Two-way Opportunistic Relaying Systems: Performance and Optimization in Rayleigh Fading Environments

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Abstract—This paper derives the error probability of a two-way relay-based communication system with opportunistic selection in Rayleigh fading environments. We consider a two-way wireless communication system where two nodes, acting as sources, are unable to exchange data directly and, thus, proceed via \( L \) intermediate relay terminals. We investigate a single-relay selection scheme wherein only the best among the \( L \) relays is chosen. Since the communication is two-way, the analyzed scheme aims at optimizing the worse performance of the two communication tasks between the pair of users. In particular, we derive closed-form expressions of the probability density functions (PDFs) of the signal-to-noise ratios (SNRs) at both users. This result is then used to derive novel closed-form expressions for the average symbol error probabilities (ASEPs) at both users’ nodes. Results on the asymptotic error probability in the high-SNR range completes the performance evaluation part. Then, we provide closed-form expressions for the transmit power optimization at the two source nodes before corroborating the analysis with numerical results.

Index Terms—Two-way communication, opportunistic relaying, performance analysis, symbol error probability, power allocation, Rayleigh fading.

I. INTRODUCTION

The implosion of wireless telecommunications has never been so spectacular: its use has spread to all possible fields and became a compulsory component of our everyday life. The new technology components and devices have become complex and more demanding in terms of resources, and the study of the performance and capacity of such elements have become of vital importance. Indeed, the determination of the performance metrics for such systems is a cornerstone in the design of quality-of-service (QoS) oriented systems.

Following its proposal in [1], the relaying concept gained a huge interest. Numerous improvements have been realized and multiple techniques conceived to improve the reliability and performance of relay-based systems. Simple relaying protocols, such as amplify-and-forward (AF) and decode-and-forward (DF) have been proposed [2] and used to study the capacity and performance of relay-based systems in conventional environments [3] as well as in the emerging cognitive radio and spectrum-sharing environments [4].

The first relaying schemes proposed were spectrum and time inefficient, as the signal sent by the source is relayed to the destination by all the relay nodes on different orthogonal time-slots (TS) and the destination node has to use further processing such as selection combining or maximum ratio combining to decode the relayed signals.

Recently, more efficient protocols, such as opportunistic relaying (OR) [5] and partial relay selection [6], have been proposed to select the “best” relay which is the unique one that will relay the signal to the destination. These improvements allow to save both bandwidth and time.

Consider now that we have two source nodes—rather than one source node and one destination node—trying to exchange data: a configuration that is referred to as two-way [7], [8]. When combined with relaying, several protocols can be considered: (i) four-phase process, where each source transmits during a TS and then a relay forwards data to the opposite source during two TSs; (ii) three-phase process similar to the previous one during the sources transmission but where the relay combines the received signals and broadcasts to the opposite source nodes during the third TS; and (iii) two-phase protocol, where both sources transmit during the same TS and the relay combines and forwards the message during the second TS. The three- TS technique is less complex than the two-TS one while providing close performance in terms of error rate [9].

Recently, the combination of two-way communication and relaying protocols has been investigated for a large panel of performance metrics such as error probability, outage probability and capacity. Authors in [10] studied the error rate of a relay-based communication system in Rayleigh fading environment, by providing closed-form expressions for the probability density functions (PDFs) of the signal-to-noise ratios (SNRs) at both end nodes and using it to evaluate the system’s performance. The two-way system in [11] is simplified considering a single AF relay in Nakagami-\( m \) fading environment, while the authors in [12] studied the outage performance for a two-way AF relay-based system but with \( L \) relay terminals among which a single one is selected by means of OR. A performance analysis framework has also been developed in [13] and pushed forward in [14] by also optimizing the power allocation in cognitive two-way relaying networks.

In this paper, we consider a two-way DF relay-based wireless communication network in a Rayleigh fading envi-
We consider the system model shown in Fig. 1, which involves \( L + 2 \) terminals: two source nodes, denoted \( S_1 \) and \( S_2 \), and \( L \) intermediate relaying terminals labeled \( R_i \), \( i = 1, 2, \ldots, L \). The source nodes want to communicate with each other but are unable to do so directly because of deep fading on the direct link and, thus, are getting help by the relaying terminals. The system functions using time-division multiple access (TDMA) technique and channel state information (CSI) is supposed to be available at the source nodes by a network monitor [5].

