Filter Design for Linear Discrete-Time Systems with Unknown Disturbances and Quantized Measurements

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Abstract—In this paper, an unknown input filter is proposed for discrete-time linear systems with quantized measurements. The approach uses a slightly modified model of the system to estimate both the system states and the unknown input simultaneously. Two well-known quantization methods, namely linear and logarithmic, are studied and a specific design procedure is derived for each case. For the linear case, the quantization error is modeled as measurement noise and for the logarithmic case, it is modeled as a norm-bounded uncertainty. In the end, simulation results are used to illustrate the effectiveness of the proposed filter.

I. INTRODUCTION

Existence of unknown disturbances in the state space equations has been an area of constant research in the filtering problem. Unknown disturbances can lead to considerable performance deterioration which can become accentuated when measurements are transmitted over a communication channel and thus are subject to uncertain delay, packet dropouts and quantization.

Unknown input filter design under uncertain delays has been extensively studied in the literature. [1], [2], [3]. However, the same problem under quantization effects is still relatively unexplored. The problem of unknown input filter design, itself has been an active research topic since late 60’s, [4]. Numerous approaches have been introduced since those early days. In [5] the authors design an unknown input observer with the assumption that the C matrix has a specific structure. The idea is to use a similarity transformation to partition the states into two groups such that only the second group is directly affected by the disturbance. Then a conventional observer is designed for the first partition and the other partition is calculated using the estimated first partition. In this method, the unknown input is not estimated. In [6] the problem of unknown input observer design is discussed in detail for generalized state space models. [7] introduces an optimal unknown input filter with a similar form to the well-known Kalman filter. Authors of [8], assuming the boundedness of the unknown input, propose a full-order observer with the same state parameters as the original continuous-time linear system. This work was later extended in [9] to reduced order observers. [11] proposes a reduced order dynamic observer with an $H_{\infty}$ performance measure. In [12] a new dynamic framework, based on the one given in [13], is used to design an $H_{\infty}$ filter for Lipschitz nonlinear systems with unknown inputs. For more works in this area, the reader is referred to [14]-[18] and the references therein.

In this work, a new approach for the design of unknown input filters for discrete-time linear systems with quantized measurements is proposed. The main idea is based on an earlier work of the authors’ in [18], where a novel unknown input observer is introduced. In that work, the design problem assumes that the measurements are not quantized and also there is no noise on the states and the measurements. Whereas, in this work both states and measurements are noisy and most importantly the measurements undergo quantization. Two common types of quantization, namely linear and logarithmic, are studied and the design problem is formulated and solved for each case. The LMI-based solution provides simultaneous state and input estimation as it can be very crucial in fault related applications. Finally, the proposed filter is simulated for both cases to illustrate the effectiveness of the solution.

The rest of the paper is organized as follows. In section II plant and filter models are given. Section III presents the design procedure for both types of quantization. In section IV, simulation results are given and finally section V concludes the paper.

II. PLANT AND FILTER MODELS

Consider the following linear system:

$$x(k+1) = Ax(k) + B_1 u(k) + B_2 d(k-1) + B_3 w(k)$$
$$y(k) = Cx(k) + v(k)$$
$$z(k) = Hx(k)$$

where $x_{n\times1}$ is the state vector; $y_{p\times1}$ represents the measured outputs; $z_{r\times1}$ is the vector to be estimated; $u_{m_1 \times 1}$ is the known input; $d_{m_2 \times 1}$ is the unknown low-frequency disturbance input; $w_{m_3 \times 1}$ and $v_{p \times 1}$ are the state and measurement noise inputs, respectively, and $A$, $B_1$, $B_2$, $B_3$, $C$, $H$ are the state space matrices of the model.

