bmatrix} C_{11} - 1 \\ C_{12} \end{bmatrix}$$

$$+ k_4(\theta_{4\text{initail}} + \alpha_4 - \Omega_4) \begin{bmatrix} C_{21} \\ C_{22} \end{bmatrix} + k_5((\theta_{5\text{initail}} + \beta - \Omega_5) - (\theta_{4\text{initail}} + \alpha_4 - \Omega_4)) \begin{bmatrix} C_{11} - C_{21} \\ C_{12} - C_{22} \end{bmatrix}$$

rewritten total spring torque $T_{spring}$ in terms of relative displacement $\alpha_2, \beta, \alpha_4$ and divide to spring torque respect to angular displacement $T_{spring,\alpha}$ and linear displacement $T_{spring,k}$

$$T_{spring,\alpha} = a_1 \alpha + b_1 \beta + c_1 \gamma + d_1$$

$$T_{spring,k} = a_2 \alpha + b_2 \beta + c_2 \gamma + d_2$$

Equation as follow state that preload $\Omega_i$, and spring constant $k_i$ will determine design variables include $\{a_j, b_j, c_j, d_j\}, j=1,2$ to formulate $T_{spring,\alpha}$ and $T_{spring,k}$ respectively.

For rotational component,

$$\begin{bmatrix} a_1 \\ b_1 \\ c_1 \\ d_1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ (\theta_{2\text{initail}} - \Omega_2) \end{bmatrix} = \begin{bmatrix} -(C_{11} - 1) \\ (C_{11} - 1) \\ 0 \\ (\theta_{2\text{initail}} - \Omega_2)(C_{21} - C_{22}) \end{bmatrix}$$

$$\begin{bmatrix} k_2 \\ k_3 \\ k_4 \\ k_5 \end{bmatrix}$$

(4-19)

Where

$$P_1 = ((\theta_{3\text{initail}} - \Omega_3) - (\theta_{2\text{initail}} - \Omega_2)) (C_{11} - 1)$$

$$P_2 = ((\theta_{5\text{initail}} - \Omega_5) - (\theta_{4\text{initail}} - \Omega_4))(C_{11} - C_{21})$$
And translational component,

\[
\begin{bmatrix}
 a_2 \\
 b_2 \\
 c_2 \\
 d_2 
\end{bmatrix} =
\begin{bmatrix}
 -C_{12} & 0 & 0 \\
 C_{12} & 0 & (C_{12} - C_{22}) \\
 0 & C_{22} & -(C_{12} - C_{22}) \\
 P_3 & (\theta_{\text{initail}} - \Omega_i) C_{22} & P_4 
\end{bmatrix}
\begin{bmatrix}
 k_3 \\
 k_4 \\
 k_5 
\end{bmatrix}
\]  

(4-20)

\[
P_3 = ((\theta_{3\text{initail}} - \Omega_3) - (\theta_{2\text{initail}} - \Omega_2)) C_{12}
\]

\[
P_4 = ((\theta_{5\text{initail}} - \Omega_5) - (\theta_{4\text{initail}} - \Omega_4))(C_{12} - C_{22})
\]

Instead of using static precision points solve for specific solution[], an optimization method is applied. Following formulation utilized optimization approach of least-square to minimize the discrepancy between desired torque profile and generated synthesis results from static synthesis in previous equation. The objective function split to two parts which evaluate discrepancy between desired and actual of rotational and translational components respectively. Furthermore, design variables are defined as \( \Omega_2, \Omega_3, \Omega_4, \Omega_5, k_2, k_3, k_4, k_5 \), or formulate \( k_2, k_3, k_4, k_5 \) as free choice. This will give designer freedom to determine using specific commercial torsion spring or not.

The optimization problem can be state as follow:

\[
\min_{k_2, k_3, k_4, k_5} \sum_{i=1}^{N} \left( T_{\text{joint}}^{\text{des}} - T_{1}^{\text{act}} \right)^2 + \left( T_{\text{adjust}}^{\text{des}} - T_{2}^{\text{act}} \right)^2
\]  

(4-21)
Where

\[ T_{1act}^{\text{act}} = T_{\text{Ext,1}} + a_1\alpha + b_1\beta + c_1\gamma + d_1 \]

\[ T_{2act}^{\text{act}} = T_{\text{Ext,2}} + a_2\alpha + b_2\beta + c_2\gamma + d_2 \]  \hspace{1cm} (4-22)

**Type II Adjustable Follower Link**

Due to the kinematic formulation is different from adjustable crank, virtual displacement, external torque needs to be modified. The derivation of adjustable follower static synthesis is shown as follows.