The communication process occurs over 3 TSs. In the TS-1, \( S_1 \) broadcasts its data, which is collected by the relay nodes. The signal \( y_{R_1}^{(1)} \) received by the relay nodes (\( R_i \)'s) during TS-1 can be written as

\[
y_{R_i}^{(1)} = h_i \sqrt{E_{S_1}} X_{S_1} + z_{R_i},
\]

where \( h_i \) is the complex Gaussian channel gain, i.e., \( h_i \sim CN(0, \Omega_h) \), \( X_{S_1} \) and \( E_{S_1} \) are, respectively, the transmit signal and its average power by the source node \( S_1 \), and \( z_{R_i} \) is the additive white Gaussian noise (AWGN) with mean zero and variance \( N_0 \).

The source node \( S_2 \) operates the same way during the second TS. The signal \( y_{R_i}^{(2)} \) received from the source node \( S_2 \) during TS-2 can be written as:

\[
y_{R_i}^{(2)} = g_i \sqrt{E_{S_2}} X_{S_2} + w_{R_i},
\]

where \( X_{S_2} \) and \( E_{S_2} \) are the transmit signal and its average power of source node \( S_2 \), \( g_i \) is the complex Gaussian channel gain, \( g_i \sim CN(0, \Omega_g) \), and \( w_{R_i} \) is the AWGN with zero mean and variance \( N_0 \).

Let us denote the SNR on the different branches as \( \gamma_{h_i} = \frac{E_{S_1}}{N_0} \sigma_h^2 \) and \( \gamma_{g_i} = \frac{E_{S_2}}{N_0} \sigma_g^2 \), for \( i = 1, 2, \ldots, L \). Then, the PDF of the SNR on the different branches can be written as:

\[
p_{\gamma_{h_i}}(x) = \frac{1}{\gamma_h} e^{-x/\gamma_h},
\]

\[
p_{\gamma_{g_i}}(x) = \frac{1}{\gamma_g} e^{-x/\gamma_g},
\]

where \( \gamma_h = \frac{E_{S_1}}{N_0} \Omega_h \) and \( \gamma_g = \frac{E_{S_2}}{N_0} \Omega_g \).

During TS-3, we form a decoding set \( C \) composed of the relay nodes that were able to decode the signals received from both \( S_1 \) and \( S_2 \). We denote the cardinality of the decoding set \( C \) as \(|C|\). For the propose of spectrum and time efficiency, only the best relay (labeled \( R_b \)) among the set \( C \), with respect to the channel coefficients \( h_i \) and \( g_i \), is selected. The relay \( R_b \) decodes then re-encodes the data before forwarding it to the source nodes \( S_1 \) and \( S_2 \). The best relay, with index \( b \), is selected according to the following rule:

\[
R_b = \arg \min_{i=1,2,\ldots,|C|} \{ \gamma_{h_i}, \gamma_{g_i} \}.
\]
Thus, the signal received during TS-3 at nodes $S_1$ and $S_2$ can be written as
\begin{align}
\hat{y}^{(3)}_{S_1} &= h_b \sqrt{E_R} X_R + z_{S_1}, \\
\hat{y}^{(3)}_{S_2} &= g_b \sqrt{E_R} X_R + z_{S_2},
\end{align}

where $E_R$ and $X_R$ are the average power and transmit signal of the selected node $R_b$, $z_{S_1}$ and $z_{S_2}$ are the AWGN terms with mean zero and variance $N_0$. As each one of the source nodes knows its own signal, both can cancel the self-interference and use maximum likelihood (ML) to detect the desired signals $X_{S_2}$ and $X_{S_1}$, respectively:
\begin{align}
\hat{y}^{(3)}_{S_1} &= X_{S_1}, \\
\hat{y}^{(3)}_{S_2} &= X_{S_2}.
\end{align}

Since the analysis at both users, i.e., $S_1$ and $S_2$, is similar, in the next section we only consider node $S_2$. For the considered node, we derive a closed-form expression for the PDF of the SNR and use this expression to study the average symbol error probability (ASEP) for different modulation schemes.