We define the new state variable $\bar{x}$ as

$$\bar{x}(k+1) = x(k+1) - B_2 d(k)$$

Substituting (2) in (1), we rewrite the plant model as follows:

$$\bar{x}(k+1) = A\bar{x}(k) + B_1 u(k) + AB_2 d(k-1) + B_3 w(k)$$
$$y(k) = C\bar{x}(k) + CB_2 d(k-1) + v(k)$$
$$z(k) = H\bar{x}(k) + HB_2 d(k-1)$$

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We will use the revised plant model given in (3) to design our filter. Consider the following filter model,

\[
\bar{x}_F(k+1) = \ldots (14)
\]

where \(\xi(k) = \begin{bmatrix} X(k)^T \omega(k)^T \end{bmatrix}^T\). In order to establish an H\(_\infty\) bound on the effects of the unwanted noise inputs, we will use the revised plant model given in (3) to design our filter. Consider the following filter model,

\[
\bar{x}_F(k+1) = \ldots (14)
\]

where \(\xi(k) = \begin{bmatrix} X(k)^T \omega(k)^T \end{bmatrix}^T\). In order to establish an H\(_\infty\) bound on the effects of the unwanted noise inputs,

\[
\Xi_t = \begin{bmatrix} \Xi_{11} & 0 \\ 0 & \Xi_{12} \end{bmatrix}
\]

\[
\Xi_{11} = \begin{bmatrix} H^TH - P_e & H^THB_2 \\ B_2^TH^TH & B_2^TH^TB_2 - P_d \end{bmatrix}
\]

\[
\Xi_{12} = \text{diag}\{-\mu_w^2 I, -\mu_q^2 I, -\mu_d^2 I\}
\]

where \(\bar{x}_F\) is the n \times 1 state vector of the filter; \(\bar{y}_F\) is the p \times 1 estimated measurement vector; \(z_F\) is the r \times 1 estimated vector; \(L\) is the static filter parameter to be designed; and \(d_F\) is the m_2 \times 1 estimated disturbance vector, which will follow a stable adaptive law to track the unknown disturbance.

III. FILTER DESIGN APPROACH

In this section, we first introduce the \(\ell_{2e}\) norm and then discuss the unknown input filter design under both linear and logarithmic quantization methods.

**Definition 3.1:** Given a sequence of real vectors \(x(0), x(1), x(2), \ldots\), we define the \(\ell_{2e}\) norm as defined as

\[
\|x\|_{2e} = \sum_{k=0}^{\infty} e(k)^T x(k) \quad \text{and} \quad e \in \ell_{2e}
\]

where \(e\) is the range of each quantization level. For this quantizer, the real measurement is related to its quantized version by the following equation,

\[
Q(y) = y + \Delta y_q \quad |\Delta y_q| \leq \frac{\tau}{2}
\]

As one can see from (6), the quantization error is modelled as an additive noise to the real measurement. The following theorem, formulates the proposed adaptive approach.

**Theorem 3.1:** Consider

- the linear system in (1) and the linear filter in (4), with \(d_F\) being updated by

\[
d_F(k) = d_F(k-1) + \Gamma B_2^T C^T \epsilon(k)
\]

where \(\epsilon(k) = Q(y(k)) - y_F(k)\) and \(Q(y)\) represents the quantized measurements via the linear quantizer given in (5),

- the \(H\(_\infty\)\) attenuation gains \(\mu_w, \mu_q, \mu_d\), which respectively bound the effects of state noise, measurement noise plus quantization error, and disturbance variations on the estimation error.

Then the filtering problem with disturbance estimator has a solution if there exist diagonal matrices \(P_d, G_d > 0\), matrices \(P_e > 0\) and \(G_e\) satisfying the following LMI:

\[
\begin{bmatrix} \Xi_1 & \Xi_2 \\ \ast & -P \end{bmatrix} < 0
\]

where

\[
P = \text{diag}\{P_e, P_d\}
\]

\[
\Xi_1 = \begin{bmatrix} \Xi_{11} & 0 \\ 0 & \Xi_{12} \end{bmatrix}
\]

\[
\Xi_{11} = \begin{bmatrix} H^TH - P_e & H^THB_2 \\ B_2^TH^TH & B_2^TH^TB_2 - P_d \end{bmatrix}
\]

\[
\Xi_{12} = \text{diag}\{-\mu_w^2 I, -\mu_q^2 I, -\mu_d^2 I\}
\]

Once solved, the filter parameters are calculated as \(L = P_e^{-1} G_e\) and \(\Gamma = P_d^{-1} G_d\).

