A Power Assignment Scheme for Improving Outage Probability in HSDPA

Raymond Kwan†, Cyril Leung††, and Jie Zhang†
† University of Bedfordshire,
Park Square, Luton, United Kingdom, LU1 3JU
††Department of Electrical and Computer Engineering
The University of British Columbia
Vancouver, B.C., Canada V6T 1Z4
Emails: raymond.kwan@beds.ac.uk, cleung@ece.ubc.ca, jie.zhang@beds.ac.uk

Abstract—Bit or frame error rates are commonly used as performance measures in wireless communication systems. However, in emerging applications such as voice over IP (VoIP), the bit rate outage probability is often a more useful performance measure. In this paper, an accurate method is proposed for approximating the probability distribution of the downlink received signal-to-interference ratio (SIR) in the High Speed Downlink Packet Access (HSDPA) channel of a 3GPP network. Based on this distribution, a power adjustment scheme is proposed to minimize the weighted sum of bit rate outage probabilities for multiple users. It is shown that the proposed method can greatly improve outage probability fairness without incurring a very large degradation in throughput.

I. INTRODUCTION

In a wireless communication network, it is desirable to maximize both the system throughput and the quality of service (QoS) level experienced by users. Given limited resources, the trade-off between system throughput and user QoS presents a difficult challenge to network operators, and is the subject of intense study [1].

One effective way for improving the spectral efficiency in a wireless communication system is the use of adaptive modulation and coding (AMC). A higher order modulation provides a better spectral efficiency at the expense of a worse error rate performance. A lower rate channel coding generally provides a better error rate performance at the cost of a poorer spectral efficiency. Thus, with a proper combination of the modulation order and channel coding rate, it is possible to design a set of modulation and coding schemes (MCS), from which a selection is made in an adaptive fashion in each transmission-time interval (TTI) so as to maximize system throughput under varying channel conditions. A common requirement is that the probability of erroneous decoding of a Transmission Block be kept below some threshold value [2].

One notable benefit of using AMC with constant power is to improve spectral efficiency. While one of the most common design objectives is to maximize the average assigned bit rate, those involving the (outage) probability that the assigned bit rate for a user drops below some target value are relatively unexplored. For some delay-sensitive applications, it may be advantageous to minimize the bit rate outage probability instead of maximizing the average bit rate. Since the bit rates are to be adjusted depending on the user channel qualities, which are highly location-dependent, the bit rate outage naturally plays an important role in network planning and dimensioning [3].

The main difficulty in using such a performance measure is due to the requirement of a complete statistical distribution of the channel quality, which is typically unknown in practice. In this paper, a simple method for adjusting the transmit power in order to minimize the aggregate bit rate outage probability for multiple users is proposed. Such a scheme can provide an extra degree of freedom in network tuning and QoS differentiation. The proposed scheme is applied to an HSDPA system [1], [2], [4] which uses AMC and orthogonal multicodes.

II. SYSTEM MODEL

Let N be the number of HSDPA users, and $P_i$ be the downlink transmit power to user $i$ from a base station (BS). Also, let $h_i$ be the path gain between BS and user $i$, and $I_i$ be the total received interference and noise at the user $i$. The downlink received SIR for user $i$ is given by

$$\gamma_i = \frac{h_i P_i}{I_i}, \quad i = 1, \ldots, N,$$

where

$$\sum_{i=1}^{N} P_i \leq P_T.$$}

and $P_T$ is the total power allocated for HSDPA. Upon receiving the SIR fed-back from user $i$, the BS decides on the most appropriate combination of MCS and number of multicodes for user $i$. This combination reflects the bit rate that the channel quality $\gamma_i$ can support. Let $R_i(\gamma_i)$ be the bit rate that can be supported with an SIR value of $\gamma_i$. Also, let $F_{\Gamma_i}(\gamma_i)$ be the cumulative distribution function (cdf) of $\Gamma_i$, and $T_i$ be the SIR threshold (corresponding to the minimum bit rate target $R_i^{(\text{min})}$) below which an outage is said to occur. The outage probability $P_i^\text{out}$ is then given by