### III. Performance of the System

In this section, we study the performance of the system presented in Section II by deriving closed-form expression for the ASEP at both $S_1$ and $S_2$. Since we consider a two-way communication system, we reduce our derivations to node $S_2$ when it receives data from the source $S_1$. Similar results, i.e., for node $S_1$, can be deduced when $S_2$ is considered as source. For this purpose, let us first consider the following integral:
\begin{equation}
\mathcal{H}(a, b, c) = \int_0^{+\infty} x^{a-1} e^{-bx} Q \left(\sqrt{cx}\right) dx.
\end{equation}

A closed-form expression for $\mathcal{H}(a, b, c)$ is given by [15, Appendix 5A]:

(i) for a non-integer,
\begin{equation}
\mathcal{H}(a, b, c) = \frac{\Gamma \left(a + \frac{1}{2}\right)}{\Gamma \left(a + 2b\right)} \frac{2 \sqrt{\pi}}{2a+1/2} \left(1 - \mu(b, c)\right)
\end{equation}

\begin{equation}
\times \frac{2a}{2b} \left(1 + \frac{1}{2a+1/2}\right); \quad (11)
\end{equation}

(ii) for a integer,
\begin{equation}
\mathcal{H}(a, b, c) = \frac{\Gamma(a)}{2^{a-1}} \left[1 - \mu(b, c)\right]
\end{equation}

\begin{equation}
\times \sum_{i=0}^{a-1} \frac{\binom{2i}{i}}{4^i} \left(\frac{1 - \mu(b, c)}{4}\right)^i, \quad (12)
\end{equation}

with $\mu(b, c)$ defined by $\mu(b, c) = \sqrt{c/(c+2b)}$.

### A. Average Symbol Error Probability

For gray-coded modulation schemes, the instantaneous SEP for a given SNR $\gamma$ of the channel reduces to:
\begin{equation}
\text{SEP}(\gamma) = \alpha Q \left(\sqrt{\beta \gamma}\right),
\end{equation}

where $\alpha$ and $\beta$ are parameters that depend on the chosen gray-coded modulation scheme. The ASEP is defined as:
\begin{equation}
\text{ASEP}(\gamma) = \int_0^{+\infty} \text{SEP}(\gamma) p_t(\gamma) d\gamma.
\end{equation}

Therefore, we need to determine the PDF of the SNRs $\gamma_{R_b}$ and $\gamma_{S_b}$ pertaining to the links between the chosen relay and the source nodes. We recall that the best relay $R_b$ is selected among the decoding set $C$. Therefore, using the total probability theorem, the PDF for the link $R_b - S_2$ can be written as:
\begin{equation}
p_{\gamma_{R_b}}(x) = \Pr \{ |C| = 0 \} \delta(x) + \sum_{t=1}^{L} \Pr \{ |C| = t \} p_{\gamma_{R_b}}(x|t),
\end{equation}

where $\delta(x)$ is the Dirac function and $p_{\gamma_{R_b}}(x|t)$ is the conditional PDF of the best-link second hop knowing that $|C| = t$. The probability $\Pr \{ |C| = t \}$ for all values $t = 0, 1, \ldots, L$ can be written as
\begin{equation}
\Pr \{ |C| = t \} = \binom{L}{t} (P_{\text{off}})^{L-t} (1 - P_{\text{off}})^t.
\end{equation}

Knowing the results of the integral $\mathcal{H}$, we can easily see that:
\begin{equation}
P_{S_1}(e) = \frac{\alpha}{\gamma_{h}} \mathcal{H} \left(1, \frac{1}{\gamma_{h}}, \beta\right)
\end{equation}

\begin{equation}
\text{and}
\end{equation}

\begin{equation}
P_{S_2}(e) = \frac{\alpha}{\gamma_{g}} \mathcal{H} \left(1, \frac{1}{\gamma_{g}}, \beta\right).
\end{equation}

The SNR PDF of the best-link second-hop $R_b - S_2$, knowing that the decoding set $C$ has cardinality $|C|$, can be written as given in Eq. (20) shown at the top of next page, where $\tau$ is the harmonic mean given by $\tau = \frac{\gamma_{h} + \gamma_{g}}{2}$. The derivation proof and symbols used follow the approach in [10]. Combining the results’ equations from (15) to (20), we obtain the unconditioned PDF $p_{\gamma_{R_b}}$ of the best-link second hop, which can be extended to $p_{\gamma_{S_b}}$ by interchanging $\gamma_h$ with $\gamma_g$.