**Proof:** Using the state error \(\bar{e} = \bar{x} - \bar{x}_F\) and the disturbance estimation error \(\bar{d} = d - d_F\), we can simplify \(\epsilon\) as follows:

\[
\epsilon(k) = C\bar{e}(k) + CB_2\bar{d}(k-1) + \Delta y(k)
\]

where \(\Delta y = v + \Delta y_q\). Now using (10) the estimation error system can be written as

\[
\bar{e}(k+1) = (A - LC)\bar{e}(k) + (A - LC)B_2\bar{d}(k-1)
\]

\[
+ B_2w(k) - L\Delta y(k)
\]

\[
\bar{d}(k) = -\Gamma B_2^T C^T \bar{e}(k) - \Gamma B_2^T C^T \Delta y(k)
\]

\[
+ (I - \Gamma B_2^T C^T C B_2)\bar{d}(k-1) + \Delta d(k)
\]

\[
\epsilon(k) = z(k) - z_F(k) = H\bar{e}(k) + HB_2\bar{d}(k-1)
\]

where \(\Delta d(k) = d(k) - \bar{d}(k-1)\). Augmenting \(\bar{e}\) and \(\bar{d}\) as \(X(k) = [\bar{e}(k)^T, \bar{d}(k-1)^T]^T\) and defining \(\omega = [u^T, \Delta y^T, \Delta d'^T]^T\), the augmented error model will be given as:

\[
X(k+1) = A X(k) + B \omega(k)
\]

\[
\epsilon(k) = C X(k)
\]

where

\[
A = \begin{bmatrix} A - LC & (A - LC)B_2 \\ -\Gamma B_2^T C^T C & I - \Gamma B_2^T C^T C B_2 \end{bmatrix}
\]

\[
B = \begin{bmatrix} B_2 & -L \\ 0 & -\Gamma B_2^T C^T \end{bmatrix}
\]

\[
C = \begin{bmatrix} H & HB_2 \end{bmatrix}
\]

To analyze the stability of the augmented system, the following Lyapunov function is used,

\[
V(k) = X(k)^T P X(k)
\]

The forward difference of this Lyapunov function can be written as

\[
\Delta V(k) = \xi(k)^T [A^T P + P A^T + [-P & 0] \xi(k)
\]

where \(\xi(k) = [X(k)^T, \omega(k)^T]^T\). In order to establish an \(H\(_\infty\)\) bound on the effects of the unwanted noise inputs,
effects of the quantization error and also the effects of the variations of the unknown disturbance \(i.e. \Delta d\), we define

\[
J \triangleq \sum_{k=0}^{s} \{ \varepsilon(k)^T \varepsilon(k) - \omega(k)^T \mu \omega(k) \} 
\]  
(15)

where \(s\) is a finite integer and \(\mu = \text{diag}\{\mu_w, \mu_v, \mu_d\}\). Adding (13) to the right hand side of (15), we get

\[
J < \sum_{k=0}^{s} \{ \varepsilon(k)^T \varepsilon(k) - \omega(k)^T \mu \omega(k) + \Delta V(k) \} = \sum_{k=0}^{s} \tilde{J}_k
\]

(16)

Now, if we design our filter such that \(\tilde{J}_k \leq 0\), we can conclude that \(J \leq 0\). Substituting (11) and (14) in (16), we have

\[
\tilde{J}_k \leq \xi^T (\Omega_1 + \Omega_2 \Xi \Omega_2^T) \xi 
\]

(17)

where

\[
\Omega_1 = \begin{bmatrix} C^T C - P & 0 \\ 0 & -\mu^T \mu \end{bmatrix}, \quad \Omega_2 = \begin{bmatrix} A^T P \\ B^T P \end{bmatrix}
\]

(18)

Now if we define \(G_e = P_e L\) and \(G_d = P_d \Gamma\), we can rewrite (17) as follows:

\[
\tilde{J}_k \leq \xi^T (\Xi_1 + \Xi_2 \Xi_2^T) \xi 
\]

(19)

where \(\Xi_1, \Xi_2\) are as given in (9).

