

Space-time attributes of physical objects and the laws of space-time physics

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Abstract

Physical time intervals are attributes of single physical object whereas physical space intervals are a relational attribute of two physical objects. Some consequences of the breaking of the space-time exchange symmetry inherent in the Lorentz transformation following from the above distinction are investigated. In particular, it is shown that the relativity of simultaneity and length contraction effects which naively follow from space-time symmetry of the Lorentz transformation do not occur. Seven laws describing the relation between observations of space intervals, time intervals and velocities in different reference frames are given. Only two of these laws are respected by conventional special relativity theory.

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1 Introduction

The purpose of the present paper is an in-depth discussion of the operational physical meanings of the symbols in the Galilean equation of motion of a physical object:

$$\Delta x = v\Delta t \tag{1.1}$$

relating space intervals, Δx , time intervals, Δt , and velocity, v . The general case, for small values of Δx and Δt , where v is time-dependent, as well as uniform motion where $v = \text{constant}$ will be considered. The specific question that is addressed is the attribution of the symbols Δx , Δt and v to one or more discrete physical objects A,B,... . The symbol C is used to denote a clock, which, as discussed below, is the class of self-age-recording objects.

The plan of the paper is as follows. The following two sections discuss the definitions of physical time intervals and space intervals respectively, and how they may be measured. In Section 4 the different concepts of frame velocity and relative velocity are defined. Section 5 considers the initial configuration of the ‘twin paradox’ thought experiment, and shows by application of the Principle of Sufficient Reason (or alternatively, by *reductio ad absurdum*) the invariance of a length interval measured in two different inertial frames. After recalling the important concepts of ‘primary’ and ‘reciprocal’ space-time experiments and ‘base’ and ‘travelling’ frames, as introduced in Refs. [1, 2], Section 6 analyses the different time dilatation effects in primary and reciprocal experiments. The formulae obtained demonstrate the spurious nature of the ‘relativity of simultaneity’ effect of standard special relativity theory (SSRT). In Section 7, transformation formulae, between two inertial frames, of the relative velocity of two objects are obtained. In the concluding Section 8, the conclusions of the paper are summarised in seven laws of space-time physics, five of which are not respected by the predictions of SSRT.

2 Temporal Intervals, Δt

The crucial point to be noticed here is that, in general, a time interval (unlike a space interval) is *an attribute of a single physical object*. This concept is a commonsense one familiar in the guise of the age of any living creature or plant, which is the time interval between the present and the birth date of any animal or the germination date of any plant¹.

In order to make quantitative physical statements concerning age or time intervals, it is necessary to introduce the concept of a clock, which is defined as *a physical object that records an interval of its own age*. In order to do this the object must furnish, at any instant, 1, a number t_1 , conventionally termed the *epoch* of the instant. The time interval between instants 1 and 2, is then defined as the difference of the epochs of 1 and 2:

$$\Delta t_{21} \equiv t_2 - t_1 \tag{2.1}$$

¹It would be more consistent to specify the age of an animal as the time interval between conception and the present, but this is not the social convention.

A good example of a self-age-recording object is a radioactive clock, as suggested by Langevin [3]. This is a ponderable object (a radioactive ‘source’) that is doped at epoch t_0 with N_0 atoms of a radioactive substance of known mean decay time τ_D . In virtue of the radioactive decay law, the age, a_1 , of the source at epoch t_1 when N_1 atoms remain undecayed is:

$$a_1 \equiv t_1 - t_0 = \tau_D \ln \left(\frac{N_0}{N_1} \right) \quad (2.2)$$

As pointed out by Langevin, a practical way to measure a_1 (since it is evidently not possible to count N_1 , even if the value of N_0 is experimentally controllable by chemical means) is to place the source in the same detector at the different epochs t_0 and t_1 and measure the corresponding activities A_0 and A_1 (the numbers of disintegrations per unit of time recorded). The age of the source is then given by the relation

$$a_1 = \tau_D \ln \left(\frac{A_0}{A_1} \right) \quad (2.3)$$

Evidently, because of the statistical nature of the decay process, the age determined by (2.3) is subject to a statistical uncertainty that depends on the value of N_0 and the efficiency of the detector. The ‘true’ value, still, however, dependent of the experimental uncertainty in τ_D , is given by (2.2) or (2.3) in the limit $N_0 \rightarrow \infty$. Consideration of such radioactive clocks makes quite transparent the absurdity of certain predictions of SSRT [1, 4].

