Journal of Systems Engineering and Electronics
Vol. 18, No. 4, 2007, pp.858–864
Available online at www.sciencedirect.com
ScienceDirect
www.sciencedirect.com/science/journal/10044132

Delay-independent robust guaranteed-cost control for uncertain linear neutral systems*

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(Received October 3, 2006)

Abstract: This article concerns the delay-independent guaranteed-cost control problem via memoryless state feedback for a class of neutral-type systems with structural uncertainty and a given quadratic cost function. New delay-independent conditions for the existence of the guaranteed-cost controller are presented in the term of LMIs. An algorithm involving optimization is proposed to design a controller achieving an optimal guaranteed-cost, such that, the system can be stabilized for all admissible uncertainties. A numerical example is provided to illustrate the feasibility of the proposed method.

Keywords: neutral system, guaranteed cost control, LMI method, delay-independent.

1. Introduction

As time-delay is often a source of instability in many engineering systems, considerable attention has been paid to the problem of stability analysis and controller synthesis for time-delay systems. With the advance of robust control theory, a number of robust stabilization methods were proposed for uncertain time-delay systems[1−5]. Especially, in recent years, the stability analysis and feedback stabilization problems for various neutral-type systems have attracted the attention of many authors[6−13], and the references therein. The systems that can be described by neutral-type systems include, but are not limited to, lumped parameter networks interconnected by transmission lines, systems of a turbojet engine, infeed grinding, and continuous induction heating of a thin moving body[14].

Guaranteed-cost control for a class of uncertain neutral systems was considered in Ref. [15]. However, the control problem of the neutral system with timedelay in derivative terms and with uncertainties in the control input term was not, still involved and discussed. Therefore, it is necessary to investigate the guaranteed-cost problem for the class of neutral systems. In this article, a delay-independent condition is presented, to maintain the stabilization of the uncertain neutral system as also an adequate level of performance, which improves the results of known references. This article is organized as follows: In Section 2, a robustness condition is given for the closed-loop system. In Section 3, a designing approach of the guaranteed-cost controller is derived in terms of linear matrix inequalities. These matrix inequalities can be easily solved by various efficient convex optimization algorithms. Finally, an example is given to illustrate the superiority of the proposed design method.

2. Problem statements

Consider the following uncertain neutral systems

$$\dot{x}(t) - \tilde{G}\dot{x}(t-\tau) = \tilde{A}x(t) + \tilde{A}_1 x(t-\tau) + \tilde{B}u(t) \quad (1)$$

with the initial condition function

$$x(t_0+\theta) = \varphi(\theta), \quad \forall\theta \in [-\tau,\ 0] \quad (2)$$

* This project was supported by the Natural Science Basic Research Plan in Shaanxi Province of China (2006A13) and the Foundation of Research Project of Educational Department of Shaanxi Province (06JK149).

where, $x \in R^n$ is the state vector, $u(t) \in R^m$ is the input vector, $\tau > 0$ is a scalar representing the delay in the system, and $\phi(\cdot)$ is the given continuously differentiable function on [-τ, 0]. The coefficient matrices $\tilde{A} = A + \Delta A(t)$, $\tilde{A}_1 = A_1 + \Delta A_1(t)$, $\tilde{B} = B + \Delta B(t)$, $\tilde{G} = G + \Delta G(t)$. A, A_1, G, and B are real constant matrices, $\Delta A(t), \Delta A_1(t), \Delta G(t)$, and $\Delta B(t)$ are uncertain time-varying matrices described by

$$\left.\begin{array}{ll} ||A|| \leqslant \delta_A, & ||A_1|| \leqslant \delta_{A1} \\ ||G|| \leqslant \delta_G, & ||B|| \leqslant \delta_B \end{array}\right\} \tag{3}$$

where, δ_A, δ_{A1}, δ_G and δ_B are positive constants.

Associated with the system (1) is the following quadratic cost function

$$J = \int_0^\infty [x^{\mathrm{T}}(t)Qx(t) + u^{\mathrm{T}}(t)Ru(t)]\mathrm{d}t \tag{4}$$

where, $Q \in R^{n\times n}$ and $R \in R^{m\times m}$ are the given positive- definite matrices.

