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Economical production and transshipment policy for coordinating multiple production sites

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In this article, we study the coordination mechanism dealing with a production–transshipment policy across the multiple regions supplying multiple products. It is assumed that each production site has its own dedicated demand region consuming multiple products. The main concern is how to determine both the production quantity and the lot-apportioning policy while minimising the relevant supply chain cost. This decision issue is formulated as a non-linear mathematical model to determine several relevant decision variables. We propose the solution procedure for deriving the production–transshipment policy minimising the overall supply chain cost.

Keywords: multiple production sites; inventory; transshipment policy; lot apportioning policy

1. Introduction

Global competition necessitates the establishment of stable managerial framework integrating the global supply chain network coordinating multiple production sites for satisfying the demands from multiple demand regions. In order to be responsive to the changes, robust operational policies need to be developed by integrating and coordinating the supply chain networks involving multiple products manufactured in multiple sites. Each production site may have its dedicated demand region such that the corresponding demand from this dedicated region might be preferably fulfilled by the pre-arranged production site. However, in case the production site does not have enough capacity, other production sites should make up for the unfulfilled demand. In this article, we consider the supply chain configuration involving multiple production sites and multiple demand regions. Each production site has its own dedicated demand region(s) and these multiple production sites should cooperate with others to satisfy the total demands across the multiple demand regions while minimising the overall supply chain cost.

This article is organised as follows. Following Section 1, the relevant previous studies are reviewed in Section 2. In Section 3, we describe the operational behaviours in the supply chain configuration, formulate it as a mathematical model and then derive several key properties of optimal solution. In Section 4, we propose the solution procedure utilising the solution properties derived in Section 3. An example is provided in Section 5, followed by concluding remarks in Section 6.

2. Literature review

The general schemes of supply chain integration are typically classified into two distinctive modes, i.e. vertical integration and horizontal integration. Studies dealing with vertical integration have focused on interaction between supply chain stakeholders. For example, the joint production–delivery policy between supplier and retailer can be regarded as a typical example for the vertical integration issues (Hoque 2008; Kwartumachai and Hop 2005; Kim, Hong, and Chang 2006; Lu 1995; Nori and Sarker 1996; Viswanathan and Piplani 2001). Basically, these studies have aimed at determining the optimal material flows across the supply chain partners.

Ramani and Narayanan (1992) extended the Economic Lot Scheduling Problem (ELSP) model for a just-in-time system and analysed the effect of variations in setup costs/times and capacity on the overall profits. Banerjee and Burton (1994) proposed a coordinated inventory replenishment policy for a single vendor and multiple buyers assuming that all buyers have an identical cycle time for each buyer and that the production cycle length is an integer multiple of the common replenishment cycle. Nori and Sarker (1996) investigated the cyclic scheduling problem for multiple products in a single production facility under a just-in-time policy. Parija and Sarker (1999) proposed an...
ordering policy for raw materials, while minimising the supply chain cost, to satisfy the customer who demands a predetermined quantity of finished products at a fixed interval of time. Hong and Hayya (1992) developed the procedure for splitting a large order quantity into multiple shipments in a single sourcing environment. Additionally, they considered the supplier selection and order sizing problems in a multiple-sourcing environment. Herer, Rosenblatt, and Hefter (1996) presented the single-sink fixed-charge transpor-
tation problem with a single product and multiple suppliers on a basis of EOQ policy. They developed an enumeration procedure to determine the supplier allocation ratios minimising the total cost including the periodic purchasing cost, ordering cost, inventory carrying cost and supplier management cost. A lot of variant problems can be classified according to the types of supply chain integration (Hsu and El-Najdawi 1990; Khouja 1997; Khoury, Abboud, and Tannous 2001; Hahm and Yano 1995; Wagner and Davis 2002; Ouenniche and Docter 1998, 2001; Jensen and Khouja 2004).