More conventional clocks are based on periodic physical processes such as the orbital period of a planet, the oscillation period of a pendulum, balance wheel or quartz crystal, or the frequency, ν_γ , associated via the Planck-Einstein relation $E_\gamma = h\nu_\gamma$ with the energy of a photon E_γ emitted in a transition between two well-defined atomic energy levels. There is therefore an irreducible connection between the concept of a physical time interval and the time dependence or frequency (exponential decay law, repeated periodic motion, or the proportionality of the energy and frequency of photons in quantum mechanics) of definite physical processes. This means that there is no conceptual ambiguity in the operational definition of a physical time interval.

Although time, as specified by the epoch number $t(C)$ of a clock C, is a fundamental attribute of C, it can be extended to every member of an array of clocks C_1, C_2, \dots which are at rest relative to each other, by introducing the further concepts of a *reference frame* F, and *frame time* t_F . The frame F contains a system of spatial coordinates that specifies the relative spatial positions of C_1, C_2, \dots , and is the the common proper frame of any of the clocks, which may be in arbitrary (uniform or accelerated) motion relative to an inertial observer. If, at any instant in the frame F, the epochs recorded by C_1, C_2, \dots are the same:

$$t(C_1) = t(C_2) = \dots \equiv t_F \quad (2.4)$$

the array of clocks is said to be *synchronised*². In the case that F is an inertial frame, the clocks may be mutually synchronised using Einstein’s well known light-signal procedure [5] that relies on the assumption of the isotropy of the speed of light. More general synchronisation procedures, not using light signals in free space, and applicable also to accelerated frames, are described in Ref. [6].

²To be contrasted with a *synchronous* array of clocks which run at the same speed but may indicate different epochs.

Of great importance for the following discussion is the concept of a *corresponding epoch* in two different reference frames F and F' . This is defined as follows:

A corresponding epoch of two frames F and F' in relative motion is one for which one or more objects at rest in F are seen to be spatially contiguous with one or more objects in at rest in F' by observers in both frames .

In the notation for space intervals to be introduced in the following section, if

$$\Delta x(A, A', t_F) = \Delta x'(A, A', t_{F'}) = 0 \quad (2.5)$$

then t_F and $t_{F'}$ are *corresponding epochs*. If it happens that $t_F = t_{F'}$ in Eq. (2.5) then *clocks in the frames F and F' are said to be synchronised*. In the case that F and F' are inertial frames S and S' and A and A' are placed at the origins of spatial coordinates in S and S' respectively, the condition (2.5) is identical to Einstein's definition of local clock synchronisation between two inertial frames [5]. Eq. (2.5) also corresponds to the 'system external' synchronisation of clocks in different reference frames of Mansouri and Sexl [7].

3 Spatial Intervals, Δx

In the following, only one-dimensional spatial separations and motion are considered. Because of the assumed isotropy of three-dimensional space no loss of generality is occasioned for the problems considered in the present paper.

In contrast to a time interval, a spatial interval is *an attribute of two distinct physical objects*³. It follows from this that a spatial interval at a given epoch t_F in a frame F requires two labels A and B corresponding to the related physical objects in its definition: $\Delta x(A, B, t_F)$, where the spatial coordinate x is defined in the frame F , in contrast to a time interval which requires only one label to specify it: $\Delta t(A)$, for a particular object, A , or t_F for a particular frame, F . It is this difference of the attribution of temporal and spatial intervals to physical objects that breaks the mathematical space-time exchange symmetry [8] of the Lorentz transformation equations⁴ in their application to actual physical problems involving the description of synchronised clocks.

For a particular spatial coordinate, x , in the frame F , the definition of the spatial separation of A and B is

$$\Delta x(A, A', t_F) \equiv x(A, t_F) - x(B, t_F) \quad (3.1)$$

The interval Δx is invariant with respect to different choices of coordinate origin, equivalent to the coordinate transformation: $x \rightarrow x + C$, where C is an arbitrary constant.

³In the case when the spatial interval corresponds to a dimension of an extended physical object, it is necessary to introduce localised portions of the object situated at the ends, that play the role of two discrete separated objects. In one dimension, any extended object has two and only two 'ends', the distance between which defines the size of the object in this dimension. In the case of redundant definition of the ends (e.g. a rectangular bar) any two localised 'end objects' can be used to specify the length of the bar.