Kinematic formulation:

\[ Z_2 \cos \theta_2 + Z_3 \cos \theta_3 = Z_1 \cos \theta_1 + \eta Z_4 \cos \theta_4 \]  \hspace{1cm} (4-23)

\[ Z_2 \sin \theta_2 + Z_3 \sin \theta_3 = Z_1 \sin \theta_1 + \eta Z_4 \sin \theta_4 \]

Virtual displacement of adjustable follower, angular displacement is \( \delta\alpha \) and linear displacement is \( \delta\eta \)

\[
\begin{bmatrix}
\delta\beta \\
\delta\gamma
\end{bmatrix} =
\begin{bmatrix}
-Z_3 \sin(\theta_3 + \beta) & \eta Z_4 \sin(\theta_4 + \gamma) \\
Z_3 \cos(\theta_3 + \beta) & -\eta Z_4 \cos(\theta_4 + \gamma)
\end{bmatrix}^{-1}
\begin{bmatrix}
Z_2 \sin(\theta_2 + \alpha) \\
-Z_2 \cos(\theta_2 + \alpha)
\end{bmatrix}
\begin{bmatrix}
Z_4 \cos(\theta_4 + \gamma)
\end{bmatrix}
\begin{bmatrix}
\delta\alpha \\
\delta\eta
\end{bmatrix}
\]  \hspace{1cm} (4-24)

Simplify the notation as

\[
\begin{bmatrix}
\delta\beta \\
\delta\gamma
\end{bmatrix} =
\begin{bmatrix}
C_{11} & C_{12} \\
C_{21} & C_{22}
\end{bmatrix}
\begin{bmatrix}
\delta\alpha \\
\delta\eta
\end{bmatrix}
\]  \hspace{1cm} (4-25)
the External load $F_x, F_y, M_Z$ and weight of links $F_{Weight}$

$$
\sum F_x \delta x = F_x [A_1 \quad A_2] [\delta \alpha] \quad \sum F_y \delta y = F_y [A_3 \quad A_4] [\delta \alpha] \\
\sum M_Z \delta \phi = M_Z [C_{11} \quad C_{12}] [\delta \alpha] \\
\sum F_{Weight} \delta y_g = m_2 g [A_5 \quad 0] [\delta \alpha] + m_3 g [A_6 \quad A_7] [\delta \alpha] + m_4 g [A_8 \quad A_9] [\delta \alpha] 
$$

(4-26)

Similarly, $A_i, i=\{1,2,3, ... 9\}$ are express as follows

$A_1 = -Z_2 \sin(\theta_2 + \alpha) - Z_6 \sin(\theta_3 + \beta) C_{11}$
$A_2 = -Z_6 \sin(\theta_3 + \beta) C_{12}$

$A_3 = Z_2 \cos(\theta_2 + \alpha) + Z_6 \cos(\theta_3 + \beta) C_{11}$
$A_4 = Z_6 \cos(\theta_3 + \beta) C_{12}$

$A_5 = \frac{Z_2}{2} \cos(\theta_2 + \alpha)$
$A_6 = Z_2 \cos(\theta_2 + \alpha) + \frac{Z_6}{2} \cos(\theta_3 + \beta) C_{11}$

$A_7 = \frac{Z_6}{2} \cos(\theta_3 + \beta) C_{12}$
$A_8 = \eta \frac{Z_4}{2} \cos(\theta_4 + \gamma) C_{21}$

$A_9 = \eta \frac{Z_4}{2} \cos(\theta_4 + \gamma) C_{21} + \frac{Z_4}{2} \cos(\theta_4 + \gamma)$

(4-27)

The formulation of spring torque and optimization are same as previous equation in adjustable crank. The only term needs to be modified is External load.
Type III. Adjustable coupler Link

The formulation of adjustable coupler is very similar as conventional four bar mechanism, except the external torque are slightly different.

The kinematic formulation and Virtual displacement are same as conventional four bar

\[
\begin{bmatrix}
\delta \theta_3 \\
\delta \theta_4 \\
\end{bmatrix}
= \begin{bmatrix}
\frac{Z_2 \sin(\theta_4 - \theta_2)}{Z_3 \sin(\theta_3 - \theta_4)} \\
\frac{Z_2 \sin(\theta_3 - \theta_2)}{Z_4 \sin(\theta_3 - \theta_4)} \\
\end{bmatrix} \delta \theta_2
\]

\[
\begin{bmatrix}
\delta \theta_2 \\
\delta \theta_3 \\
\delta \theta_4 \\
\end{bmatrix}
= \begin{bmatrix}
\frac{1}{Z_3 \sin(\theta_3 - \theta_4)} \\
\frac{Z_2 \sin(\theta_3 - \theta_2)}{Z_4 \sin(\theta_3 - \theta_4)} \\
\end{bmatrix} \delta \theta_2
= \begin{bmatrix}
C_1 \\
C_2 \\
\end{bmatrix} \delta \theta_2
\] (4-28)

the External load \( F_x, F_y, M_Z \) and weight of links \( F_{Weight} \)

\[
\sum F_x \delta x = F_x [A_1 \quad A_2] [\delta \alpha] \\
\sum F_y \delta y = F_y [A_3 \quad A_4] [\delta \alpha] \\
\sum M_Z \delta \phi = M_Z [C_1] [\delta \alpha]
\]

\[
\sum F_{Weight} \delta y_g = m_2 g [A_5 \quad 0] [\delta \alpha] + m_3 g [A_6 \quad A_7] [\delta \alpha] + m_4 g [A_8 \quad 0] [\delta \alpha]
\] (4-29)