Studies dealing with horizontal integration have investigated the issues for operational integration among supply chain participants located at the same stage. Typical problems or issues under the framework of horizontal integration can be enumerated as capacity pooling among facilities, transshipment policy among supply chain facilities, and so on. Jain (2007) studied the beneficial effects of capacity pooling. Yang and Schrage (2009) investigated the inventory management issue when substitutions among products are allowed. Olsson (2009) studied the issue of lateral shipment with two identical locations in a single-echelon inventory system. Minner and Silver (2005) considered the business situation where an organisation has more than one outlet, where stock is held to meet customer demand. They provided a model that evaluating the effect of two extreme policies such as no transshipment and full transshipment between two stocking points. Kim, Hong, and Lee (2005) introduced a production system where parallel production facilities produce multiple products. They proposed the solution procedure for determining the production lot size for each product, lot-apportioning policy among multiple facilities. This kind of horizontal integration issue also can be seen in a manufacturing system with parallel machines. Torabi, Karimi, and Fatemi Ghomi (2005) and Torabi, Fatemi Ghomi, and Karimi (2006) studied the ELSP in deterministic job shops where the planning horizon is finite and fixed by management. They proposed an efficient enumeration method to determine the optimal solution yielding both machine assignment/sequencing problem and lot-sizing/scheduling problem.

In this article, we investigate the issue of coordinating multiple production sites in which lateral transshipments are allowed among production sites. The objective is to determine the production quantity at each production site and transshipment quantity among production sites to satisfy the given demand while minimising the overall operating cost of the supply chain.

3. Model formulation

3.1. Problem description

For a more generalised model, we assume that the supply chain is comprised of multiple production sites and each production site is manufacturing multiple products. The demand for each product at a certain region is known and constant over time and each production site may have its dedicated demand region(s) (see Figure 1 for the configuration of the supply chain). Transshipment of products is permitted between production sites due to cost or capacity consideration.

All production sites have a common cycle to operate the production–transshipment policy across the supply chain. Additionally, it is assumed that no items of production lot \( q_{ij} \) are used to meet demand of product-\( i \) before the whole lot is produced. The inventory patterns at production sites can be depicted as shown in Figure 2.

3.2. Mathematical model

Following notations are being used hereafter:

- **Given parameters:**
  - \( m \) number of products, where \( M = \{ i | 1 \leq i \leq m \} \)
  - \( n \) number of regions (or sites), where \( N = \{ j | 1 \leq j \leq n \} \)
  - \( i \) index for products, where \( i \in M \)
  - \( j,k \) index for regions (or sites), where \( j,k \in N \)
  - \( s_{ij} \) set-up cost per setup for product-\( i \) at production site-\( j \)

Figure 1. A schematic demand region linked to the production site.

Figure 2. A schematic view of supply chain operating multiple production/demand sites.
Figure 2. An exemplar inventory pattern at each production sites (m = 2, n = 3).

- $h_{ij}$: annual inventory carrying cost for product-$i$ at production site-$j$
- $t_i$: transshipment cost per unit product-$i$
- $p_{ij}$: annual production rate for product-$i$ at production site-$j$
- $d_{ij}$: annual pre-arranged demand rate for product-$i$ at demand region-$j$
- $w_{ij}$: opportunity cost incurred by facility idle time at production site-$j$

**Decision variables:**
- $T$: common cycle for all production sites, where $T \geq 0$. 
- $q_{ij}$: production lot size for product-$i$ at production site-$j$ within a common cycle time, where $q = \{q_{ij} : i \in \mathbb{M}, j \in \mathbb{N} \}$ and $q_{ij} \geq 0$.
- $\beta_{ij}$: allocated demand rate for product-$i$ at production site-$j$ within a cycle time $T$, where $q_{ij} = \beta_{ij}T$ and $\beta = \{\beta_{ij} : i \in \mathbb{M}, j \in \mathbb{N} \}$.
- $\lambda_{jk}$: apportioning ratio of product-$i$ manufactured at production site-$j$ to demand region-$k$, where $\lambda = \{\lambda_{jk} : i \in \mathbb{M}, j \in \mathbb{N} \}$ and $0 \leq \lambda_{jk} \leq 1$.

Above all, to guarantee the relevance of this problem for satisfying the demand from total available production capacity, the feasibility condition of $\sum_{i \in \mathbb{M}} (\sum_{k \in \mathbb{N}} d_{ik} / \sum_{j \in \mathbb{N}} p_{ij}) \leq 1$ should be satisfied. The total relevant cost is comprised of the cost factors such as production set-up cost ($s_{ij}$), inventory holding cost ($h_{ij}$), transshipment cost ($t_i$) and opportunity cost incurred due to idle time ($w_{ij}$). Equation (1.2) provides this operational constraint keeping the feasible common cycle for a certain production site-$j$, where $j \in \mathbb{N}$.