$$P_i^\text{out} = P\left(R_i(\gamma_i) \leq R_i^{(\text{min})}\right),$$

$$= P\left(\Gamma_i \leq T_i\right),$$

$$= F_{\Gamma_i}(T_i).$$
III. DOWNSIGHT POWER ASSIGNMENT SCHEME

A. Problem Formulation

Let $\Phi = [\phi_1 \phi_2 \ldots \phi_N]$ be an $N$-dimensional vector such that the BS allocates an adjusted power of $\phi_i P_i$ to user $i$ instead of $P_i$. Then the SIR for user $i$ after the power adjustment is $\gamma_i' = \phi_i \gamma_i$, and the corresponding SIR outage probability for user $i$ becomes

$$P_i^{out} = F_{\gamma_i'} \left( \frac{T_i}{\phi_i} \right).$$

The optimization problem can now be written as

$$\min_{\Phi} \sum_{i=1}^{N} w_i F_{\gamma_i'} \left( \frac{T_i}{\phi_i} \right),$$

subject to

$$\sum_{i=1}^{N} \phi_i P_i \leq P_T.$$  \hspace{1cm} (8)

In (7), $w_i \in \mathcal{R}^+$ are positive weights which indicate the relative importance of the users. Without loss of generality, let $P_1 = P_2 = \ldots = P_N = P_T / N$. Then (8) reduces to

$$\sum_{i=1}^{N} \phi_i \leq N.$$  \hspace{1cm} (9)

**Theorem 1:** There is an optimal solution to the optimization problem in (7) which lies on the hyperplane

$$\sum_{i=1}^{N} \phi_i = N.$$  \hspace{1cm} (10)

**Proof:** Assume that there is an optimal solution $\{\phi_1^*, \phi_2^*, \ldots, \phi_N^*\}$ with $\sum_{i=1}^{N} \phi_i^* < N$. Then at least one of $\{\phi_1^*, \phi_2^*, \ldots, \phi_N^*\}$, say $\phi_i^*$, can be increased to $\phi_i^+$ until $\phi_i^* = N$. Since $\phi_i^* < \phi_i^+$, $F_{\gamma_i'} \left( \frac{T_i}{\phi_i} \right) \leq F_{\gamma_i'} \left( \frac{T_i}{\phi_i^*} \right)$ and thus $\{\phi_1^+, \phi_2^*, \ldots, \phi_N^*\}$ is also an optimal solution. \n
Theorem 1 allows the optimization problem in (7) with constraint (9) to be solved using the method of Lagrange multipliers, with the Lagrangian

$$L(\Phi, \lambda) = \sum_{i=1}^{N} w_i F_{\gamma_i'} \left( \frac{T_i}{\phi_i} \right) + \lambda \sum_{i=1}^{N} \phi_i.$$  \hspace{1cm} (11)

Setting the partial derivative of the Lagrangian with respect to $\phi_i$ to 0 yields

$$w_i T_i f_{\gamma_i'} \left( \frac{T_i}{\phi_i} \right) = \lambda, \ i = 1, 2, \ldots, N$$  \hspace{1cm} (12)

where $f_{\gamma_i'}(.)$ is the probability density function (pdf) of $\Gamma_i$. The adjustment factors $\{\phi_i\}$ can then be obtained using (10) and (12). Depending on the nature of $F_{\gamma_i'}(.)$, the solution to (11) can be obtained analytically or numerically using a standard mathematical package such as MATLAB™ or MAPLE™.

B. Approximation of $F_{\gamma_i'}(\gamma)$

The proposed power assignment scheme requires knowledge of the cdf $F_{\gamma_i'}(.)$ for user $i$, which may not be available in practice. However, it is possible to obtain a reasonably accurate approximation for this distribution using the method of Pearson systems [5]. To simplify the notation, the user index "i" is omitted in the rest of this subsection.