\( A_i, i=\{1,2,3, ... 8\} \) are express as

\[
A_1 = -Z_2 \sin(\theta_2 + \alpha) - \kappa Z_6 \sin(\theta_3 + \beta) C_1 \\
A_2 = Z_6 \cos(\theta_3 + \beta)
\]

\[
A_3 = Z_2 \cos(\theta_2 + \alpha) - \kappa Z_6 \cos(\theta_3 + \beta) C_1 \\
A_4 = Z_6 \sin(\theta_3 + \beta)
\]

\[
A_5 = \frac{Z_2}{2} \cos(\theta_2 + \alpha) \\
A_6 = Z_2 \cos(\theta_2 + \alpha) + \kappa \frac{Z_6}{2} \cos(\theta_3 + \beta) A_1
\]

\[
A_7 = \frac{Z_6}{2} \sin(\theta_3 + \beta) \\
A_8 = \frac{Z_4}{2} \cos(\theta_4 + \gamma) A_2
\] (4-30)

Similarly, formulation of spring torque and optimization are same as previous equation.
4.6 Examples:

Case 1. Fixed Chassis for Adjustable Crank

Figure 4.8 (a) Optimization Results of Kinetostatic Synthesis (b) Initial and Optimized Torque Comparison
(c) Optimized Spring Constant Table

Case 2. Moving Chassis for Adjustable Crank

Figure 4.9 (a) Optimization Results of Kinetostatic Synthesis
(b) Initial and Optimized Torque Comparison

---

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<tr>
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<th>$k_4$</th>
<th>$k_5$</th>
</tr>
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<td>3.1594</td>
<td>0.74349</td>
<td>7.0257</td>
<td>9.2539</td>
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<table>
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<th>$k_4$</th>
<th>$k_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.34763</td>
<td>3.4602</td>
<td>0.5998</td>
<td>4.5985</td>
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</table>
Case 3. Fixed Chassis for Adjustable Follower

<table>
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<th>$k_4$</th>
<th>$k_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.9682</td>
<td>16.9287</td>
<td>33.8198</td>
<td>41.759</td>
</tr>
</tbody>
</table>

Figure 4.10 (a) Optimization Results of Kinetostatic Synthesis (b) Initial and Optimized Torque Comparison (c) Optimized Spring Constant Table

Case 4. Moving Chassis for Adjustable Follower

<table>
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<th>$k_4$</th>
<th>$k_5$</th>
</tr>
</thead>
<tbody>
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<td>47.9279</td>
<td>34.3447</td>
<td>14.1923</td>
<td>69.4571</td>
</tr>
</tbody>
</table>

Figure 4.11 (a) Optimization Results of Kinetostatic Synthesis (b) Initial and Optimized Torque Comparison
Case 5. Moving Chassis for Adjustable Coupler

By applying similar optimization approach in kinetostatic synthesis, corresponding torsion spring preload and spring constant are being generated and optimized. Reviewing and comparing these results, well design mechanism can reduce required spring constant by adding actuator.

Overviewing all design, adjustable crank has an optimized spring constant requirement.

<table>
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<th>(k_3)</th>
<th>(k_4)</th>
<th>(k_5)</th>
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<tbody>
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<td>32.18</td>
<td>43.78</td>
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<td>100</td>
</tr>
</tbody>
</table>

Figure 4.12 (a) Optimization Results of Kinetostatic Synthesis (b) Initial and Optimized Torque Comparison (c) Optimized Spring Constant Table

By applying similar optimization approach in kinetostatic synthesis, corresponding torsion spring preload and spring constant are being generated and optimized. Reviewing and comparing these results, well design mechanism can reduce required spring constant by adding actuator.
5. Discussion and Future Work

Discussion

In this project, we revisit and extend the systematic approach of kinematic synthesis and kinetostatic synthesis for articulated wheel subsystem. First, based on pervious design, we select the mechanism type as four bar and extent to adjustable four bar mechanism, which exploring the possibility from 1 D.O.F to 2 D.O.F. This provides a flexible motion range to tracking desired trajectory, and introduces extra actuator transfer to an active subsystem.

Second, a design objective scenario is established. Since the articulated wheel vehicle are not form high speed application, we simplify the wheel model assume it only has pure rolling motion, which does not consider about any slipping, sliding, and friction. Furthermore, two type of kinematic objective are adapt, assuming the motion have fixed chassis and moving chassis; thus, two set of precision points and trajectory are implemented. Similarly, a trajectory based-method is utilized to formulate a desired torque profile.

Third, we formulated and derived a precision point synthesis-based systematic approach to generate both convention four bar/ adjustable four bar kinematic solution for trajectory tracking problem. Sequentially, using principle of virtual work, kinetostatic synthesis is employed to generate required spring constant. We compare and exam the both kinematic and kinetostatic synthesis performance to select an optimum design.
Future work

- Develop and virtual prototype in multidynamics simulation software such as simmechanics or maplesim to examine and compare design results
- Building physical prototype of adjustable crank four bar legged wheel subsystem design and implement on mobile robot
- Develop active, semi-active, and passive type adjustable crank, by using actuator, damper and spring; evaluating the advantages and disadvantages