⁴Introducing a temporal coordinate with dimension $[L]$ according to $x_0 = ct$, the space-time Lorentz transformation equations are written as $x' = \gamma(x - \beta x_0)$, $x'_0 = \gamma(x_0 - \beta x)$. This pair of equations is invariant under exchange of space-time coordinates: $x \leftrightarrow x_0$, $x' \leftrightarrow x'_0$.

The measured spatial separation of two objects in arbitrary motion in a reference frame F is defined as follows:

If two objects A' and B' in arbitrary motion in a reference frame F are spatially contiguous with the objects A and B at rest in F at epoch t_F , the spatial separation of A' and B' at this epoch is the same as that of A and B.

This is just a statement of the methodology of ‘ruler measurement’ of the separation of two objects at a given epoch, as discussed in detail in Ref. [6]. The expression ‘spatially contiguous’ in the above definition is equivalent, in the present context, to ‘have the same x -coordinate’. As must be the case for non-colliding physical objects of non-vanishing lateral dimensions, the y and/or z coordinates of the objects may be different.

4 Frame velocity, v , and Relative Velocity, u

The *frame velocity*, $v_{A'}$ is an attribute of a single physical object, A', and depends on the frame of reference, F, in which it is defined. If the frame F is understood in the symbol v (so that, for example, the frame velocity in the frame F' is denoted by v') $v_{A'}$ depends only on the label A' $v_{A'} \equiv v(A', t_F)$. Other objects B', C', ... at rest in the proper frame F' of A', but at different spatial locations, have the same frame velocity as A' in F at any epoch:

$$v(A', t_F) = v(B', t_F) = v(C', t_F) = \dots = v(t_F) \quad (4.1)$$

If A is an object at rest at an arbitrary position in F, the definition of $v(t_F)$ in terms of the epoch t_F and the spatial interval Δx introduced above is:

$$v(t_F) \equiv \frac{d[\Delta x(A', A, t_F)]}{dt_F} = \frac{d[x(A', t_F) - x(A, t_F)]}{dt_F} = \frac{dx(A', t_F)}{dt_F} = \frac{dx(B', t_F)}{dt_F} = \dots \quad (4.2)$$

since $dx(A, t_F)/dt_F = 0$. The parameter v (now independent of t_F) is that appearing in the space-time Lorentz transformation between the inertial frames S and S':

$$x' = \gamma(v)[x(t_F) - vt_F] \quad (4.3)$$

$$t' = \gamma(v)\left[t_F - \frac{vx(t_F)}{c^2}\right] \quad (4.4)$$

where $\gamma(v) \equiv 1/\sqrt{1 - (v/c)^2}$, in which case $v = v_{A'} = \text{constant}$ where A' is any object at rest in S'.

The *relative velocity*, u , of two objects A', A'' in arbitrary motion in the frame F, unlike v , is a relational attribute of both A' and A''. It is defined in a similar manner to v in Eq. (4.2):

$$\begin{aligned} u(A', A'', t_F) &\equiv \frac{d[\Delta x(A', A'', t_F)]}{dt_F} = \frac{d[x(A', t_F) - x(A'', t_F)]}{dt_F} \\ &= \frac{dx(A', t_F)}{dt_F} - \frac{dx(A'', t_F)}{dt_F} = v(A', t_F) - v(A'', t_F) \end{aligned} \quad (4.5)$$

The velocity of A' relative to A'' in F is thus equal to the difference of their frame velocities. Note that, although the frame velocity of a ponderable physical object has, in

SSRT, the upper limit of c , the velocity of light in free space, the upper limit on u in the frame F, given by (4.5), is $2c$.

5 Frame Invariance of Length Intervals

For illustrative purposes, and to demonstrate the flaw in the standard text book interpretation of the ‘twin paradox’ thought experiment [3] the journey, at uniform frame velocity v , in the rest frame, S, of the Earth (E) of a space ship R_1 to a distant star Sirius (Si), assumed to be at rest in S, is considered.