Substituting (15) into (14) and solving the integration, the ASEP at node $S_2$, denoted $\text{ASEP}_{S_2}$, can be expressed as:
\begin{equation}
\text{ASEP}_{S_2} = \frac{\alpha}{4} \sum_{t=1}^{L} \binom{L}{t} (P_{\text{off}})^{L-t} (1 - P_{\text{off}})^t \lambda_t (\beta),
\end{equation}

where $\lambda_t (\beta)$ is given in Eq. (22) on the next page, for a decoding set with a cardinal $|C| = t$.

We note that a similar formula can be found for the ASEP at node $S_1$ by following the same procedure as for the PDF, i.e., simply by interchanging $\gamma_h$ and $\gamma_g$ where applicable.
\[ p_{\gamma_{0i}}(x|t) = \sum_{i=1}^{t} \binom{t}{i} \left( \frac{-1}{\gamma_h} \right)^{i-1} \frac{i\gamma}{\gamma_h - \gamma} \left( e^{-x/\gamma_g} - e^{-ix/\gamma} \right) + \sum_{i=1}^{t} \binom{t}{i} \left( \frac{-1}{\gamma_g} \right)^{i-1} i^i e^{-ix/\gamma}. \]  

\[ \lambda_t(\beta) = \sum_{i=1}^{t} \binom{t}{i} \left( \frac{-1}{\gamma_h} \right)^{i-1} \frac{i\gamma}{\gamma_h - \gamma} \left( H \left( 1, \frac{i}{\gamma_g}, \beta \right) - H \left( 1, \frac{i}{\gamma}, \beta \right) \right) + \sum_{i=1}^{t} \binom{t}{i} \left( \frac{-1}{\gamma_g} \right)^{i-1} i^{\frac{h}{g}} H \left( 1, \frac{i}{\gamma}, \beta \right). \]  

### IV. Power Allocation Optimization

#### A. Asymptotic Error Probability

Despite its simplicity, the expression in (21) does not provide immediate and insightful conclusions concerning the key parameters used such as \( \gamma_h, \gamma_g \) and the number of relay nodes \( L \). Next, we propose to simplify the expression of the ASEP for the high SNR range, by providing an asymptotic expression of the ASEP.

Using Theorem 1 in the Appendix of [16], \( p_{\gamma_{0i}}(x) \) can be approximated by

\[ p_{\gamma_{0i}}(x) \rightarrow \frac{L}{\gamma_g} \left( \frac{1}{2} \right)^{L-1} x^{L-1} + \text{H.O.T.}, \]  

where H.O.T. stands for high-order terms. We can also see from the definition of \( P_{\text{off}} \) in Eq. (17) that, for high SNRs, we can approximate \( P_{\text{off}} \) as:

\[ P_{\text{off}} \approx P_{S_1}(e) + P_{S_2}(e), \]  

where \( P_{S_1}(e) \approx \alpha/(2\beta \gamma_h) \) and \( P_{S_2}(e) \approx \alpha/(2\beta \gamma_g) \).

Finally, after some algebraic manipulations, the ASEP can be approximated as:

\[ \text{ASEP}_{S_2} \approx \frac{\alpha}{4} \left( \frac{\alpha}{2\beta \gamma_h} + \frac{\alpha}{2\beta \gamma_g} \right)^L + \frac{\alpha}{\gamma_g} \left( \frac{1}{2} \right)^{L-1} \frac{\Gamma(2L+1)}{2^{L+1} \beta L \Gamma(L+1)}. \]  

Assuming that we have the same transmit power \( E_S \) at all nodes and plugging the values of \( \gamma_h \) and \( \gamma_g \) into (25), we obtain:

\[ \text{ASEP}_{S_2} \approx \left[ \frac{\alpha}{4} \left( \frac{\alpha}{2\beta} \right)^L \left( \frac{1}{\Omega_h} + \frac{1}{\Omega_g} \right)^L + \frac{\alpha}{\Omega_g} \left( \frac{1}{\Omega_h} + \frac{1}{\Omega_g} \right)^{L-1} \frac{\Gamma(2L+1)}{2^{L+1} \beta L \Gamma(L+1)} \right] \left( \frac{1}{E_S/N_0} \right)^L, \]  

and we can conclude that the system reaches a diversity gain of \( L \), which is exactly the number of relay nodes available.

#### B. Power Optimization

Power consumption is a key parameter in wireless communication systems and any device involved in the communication process has to balance between a smooth data transmission and an efficient consumption of the energy.