**B. Unknown Input Filter Design under Logarithmic Quantization**

Consider the linear system given in (1) and the linear filter model given in (4). Assume that \(Q(y)\) is the vector of the logarithmically quantized measurements given by

\[
Q(y) = \begin{cases} 
\rho^j \mu & \text{if } \frac{\mu^j}{1+\delta} \leq y < \frac{\mu^j}{1-\delta} \\
0 & \text{if } y = 0 \\
-\rho \tilde{Q}(y) & \text{if } y < 0
\end{cases} 
\]

(20)

where \(j = 0, \pm1, \pm2, \ldots, 0 < \rho < 1\) is the quantization density, \(\mu\) is a scaling parameter, and

\[
\delta = (1-\rho)/(1+\rho) 
\]

(21)

For a signal quantized by (20), the quantization error is given as

\[
e_q = Q(y) - y = \Lambda y 
\]

(22)

where \(\Lambda\) is an uncertain variable (see [10]), which depends on \(y\) and is bounded by \(\delta\), i.e. \(-\delta \leq \Lambda \leq \delta\). The following theorem, formulates the proposed adaptive approach.

**Theorem 3.2:** Consider

- the linear system in (1), and the linear filter in (4), with \(d_F\) being updated by (7), and \(Q(y)\) representing the quantized measurements via the logarithmic quantizer given in (20),
- the \(H_\infty\) attenuation gains \(\mu_w, \mu_v, \mu_d\), which respectively bound the effects of state noise, measurement noise, and disturbance variations on the estimation error.

Then the filtering problem with disturbance estimator has a solution if there exist diagonal matrices \(P_d, G_d > 0\), matrices \(P_x, P_e > 0\) and \(G_e\) and scalar \(\eta > 0\) satisfying the following LMI:

\[
\begin{bmatrix} 
\Xi_1 & \Xi_2 & 0 \\
* & -\mathbb{P} & E \\
* & * & -\eta I \end{bmatrix} < 0
\]

(23)

where

\[
\mathbb{P} = \text{diag}\{P_x, P_e, P_d\} 
\]

\[
\Xi_1 = \text{diag}\{\Xi_1^1, \Xi_2^1\} 
\]

\[
\Xi_1^1 = \begin{bmatrix} -P_x + \eta C^T C & 0 & 0 \\
0 & H^T H - P_e & H^T H B_2 \\
0 & B_2^T H^T H & B_2^T H^T H B_2 - P_d \end{bmatrix} 
\]

\[
\Xi_2^1 = \text{diag}\{-\mu^3_w I, -\mu^3_v I + \eta I, -\mu^3_d I\} 
\]

\[
\Xi_2 = \begin{bmatrix} A^T P_x & 0 & 0 \\
0 & B_2^T P_x & 0 \\
0 & B_2^T C^T C B_2 G_d & 0 \end{bmatrix} 
\]

\[
\Xi_3 = \begin{bmatrix} 0 & -C^T C B_d G_d \\
0 & -CB_2 G_d \\
P_d & -B_2^T C^T C B_d G_d & 0 \end{bmatrix} 
\]

\[
E = \begin{bmatrix} 0 & -G_e \\
0 & -G_d B_2^T C^T \end{bmatrix}
\]

(24)

Once solved, the filter parameters are calculated as \(L = P_e^{-1} G_e\) and \(\Gamma = P_d^{-1} G_d\).

