We introduce and study fundamental relation of fuzzy hypersemigroups. Also we introduce the notion of a complete part of a fuzzy hypersemigroup and investigate its basic properties. Finally, we study the relationship between the fundamental relation and complete parts of a given fuzzy hypersemigroup.

**Keywords:** Fuzzy hypersemigroup, Fuzzy regular relation, Fundamental relation, Complete part, Complete closure.

## 1 INTRODUCTION

Hyperstructure theory was born in 1934 when Marty defined hypergroups, began to analysis their properties and applied them to groups, relational algebraic functions (see [11]). Since then, hundreds of papers and several books have been written on this topic. Nowadays, hyperstructures have a lot of applications to several domains of mathematics and computer science (see [1, 2, 3, 4, 6, 13, 16]) and they are studied in many countries of Europe, America and Asia. In 1971, Rosenfeld (see [14]) introduced fuzzy sets in the context of group theory and formulated the concept of a fuzzy subgroup of a group.
group. Since then, many researchers are engaged in extending the concepts of abstract algebra to the framework of the fuzzy setting (see for instance [12]).

The study of fuzzy hyperstructures is an interesting research topic of fuzzy sets. A hyperoperation assigns to every pair of elements of $H$ a nonempty subset of $H$, while a fuzzy hyperoperation assigns to every pair of elements of $H$ a nonzero fuzzy set on $H$. This idea was introduced by Corsini and Tofan [5] and then studied by Kehagias [8], for the interesting properties obtained in connections with an important hyperstructure, called join space. Recently, Sen, Ameri and Chowdhury introduced and analyzed fuzzy hypersemigroups in [15] based on a fuzzy hyperoperation. This approach followed by some researchers and they extended it to fuzzy hyperring [10], and fuzzy hypermodules [9].

As it is well known if $R$ is a strongly regular equivalence relation on a given (resp. semihypergroup) hypergroup $H$, then we can define a binary operation $\otimes$ on the quotient set $H/R$, the set of all equivalence classes of $H$ with respect to $R$, such that $(H/R, \otimes)$ consists a (resp. semigroup) group. In fact, the relation $\beta^*$ is the smallest equivalences relation such that the quotient $H/\beta^*$ is a semigroup and it is called fundamental relation of $H$. The fundamental relation $\beta^*$ plays an important role in the theory of hyperstructures and it was studied by many authors (for more see [3], [6], [13]). Now in this paper we follow the results obtained by Sen, Ameri and Chowdhury about fuzzy hypersemigroups (see [15]) to introduce and study the fundamental relation of fuzzy hypersemigroups. In this regards we characterize the fundamental relation of a given fuzzy semihypergroup and obtain its basic properties. We will proceed to introduce and study the basic properties of complete parts of fuzzy hypersemigroups. Finally, we investigate the relationship between the complete parts and fundamental relation of fuzzy hypersemigroups.

2 PRELIMINARIES

We recall some definitions and theorems from the books [3],[4] and [15], which we need them for development of our paper.

Let $H$ be a set and $P^*(H)$ be the family of all nonempty subsets of $H$ and $\circ$ a hyperoperation or join operation, that is $\circ$, is a map from $H \times H$ to $P^*(H)$. If $(a, b) \in H \times H$, its image under $\circ$ is denoted by $a \circ b$ or $ab$. A join operation can be extended to subsets of $H$ in a natural way, so that $A \circ B$ or $AB$ is given by $AB = \cup \{ab \mid a \in A, b \in B\}$. The notions $aA$ and $Aa$ are used for $\{a\}A$ and $A\{a\}$ respectively. Generally, the singleton $\{a\}$ is identified by its element $a$.

A hypergroupoid is a structure $(H, \circ)$. A hypergropoid $(H, \circ)$, which is associative, that is $x \circ (y \circ z) = (x \circ y) \circ z, \forall x, y, z \in H$ is called a semihypergroup.

**Definition 2.1.** A hypergroup is a semihypergroup such that $x \circ H = H = H \circ x$, for all $x \in H$ (reproduction axiom).
Definition 2.2. Let $H$ be a hypergroup and $K$ be a nonempty subset of $H$. Then $K$ is a subhypergroup of $H$ if itself is a hypergroup under hyperoperation $\circ$ restricted to $K$. It is clear that a subset $K$ of $H$ is a subhypergroup if and only if $a \circ K = K = K \circ a$, for all $a \in K$, under the hyperoperation on $H$, and it is denoted by $K <_H H$.