Let $\mu_n \triangleq E[(\Gamma - \mu_1)^n]$ and $\nu_n \triangleq E[\Gamma^n]$ be the $n^{th}$ central and non-central moments of the random variable $\Gamma$ respectively. Sample values of $\Gamma$ can be readily obtained from the uplink feedback channel of the user. The idea behind this method is to map $\{\mu_n\}_{n=1}^{4}$ to one of a set of families of curves, where each family corresponds to a solution of the simple differential equation

$$\frac{df(x)}{dx} = \frac{(x + a_0) f(x)}{b_0 + b_1 x + b_2 x^2}.$$  \hspace{1cm} (13)

The values of the constants $a_0, b_0, b_1, b_2$ define the families of curves. Although a total of twelve families have been tabulated in [6], only three are of practical importance. The most common (main) types of curves are known as the type I, type IV, and type VI curves, where each family corresponds to the cases $\kappa < 0$, $0 < \kappa < 1$, and $\kappa > 1$ respectively. The relationship between the moments $\{\mu_n\}_{n=1}^{4}$ and the solutions of (13) can be found in [5], [6]. In this method, a selection parameter $\kappa$ is first computed as

$$\kappa = \frac{\beta_1(\beta_2 + 3)^2}{4(4\beta_2 - 3\beta_1)(2\beta_2 - 3\beta_1 - 6)},$$  \hspace{1cm} (14)

where the terms $\beta_1 \triangleq \mu_3 / \mu_2^2$ and $\beta_2 \triangleq \mu_4 / \mu_2^2$ are the skewness and kurtosis [7] of $\Gamma$ respectively.

Depending on the value of $\kappa$, the type of the curve to be used is chosen. The method yields a simple analytical expression which can accurately approximate $F_{\gamma_i'}(.)$.

C. Quantized Feedback and its Approximation

Up to this point, the channel quality (SIR) has been assumed to take on continuous values. However, this assumption would necessitate a very large feedback channel bandwidth, which is impractical. In [8], the feedback from the mobile, also known as the channel quality indicator (CQI)⁴, can only take on non-negative integer values. The CQI is provided by the mobile via the High Speed Dedicated Physical Control Channel (HS-DPCCH). Each CQI value maps directly to a maximum bit rate⁵ that a mobile can support, based on the received channel quality and capability [9], while ensuring that the Block Error Rate (BLER) is below 10%.

⁴Note that the CQI measured by the mobile is based on the SIR of the primary common pilot channel (P-CPICH), and is different from the SIR of the High Speed Downlink Shared Channel (HS-DSCH) by a scaling factor. Without loss of generality, such a scaling factor is assumed to be unity in this paper.

⁵In this paper, the bit rate refers to the bit rate on the transport channel, which is determined by the transport block size and the duration of the transmission-time-interval (TTI). They are, in turn, defined by the appropriate combination of MCS and the number of multicodecs.
While the mapping between the CQI and the SIR is not specified in [8], this issue has been addressed in a number of proposals [10]-[12]. Recently, a mapping has been proposed in which the system throughput is maximized while the BLER constraint is relaxed [13]. Let $q_i$ be the value of the CQI for user $i$, and $\tilde{\gamma}_i = 10 \log_{10}(\gamma_i)$ be the received SIR value in dB. According to [10], [11], [13], such a mapping can generally be expressed as a piece-wise linear function

$$q_i = \begin{cases} 0 & \tilde{\gamma}_i \leq t_0 \\ [c_1 \tilde{\gamma}_i + d_1] & t_0 < \tilde{\gamma}_i \leq t_1 \\ q_{\text{max}} & \tilde{\gamma}_i > t_1 \end{cases}$$