**Proof:** Using the state error \(\bar{e} = \tilde{x} - \hat{x}_F\) and the disturbance estimation error \(\hat{d} = d - d_F\), and also using \(Q(y) = y + \Lambda y\) one can rewrite (2) as

\[
\varepsilon(k) = C \tilde{e}(k) + C B_2 \tilde{d}(k - 1) + v(k) + \Lambda C x(k) + \Lambda v(k)
\]

(25)

Now using (25) the error dynamics can be written as

\[
\bar{e}(k + 1) = (A - LC)\tilde{e}(k) + (A - LC)B_2 \tilde{d}(k - 1) + B_3 w(k) - L v(k) - LA \tilde{C} \tilde{e}(k) - LA v(k)
\]

\[
\bar{d}(k) = -G_2^T C^T C \varepsilon(k) + (I - \Gamma B_2^T C^T C B_2) \tilde{d}(k - 1) - \Gamma B_2^T C^T v(k) - \Gamma B_2^T C^T \Lambda C x(k) - \Delta d(k)
\]

\[
\varepsilon(k) = H \bar{e}(k) + H B_2 \tilde{d}(k - 1) 
\]

(26)

where \(\Delta d(k) = \tilde{d}(k) - d(k - 1)\). Augmenting the state variables of the plant and error models as \(X = [x^T \bar{e}^T \tilde{d}_F^T]^T\) and defining \(\omega = [w^T v^T \Delta d_F]^T\), the augmented plant-error model will be given as:

\[
X(k + 1) = (A + \Delta A)X(k) + (B + \Delta B)\omega(k)
\]

\[
\varepsilon(k) = CX(k)
\]

(27)
where
\[
A = \begin{bmatrix}
A & 0 & 0 \\
0 & A - LC & (A - LC)B_2 \\
0 & -\Gamma B_2^T C & I - \Gamma B_2^T C B_2
\end{bmatrix},
\]
\[
\Delta A = \begin{bmatrix}
0 & 0 & 0 \\
-LLC & 0 & 0 \\
-\Gamma B_2^T C & 0 & 0
\end{bmatrix},
\]
\[
B = \begin{bmatrix}
B_1 \\
B_3 \\
0
\end{bmatrix},
\]
\[
\Delta B = \begin{bmatrix}
0 & -LA & 0 \\
0 & 0 & 0 \\
0 & -\Gamma B_2^T C & 0
\end{bmatrix},
\]
\[
C = \begin{bmatrix} 0 & H & HB_2 \end{bmatrix}
\]

To analyze the stability of the augmented system, the following Lyapunov function is used,
\[
V(k) = X(k)^T P X(k)
\]  
(28)

where \(P = \text{diag}\{P_x, P_v, P_d\}\) and \(P_x, P_v, P_d > 0\). The forward difference of this Lyapunov function can be written as follows:
\[
\Delta V(k) = \xi^T \left[ \left( A + \Delta A \right)^T \left( B + \Delta B \right) \right] P \left( A + \Delta A \right)^T
\]
\[
+ \left[ \begin{array}{c} -P \ 0 \\
0 & 0 \end{array} \right] \xi
\]  
(29)

where \(\xi(k) = [X(k)^T \ \omega(k)^T]^T\). The following lemma will provide us with tools to handle the uncertain terms \(\Delta A\) and \(\Delta B\).

**Lemma 3.1:** Let \(A, E, F, \Lambda\) and \(P\) be real matrices of appropriate dimensions with \(P > 0\) and \(\Lambda \text{ satifying } \Lambda^T \Lambda \leq I\). Then for any scalar \(\eta > 0\) satisfying \(P^{-1} - \eta^{-1} EE^T > 0\), we have
\[
(\Lambda + EAF)^T P (\Lambda + EAF) \leq A^T (P^{-1} - \eta^{-1} EE^T)^{-1} A + \eta FT^T F
\]