Definition 2.3. [15] Let $S$ be a non-empty set. $F(S)$ denotes the set of all fuzzy subsets of $S$. A fuzzy hyperoperation on $S$ is a mapping $\circ : S \times S \mapsto F(S)$ written as $(a, b) \mapsto a \circ b$. $S$ together with a fuzzy hyperoperation $\circ$ is called a fuzzy hypergropoid.

Definition 2.4. [15] A fuzzy hypergropoid $(S, \circ)$ is called a fuzzy hypersemigroup if for all $a, b, c \in S$, $(a \circ b) \circ c = a \circ (b \circ c)$, where for any fuzzy subset $\mu$ of $S$ 

$$ (a \circ \mu)(r) = \begin{cases} \bigvee_{t \in S} ((a \circ t)(r) \wedge \mu(t)), & \text{if } \mu \neq 0 \\ 0, & \text{otherwise} \end{cases} $$

$$ (\mu \circ a)(r) = \begin{cases} \bigvee_{t \in S} (\mu(t) \wedge (t \circ a)(r)), & \text{if } \mu \neq 0 \\ 0, & \text{otherwise} \end{cases} $$

for all $r \in S$.

Definition 2.5. [15] Let $\mu, \nu$ be two fuzzy subsets of a fuzzy hypergropoid $(S, \circ)$, then we define $\mu \circ \nu$ by $(\mu \circ \nu)(t) = \bigvee_{p,q \in S} ((\mu(p) \wedge (p \circ q)(t)) \wedge (\nu(q)))$, for all $t \in S$.

Definition 2.6. [15] A fuzzy hypersemigroup $(S, \circ)$ is called a fuzzy hypergroup if $\chi_a \circ S = S \circ \chi_a = \chi_S$, for all $x \in S$.

Example 2.7. [15] Define a fuzzy hyperoperation $\circ$ on a non-empty set $S$ by $a \circ b = \chi_{\{a, b\}}$ for all $a, b \in S$, where $\chi_{\{a, b\}}$ denotes the characteristic function of the set $\{a, b\}$. Then $(S, \circ)$ is a fuzzy hypersemigroup as well as fuzzy hypergroup.

Theorem 2.8. [15] Let $(S, \circ)$ be a fuzzy hypersemigroup. Then $\chi_a \circ \chi_b = \chi_{a \circ b}$, for all $a, b \in S$.

Definition 2.9. Let $(S, \ast)$ and $(T, \circ)$ be two fuzzy hypersemigroups. A mapping $\phi : S \mapsto T$ is said to be a

(i) fuzzy homomorphism if $\phi(a \ast b) \leq \phi(a) \circ \phi(b)$, $\quad (a, b \in S)$.

(ii) fuzzy good homomorphism if $\phi(a \ast b) = \phi(a) \circ \phi(b)$, $\quad (a, b \in S)$. 


Definition 2.10. (see [15]) Let $\rho$ be an equivalence relation on a fuzzy hypersemigroup $(S, \circ)$ and let $\mu, \nu$ be two fuzzy subsets on $(S, \circ)$. If $\mu(a) > 0$ implies there exists $b \in S$ such that $\nu(b) > 0$ and $apb$ and if $\nu(x) > 0$ implies there exists $y \in S$, such that $\mu(y) > 0$ and $xpy$, then we say that $\mu \rho \nu$. If for all $x \in S$ such that $\mu(x) > 0$ and for all $y \in S$ such that $\nu(y) > 0$, $x \rho y$, we say that $\mu \rho \nu$.

Definition 2.11. (see [15]) An equivalence relation $\rho$ on a fuzzy hypersemigroup $(S, \circ)$ is said to be a fuzzy (strongly) regular relation on $(S, \circ)$ if $a \rho b, a' \rho b'$ implies $a \circ b \rho a' \circ b'$.

3 FUNDAMENTAL RELATION

Let $S$ be a fuzzy hypersemigroup. For every map $f : S \to S'$ where $S'$ is a semigroup, we define $f(a \circ b) = \{f(t) : (a \circ b)(t) > 0\}$. By this, the map $f : S \to S'$ is a homomorphism if $\forall a, b \in S : f(a \circ b) \subseteq f(a) \circ f(b)$.

Remark. If $S$ is a fuzzy hypersemigroup and $S'$ a semigroup, every homomorphism of $S$ in $S'$ is good.