Definition 1 The uncertain neutral system of Eq. (1) (setting $u(t) = 0$) is said to be quadratically stable, if there exists an associated Lyapunov function

$$V(t, x_t, \dot{x}_t) = x^{\mathrm{T}}(t)Px(t) + \int_{t-\tau}^{t} x^{\mathrm{T}}(\theta)Mx(\theta)\mathrm{d}\theta + \int_{t-\tau}^{t} \dot{x}^{\mathrm{T}}(\sigma)N\dot{x}(\sigma)\mathrm{d}\sigma \tag{5}$$

for all admissible uncertainties such that

$$\begin{bmatrix} -N^{-1} & \tilde{A} & \tilde{A}_1 & \tilde{G} \\ * & P\tilde{A} + \tilde{A}^{\mathrm{T}}P + M & P\tilde{A}_1 & P\tilde{G} \\ * & * & -M & 0 \\ * & * & * & -N \end{bmatrix} < 0 \tag{6}$$

that is

$$\begin{bmatrix} P\tilde{A} + \tilde{A}^{\mathrm{T}}P + M & P\tilde{A}_1 & P\tilde{G} \\ * & -M & 0 \\ * & * & -N \end{bmatrix} - \begin{bmatrix} \tilde{A}^{\mathrm{T}} \\ \tilde{A}_1^{\mathrm{T}} \\ \tilde{G}^{\mathrm{T}} \end{bmatrix} N \begin{bmatrix} \tilde{A} & \tilde{A}_1 & \tilde{G} \end{bmatrix} < 0 \tag{6$'$}$$

where, P, M, N, and N are positive-definite matrices. The symbol "*" in the matrix represents the elements below the main diagonal of a symmetric partitioned matrix.

Definition 2 For the uncertain neutral system (1) and the quadratic cost function (4), if there exists a control law $u^*(t)$ and a positive number J^*, such that, for all admissible uncertainty, the closed-loop system (8) is quadratically stable and the closed-loop value of the cost function (4) satisfies $J \leqslant J^*$, then $J^*$is said to be the guaranteed-cost and $u^*(t)$ is said to be a guaranteed-cost control law of the system (1) and cost function (4).

The objective here is to design a memoryless state feedback controller for the system (1) as

$$u(t) = Kx(t) \tag{7}$$

such that, the resulting closed-loop system given by

$$\dot{x}(t) - \tilde{G}\dot{x}(t-\tau) = (\tilde{A} + \tilde{B}\mathrm{K})x(t) + \tilde{A}_1 x(t-\tau) \tag{8}$$

is quadratically stable, and the closed-loop value of the cost function (4) satisfies $J \leqslant J^*$, where K is a gain matrix to be designed, J^* is some specified constant.

Throughout the article, the following well-known fact is required.

Lemma 1 Let $\forall\ x \in R^n$, $y \in R^m$, and let X and Y be the given real matrices with appropriate dimensions

$$X^{\mathrm{T}}XYy + x^{\mathrm{T}}Y^{\mathrm{T}}X^{\mathrm{T}}y \leqslant \delta x^{\mathrm{T}}XX^{\mathrm{T}}x + \delta^{-1}y^{\mathrm{T}}Y^{\mathrm{T}}Yy$$

for any $\delta > 0$.

3. Main results

First, the closed-loop system (8) is considered, with the performance index

$$J = \int_0^\infty x^{\mathrm{T}}(t)(Q + \mathrm{K}^{\mathrm{T}}B^{\mathrm{T}}RBK)x(t)\mathrm{d}t \tag{9}$$

Theorem 1 If there exist matrices $X > 0$, $\bar{M} > 0$, $\bar{N} > 0$, and matrix Y, scalar $\in_i > 0$ $(i = 1, 2, \cdots, 8)$ such that the LMI

$$\begin{bmatrix} -\bar{N} & Z & A_1\bar{M} & G\bar{N} & \varOmega_{15} & 0 & 0 \\ * & Z+Z^{\mathrm{T}} & A_1\bar{M} & G\bar{N} & \varOmega_{25} & 0 & \varOmega_{27} \\ * & * & -\bar{M} & 0 & 0 & \varOmega_{36} & 0 \\ * & * & * & -\bar{N} & 0 & \varOmega_{46} & 0 \\ * & * & * & * & -\varOmega_{55} & 0 & 0 \\ * & * & * & * & * & -\varOmega_{66} & 0 \\ * & * & * & * & * & * & -\varOmega_{77} \end{bmatrix} < 0 \quad (10)$$