At the start of the journey, when R_1 and the Earth are aligned, two other spaceships R_2 and R_3 aligned with Sirius are introduced. R_2 is at rest in S while R_3 has a certain frame velocity w , to be determined below, such that in the proper frame, S' of R_1 , R_3 has velocity v in the direction of the Earth. In Fig. 1a is shown the disposition of objects in S at the start of the journey. The following space-interval equalities hold in S at the epoch shown in Fig. 1a:

$$\Delta x(\text{Si}, \text{E}) = \Delta x(\text{R}_2, \text{E}) = \Delta x(\text{R}_3, \text{E}) = \Delta x(\text{Si}, \text{R}_1) = \Delta x(\text{R}_2, \text{R}_1) = \Delta x(\text{R}_3, \text{R}_1) \quad (5.1)$$

Similarly, in the rest frame of R_1 at the corresponding epoch:

$$\Delta x'(\text{Si}, \text{E}) = \Delta x'(\text{R}_2, \text{E}) = \Delta x'(\text{R}_3, \text{E}) = \Delta x'(\text{Si}, \text{R}_1) = \Delta x'(\text{R}_2, \text{R}_1) = \Delta x'(\text{R}_3, \text{R}_1) \quad (5.2)$$

Attention is now fixed on the spatial configuration of R_1 and R_2 in S at this epoch (Fig. 1b) and that of R_1 and R_3 in S' at the corresponding epoch (Fig. 1c). From Eq. (5.1) the separation of R_1 and R_2 in Fig. 1b is $\Delta x \equiv \Delta x(\text{Si}, \text{E})$ while from Eq. (5.2) that of R_1 and R_3 in Fig. 1c is $\Delta x' \equiv \Delta x'(\text{Si}, \text{E})$.

Inspection of Fig. 1b and Fig. 1c reveals perfect symmetry between the two kinematical configurations. In fact they are related to each other by the parity operation — reflection in the plane midway between the Earth and Sirius perpendicular to the directions of motion of R_1 and R_3 . In these circumstances what possible physical effect, depending only on the value of the velocity, v , could result in a *difference* between Δx and $\Delta x'$? There is none. It therefore follows on application of Leibnitz’ Principle of Sufficient Reason that $\Delta x = \Delta x'$.

The same conclusion is reached by applying to the configurations of Fig. 1b and 1c the relativistic reciprocity relation⁵. used by Ignatowsky [9] to derive the Lorentz transformation, as cited by Pauli [10]⁶:

The contraction of length at rest in S' and observed in S is equal to that of lengths at rest in S and observed from S' .

⁵Pauli’s reciprocity relation is a special case of the Measurement Reciprocity Postulate (MRP) [6, 11, 12] which is a simple, purely kinematical, form of the Special Relativity Principle from which the Lorentz transformation may be derived without consideration of light signals or classical electromagnetism [6, 9, 11]

⁶Pauli’s frame labels K and K’ are replaced by S and S’ for conformity with the notation of the present paper.