If $\rho$ is an equivalence relation on a fuzzy hypersemigroup $(S, \circ)$, then we consider the following hyperoperation on the quotient set $S/\rho$ as follows: for every $a \rho, b \rho \in S/\rho$ define

$$a \rho \oplus b \rho = \{c \rho : (a' \circ b')(c) > 0, apa', bpb'\}$$

Theorem 3.1. Let $(S, \circ)$ be a fuzzy hypersemigroup and $\rho$ be an equivalence relation on $S$. Then

i) the relation $\rho$ is a fuzzy regular relation on $(S, \circ)$ iff $(S/\rho, \oplus)$ be a hypersemigroup.

ii) the relation $\rho$ is a fuzzy strongly regular relation on $(S, \circ)$ iff $(S/\rho, \oplus)$ be a semigroup.

iii) if $f : S \to S'$ is homomorphism and $S'$ is a semigroup, then the equivalence associated with $f$ is fuzzy strongly regular.

Proof. Straight forward.

Definition 3.2. Let $(S, \circ)$ be a fuzzy hypersemigroup. The fundamental relation on $(S, \circ)$ is the smallest equivalence relation $\rho$ on $S$ such that the quotient structure $(S/\rho, \oplus)$ be a semigroup.

Definition 3.3. Let $(S, \circ)$ be a fuzzy hypersemigroup. We define the relation $\alpha$ on $S$ in the following way:
Also take \( tk' \) a \( \alpha \) there we obtain

\[
0 \text{ and } \left( \alpha \right)
\]

It is clear that \( \alpha \) is symmetric. Define for any \( a \in S \), \( a(a) = (\chi_{\alpha})(a) = 1 \), thus the relation \( \alpha \) is also reflexive. We take \( \alpha^* \) to be the transitive closure of \( \alpha \). Then \( \alpha^* \) is an equivalence relation on \( H \).

**Theorem 3.4.** The relation \( \alpha^* \) is the fundamental relation on fuzzy hypersemigroup \( (S, \circ) \).

**Proof.** First, we prove the quotient set \( S/\alpha^* \) is a semigroup. The hyperoperation \( \oplus \) in \( S/\alpha^* \) is defined as follows

\[
\alpha^*(a) \oplus \alpha^*(b) = [\alpha^*(c) | (a' \circ b')(c) > 0, a' \alpha^*a, b' \alpha^*b]
\]

Take \( a' \in \alpha^*(a), b' \in \alpha^*(b) \). Then we have

\[
a' \alpha^*a \Leftrightarrow \exists x_1, \ldots, x_{m+1} \in S, x_1 = a', x_{m+1} = a \text{ and finite products}
\]

\[
u_1, \ldots, v_n : v_j(1) > 0 \text{ and } v_j(y_j+1) > 0 \forall j = 1, \ldots, n.
\]

Sine for every \( t \in S \), \( (u \circ v)(t) = \vee\{u(r) \land v(s) | r, s \in S, (r \circ s)(t) > 0\} \), therefore we obtain

\[
\forall t \in S \text{ s.t. } (x_1 \circ y_1)(t) > 0, (x_{i+1} \circ y_1)(t) > 0 : (u_i \circ v_1)(t) > 0 \quad (i = 1, \ldots, m-1)
\]

\[
\forall t \in S \text{ s.t. } (x_{m+1} \circ y_j)(t) > 0, (x_{m+1} \circ y_{j+1})(t) > 0 : (u_m \circ v_j)(t) > 0 \quad (j = 1, \ldots, n)
\]

Set \( u_i \circ v_1 = t_i(i = 1, \ldots, m - 1) \) and \( u_m \circ v_j = t_{m+j-1}(j = 1, \ldots, n) \), therefore \( t_k \) is a finite products of elements of \( S \) for all \( k \in \{1, \ldots, m + n - 1\} \). Now, pick up any elements \( z_k, \ldots, z_{m+n} \in S \) such that \( (x_i \circ y_1)(z_k) > 0 \) \( i = 1, \ldots, m \) and \( (x_{m+1} \circ y_{j+1})(z_{m+n}) > 0 \) \( j = 1, \ldots, n \) and using by the relation (1) we have

\[
t_i(z_k) > 0 \text{ and } t_i(z_{k+1}) > 0 \text{ for } k = 1, \ldots, m + n - 1.
\]

So, every element \( z_k \) such that \( (x_i \circ y_1)(z_k) = (a' \circ b')(z_k) > 0 \) is \( \alpha^* \) equivalent to every element \( z_{m+n} \) that \( (x_{m+1} \circ y_{j+1})(z_{m+n}) = (a \circ b)(z_{m+n}) > 0 \). Thus \( \alpha^*(a) \oplus \alpha^*(b) \) is singleton so we can write

\[
\alpha^*(a) \oplus \alpha^*(b) = \alpha^*(c) \quad \text{for all } c \text{ that } (a' \circ b')(c) > 0, a' \alpha^*a, b' \alpha^*b
\]