where

$$Z = AX + BY$$

$$\varOmega_{15} = \begin{bmatrix} \varepsilon_1 I & \varepsilon_2 I & \varepsilon_3 I & \varepsilon_4 I & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\varOmega_{25} = \begin{bmatrix} 0 & 0 & 0 & 0 & \varepsilon_5 I & \varepsilon_6 I & \varepsilon_7 I & \varepsilon_8 I \end{bmatrix}$$

$$\varOmega_{55} = \mathrm{diag}\{\varepsilon_1 I,\ \varepsilon_2 I,\ \varepsilon_3 I,\ \varepsilon_4,\ \varepsilon_5 I,\ \varepsilon_6 I,\ \varepsilon_7 I,\ \varepsilon_8 I\}$$

$$\varOmega_{36} = \begin{bmatrix} \delta_{A_1} I & \delta_{A_1} I & 0 & 0 \end{bmatrix}$$

$$\varOmega_{46} = \begin{bmatrix} 0 & 0 & \delta_G I & \delta_G I \end{bmatrix}$$

$$\varOmega_{66} = \mathrm{diag}\{\varepsilon_3 I,\ \varepsilon_7 I,\ \varepsilon_4 I,\ \varepsilon_8 I\}$$

$$\varOmega_{27} = \begin{bmatrix} \delta_A X & \delta_A X & \delta_B Y^{\mathrm{T}} & \delta_B Y^{\mathrm{T}} & X & X & Y^{\mathrm{T}} \end{bmatrix}$$

$$\varOmega_{77} = \mathrm{diag}\{\varepsilon_1 I,\ \varepsilon_5 I,\ \varepsilon_2 I,\ \varepsilon_8 I,\ \bar{M},\ Q^{-1},\ R^{-1}\}$$

then $u(t) = YX^{-1}x(t)$ is a guaranteed-cost control law of the system (1), and an upper bound of the quadratic cost function J is decided by

$$J^* = x^{\mathrm{T}}(0)X^{-1}x(0) + \int_{-\tau}^{0} x^{\mathrm{T}}(s)\bar{M}^{-1}x(s)\mathrm{d}s + \int_{-\tau}^{0} \dot{x}^{\mathrm{T}}(s)\bar{N}^{-1}\dot{x}(s)\mathrm{d}s \quad (11)$$

Proof From Definition 1, it is known that the unforced system of inequality (10) is said to be quadratically stable, only if inequality (6) holds for all the admissible uncertainties on the Lyapunov function (5). therefore, the left side of inequality (6) is divided into two parts: the scalar part Φ and the uncertain part $\Delta\varPhi$, where

$$\varPhi = \begin{bmatrix} -N^{-1} & \hat{Z} & A_1 & G \\ * & P\hat{Z} + \hat{Z}^{\mathrm{T}}P + M & PA_1 & PG \\ * & * & -M & 0 \\ * & * & * & -N \end{bmatrix}$$

$$\Delta\varPhi = \begin{bmatrix} 0 & \Delta\hat{Z} & \Delta A_1 & \Delta G \\ * & P\Delta Z + \Delta Z^{\mathrm{T}}P & P\Delta A_1 & P\Delta G \\ * & * & 0 & 0 \\ * & * & * & 0 \end{bmatrix}$$

with $\hat{Z} = A + BK$, $\Delta\hat{Z} = \Delta A + \Delta BK$

For the arbitrary nonzero vector

$$\xi = \begin{bmatrix} \xi_1^{\mathrm{T}} & \xi_2^{\mathrm{T}} & \xi_3^{\mathrm{T}} & \xi_4^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}}$$