For the demand rate ($d_{ij}$) of product-$i$ from demand region-$j$, we introduce $\beta_{ij}$ to represent the allocated demand rate in a given cycle time ($T$). Hence, regardless of the demand at region-$j$, the value of $\beta_{ij}$ can be regarded as an actual demand necessitating product-$i$ manufactured at production site-$j$, which can be consumed at both its corresponding demand region and other demand regions if needed. Hence, according to the values of both $\beta_{ij}$ and $d_{ij}$, there can be three distinctive cases indicating the supply mode for product-$i$ at the production site-$j$: surplus supply (i.e. $\beta_{ij} > d_{ij}$), target supply (i.e. $\beta_{ij} = d_{ij}$) and slack supply (i.e. $\beta_{ij} < d_{ij}$). However, for a certain product-$i$, the equality condition between total supply and total demand across the supply chain, i.e. $\sum_{j \in \mathbb{N}} \beta_{ij} = \sum_{k \in \mathbb{N}} d_{ik}$. All, should be satisfied to fulfill all demands from multiple demand regions. Equation (1.3) represents that the total supply quantity for a specific product-$i$ over multiple production sites should be equal to the total demand quantity across the supply chain.

The mathematical model for this problem can be formulated as follows:

Minimise

$$ TRC(T, \lambda) = \frac{\sum_{i \in \mathbb{M}} \sum_{j \in \mathbb{N}} s_{ij} + \frac{h_{ij} q_{ij}}{T} + t_i q_{ij} (1 - \lambda_{ij}) + \sum_{j \in \mathbb{N}} w_{ij} (T - \sum_{i \in \mathbb{M}} \frac{q_{ij}}{p_{ij}}) }{T} $$

(1.1)
Subject to
\[ \sum_{i \in M} \left( \frac{q_{ij}}{p_{ij}} \right) \leq T, \quad \forall j \]
(1.2)
\[ \sum_{j \in \text{N}} q_{ij} = \sum_{k \in \text{N}} d_k T, \quad \forall i \]
(1.3)
\[ \sum_{k \in \text{N}} \lambda_{ik}^j = 1, \text{ where } 0 \leq \lambda_{ik}^j \leq 1, \quad \forall i, j \]
(1.4)

Note that Equation (1.1) gives the objective function considering several relevant cost factors; and Equation (1.4) means that the whole production lot \( q_{ij} \) for product-\( i \) at production site-\( j \), should be apportioned among demand regions.

Let us consider the \( j \)th region manufacturing the \( j \)th product. And then, the demand requirement within a given cycle time \( T \) for a product at a certain demand region-\( j \) (i.e. \( d_j T \)) can be fulfilled by utilising multiple production sites including its dedicated production site-\( j \), i.e. \( \sum_{k \in \text{N}} q_{ik} \lambda_{kj}^j = d_j T \). This constraint also represents the situation in which the given demand for product-\( i \) at region-\( j \) can be satisfied by utilising the lateral shipment from other production sites not only by its own dedicated production site-\( j \). Using both this equality constraint and the definition of production lot size \( (q_{ij}) \) satisfying the allocated amount within a cycle, i.e. \( q_{ij} = \beta_{ij} T \), for \( i \in \text{M} \) and \( j \in \text{N} \), we can derive the following equation:

\[
\begin{align*}
\beta_{i1} \lambda_{i1}^j + \beta_{i2} \lambda_{i2}^j + \cdots + \beta_{in} \lambda_{in}^j &= d_j, \quad \forall i, j
\end{align*}
\]
(2)

For a certain product-\( i \), if the supply capacity is less than its own demand requirement at a certain region-\( j \), i.e. \( \beta_{ij} < d_j \), then the demand of region-\( j \) borrow some portion of its unfulfilled demand from other sites, the region-\( j \) cannot supply to other demand regions, i.e. \( \lambda_{ik}^j \lambda_{kl}^j = 0 \) if \( j \neq k \neq l \).

To consider these operational constraints, we induce the two sets of Lagrangian multipliers as follows:

\( \mu \), a set of Lagrangian multiplier for cycle time feasibility at production site-\( j \), where \( \mu = \{ \mu_j | j \in \text{N} \} \) and \( \kappa \), a set of Lagrangian multiplier for balancing between total supply and total demand for product-\( i \), where \( \kappa = \{ \kappa_i | i \in \text{M} \} \).