Moreover, \( \alpha^*(x) \oplus (\alpha^*(y) \oplus \alpha^*(z)) = \alpha^*(x) \oplus \alpha^*(k) = \alpha^*(l) \), where \( (y \circ z)(k) > 0 \) and \( (x \circ k)(l) > 0 \) therefore \( 0 < (x \circ (y \circ z))(l) = ((x \circ y) \circ z)(l) = \bigvee_{p \in S} [(x \circ y)(p) \land (p \circ z)(l)] \), thus there exists \( p \in S \) such that \( \alpha^*(l) = \alpha^*(p) \oplus \alpha^*(c) = \alpha^*(a) \oplus \alpha^*(b) \).
Theorem 4.2. Therefore \( \rho \) by (3.1) \( \rho \) is fuzzy strongly regular relation we can show \( \alpha^* \leq \rho \). If \( x \neq y \) then there exists \( n \in \mathbb{N} \), and \( a_1, \ldots, a_n \in S \) for which \( \prod_{i=1}^{n} a_i(x) > 0 \) and

\[
\prod_{i=1}^{n} a_i(y) > 0;
\]

Since \( \rho \) is fuzzy regular relation we have that \( x \rho y \), hence \( \alpha \leq \rho \), which implies \( \alpha^* \leq \rho \). \( \square \)

Example. Let \( S = \{a, b, c, d\} \). Consider fuzzy hyperoperation \( \circ \) on \( S \), which is defined by:

\[
(a \circ a)(b) = (a \circ a)(c) = 0.1, \quad (a \circ b)(b) = (a \circ c)(b) = (a \circ d)(b) = (b \circ a)(b) = (b \circ c)(b) = (b \circ d)(b) = (c \circ a)(b) = (c \circ b)(b) = (c \circ d)(b) = (d \circ a)(b) = (d \circ b)(b) = (d \circ c)(b) = (d \circ d)(b).
\]

Therefore the relation \( \alpha \) is complete part.

\[\alpha^* = \{(a, a), (b, b), (c, c), (b, c), (b, d), (c, d)\} \text{ and } S/\alpha^* = \{\{a\}, \{b, c, d\}\}.\]

We proved that the operation \( \alpha^* \) in \( S/\alpha^* \) is associative. Therefore \( (S/\alpha^*, \oplus) \) is a semigroup.

\[\alpha^* \circ (x \oplus \alpha^*(y)) \oplus \alpha^*(z).\]

4 COMPLETE PARTS

Definition 4.1. Let \( (S, \circ) \) be a fuzzy hypersemigroup and \( A \) be a non empty subset of \( S \). \( A \) is called complete part if the following implication is valid:

\[\forall n \in \mathbb{N}, \forall (x_1, \ldots, x_n) \in S^n, \text{ if } \exists u \in A \text{ such that } \prod_{i=1}^{n} x_i(u) > 0 \text{ then } \]

\[\forall y \in S \setminus A : \prod_{i=1}^{n} x_i(y) = 0 \]

Theorem 4.2. Let \( \rho \) be a fuzzy strongly regular relation on a fuzzy hypersemigroup \( (S, \circ) \), \( z \in S \) and \( \rho(z) \) be the class of \( z \) module \( \rho \). Then \( \rho \) is a complete part.
Theorem 4.4. Let $S$ be a non-empty subset of fuzzy hypersemigroup $(S, \circ)$. Consider fuzzy hyperoperation $\circ$ on $S$, which is defined by 

$$(a \circ b)(a) = (b \circ b)(a) = (c \circ c)(a) = 0.5, \quad (a \circ b)(b) = (b \circ a)(b) = (c \circ c)(b) = 0.1, \quad (a \circ c)(c) = (b \circ b)(b) = (c \circ a)(a) = 0.7$$

and 

$$(a \circ a)(b) = (a \circ a)(c) = (a \circ b)(a) = (a \circ b)(c) = (a \circ c)(a) = (a \circ c)(b) = (b \circ a)(a) = (b \circ a)(c) = (b \circ b)(b) = (b \circ c)(a) = (b \circ c)(c) = (c \circ a)(a) = (c \circ a)(b) = (c \circ b)(c) = (c \circ c)(b) = (c \circ c)(c) = 0.$$ 

Let $\rho = (\{a, b\}, \{b, c\}, \{a, c\}, \{a, c\})$. It is routine to verify that $\rho$ is a fuzzy strongly regular relation, therefore $\{a, c\}$ is a complete part.