(6) holds if and only if

$$\xi^{\mathrm{T}}(\varPhi + \Delta\varPhi)\xi < 0 \quad (12)$$

By Lemma 1, it follows that

$$\begin{aligned} \xi^{\mathrm{T}}\Delta\varPhi\xi = {} & 2\xi_1^{\mathrm{T}}\Delta A\xi_2 + 2\xi_1^{\mathrm{T}}\Delta BK\xi_2 + 2\xi_1^{\mathrm{T}}\Delta A_1\xi_3 + \\ & 2\xi_1^{\mathrm{T}}\Delta G\xi_4 + 2\xi_2^{\mathrm{T}}P\Delta A\xi_2 + 2\xi_2^{\mathrm{T}}P\Delta BK\xi_2 + \\ & 2\xi_2^{\mathrm{T}}P\Delta A_1\xi_3 + \\ & 2\xi_2^{\mathrm{T}}P\Delta G\xi_4 \leqslant (\varepsilon_1+\varepsilon_2+\varepsilon_3+\varepsilon_4)\xi_1^{\mathrm{T}}\xi_1 + \\ & (\varepsilon_4+\varepsilon_5+\varepsilon_6+\varepsilon_8)\xi_2^{\mathrm{T}}P^2\xi_2 + \xi_2^{\mathrm{T}}[(\varepsilon_1^{-1}+\varepsilon_5^{-1})\delta_A^2 I + \\ & (\varepsilon_2^{-1}+\varepsilon_6^{-1})\delta_B^2 K^{\mathrm{T}}K]\xi_2 + \xi_3^{\mathrm{T}}(\varepsilon_3^{-1}+\xi_7^{-1})\delta_{A_1}^2\xi_3 + \\ & \xi_4^T(\varepsilon_4^{-1}+\xi_8^{-1})\delta_G^2\xi_3 = \xi^{\mathrm{T}}\varTheta\xi \end{aligned}$$

where

$$\varTheta = \mathrm{diag}\{\varTheta_{11};\varTheta_{22};\varTheta_{33};\varTheta_{44}\}$$

with

$$\varTheta_{11} = (\varepsilon_1+\varepsilon_2+\varepsilon_3+\varepsilon_4)\mathrm{I}$$

$$\varTheta_{22} = (\varepsilon_4+\varepsilon_5+\varepsilon_6+\varepsilon_8)P^2 + (\varepsilon_1^{-1}+\varepsilon_5^{-1})\delta_A^2\mathrm{I} + (\varepsilon_2^{-1}+\varepsilon_6^{-1})\delta_B^2 K^{\mathrm{T}}K$$

$$\varTheta_{33} = (\varepsilon_3^{-1}+\xi_7^{-1})\delta_{A_1}^2 I$$

$$\varTheta_{44} = (\varepsilon_4^{-1}+\xi_8^{-1})\delta_G^2 I$$

Hence, only if $\varPhi + \varTheta < 0$ is true, then inequality (12) holds.

On the other hand, Pre- and postmultiplying the matrix of the left side in inequality (10) by W^{T} and W, where $W = \mathrm{diag}\{I,P,I,I,I,I,I\}(X = P^{-1})$, the matrix inequality (10) is equivalent to

$$\begin{bmatrix} -\bar{N} & Z & A_1 & G & \Omega_{15} & 0 & 0 \\ * & Z+Z^{\mathrm{T}} & PA_1 & PG & \bar{\Omega}_{25} & 0 & \bar{\Omega}_{27} \\ * & * & -\bar{M} & 0 & 0 & \Omega_{36} & 0 \\ * & * & * & -\bar{N} & 0 & \Omega_{46} & 0 \\ * & * & * & * & -\Omega_{55} & 0 & 0 \\ * & * & * & * & * & \Omega_{66} & 0 \\ * & * & * & * & * & * & -\Omega_{77} \end{bmatrix} < 0 \tag{13}$$

where

$$K = YK^{-1}, Z = AX + BY$$

$$\bar{\Omega}_{25} = \begin{bmatrix} 0 & 0 & 0 & 0 & \varepsilon_5 P & \varepsilon_6 P & \varepsilon_7 P & \varepsilon_8 P \end{bmatrix}$$

$$\bar{\Omega}_{27} = \begin{bmatrix} \delta_A & \delta_A & \delta_B K^{\mathrm{T}} & \delta_B K^{\mathrm{T}} & I & I & K^{\mathrm{T}} \end{bmatrix}$$