In order to be able to use the results of lemma 3.1, we need to rewrite the uncertain terms \(\Delta A\) and \(\Delta B\) such that they are compatible with the lemma. Thus,
\[
P \left[ \Delta A \ \Delta B \right] = P \begin{bmatrix}
0 & -L & 0 \\
-\Gamma B_2^T C & 0 & 0
\end{bmatrix} \Lambda \begin{bmatrix}
C & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}
\]
\[
\triangleq EAF
\]  
(30)

with \(\Lambda\) satisfying \(\Lambda^T \Lambda \leq \delta_{\text{max}} I\), where \(\delta_{\text{max}} = \max \delta_j \ j = 1, \ldots, p\). Based on the results of lemma 3.1, the following inequality holds:
\[
\begin{bmatrix}
A^T P + \Delta A^T P \\
B^T P + \Delta B^T P
\end{bmatrix} \leq \begin{bmatrix}
A^T P + \Delta A^T P \\
B^T P + \Delta B^T P
\end{bmatrix} \left[ P^{-1} \right] \left[ \begin{array}{c} A^T P \ 0 \\
B^T P \ 0 \end{array} \right] + \eta FT^T F
\]  
(31)

Substituting (31) in (29), we get
\[
\Delta V(k) \leq \xi^T \left[ \begin{array}{c} A^T P \ \Delta A^T P \\
B^T P + \Delta B^T P
\end{array} \right] \left( P^{-1} \right) \left[ \begin{array}{c} A^T P \ 0 \\
B^T P \ 0 \end{array} \right] + \eta FT^T F
\]

The next step is to establish an \(H_{\infty}\) bound on the effects of the unwanted noise inputs and also the effects of the variations of the unknown disturbance \(i.e. \Delta d\). To this end, we define
\[
J \triangleq \sum_{k=0}^{s} \{ \varepsilon(k)^T \varepsilon(k) - \omega(k)^T \mu^T \mu \omega(k) \}
\]  
(33)

where \(s\) is a finite integer and \(\mu = \text{diag}\{\mu_w, \mu_v, \mu_d\}\). Adding (28) to the right hand side of (33), we get
\[
J \leq \sum_{k=0}^{s} \{ \varepsilon(k)^T \varepsilon(k) - \omega(k)^T \mu^T \mu \omega(k) + \Delta V(k) \} = \sum_{k=0}^{s} \tilde{J}_k
\]  
(34)

Similar to the case with linear quantization, if we design our filter such that \(\tilde{J}_k \leq 0\), we can conclude that \(J \leq 0\). Using (26) and (32), we can rewrite \(\tilde{J}_k\) as
\[
\tilde{J}_k \leq \xi^T \left( \Omega_1 - \Omega_2 \Omega_3^{-1} \Omega_2^T \right) \xi
\]  
(35)

where
\[
\Omega_1 = \begin{bmatrix} C^T C - P & 0 \\
0 & -\mu^T \mu \end{bmatrix} + \eta FT^T F
\]
\[
\Omega_2 = \begin{bmatrix} A^T P \ 0 \\
B^T P \ 0 \end{bmatrix}, \quad \Omega_3 = -P + \eta^{-1} EE^T
\]  
(36)

Now if we define \(G_e = P_a L\) and \(G_d = P_a \Gamma\), we can rewrite (35) as follows:
\[
\tilde{J}_k \leq \xi^T \left( \Xi_1 + \Xi_2 \Xi_2^{-1} \Xi_1^T + E(\eta I)^{-1} EE^T \right) \xi
\]  
(37)

where \(\Xi_1\) and \(\Xi_2\) are as given in (24).

Finally, one can show using Schur’s Complement that (37) holds true if the LMI given in (23) is satisfied. ■

**Remark.** Although both of the mentioned quantizers have their own advantages, from author’s perspective a linear quantizer is preferred for two reasons: first, due to the nature of logarithmic quantization, stability of the plant is a necessary condition in the design process, and second, in setpoint tracking all the advantages of a logarithmic quantizer can disappear unless a dynamic element is added to update the center of the quantizer.