Example. Let $S = \{a, b, c\}$. Consider fuzzy hyperoperation $\circ$ on $S$, which is defined by 

$$(a \circ a)(a) = (b \circ b)(a) = (c \circ c)(a) = 0.5, \quad (a \circ b)(b) = (b \circ a)(b) = (c \circ c)(b) = 0.1, \quad (a \circ c)(c) = (b \circ b)(b) = (c \circ a)(a) = 0.7$$

and 

$$(a \circ a)(b) = (a \circ a)(c) = (a \circ b)(a) = (a \circ b)(c) = (a \circ c)(a) = (a \circ c)(b) = (b \circ a)(a) = (b \circ a)(c) = (b \circ b)(b) = (b \circ c)(a) = (b \circ c)(c) = (c \circ a)(a) = (c \circ a)(b) = (c \circ b)(c) = (c \circ c)(b) = (c \circ c)(c) = 0.$$ 

Let $\rho = (\{a, b\}, \{b, c\}, \{a, c\}, \{a, c\})$. It is routine to verify that $\rho$ is a fuzzy strongly regular relation, therefore $\{a, c\}$ is a complete part.

Definition 4.3. Let $A$ be a non-empty subset of fuzzy hypersemigroup $(S, \circ)$. The intersection of the complete parts of $S$ and contain $A$ is called the complete closure of $A$ in $S$, it will be denoted by $\partial(A)$.

Theorem 4.4. Let $A$ be a non-empty subset of fuzzy hypersemigroup $(S, \circ)$. Set 

$K_1(A) = A,$ 

$K_{n+1}(A) = \{x \in S | \exists (p \in \mathbb{N}, (h_1, \ldots, h_p) \in S^p, y \in K_n(A)) : \prod_{i=1}^p h_i(x) > 0, \prod_{i=1}^p h_i(y) > 0\}$ 

Let $K(A) = \bigcup_{n \geq 1} K_n(A)$. Then $\partial(A) = K(A)$.

Proof. We will prove:
(i) $K(A)$ is complete, 
(ii) if $B \supset A$ and $B$ is complete, then $B \supset K(A)$.

(i) If $\exists x \in K(A)$ such that $\prod_{i=1}^p x_i(x) > 0$, then $\exists n \in \mathbb{N}$ such that $x \in K_n(A)$ and 

$\prod_{i=1}^p x_i(x) > 0$. Now if $\exists y \in S \setminus K(A)$ such that $\prod_{i=1}^p x_i(y) > 0$ then $y \in K_{n+1}(A)$. 

It is contradict with $y \in S \setminus K(A)$.

(ii) $B \supset A = K_1(A)$. Suppose $B \supset K_n(A)$ and we prove that this implies $B \supset K_{n+1}(A)$. Suppose $B \supset K_{n+1}(A)$. Suppose $z \in K_{n+1}(A)$, $p \in \mathbb{N}$ exists and $(x_1, \ldots, x_p) \in H^p$ exists such that $\prod_{i=1}^p x_i(z) > 0$ and $\prod_{i=1}^p x_i(k) > 0$ such that $k \in K_n(A)$. Then $B \supset K_n(A)$.
implies $k \in B$ and $B$ is complete, therefore $\forall y \in S \setminus B : \prod_{i=1}^{p} x_i(y) = 0$. Thus

$$\prod_{i=1}^{p} x_i(z) > 0$$
implies that $z \in B$. $\square$

**Lemma 4.5.** i) $\forall n \geq 2$, $\forall x \in S$, $K_n(K_2(x)) = K_{n+1}(x)$

ii) $x \in K_n(y) \iff y \in K_n(x)$.