Using Schur complement, the inequality (13) is equivalent to

$$\begin{bmatrix} -\bar{N}+\Theta_{11} & Z & A_1 & G \\ * & \Lambda & PA_1 & PG \\ * & * & -\bar{M}^{-1}+\Theta_{33} & 0 \\ * & * & * & -\bar{N}^{-1}+\Theta_{44} \end{bmatrix} < 0 \tag{14}$$

where

$$\Lambda = P(A+BK) + (A+BK)^{\mathrm{T}}P + \bar{M}^{-1} + Q + K^{\mathrm{T}}RK + \Theta_{22}$$

Let $M = \bar{M}^{-1}$, $N = \bar{N}^{-1}$, then the matrix inequality (14) implies that

$$\Phi + \Theta < 0$$

Therefore, the neutral system (1) is quadratically stabilizabled via the control law $u(t) = YX^{-1}x(t)$.

In supervene to prove that the quadratic cost function (9) satisfies $J \leqslant J^*$. For the Lyapunov function (6), a derivative on the time variable is taken along the solution of (8), where the positive-definite matrices $P, M, N (P = X^{-1}, N = \bar{N}^{-1})$ are made certain by inequality (10).

$$\begin{aligned} &\dot{V}(t, x_t, \dot{x}_t) < -x^{\mathrm{T}}(t)(Q + K^{\mathrm{T}}RK)x(t) + \\ &\zeta^{\mathrm{T}}(t)\Xi\zeta(t) \ < -x^{\mathrm{T}}(t)(Q + K^{\mathrm{T}}RK)\mathrm{x}(t) \ < 0 \end{aligned} \tag{15}$$

where

$$\zeta(t) = \begin{bmatrix} x^{\mathrm{T}}(t) & x^{\mathrm{T}}(t-\tau) & \dot{x}^{\mathrm{T}}(t-\tau) \end{bmatrix}^{\mathrm{T}}$$

$$\Xi = \begin{bmatrix} P\tilde{A} + \tilde{A}^{\mathrm{T}}P + M + Q + K^{\mathrm{T}}RK & P\tilde{A}_1 & P\tilde{G} \\ * & -M & 0 \\ * & * & -N \end{bmatrix} - \begin{bmatrix} \tilde{A}^{\mathrm{T}} \\ \tilde{A}_1^{\mathrm{T}} \\ \tilde{G}^{\mathrm{T}} \end{bmatrix} N \begin{bmatrix} \tilde{A} & \tilde{A}_1 & \tilde{G} \end{bmatrix}$$

Integrating both sides of the inequality (15) from 0 to T leads to

$$\begin{aligned} &\int_0^{\mathrm{T}} x^{\mathrm{T}}(t)(Q + K^{\mathrm{T}}RK)x(t)\mathrm{d}t < \\ &V(0, x_0, \dot{x}_0) - V(t, x_T, \dot{x}_T) < \\ &x^{\mathrm{T}}(0)Px(0) - x^{\mathrm{T}}(T)Px(T) + \int_{-\tau}^{0} x^{\mathrm{T}}(s)Mx(s)\mathrm{d}s - \\ &\int_{\mathrm{T}-\tau}^{\mathrm{T}} x^{\mathrm{T}}(s)Mx(s)\mathrm{d}s + \int_{-\tau}^{0} \dot{x}^{\mathrm{T}}(s)N\dot{x}(s)\mathrm{d}s - \\ &\int_{\mathrm{T}-\tau}^{\mathrm{T}} \dot{x}^{\mathrm{T}}(s)N\dot{x}(s)\mathrm{d}s \end{aligned}$$

As the closed-loop system (8) is asymptotically stable, when $T \rightarrow \infty$

$$x^{\mathrm{T}}(t)Px(t) \rightarrow 0, \quad \int_{T-\tau}^{\mathrm{T}} x^{\mathrm{T}}(s)Mx(s)\mathrm{d}s \rightarrow 0, \quad \int_{T-\tau}^{\mathrm{T}} \dot{x}^{\mathrm{T}}(s)N\dot{x}(s)\mathrm{d}s \rightarrow 0$$

Therefore, the following is obtained

$$\begin{aligned} &\int_0^{\infty} x^{\mathrm{T}}(t)(Q + K^{\mathrm{T}}RK)x(t)\mathrm{d}t < V(0, x_0, \dot{x}_0) < \\ &x^{\mathrm{T}}(0)Px(0) + \int_{-\tau}^{0} x^{\mathrm{T}}(s)Mx(s)\mathrm{d}s + \\ &\int_{-\tau}^{0} \dot{x}^{\mathrm{T}}(s)N\dot{x}(s)\mathrm{d}s \ \underline{\underline{\mathrm{def}}}\ J* \end{aligned}$$

This completes the proof.