As defined above, note that \( \mu_j \)'s are Lagrangian multipliers introduced to satisfy the feasibility condition for a common cycle time at each production site, which is described in Equation (1.2). Additionally, we make use of additional Lagrangian multipliers \( \kappa_i \)'s to keep the equality constraint for balancing the total supply and total demand for product-\( i \), i.e. \( \sum_{j \in \text{N}} \beta_{ij} = \sum_{k \in \text{N}} d_k \). From Equation (2), we can rewrite \( \beta_{ij} \lambda_{ik}^j \) as \( d_j - \sum_{k \in \text{N}, k \neq j} \beta_{ik} \lambda_{kj}^j \). And then, the value of \( \beta_{ij} \lambda_{ik}^j \) can be regarded as the allocated quantities in a year for fulfilling the corresponding demand of product-\( i \) for its dedicated demand region-\( j \), which is originally manufactured at the production site-\( j \). Correspondingly, the consumed quantity of product-\( i \) at its dedicated demand region-\( j \) in case that the production site-\( j \) makes the product-\( i \) as much as \( q_{ij} \), i.e. \( \beta_{ij} \lambda_{ik}^j \), where \( q_{ij} = \beta_{ij} T \), can be described by \( d_j T - \sum_{k \in \text{N}, k \neq j} \beta_{ik} \lambda_{kj}^j \).

From these properties and Equation (2), we can re-state the objective function using a set of Lagrangian multipliers as seen below.

\[
\begin{align*}
TRC(T, \beta, \lambda; \mu, \kappa) &= \frac{S}{T} + \left( \sum_{i \in \text{M}} \sum_{j \in \text{N}} \left( \frac{h_{ij} \beta_{ij}^2}{2p_{ij}} \right) 
+ \sum_{j \in \text{N}} \mu_j \left( \sum_{i \in \text{M}} \beta_{ij}^2 - 1 \right) \right) + \sum_{j \in \text{N}} \left( T + \sum_{i \in \text{N}} w_j \right) 
+ \sum_{i \in \text{M}} \sum_{j \in \text{N}} \left( \left( t_i - \frac{w_j}{p_{ij}} \right) \beta_{ij} - t_i \right) 
\times \left( d_j - \sum_{k \in \text{N}, k \neq j} \beta_{ik} \lambda_{kj}^j \right) 
+ \sum_{i \in \text{M}} \kappa_i \left( \sum_{j \in \text{N}} \beta_{ij} - \sum_{k \in \text{N}} d_k \right),
\end{align*}
\]
(3)

3.3. Determination of the values of \( T, \beta, \mu \) and \( \kappa \)

The optimal cycle time can be determined by the first-order optimality condition, i.e. \( \frac{\partial TRC(T, \beta, \lambda; \mu, \kappa)}{\partial T} = 0 \) and it is given by

\[
T^*(\beta; \mu) = \frac{S}{\sqrt{\sum_{i \in \text{M}} \sum_{j \in \text{N}} \left( \frac{h_{ij} \beta_{ij}^2}{2p_{ij}} \right) + \sum_{j \in \text{N}} \mu_j \left( \sum_{i \in \text{M}} \beta_{ij}^2 - 1 \right)}}
\]
(4)

From Equation (4), we note that the lot-apportioning policy (\( \lambda \)) has no influence on the common cycle itself. By putting \( T^*(\beta; \mu) \) in Equation (4) into Equation (3), we can re-arrange the objective function as follows:

\[
\begin{align*}
TRC(T^*(\beta; \mu), \beta, \lambda; \mu, \kappa) &= \frac{S}{\sqrt{\sum_{i \in \text{M}} \sum_{j \in \text{N}} \left( \frac{h_{ij} \beta_{ij}^2}{2p_{ij}} \right) + \sum_{j \in \text{N}} \mu_j \left( \sum_{i \in \text{M}} \beta_{ij}^2 - 1 \right)}} 
+ \sum_{j \in \text{N}} w_j + \sum_{i \in \text{M}} \sum_{j \in \text{N}} \left( t_i - \frac{w_j}{p_{ij}} \right) \beta_{ij}. 
\end{align*}
\]
(5)
From the first-order optimality condition, i.e. 
\[ \frac{\partial \text{TRC}(\beta, \mu, \lambda, \pi, \mathbf{s})}{\partial \beta_{ij}} = 0, \]
we obtain the optimal value of \( \beta_{ij} \) as follows (refer to the Appendix for details): 
\[ \beta_{ij} = \frac{w_j - (t_j + \kappa_j)}{T^*(\beta, \pi)n_{ij}} - \frac{\mu_j}{n_{ik}}, \quad \forall i, j \]  