**Proof.**

i) $K_2(K_2(x)) = \{ z \mid \exists (q \in \mathbb{N}, (a_1, \ldots, a_q) \in H^q, y \in K_2(x)) : q \prod_{i=1}^{q} a_i(z) > 0, \prod_{i=1}^{q} a_i(y) > 0 \}$

$= K_3(x)$. We now proceed by induction, suppose $K_{n-1}(K_2(x)) = K_n(x)$, then

$K_n(K_2(x)) = \{ z \mid \exists (q \in \mathbb{N}, (a_1, \ldots, a_q) \in H^q, t \in K_{n-1}(K_2(x))) : q \prod_{i=1}^{q} a_i(z) > 0, \prod_{i=1}^{q} a_i(t) > 0 \}$

$= \{ z \mid \exists (q \in \mathbb{N}, (a_1, \ldots, a_q) \in H^q, t \in K_n(x)) : q \prod_{i=1}^{q} a_i(z) > 0, \prod_{i=1}^{q} a_i(t) > 0 \} = K_{n+1}(x)$

ii) We also prove this by induction. It is clear that $x \in K_2(y) \iff y \in K_2(x)$. Suppose $x \in K_{n-1}(y) \iff y \in K_{n-1}(x)$. Let $x \in K_n(y)$, then $\exists q \in \mathbb{N}, (a_1, \ldots, a_q) \in H^q, s \in K_{n-1}(y)$ such that $\prod_{i=1}^{q} a_i(x) > 0$ and $\prod_{i=1}^{q} a_i(s) > 0$ thus $s \in K_2(x)$ is obtained. From $s \in K_{n-1}(y)$ we have $y \in K_{n-1}(K_2(x)) = K_n(x)$. $\square$

**Theorem 4.6.** The relation $xKy \iff x \in \partial\{y\}$ is an equivalence.

**Proof.** $K$ is clearly reflexive. Now let $xKy$ and $yKz$ be. Since $y \in \partial\{z\}$ then $\partial\{y\} \subset \partial\{z\}$, therefore $x \in \partial\{z\}$ and $xKz$. The simmetricity of $K$ follows in a direct way from the previous lemma. $\square$

In the next theorem, we see the relationship between fundamental relation and complete parts in fuzzy hypersemigroups.

**Theorem 4.7.** $\forall (x, y) \in S^2$, we have $xKy \iff x\alpha^*y$.

**Proof.** From $x\alpha y$, $\exists n \in \mathbb{N}, (a_1, \ldots, a_n) \in S^n$ such that $\prod_{i=1}^{n} a_i(x) > 0$ and
\[ \prod_{i=1}^{n} a_i(y) > 0, \text{ thus } x \in K_2(y), \text{ therefore } x \in K(y), \text{ from which } \alpha \subset K, \text{ and for this reason } \alpha^* \subset K. \]

On the converse if \( xK_y, n \in \mathbb{N} \) exists such that \( x \in K_{n+1}(y) \), from this we have a product \( \prod_{i=1}^{m_1} z_i^1 \) and \( x_1 \in K_n(y) \) exists such that \( \prod_{i=1}^{m_1} z_i^1(x) > 0 \) and \( x_1 \alpha x_1 \). From \( x_1 \in K_n(y) \) implies that there exists a product \( \prod_{i=1}^{m_2} z_i^2 \) and \( x_2 \in K_{n-1}(y) \) such that \( \prod_{i=1}^{m_2} z_i^2(x_1) > 0 \) and \( \prod_{i=1}^{m_2} z_i^2(x_2) > 0, \text{ therefore } x_1 \alpha x_2. \]

So as a consequence we have \( x_n \in K_{n-(n-1)}(y) \) such that \( \prod_{i=1}^{m_n} z_i^n(x_n) > 0 \) therefore \( x_n \alpha x_{n-1}. \) Thus \( x_n \in K_1(y) = \{ y \}, \text{ from which } x_n = y. \) For this reason \( x_1 \alpha x_1 \ldots \alpha x_n = y. \) Thus \( K \subset \alpha^*. \)

**Theorem 4.8.** Let \( (S, \circ) \) be a fuzzy hypersemigroup, \( \phi_S : S \rightarrow S/K \) the canonical projection. If \( (S', \circ') \) is a semigroup and \( f : S \rightarrow S' \) is a homomorphism, then a homomorphism \( g : S/K \rightarrow S' \) exists such that \( g\phi_S = f. \)

**Proof.** It is enough to show that \( g\phi_S(x) = f(x) \). \( g \) is well defined, since \( \phi_S(x) = \phi_S(y) \Rightarrow xK_y, \) then from theorem 3.1 follows that \( f(x) = f(y) \), and \( g \) is a homomorphism because \( \forall (x, y) \in S^2, \forall u \in S \) such that \( (x \circ y)(u) > 0, \) we have \( g(\phi_S(x) \oplus \phi_S(y)) = g\phi_S(x \circ y) = g\phi_S(u) = f(u) = f(x \circ y) = f(x) \circ f(y) = g\phi_S(x) \circ' g\phi_S(y). \)

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