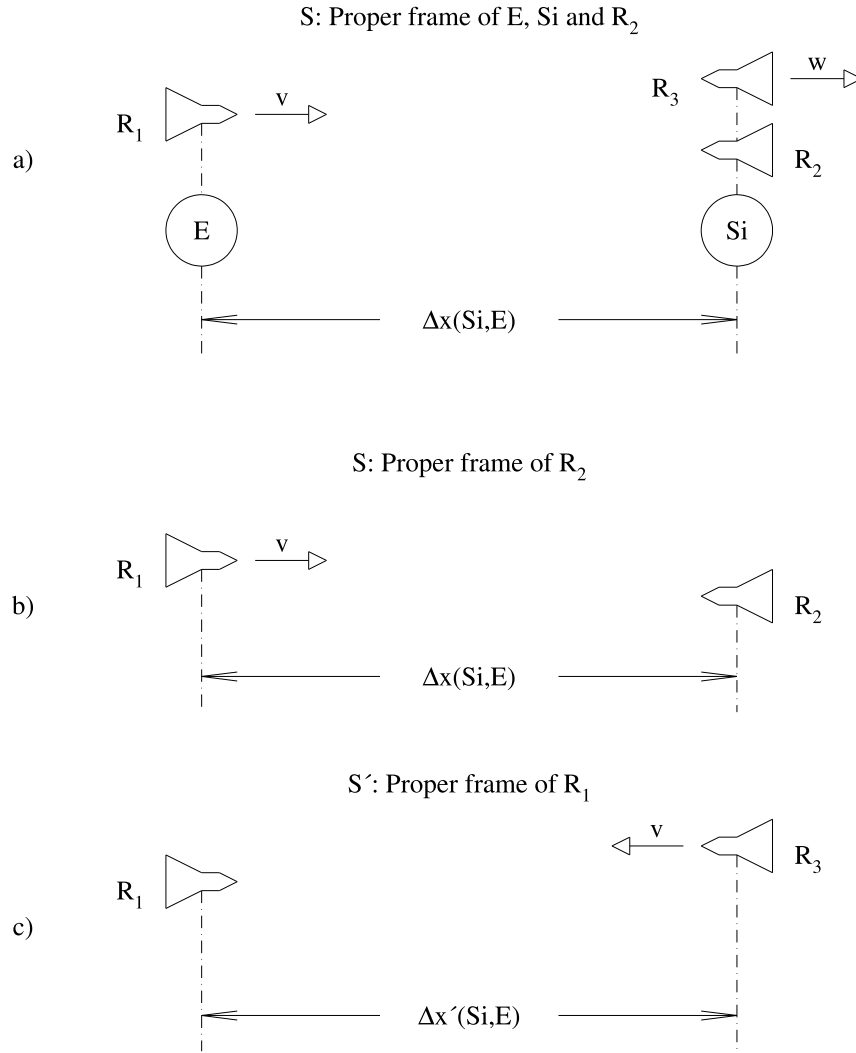


Figure 1: *Spatial and kinematical configurations of the Earth (E) the star Sirius (Si) and the spaceships R₁, R₂ and R₃ at the beginning of the voyage of R₁ to Sirius in the ‘twin paradox’ thought experiment. a), b) in the rest frame, S, of E, Si and R₂; c) in the rest frame, S’, of R₁. The perfect symmetry of the configurations of R₁ and R₂ in b) and R₁ and R₃ in c) together with the Principle of Sufficient Reason requires that $\Delta x'(Si, E) = \Delta x(Si, E)$.*

It the velocity-dependent contraction factor is denoted by $\alpha(v)$, the first condition applied to the configurations of Fig. 1b and 1c gives:

$$\alpha(v)\Delta x' = \Delta x \quad (5.3)$$

while the second gives:

$$\alpha(v)\Delta x = \Delta x' \quad (5.4)$$

Combining (5.3) and (5.4),

$$\alpha(v)\Delta x' = \alpha(v)[\alpha(v)\Delta x] = \alpha(v)^2\Delta x = \Delta x \quad (5.5)$$

So either $\Delta x' = \Delta x = 0$ or $\alpha(v)^2 = 1$. In the present case since Δx and $\Delta x'$ are both non-zero and positive it follows that $\alpha(v) = 1$ so that

$$\Delta x' = \Delta x \quad (5.6)$$

In the interpretation of the twin paradox in SSRT it is instead assumed that ‘length contraction’ occurs in an asymmetric manner in the rest frame of R_1 so that $\Delta x' = \Delta x/\gamma(v)$. The erroneous nature of this prediction and the absurd consequences that it entails are discussed at length in Ref. [1]. If $\alpha(v)$ is identified with $\gamma(v)$, Eqs. (5.3) and (5.4) require that $v = 0$, contradicting the initial assumption of Fig.1 that $v > 0$. If the reciprocity relation quoted by Pauli holds, the identification of the contraction factor with $\gamma(v)$ is thus excluded by *reductio ad absurdum*⁷.

The calculation above considered only spatial intervals in two different inertial frames, but the same invariance of spatial intervals at corresponding epochs is found for two frames each undergoing arbitrary accelerated motion since the above reasoning is valid in the comoving inertial frames at the corresponding epoch. See Ref. [12] for a discussion of the spatial interval between two objects undergoing identical accelerated motion.

6 Time Dilatation and Reciprocal Experiments

This section specialises to the case of inertial frames S and S' in the standard configuration where S' moves with velocity v along the positive x -axis and the x - and x' -axes are parallel. The space-time Lorentz transformation (LT) gives a relation between time intervals Δt in S and $\Delta t'$ in S' as registered by clocks at rest in these frames. Two physically independent and reciprocal experiments are possible [1, 2]. In the first *primary* experiment a clock at rest in S' is observed from S and compared with a clock at rest in the latter frame. In the second *reciprocal* experiment a clock at rest in S is observed from S' and compared with clock, like the observer, at rest in S'. The LT equations are completely different in these two *independent experiments*. In the primary experiment only the spatial coordinates of the clock at rest in S' appear, whereas in the reciprocal experiment only those of the clock at rest in S are used. In the primary experiment the position of the clock at rest in S is arbitrary, while in the reciprocal experiment the position

⁷This argument was previously given in Ref. [13].

of the clock at rest in S' is arbitrary, and the spatial coordinates of these clocks do not appear in the LT equations.