**IV. SIMULATION RESULTS**

In this section, we simulate the proposed unknown input filters and then compare the results. We consider a stable system to which a constant disturbance with the amplitude of 2 is added at \(t = 3\) sec and then it grows linearly till \(t = 4\) sec where it reaches the amplitude of 4. For both examples, it is assumed that \(w\) and \(v\) are white noise inputs with the standard deviations of 0.5 and 0.1, respectively.
Example. Consider the following stable linear system with the sampling time $T_s = 0.01$ sec,

$$
x(k+1) = \begin{bmatrix} 0.9323 & 0.0185 \\ -0.0092 & 0.9138 \end{bmatrix} x(k) + \begin{bmatrix} 0.1 \\ 0 \end{bmatrix} u(k) + \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix} d(k) + \begin{bmatrix} 0.0098 \\ 0.0095 \end{bmatrix} w(k)
$$

$$
y(k) = \begin{bmatrix} 2 & 0 \end{bmatrix} x(k) + v(k)
$$

$$
z(k) = \begin{bmatrix} 0 & 1 \end{bmatrix} x(k)
$$

(38)

Depending on the type of the employed quantizer, we have the following two cases:

Case 1: A linear quantizer with a range of $(-17, 17)$ and 128 levels is employed. Choosing the $\mathcal{H}_\infty$ bounds as $\mu_w = \mu_v = \mu_q = 0.5$, $\mu_d = 1$ the filter parameters are calculated as

$$
L = \begin{bmatrix} 0.1931 \\ 0.2601 \end{bmatrix} \quad \Gamma = 0.9915
$$

(39)

Case 2: A logarithmic quantizer with $y_0 = 17$, $\rho = 0.94$ and 128 levels is employed. Choosing the $\mathcal{H}_\infty$ measures as $\mu_w = \mu_v = 0.5$, $\mu_d = 1$ the filter parameters are calculated as

$$
L = \begin{bmatrix} 0.1394 \\ 0.1319 \end{bmatrix} \quad \Gamma = 0.7775
$$

(40)

In both cases a 7-bit data packet can be used for every transmission. Figures 1, 2 and 3 show the estimated signal $z_F$ and $d_F$ in case 1. As seen in these figures, the proposed filter tracks the desired signal $z_F$ even in the presence of the unknown disturbance, whereas the conventional observer leads to a steady state error when the disturbance is applied. The designed filter also estimates the unknown disturbance input online, which can be crucial in faults detection applications over the network.

Fig. 1: Real signal $z$ and its estimate $z_F$ for a Luenberger observer in case 1

Figures 4, 5 and 6 show the estimated signal $z_F$ and $d_F$ in case 2. Similar to case 1, the proposed filter operates more efficient compared to the conventional observer. However, looking at the estimated unknown input $d_F$, one may wonder why there is a considerable error between $d$ and $d_F$. In order to answer this question, we need to take a closer look at the operation of a logarithmic quantizer. This quantizer concentrates most of its levels around a specific value (0 in our example) and the farther we go from this value, the less concentrated the levels are. As a result, the quantization error increases as we get closer to the edge of the quantizer. In this example, the unknown disturbance drives the measurement to about 14, where the quantization error is about 1. Consequently, the error generated by the quantization reflects itself as a steady state error in $d_F$ and a noise in $z_F$.

V. CONCLUSION

In this work, unknown input filter design under quantized measurements was studied. The designed filter was based on a simple variable change in a way that the new state variable was affected by the value of the unknown disturbance from two samples before. Two common types of quantization were taken into account: Linear and Logarithmic. For the first one,
the quantization error was modelled as measurement noise and an $H_\infty$ approach was taken to bound the compromising effects on the estimation error. For logarithmic quantization, the quantization error was modelled as model uncertainty and then a robust approach was given to handle the undesirable effects. Finally, both filters were simulated for an example system and it was seen that state tracking happens for both cases, however, the unknown input estimate for the logarithmic case is accompanied with an steady state error.

REFERENCES