(15)

where $\{c_1, d_1, t_0, t_1\}$ are model and mobile capability dependent constants, and $[\cdot]$ is the floor function. From (15), it is clear that $\tilde{\gamma}_i$ cannot be recovered exactly based on the value of $q_i$ alone due to the quantization. It is important to note that the region $t_0 < \tilde{\gamma}_i < t_1$ is the operating region for the purpose of link adaptation and it should be large enough to accommodate SIR variations encountered in most practical scenarios [1]. In a well designed system, the probability that $\tilde{\gamma}_i$ lies outside this range should be small\(^1\). As part of the proposed procedure, $\tilde{\gamma}_i$ can be approximated as

$$\tilde{\gamma}_i = \tilde{\gamma}_i^{(l)} + \left(\tilde{\gamma}_i^{(u)} - \tilde{\gamma}_i^{(l)}\right) \xi,$$

(16)

where

$$\tilde{\gamma}_i^{(l)} = \frac{q_i - d_1}{c_1},$$

(17)

$$\tilde{\gamma}_i^{(u)} = \frac{q_i + 1 - d_1}{c_1},$$

(18)

and $\xi$ follows a uniform distribution, i.e. $\xi \sim U(0, 1)$. Note that this approximation assumes that $\tilde{\gamma}_i$ is uniformly distributed between $\tilde{\gamma}_i^{(l)}$ and $\tilde{\gamma}_i^{(u)}$ for a given value of $q_i$.

When $q_i = 0$ and $q_i = q_{\text{max}}$, $\tilde{\gamma}_i$ can be simply approximated as $t_0$ and $t_1$ respectively, or more generally as $t_0 - \xi_0$ and $t_1 + \xi_1$ respectively, with $\xi_0$ and $\xi_1$ following certain pre-defined distributions. Finally, the estimated value of $\gamma_i$, $\tilde{\gamma}_i = 10^{\tilde{\gamma}_i/10}$, can be used to obtain the moments needed to approximate the distribution as described in section III-B.

IV. Numerical Results

For illustration purposes, let the parameter values in (15) be $t_0 = -4.5$, $t_1 = 25.5$, $c_1 = 1$, $d_1 = 4.5$, and $q_{\text{max}} = 30$; these values can be obtained from [10], assuming the user mobiles are of category 10 (i.e. have a wide CQI range) as defined in [8]. Also, let $\Gamma_i, i = 1, \ldots, N$ follow a Gamma distribution, with pdf given by

$$f_{\Gamma_i}(\gamma) = \begin{cases} \left(\frac{\alpha_i}{\Gamma_i}\right)^\alpha_i \gamma^{\alpha_i - 1} \exp\left(-\frac{\gamma}{\Gamma_i}\right) & \gamma \geq 0 \\ 0 & \gamma < 0 \end{cases}$$

(19)

where $\Gamma_i(\cdot)$ is the Gamma function, $\alpha_i$ is the fading figure, and $\Gamma_i$ is the mean of $\Gamma_i$.

Fig. 1 shows the cdf’s of the simulated actual and approximated SIRs based on discrete CQI feedback from two users, with $\alpha_1 = 6.5$, $\Gamma_1 = 4.5$ dB, $\alpha_2 = 2.25$, $\Gamma_2 = 12$ dB. Note that the approximated SIR is based on the method described in section III-C, using only knowledge of the CQI feedback. It can be seen that the approximate cdf’s are very close to the actual received SIR cdf’s.

Fig. 2 shows the cdf of the actual simulated SIR for each user together with the Pearson approximated cdf based on discrete CQI feedback from the two users, with $\alpha_1 = 6.5$, $\Gamma_1 = 4.5$ dB, $\alpha_2 = 2.25$, $\Gamma_2 = 12$ dB. The results show that the Pearson method described in Section III-B provides a reasonably accurate cdf. For an outage threshold, $T_i$, of 2.5 dB, it can be seen that the outage probabilities (QoS) for the two users are very different.