In the primary experiment an interval on the world line of a clock C' at rest at an arbitrary position in S' is given by Eq. (1.1) as

$$\Delta x(C') = v\Delta t(C)_B \quad (6.1)$$

where C is a clock at rest at an arbitrary position in S. The subscript *B* on $\Delta t(C)_B$ stands for *base frame* and indicates that the time interval is recorded by a clock at rest in the same frame, S, as the the observer. The corresponding time interval in S' is given by the time LT as:

$$\Delta t'(C')_T = \gamma(v) \left[\Delta t(C)_B - \frac{v\Delta x(C')}{c^2} \right] \quad (6.2)$$

The subscript *T* on $\Delta t'(C')$ stands for *Travelling frame* indicating that the clock is in motion relative to the observer at rest in the base frame S. Thus, in the primary experiment, S' is the travelling frame. Combining (6.1) and (6.2) gives the Time Dilatation (TD) relation for the primary experiment:

$$\Delta t(C)_B = \gamma(v)\Delta t'(C')_T \quad (6.3)$$

In the reciprocal experiment where, by definition, the clock C, at an arbitrary position in S, moves with speed *v* parallel to the negative *x'*-axis in S', an interval of the world line of C is given by Eq. (1.1) as:

$$\Delta x'(C) = -v\Delta t'(C')_B \quad (6.4)$$

where the clock C' is at an arbitrary position in S'. Combining the appropriate time LT:

$$\Delta t(C)_T = \gamma(v) \left[\Delta t'(C')_B + \frac{v\Delta x(C)}{c^2} \right] \quad (6.5)$$

with (6.4) gives the TD relation for the reciprocal experiment:

$$\Delta t'(C')_B = \gamma(v)\Delta t(C)_T \quad (6.6)$$

Notice that since the quantities represented by the symbols $\Delta t(C)_B$, $\Delta t(C)_T$, $\Delta t'(C')_B$ and $\Delta t'(C')_T$ are physically distinct there is no antinomy between Eqs. (6.3) and (6.6) and the condition $\gamma(v) > 1$, as there is between Eqs. (5.3) and (5.4) and the 'length contraction' condition $\alpha(v) = \gamma(v) > 1$ ⁸. This shows clearly that the apparent space-time symmetry of the LT equations breaks down when they are applied to actual physical problems and care is taken over the precise operational meaning of the symbols for spatial and temporal intervals that appear in the equations.

Since the spatial positions of the clocks C and C' in the TD relations (6.3) and (6.6) are arbitrary, it is an immediate consequence that clocks in S and S', once synchronised

⁸It was the failure to notice that $\Delta t(C)_B$ is not the same physical quantity as $\Delta t(C)_T$ and $\Delta t'(C')_B$ is not the same physical quantity as $\Delta t'(C')_T$ that led Dingle to wrongly conclude, by a *reductio ad absurdum* argument, similar to that of Eqns(5.3) and (5.4) above, that SSRT was self-contradictory and therefore untenable [14].

at a corresponding epoch, remain so at all later epochs. Suppose that C'_1 and C_1 are synchronised so that:

$$t(C_1)_B = \gamma(v)t'(C'_1)_T \quad (6.7)$$

which implies that $t'(C'_1)_T = 0$ when $t(C_1)_B = 0$, and that C'_2 and C_2 are similarly synchronised so that:

$$t(C_2)_B = \gamma(v)t'(C'_2)_T \quad (6.8)$$

If now, it so happens that C_1 and C_2 , at rest in S , were previously synchronised so that

$$t(C_1)_B = t(C_2)_B \equiv t_B \quad (6.9)$$

it follows from (6.8) and (6.9) that

$$t'(C'_1)_T = t'(C'_2)_T \equiv t'_T = t_B/\gamma(v) \quad (6.10)$$

C'_1 and C'_2 —clocks at arbitrary positions in S' — are then also synchronised at any epoch —there is no ‘relativity of simultaneity’ effect.