4. Design of guaranteed guaranteed cost controller

Theorem 1 A method of designing a state feedback guaranteed-cost controller is presented. The problem is to determine whether the problem is feasible or not. It is called the feasibility problem. Also, the solutions for the problem can be found by a quasi-convex optimization problem. The following theorem provides a method of selecting a controller minimizing the upper bound of the guaranteed-cost Eq. (11).

Theorem 2 Consider the system (1) with the cost function (5). If the following optimizing problem

$$\min_{X>0,Y,\bar{M}>0,\bar{N}>0,\mu>0,\alpha>0,\Gamma_1>0,\Gamma_2>0}\{\alpha+\mathrm{tr}\{\Gamma_1)+\mathrm{tr}(\Gamma_2)\} \tag{16}$$

is subject to

(1) matrix inequality (10),

(2) $\begin{bmatrix} -\alpha & x^{\mathrm{T}}(0) \\ x(0) & -X \end{bmatrix} < 0,$

(3) $\begin{bmatrix} -\Gamma_1 & \Sigma_1^{\mathrm{T}} \\ \Sigma_1 & -\bar{M} \end{bmatrix} < 0,$

(4) $\begin{bmatrix} -\Gamma_2 & \Sigma_2^{\mathrm{T}} \\ \Sigma_2 & -\bar{N} \end{bmatrix} < 0,$

has a solution set $\{X > 0,\ Y,\ \bar{M} > 0,\ \bar{N} > 0,\ \mu > 0,\ \alpha > 0,\ \Gamma_1 > 0,\ \Gamma_2 > 0\}$, then the control law $u(t) = YX^{-1}x(t)$ is an optimal robust guaranteed-cost control law, which ensures the minimization of the guaranteed-cost Eq. (11) for the neutral system (1), where $\int_{-\tau}^{0} x(\theta)x^{\mathrm{T}}(\theta)\mathrm{d}\theta = \Sigma_1^{\mathrm{T}}\Sigma_1$ and $\int_{-\tau}^{0} \dot{x}(\theta)\dot{x}^{\mathrm{T}}(\theta)\mathrm{d}\theta = \Sigma_2^{\mathrm{T}}\Sigma_2$.

Proof By Theorem 2, (1) in formula (16) is clear. Also, it follows from Schur completement that (2), (3), and (4) in formula (16) are equivalent to $x^{\mathrm{T}}(0)X^{-1}x(0) < \alpha$, $\Sigma_1^{\mathrm{T}}\bar{M}^{-1}\Sigma_1 < \Gamma_1$, $\Sigma_2^{\mathrm{T}}\bar{N}^{-1}\Sigma_2 < \Gamma_2$, respectively. On the other hand

$$\int_{-\tau}^{0} x^{\mathrm{T}}(\theta)\bar{M}^{-1}x(\theta)\mathrm{d}\theta = \int_{-\tau}^{0} \mathrm{tr}(x^{\mathrm{T}}(\theta)\bar{M}^{-1}x(\theta))\mathrm{d}\theta = \mathrm{tr}(\Sigma_1^{\mathrm{T}}\bar{M}^{-1}\Sigma_1) < \mathrm{tr}(\Gamma_1)$$

$$\int_{-\tau}^{0} \dot{x}^{\mathrm{T}}(\theta)\bar{N}^{-1}\dot{x}(\theta)\mathrm{d}\theta = \int_{-\tau}^{0} \mathrm{tr}(x^{\mathrm{T}}(\theta)\bar{N}^{-1}x(\theta))\mathrm{d}\theta = \mathrm{tr}(\Sigma_2^{T}\bar{N}^{-1}\Sigma_2) < \mathrm{tr}(\Gamma_2)$$

Hence, it follows from formula (16) that

$$J* < \alpha + \mathrm{tr}(\Gamma_1) + \mathrm{tr}(\Gamma_2)$$

Thus, the minimization of α + tr(Γ_1) + tr(Γ_2) implies the minimization of the guaranteed-cost for the system (1). And this quasiconvex optimization problem guarantees that a global optimum, when it exists, is reachable.