Using the power adjustment scheme described in section III, the new cdf’s, optimized for an outage threshold of $T_1 = T_2 = 2.5$ dB, are shown in Fig. 3 assuming $w_1 = w_2 = 1$. It can be seen that the two users now achieve very similar outage performances. It should be noted that the thresholds $T_1$ and $T_2$ as well as the weights $w_1$ and $w_2$ need not be equal. Their values can be selected as a means to providing additional degrees of freedom in resource allocation depending on specific applications and requirements.

Fig. 4 shows the objective function in (7) as a function of $\phi_1$ and $\phi_2$. The function is minimum at $\phi_1 = 1.411$ and $\phi_2 = 0.589$. So in this case, more power is allocated to user 1 at the expense of user 2.

For another illustration, let both mobiles be of category 10 capability [9] and $T_1 = T_2 = 2.5$ dB. Also, without loss of generality, let the relationship between the CQI and SIR be based on [10]. Thus, the bit rate outage thresholds for both users are $R_1^{(\text{min})} = R_2^{(\text{min})} = 0.325$ Mbps. With $w_1 = w_2 = 1$, the average bit rates for users 1 and 2 are obtained to be 0.53 Mbps and 1.9 Mbps in the absence of the proposed power adjustment, and 0.73 Mbps and 1.37 Mbps with the adjustment. On the other hand, the outage probabilities for users 1 and 2 are 0.16 and 0.015 in the absence of the proposed power adjustment, and 0.048 and 0.043 with the adjustment. Thus, although the average aggregate bit rate is decreased by 12% with the proposed power adjustment scheme, the aggregate outage probability is decreased by 48%. The proposed power adjustment scheme is very effective at balancing the user outage probabilities. Figs. 5 and 6 show the cdf’s of user bit rates using the Pearson approximated SIR based on discrete CQI without and with the power adjustment respectively.

V. Conclusion

The bit rate outage probability was studied as an alternative performance measure to the average bit rate for wireless systems. A method was described for approximating the distribution of the downlink received SIR based on the CQI fed back in HSDPA. Numerical results show that this approximation is

\(^1\)In this paper, it is assumed that the total number of Orthogonal Variable Spreading Factor (OVSF) codes are not limited, i.e. the choice of the CQI for one user is not affected by that of another user due to code shortage. In practice, limited number of codes may affect the values of $t_1$ and $q_{\text{max}}$ in (15), and such an issue is beyond the scope of this paper.
quite accurate. A power adjustment scheme was then proposed in which the weighted sum of bit rate outage probabilities for multiple users is minimized. It is found that the method can substantially improve outage probability fairness without incurring a very large average bit rate degradation.

REFERENCES


Acknowledgment: This work was supported in part by the Natural Sciences and Engineering Research Council (NSERC) of Canada under Grant OGP0001731, by an NSERC Post-Doctoral Fellowship, by the UBC PMC-Sierra Professorship in Networking and Communications and by a Marie Curie Post-Doctoral Fellowship.
Fig. 3. cdf’s of the actual simulated SIR and the Pearson approximated distribution of approximated SIR based on discrete CQI feedback with power adjustment. User 1: $\alpha_1=6.5$, $\Gamma_1=4.5$ dB; User 2: $\alpha_2=2.25$, $\Gamma_2=12$ dB, assuming $w_1 = w_2 = 1$.

Fig. 4. The objective function in (7) as a function of $\phi_1$ and $\phi_2$. The minimum occurs at $\phi_1 = 1.411$ and $\phi_2 = 0.589$.

Fig. 5. cdf’s of user bit rates with Pearson approximated SIR based on discrete CQI feedback with no power adjustment. User 1: $\alpha_1=6.5$, $\Gamma_1=4.5$ dB; User 2: $\alpha_2=2.25$, $\Gamma_2=12$ dB.

Fig. 6. cdf’s of user bit rates with Pearson approximated SIR based on discrete CQI feedback with power adjustment. User 1: $\alpha_1=6.5$, $\Gamma_1=4.5$ dB; User 2: $\alpha_2=2.25$, $\Gamma_2=12$ dB, assuming $w_1 = w_2 = 1$. 