7 Relative Velocity Transformation Formulae

A transparent way to derive the transformation formula, between two inertial frames, of the relative velocity of two objects in uniform motion is by consideration of the outward journey of R_1 from Earth to Sirius in the twin paradox experiment shown in Fig. 1. In order to deduce the value of the velocity, w , of R_3 in Fig. 1a, it is convenient to consider a configuration where R_1 and R_3 are aligned at the beginning of the journey as shown in Fig. 2a. The velocity of R_1 relative to R_3 has the same value $v - w$ in Fig 1a and Fig. 2a.

The configuration of the objects at the end of the journey when $T \equiv t = \gamma(v)t' \equiv \gamma(v)T'$ as given by the TD relation (6.3), are shown in the frame S (the proper frame of the Earth and Sirius) in Fig. 2b, and in S' (the proper frame of R_1) in Fig. 2c. Denoting the velocity of R_3 relative to R_1 in S' by $u'(R_3, R_1)$ and that of the Earth relative to R_1 in the same frame by $u'(E, R_1)$, the geometry of Figs. 2a and 2b gives the relations:

$$T = \frac{\Delta x(R_1, E, T)}{v} = \frac{\Delta x(R_1, R_3, T)}{v - w} \quad (7.1)$$

$$T' = \frac{\Delta x'(R_1, E, T')}{u'(E, R_1)} = \frac{\Delta x'(R_1, R_3, T')}{u'(R_3, R_1)} \quad (7.2)$$

Since

$$\Delta x'(R_1, E, T') = \Delta x(R_1, E, T) \quad (7.3)$$

$$\Delta x'(R_1, R_3, T') = \Delta x(R_1, R_3, T) \quad (7.4)$$

$$T = \gamma(v)T' \quad (7.5)$$

(7.1) and (7.2) give the relative velocity transformation formulae:

$$u'(E, R_1) = \gamma(v)v = \gamma u(R_1, E) \quad (7.6)$$

$$u'(R_3, R_1) = \gamma(v)(v - w) = \gamma(v)u(R_1, R_3) \quad (7.7)$$

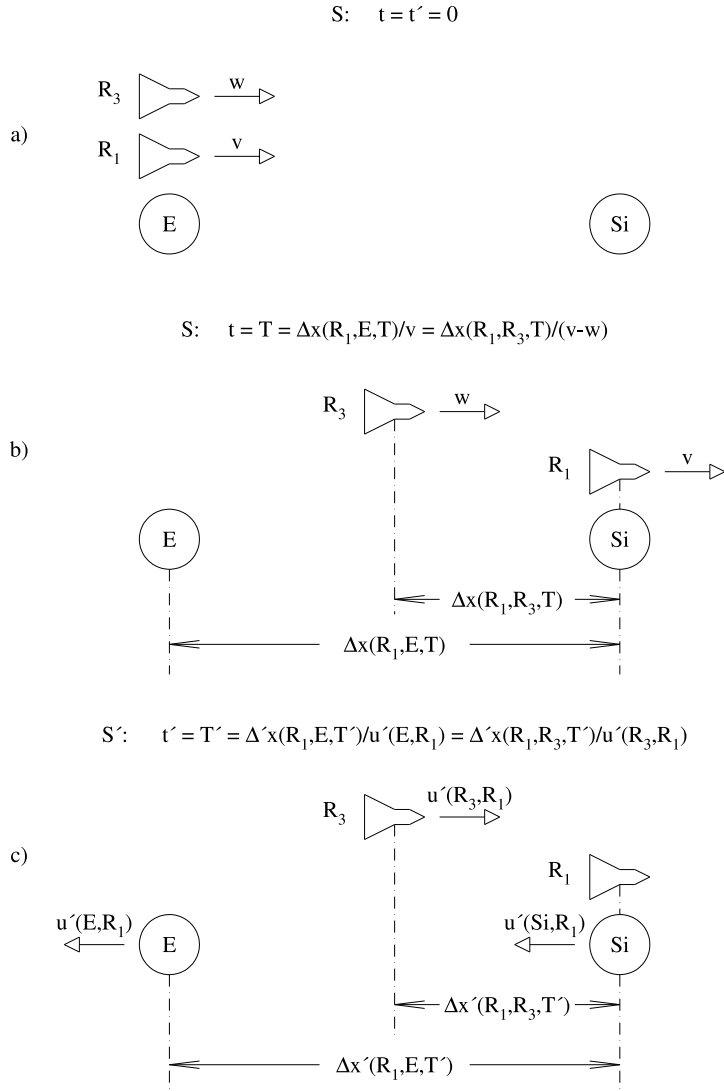


Figure 2: *Spatial and kinematical configurations in the outward journey of R_1 from the Earth (E) to Sirius (Si) in the twin paradox thought experiment. a) beginning of the journey in the frame S; b) end of journey in the frame S; c) end of journey in the rest frame, S' , of R_1 .*

Note that (7.6) is a special case of (7.7) when $w = 0$, so that R_3 is at rest relative to the Earth. The change in the order of the labels on the right and left sides of (7.6) and (7.7) corresponds to the change in direction of the relative velocity vector in the transformation from S to S' .