(6)

From Equation (6), it can be noted that the value of allocated demand size \( \beta_{ij} \) in a common cycle for product-i at production site-j becomes large in the following cases: when either the holding cost \( (h_j) \) or transshipment cost \( (t_i) \) becomes small and when the opportunity cost \( (w_j) \) caused by an idle time at site-j becomes large. Furthermore, by putting \( \beta_{ij} \) in Equation (6) into the equality condition of \( \sum_{i \in N} \beta_{ij} = \sum_{k \in N} d_{ik} \), we obtain the Lagrangian multiplier \( \kappa_i \) as follows:

\[ \kappa_i = \sum_{i \in N} \left( \frac{w_i - (t_i + \kappa_i) p_i}{T^*(\beta, \pi)n_{ij}} - \frac{\mu_i}{n_{ik}} - d_{ij} \right), \quad \forall i \]  

(7)

For the Lagrangian multipliers, the first-order optimality condition for them, i.e. 
\[ \frac{\partial \text{TRC}(\beta, \mu, \lambda, \pi, \mathbf{s})}{\partial \mu_{ij}} = 0 \quad \text{and} \quad \frac{\partial \text{TRC}(\beta, \mu, \lambda, \pi, \mathbf{s})}{\partial \lambda_{ik}} = 0, \]
we obtain the value of \( \mu_{ij} \) satisfying the condition of \( \sum_{i \in M} \mu_{ij} = 1 \) at production site-j as follows:

\[ \mu_{ij} = \max \left\{ \frac{\sum_{i \in M} \left( \frac{w_j - (t_j + \kappa_j) p_j}{T^*(\beta, \pi)n_{ij}} - 1 \right)}{\sum_{i \in M} \left( \frac{1}{n_{ik}} \right)}, 0 \right\}, \quad \forall j \]  

(8)

3.4. Determination of production-lot apportioning policy, i.e. \( \lambda \)

Assume that the allocated demand (i.e. \( \beta_{ij} \)) for a certain product-i at production site-j is given. As aforementioned, using the values of \( \beta \), we might classify the supply status of production site-j for product-i compared to the pre-arranged demand (i.e. \( d_{ij} \)) at this site, i.e. \( \beta_{ij} > d_{ij} \) or \( \beta_{ij} < d_{ij} \). From this categorisation, we can infer the following: if the inequality of \( \beta_{ij} > d_{ij} \) is satisfied, then the production site-j produces more than its own pre-arranged demand and then allocates its surplus quantity to other demand regions. Furthermore, if the result of \( \beta_{ij} < d_{ij} \) occurs, then the unfilled demand at site-j can be satisfied only from other production sites. In this article, we assume that the surplus finished goods at a certain production site are apportioned to other demand regions according to the ratio of slack quantity of the corresponding demand region to total unfulfilled demand for product-i.

3.4.1. Procedure to determine the values of \( \lambda \) for product-i

Step 1: Using the following two equations, calculate slack quantity \( (SQ_{ik}) \) for the specific product-i at the region-k and its corresponding lot-apportioning ratios \( (\lambda_{ik}^j) \).

\[ SQ_{ik} = \max\{d_{ik} - \beta_{ik}, 0\}, \quad i \in M, \ k \in N \]  

(9.1)

\[ \lambda_{ik}^j = \min\{d_{ik}/\beta_{ik}, 1, 0\}, \quad i \in M, \ k \in N \]  

(9.2)

Step 2: By using Equation (9.3), calculate total slack quantity \( (TSQ_i) \) and its corresponding slack ratio \( (r_{ik}^j) \) for the product-i at the region-k.

\[ r_{ik}^j = SQ_{ik}/TSQ_i, \quad \text{where} \quad TSQ_i = \sum_{k \in N} SQ_{ik} \text{ and } i \in M, \ k \in N \]  