Remark 1 Ju H Park[15] investigated the guaranteed-cost problem of a class of neutral systems, however, the proposed conclusion is only applicable to the system with uncertainties, in the structure of system matrices A and A_1.

5. An example

To illustrate the design procedure of the proposed method, a numerical example has been run. Consider the following uncertain system

$$\begin{aligned}\dot{x}(t) - (G + \Delta G)\dot{x}(t-\tau) = (A + \Delta A)x(t) + \\ (A_1 + \Delta A_1)x(t-\tau) + (B + \Delta B)u(t)\end{aligned} \tag{17}$$

$\Delta A(t)$, $\Delta A_1(t)$, $\Delta G(t)$, and $\Delta B(t)$ are uncertain time-variable matrices described by inequalies (3). Where

$$A = \begin{bmatrix} -2 & -1 \\ -1 & 2 \end{bmatrix},\quad A_1 = \begin{bmatrix} 0.5 & -0.3 \\ -0.2 & 0.6 \end{bmatrix}$$

$$G = \begin{bmatrix} 0.2 & 0 \\ 0.1 & 0.2 \end{bmatrix},\quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\delta_A = 0.6,\ \delta_{A_1} = 0.2,\ \delta_G = 0.1,\ \delta_B = 0.2$$

and the initial condition of the system is as follows

$$x(t) = [0.5\mathrm{e}^t\ 0.5\mathrm{e}^t]^{\mathrm{T}},\ \ -1 \leqslant t \leqslant 0$$

Actually, when the control input is not forced on the system (17), that is, $u(t) = 0$, the system is unstable as the states of the system go to infinity, as $t\to\infty$ (see Fig. 1).

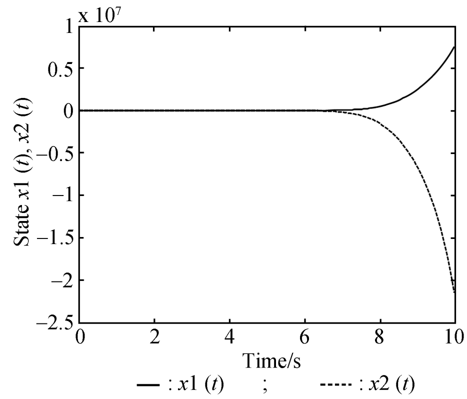

Fig. 1 State curves of the free system

Here, a memoryless state feedback controller is about to be constructed, with the form, $u(t) =$

$YX^{-1}x(t)$, for system (17), such that, the closed-loop system is quadratically stable, and a corresponding upper bound of the cost function

$$J = \int_0^\infty x_1^2(t) + x_2^2(t) + 0.1u^2(t)]\mathrm{d}t$$

is minimized. That is, $Q = i$ and $R = 0, 1I$.

From the relation (16), $\int_{-\tau}^0 x(\theta)x^{\mathrm{T}}(\theta)\mathrm{d}\theta = \Sigma_1^{\mathrm{T}}\Sigma_1$ and $\int_{-\tau}^0 \dot{x}(\theta)\dot{x}^{\mathrm{T}}(\theta)\mathrm{d}\theta = \Sigma_2^{\mathrm{T}}\Sigma_2$. It follows

$$\Sigma_1 = \begin{bmatrix} 0.238\,0 & -0.226\,8 \\ -0.226\,8 & 0.864\,4 \end{bmatrix}$$

$$\Sigma_2 = \begin{bmatrix} 0.238\,0 & 0.226\,8 \\ 0.226\,8 & 0.864\,4 \end{bmatrix}$$