The value of w in Fig. 1a in order to yield the configuration of Fig. 1c in the rest frame of R_1 is given by (7.7) as:

$$v = u'(R_3, R_1) = \gamma(v)(v - w) \quad (7.8)$$

or, transposing:

$$w = \frac{\gamma(v) - 1}{\gamma(v)}v \quad (7.9)$$

The relative velocity transformation formula (7.7) may be contrasted with the conventional special-relativistic parallel-velocity addition relation (PVAR) which gives instead the transformation of frame velocities. In the notation of Section 4, it is written:

$$v'(A'', t'_R) = \frac{v(A'', t_R) - v(A', t_R)}{1 + \frac{v(A'', t_R)v(A', t_R)}{c^2}} \quad (7.10)$$

or, setting $v(A', t_R) \equiv v$, $v(A'', t_R) \equiv w$ and $v'(A'', t'_R) \equiv w'$,

$$w' = \frac{w - v}{1 + \frac{wv}{c^2}} \quad (7.11)$$

As discussed in Refs. [1, 2], this formula connects the kinematical configurations of a primary space-time experiment and its reciprocal, not configurations observed in the two frames of the primary experiment, as in (7.7). For example, setting $w = 0$ in (7.11) gives $w' = -v$ corresponding to a transformation of between the world line segment of C' in S given by Eq. (6.1) to that of the world line segment of C in S' given by Eq. (6.4) in the reciprocal experiment.

8 Laws of Space-Time Physics

In this section some general statements concerning the physical properties of measured space and time intervals as derived in Refs. [1, 2] and the preceding sections of the present paper, are listed.

- I** The measured spatial interval between two objects at rest in the same reference frame is independent of the reference frame in which, and epoch at which, it is measured.
- II** The spatial interval between two objects in arbitrary motion at a given epoch is independent of the reference frame in which it is measured at the corresponding epoch.
- III** Clocks which are synchronised in any reference frame (inertial or accelerated) are observed to be synchronised in any other reference frame.

A ‘corresponding epoch’ of two reference frames is defined in Section 2 above.

the following definitions are introduced in describe experiments in which clocks in uniform motion are observed:

- (i) Primary Experiment An experiment in which a clock C' at rest in the Travelling Frame S' , moving with uniform velocity v in the direction of the positive x -axis in the Base Frame S , is compared with a similar clock C at rest in S .
- (ii) Reciprocal Experiment An experiment in which the clock C at rest in the Travelling Frame S , moving with uniform velocity v in the direction of the negative x' -axis in the Base Frame S' , is compared with the clock C' at rest in S' .

With these definitions, the following laws concerning primary and reciprocal experiments and observed clock rates may be stated:

IV primary and reciprocal experiments are physically independent.

V In the primary experiment, the clock C' is seen to be running slower than C by the factor $1/\gamma(v)$ by base frame observers, whereas in the travelling frame the clock C is observed be running faster than C' by the factor $\gamma(v)$.

VI In the reciprocal experiment, the clock C is seen to be running slower than C' by the factor $1/\gamma(v)$ by base frame observers, whereas in the travelling frame the clock C' is observed be running faster than C by the factor $\gamma(v)$.

VII The relative velocity of two objects moving parallel to the x' axis in the travelling frame is $\gamma(v)$ times greater than the relative velocity of the same objects in the base frame

Since the concepts of primary and reciprocal experiments are not introduced in SSRT, only the first parts of the laws V and VI (observation of TD from the base frame) are in agreement with the predictions of SSRT. The LC effect is in contradiction with I and II and RS with III. In SSRT it is assumed, in contradiction with VII, that velocities transform between base and travelling frames according to the PVAR (7.10). The latter in fact describes instead the transformation between base frame configurations in a primary experiment and its reciprocal. The origin of the spurious and correlated LC and RS effects of SSRT is explained in Refs. [4, 6, 15, 16, 17].

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