(9.3)

Step 3: Calculate the corresponding lot apportioning ratio \( (\lambda_{ik}^j) \) for the product-i from the production site-j to the region-k by using Equation (9.4).

\[ \lambda_{ik}^j = \min\left\{ \left( \frac{1 - \lambda_{ik}^j \times r_{ik}^j}{\lambda_{ik}^j}, 0, 1.0 \right) \right\}, \quad j \in N, \ k \in N - \{ j \} \]  

(9.4)

4. Solution procedure

Using the solution properties derived in Section 3, we build the following solution procedure determining the economical production–transshipment policy over multiple production sites to minimise the overall supply chain cost.

Step 0: Initialisation:

Let \( z = 1 \), \( \beta_{ij} = d_{ij} \), \( \mu_{ij} = 1 \), for all \( i, j \), where the superscript \( z \) represents the number of iterations. It is necessary to initialise the total cost to decide whether we iterate the procedure again. Hence, we set \( TRC^{(0)} = L \), where \( L \) is a sufficiently large number. Calculate the common cycle, i.e. \( T^*(\beta^{(z)}, \mu^{(z)}) \). And then, calculate the initial Lagrangian multiplier \( (\kappa^{(z)}) \) for each product.

Step 1: Calculate the Lagrangian multiplier \( (\mu^{(z)}) \) for each production site.

Step 2: Calculate the Lagrangian multiplier \( (\kappa^{(z)}) \) for each product.

Step 3: Calculate \( \beta^{(z)} \) for both each product and individual production site.

Step 4: Calculate the common production cycle \( (T^*(\beta^{(z)}, \mu^{(z)})) \) using \( \beta^{(z)} \) and \( \mu^{(z)} \). Additionally, using the values of \( \beta^{(z)} \), calculate the corresponding transshipment policy, i.e. \( \lambda^{(z)} \), using the procedure described in Section 3.4. And then, we calculate the \( TRC^{(z)} = TRC(T^{(z)}, \beta^{(z)}, \lambda^{(z)}, \mu^{(z)}) \), correspondingly.

Step 5: If the total relevant cost can be decreased, i.e. \( TRC^{(z)} < TRC^{(z-1)} \), then we update the best policy as the current new policy. And then, \( z = z + 1 \) and go to
Step 1 for next iteration. Otherwise, the previous policy is best one, and then we terminate the solution procedure.

5. Illustrative examples

In this section, we provide a numerical example illustrating the proposed solution procedure. As an illustrative example, we consider the case where two products are produced at three production sites and consumed at three demand regions, i.e. $m = 2$ and $n = 3$.

Basic operational parameters for an illustrative example are prepared as seen in Table 1.

First of all, we note that the ratios of production period within a common cycle at production site-$j$, i.e. $\frac{d_{ij}}{p_{ij}}$, where $j = 1, 2, 3$, can be evaluated as 1.057, 1.042 and 0.977, respectively. This means that both production sites do not have the enough capacity. Hence, it is definite that the transshipments from other sites should be implemented to confirm that these two sites are guaranteed to supply the demand. For reference, if we aggregate both the production capacities and demands over multiple sites, the necessary length of production period, i.e. $\frac{\sum_{j=1}^{3} (\sum_{k=1}^{3} d_{ik} / \sum_{j=1}^{3} p_{ij})}{\sum_{j=1}^{3} (\sum_{k=1}^{3} d_{ik} / \sum_{j=1}^{3} p_{ij})} = 0.90$ 1.0 1.0

Common cycle ($T$) 0.1578

As seen in Figure 3, two products have different operating scenarios. At first, the production site-3 supplies product-1 to the other two sites while demand for product-2 at site-3 can be fulfilled by borrowing product-2 from other production sites. On the other hand, production site-1 has the highest holding cost for both products, so it makes less than its given demand thereby satisfying the feasibility condition accommodating multiple products over three production sites.

6. Concluding remarks

In this article, we investigate the coordination issue dealing with the production–transshipment policy...
among multiple production sites while accommodating multiple products. A set of multiple production sites utilises the common cycle synchronising the production plan across the multiple production sites and demand regions. Additionally, the co-operation framework accommodating the aggregated demand from all demand regions should be implemented to minimise the overall supply chain cost. We develop the solution procedure to obtain the economical production–transshipment policy over multiple production sites and demand regions. Further effort is necessary to consider the generalised transshipment cost, since we assume that the transshipment cost for a specific product is constant regardless of its transshipment distance between sites. Also, the framework justifying the optimality should be developed to obtain the generalised managerial insights.