By solving the optimization problem of Theorem 2 using the LMI toolbox in Matlab, it is found that the problem is feasible, and the solutions are

$$X = \begin{bmatrix} 0.936\,3 & 0.319\,1 \\ 0.319\,1 & 0.383\,4 \end{bmatrix}, \ \bar{M} = \begin{bmatrix} 2.974\,6 & 1.566\,0 \\ 1.918\,4 & 3.077\,3 \end{bmatrix}$$

$$\bar{N} = \begin{bmatrix} 23.055\,3 & -2.895\,2 \\ -2.895\,2 & 24.084\,9 \end{bmatrix}$$

$$Y = \begin{bmatrix} 0.023\,6 & -4.122\,6 \end{bmatrix}, \quad \varepsilon_1 = 3.832\,0$$

$$\varepsilon_2 = 3.638\,2, \quad \varepsilon_3 = 1.290\,3, \quad \varepsilon_4 = 0.1018$$

$$\varepsilon_5 = 0.463\,9, \quad \varepsilon_6 = 1.145\,0 \times 10^{-6}$$

$$\varepsilon_7 = 0.156\,2, \quad \varepsilon_8 = 0.440\,6, \quad \alpha = 1.903\,3$$

$$\Gamma_1 = \begin{bmatrix} 0.976 & -0.227\,0 \\ -0.227\,0 & 0.572\,4 \end{bmatrix}$$

$$\Gamma_2 = \begin{bmatrix} 0.004\,1 & 0.009\,3 \\ -0.009\,3 & 0.031\,7 \end{bmatrix}$$

Therefore, the stabilizing optimal guaranteed-cost control law, $u(t)$, for system (16) is given by

$$u(t) = YX^{-1}x(t) = [5.149\,7 \quad -15.036\,1]x(t)$$

and the corresponding optimal guaranteed-cost of the closed-loop system is

$$J* = \alpha + tr(\Gamma_1) + tr(\Gamma_2) = 2.609\,2$$

For computer simulation, the following uncertainty is employed

$$\Delta A = \begin{bmatrix} 0.6\sin t & 0 \\ 0 & 0.6\cos t \end{bmatrix}$$

$$\Delta A_1 = \begin{bmatrix} 0.6\sin\ (t-\tau) & 0 \\ 0 & 0.6\cos\ (t-\tau) \end{bmatrix}$$

$$\Delta G = \begin{bmatrix} 0.1\cos\ (t-\tau) & 0 \\ 0 & 0.1\sin\ (t-\tau) \end{bmatrix}$$

$$\Delta B = \begin{bmatrix} 0 \\ 0.2\sin t \end{bmatrix}$$

The simulation results are given in Figs 2 and 3. In the figures, one can see that the system is indeed well stabilized.

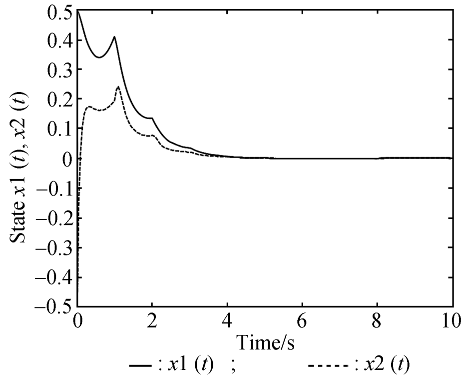

Fig. 2 State responses of the system

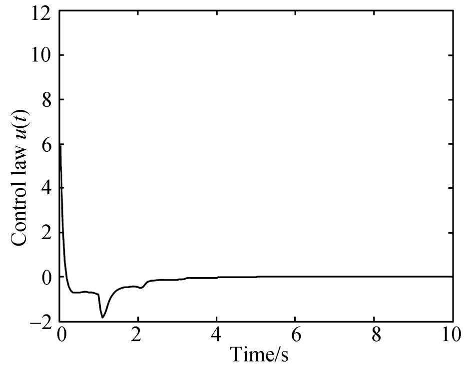

Fig. 3 Control law for the system

6. Conclusions

This article presents a solution to the optimal robust guaranteed-cost control problem, via memoryless state feedback control laws, for neutral systems with parametric uncertainty in a matrix inequality framework. A guaranteed-cost control gain is obtained through an optimization problem, which can be solved by using software such as MATLAB's LMI Control Toolbox. Finally, a simulation result is illustrated to show that the neutral system is indeed well stabilized, irrespective of uncertainty and time delay.

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