In our two-way communication system, our goal is to minimize the total ASEP of the system (as summation of the ASEP at both source nodes, \( S_1 \) and \( S_2 \)) while providing an efficient energy distribution among nodes participating in the communication process. In what follows, we consider the case where we have at least one relay able to decode both signals received from the source nodes \( S_1 \) and \( S_2 \), as the optimization problem would be useless in the case where \( |C| = 0 \).

The power optimization problem can be written as:

\[
\begin{align*}
\text{minimize} & \quad \text{ASEP}, \\
\text{subject to} & \quad E_{S_1} + E_{S_2} + E_R \leq P_{\text{total}}, \\
& \quad E_{S_1} > 0, \ E_{S_2} > 0, \ E_R > 0,
\end{align*}
\]  

where the total ASEP is given by

\[
\text{ASEP} = C \left( \frac{1}{E_{S_1} \Omega_h} + \frac{1}{E_{S_2} \Omega_g} \right)^L + D \left( \frac{\alpha \Omega_h}{\Omega_g} \right)^L.
\]  

At this point, we can distinguish two cases which we detail next.

1) Equal Transmit Powers: We consider that \( E_{S_1} = E_{S_2} = E_R = E_T \). This case is not realistic but we mention it for purposes of comparison. The constraint in Eq. (27) becomes \( 3E_T \leq P_{\text{total}} \), which gives an optimized transmit power \( E_T^{\text{opt}} = P_{\text{total}}/3 \).

2) Unequal Transmit Powers: Using the Lagrangian method and after some algebraic manipulations, we prove that the optimized transmit power at the participating nodes, namely \( S_1, S_2 \) and \( R_b \), have the followings relations:

\[
E_{S_1}^{\text{opt}} = V E_{S_2}^{\text{opt}} \quad \text{(31)}
\]

\[
E_{R_b}^{\text{opt}} = U E_{S_2}^{\text{opt}} \quad \text{(32)}
\]

where

\[
V = \sqrt{\Omega_g/\Omega_h} \quad \text{and} \quad U = \left( \frac{2L \Omega_h}{C(1 + V)L - 1} \right)^{1/L+1}. \]  

(33)
Knowing these results, we can easily prove that the optimized transmit power at the source \( S_2 \) is given by
\[
E_{S_2}^{\text{opt}} = \frac{P_{\text{total}}}{1 + U + V},
\]
while the other optimized transmit powers \( E_{S_1}^{\text{opt}} \) and \( E_R^{\text{opt}} \) are obtained using (31) and (32).

V. NUMERICAL EXAMPLES

In this section, we provide numerical results based on the analysis developed in the precedent section. We consider that \( S_1 \) and \( S_2 \) use binary frequency-shift keying (BPSK) modulation and, thus, the modulation parameters are \( \alpha = 1 \) and \( \beta = 2 \). We also use the factor \( \rho = \Omega_h/\Omega_y \), which will help interpret the obtained results as it encompasses the different comparison cases, i.e., when the per-hop average power \( \Omega_h \) is larger, equal or lower than \( \Omega_y \).

Fig. 2 shows the evolution of the ASEP of the studied system for a number of relay terminals \( L = 5 \). As ultimately expected, the performance of the system increases when \( L \) increases. For this, the system performance for lower values of the number \( L \) of relays has not been plotted. Moreover, we see that the performance of the system declines when the ratio \( \rho \) increases. This is also expected, as a higher value of \( \rho \) means that the per-hop average power \( \Omega_h \) is larger than \( \Omega_y \) and thus, less relay nodes are able to decode both signals (of the source nodes \( S_1 \) and \( S_2 \)) which results in a smaller probability of having a good link between the “best” relay \( R_0 \) and the source \( S_2 \) and, thus, a higher error rate.

Fig. 2 also shows that the ASEP for the optimized power allocation improves when the available total power budget \( P_{\text{total}} \) increases.

VI. CONCLUSION

In this paper, we have studied a two-way relay-based wireless communication system with opportunistic selection operating in Rayleigh fading environment. We used the closed-form expressions for the PDF of the SNRs pertaining to both users to derive a closed-form expression for the average symbol error probability. We also provided a good and easy approximation of the asymptotic error probability, which clearly underlined that the diversity gain of the system equals the number of relay terminals and allowed the development of the optimized powers at the transmit nodes. The derivations of both expressions provide a good cornerstone for further improvements in the design of QoS-sensitive relay-based communication networks, which is currently under investigation.

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