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**References**


Appendix: Derivation of the optimal $\beta$

Using the value of $\lambda_{ij}^*$, we can rearrange the structure of $TRC(T^*(\beta, \mu), \beta, \lambda; \mu, \kappa)$ as follows:

$$TRC(T^*(\beta, \mu), \beta, \lambda; \mu, \kappa)$$

$$= 2 \left\{ \sum_{\omega \in M} \sum_{j \in N} \left( \frac{b_{ij}}{x_{\omega j}} \right) \right\} + \sum_{j \in N} W_j$$

$$+ \sum_{\omega \in M} \sum_{j \in N} \left( \left( t_{ij} - \frac{W_j}{p_{ij}} \right) \beta_{ij} - t_i \right)$$

$$\times \left( d_j - \sum_{\omega \in M} \sum_{k \in N} \lambda_{kj}^* \beta_{ik} \right)$$

$$\times \sum_{\omega \in M} \kappa_i \left( \sum_{j \in N} \beta_{ij} - \sum_{k \in N} d_k \right)$$

We can simplify the structure of objective function using two intermediate functions, i.e. $f(\beta, \mu)$ and $g(\beta)$ as follows:

$$TRC(T^*(\beta, \mu), \beta, \lambda; \mu, \kappa)$$

$$= 2 \sqrt{f(\beta, \mu) + g(\beta, \lambda) + \sum_{j \in N} W_j}$$

$$+ \sum_{\omega \in M} \kappa_i \left( \sum_{j \in N} \beta_{ij} - \sum_{k \in N} d_k \right)$$

where

$$f(\beta, \mu) = S \left( \sum_{\omega \in M} \sum_{j \in N} \left( \frac{b_{ij}^2}{2p_{ij}} \right) \right)$$

$$+ \sum_{j \in N} \mu_j \left( \sum_{\omega \in M} \beta_{ij} - 1 \right)$$

$$g(\beta, \lambda) = \sum_{\omega \in M} \sum_{j \in N} \left( \left( t_{ij} - \frac{W_j}{p_{ij}} \right) \beta_{ij} \right)$$

$$- t_i \left( d_j - \sum_{\omega \in M} \sum_{k \in N} \lambda_{kj}^* \beta_{ik} \right)$$


And then, by the following mathematical manipulation, we can obtain the optimal value of $\beta_{ij}$.

$$\frac{\partial}{\partial \beta_{ij}} TRC(T^*(\beta, \mu), \beta, \lambda; \mu, \kappa)$$

$$= f^{-1}(\beta, \mu) \frac{\partial}{\partial \beta_{ij}} f(\beta, \mu) + \frac{\partial}{\partial \beta_{ij}} g(\beta, \lambda) + \kappa_i$$

$$= Sf^{-1}(\beta, \mu) \left( \frac{h_{ij} \beta_{ij}}{p_{ij}} + \frac{\mu_i}{p_{ij}} \right) + \left( t_i - \frac{w_j}{p_{ij}} \right) + \kappa_i$$

Finally, from the first-order optimality condition, i.e.

$$\frac{\partial}{\partial \beta_{ij}} TRC(T^*(\beta, \mu), \beta, \lambda; \mu, \kappa) = 0$$

as follows:

$$\beta_{ij} = \frac{w_j - (t_i + \kappa_i) p_{ij}}{T^*(\beta, \mu) h_{ij}} - \frac{\mu_j}{h_{ij}}$$

where $Sf^{-1}(\beta, \mu) = T^*(\beta, \mu)$

$$= T^*(\beta, \mu) \left( \frac{h_{ij} \beta_{ij}}{p_{ij}} + \frac{\mu_i}{p_{ij}} \right) + \left( t_i - \frac{w_j}{p_{ij}} \right) + \kappa_i$$ (A.3)

$$\beta_{ij} = \frac{w_j - (t_i + \kappa_i) p_{ij}}{T^*(\beta, \mu) h_{ij}} - \frac{\mu_j}{h_{ij}}$$ (A.4)